A Particle Swarm Optimization Method With Enhanced Global Search Ability for Design Optimizations of Electromagnetic Devices

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Based on the refinement successes to particle swarm optimization (PSO) methods, which include, namely, the introduction of an age variable, the proposal of new selection strategies to find the best solutions of the particle as well as for its neighbors, the design of a novel formula for velocity updating, the incorporation of an intensification search phase, and so on, an improved PSO method is presented. The experimental results reported indicate that the refined pinpointing search ability and the global search ability of the proposed method are significantly improved when compared to those of conventional PSOs.

Index Terms—**Global optimization, metaheuristic algorithm, particle swarm optimization (PSO), stochastic method.**

I. Introduction

The PARTICLE swarm optimization (PSO) algorithm is a new entrant to the family of evolutionary algorithms (EAs). It was first proposed by Kenney and Eberhart [1], [2] based on the metaphor of social behavior of birds flocking and fish schooling in search for food. Essentially, PSO is similar to EA, albeit defined in a social context rather than modeling the biological perspective of genetic algorithms (GAs). Similar to other EAs, it works with a population referred to as a swarm and each individual is called a particle, but it differs from other EAs in that the population is not manipulated through operators inspired by processes on the deoxyribonucleic acid (DNA) of humans. In PSO, each particle "flies" over the search space to look for promising regions according to its own experiences and that of the group. Consequently, the sharing of social information takes place and individuals profit from the discoveries and the previous experiences of all other particles during the search. As with other EAs, PSO has the ability to search over a wide landscape around the better solutions.

Mathematically, given a swarm of $N_{\rm popsize}$ particles, each particle $i(i \in \{1, 2, \ldots, N_{\rm popsize}\})$ is associated with a position vector $x_i = (x_1^i, x_2^i, \ldots, x_D^i)$ (D is the number of decision parameters of an optimal problem) which is a feasible solution in an optimal problem. Let the best previous position ($p_{\rm best}$ memorized in $P_{\rm best}$) that particle i has ever found to be denoted by $p_i = (p_1^i, p_2^i, \ldots, p_D^i)$ and the group's best position ever found by the neighborhood particles of the ith particle ($g_{\rm best}$ memorized in $G_{\rm best}$) is $g_i = (g_1^i, g_2^i, \ldots, g_D^i)$. At each iteration step $k+1$, the position vector of the ith particle $x_i(k+1)$ is updated by adding an increment vector $\Delta x_i(k+1)$, called velocity $v_i(k+1)$, as follows:

$$v_d^i(k+1) = v_d^i(k) + c_1 r_1 \left(p_d^i - x_d^i(k)\right) + c_2 r_2 \left(g_d^i - x_d^i(k)\right) \quad (1)$$

$$v_d^i(k+1) = \frac{v_d^i(k+1) \cdot v_d^{\max}}{\left|v_d^i(k+1)\right|}, \quad \text{if } \left|v_d^i(k+1)\right| > v_d^{\max} \quad (2)$$

$$x_d^i(k+1) = x_d^i(k) + v_d^i(k+1) \quad (3)$$

where c_1 and c_2 are two positive constants, r_1 and r_2 are two random parameters which are found uniformly within the interval [0, 1], and $v_d^{\max}$ is a parameter that limits the velocity of the particle in the dth coordinate direction.

This iterative process will continue swarm by swarm until a stop criterion is satisfied, and this forms the basic iterative process of a PSO algorithm. Moreover, on the right hand of (1), the second term represents the cognitive part of PSO as the particle changes its velocity based on its own thinking and memory. The third term of (1) corresponds to the social part to enable the particle to modify its velocity based on the social-psychological adaptation of knowledge. PSO is conceptually very simple, and can be readily implemented in a few coding lines. It requires only primitive mathematical operators and very few algorithm parameters need tuning. As a result, the PSO has attracted the attentions of many fellow researchers from different disciplines. Generally speaking, PSOs are found to be robust and fast in solving a wide range of engineering design problems [3]-[5].

However, as a newly developed optimal method, the PSO algorithm is still in its developmental infancy and further studies are necessary. For example, the original PSO had difficulties in controlling the balance between exploration and exploitation because it tends to favor the intensification search around the "better" solutions previously found [6]. In such a context, the

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PSO appears to be lacking global search ability. Also, its computational efficiency for a refined local search to pinpoint the exact optimal solution is not satisfactory [7]. Consequently, some improvements are proposed in this paper to enhance the performance of PSOs, while retaining their merits, such as conceptual and implementation simplicities, both in terms of global and refined pinpointing search abilities.

II. Modified PSO With Enhanced Performances

To enhance the refined pinpointing search ability and to strike a balance between exploration and exploitation of available PSOs, the following improvements are proposed.

A. Introduction of Age Variables

On one hand, the information sharing among particles is a blessing because the convergence speed of the particles can be enhanced by the discoveries and previous experiences of both the particles itself and that of its neighborhood. However, it can also be a demerit in optimal problems dealing with multimodal objective functions, since the diversity of particles is inevitably degraded due to information sharing. Therefore, a stagnation phenomenon may occur (i.e., all particles might be taking the same position), and the algorithm is then trapped onto a local optimum. Moreover, the values of p_{best} and g_{best} for a specific particle may be kept unchanged for several successive iterative swarms, especially toward the end of the searching process of the algorithm. To guarantee the diversity of the algorithm and, hence, enhancing its global search ability, an age variable, which represents the "age" of the members in P_{best} and G_{best}, is introduced. When a particle finds a new p_{best} or a g_{best} solution, the age variable of the specified solution will be assigned a minimum value. The age of a member in P_{best} or G_{best} will increase by an incremental value once the specified solution is selected as a p_{best} or a g_{best} solution. If the age of a solution memorized in both P_{best} and G_{best} exceeds a threshold, the solution in question will be discarded and a new one is generated randomly to replace it. In this manner, the diversity of the algorithm is enhanced.

B. Velocity Updating

As governed by (1)–(3), in a typical PSO algorithm, a particle updates its velocity according to the flying experience of its own as well as those of the group; thus, the particle is always gravitated toward a stochastically weighted average of the previous best points of itself and that of all other members in its neighborhood. Obviously, this will reduce the solution diversity of the particles in the feasible space. In order to alleviate such drawbacks, the p_{best} and g_{best} solutions of a particle are selected from the entire set of P_{best} and G_{best} using a Roulette wheel selection scheme, so as to further enhance the diversity of particles, according to the following probability:

$$p_i(k) = \frac{f_i(k)}{\sum_{j=1}^{N} f_j(k)} \quad (4)$$

where $f_i(k)$ and N are, respectively, the fitness value of the ith quantity stored and the total number of quantities in P_{best} and G_{best}.

On the other hand, since the two random parameters r_1 and r_2 are independently generated, there are cases in which they are both too large or too small. In the former case, both the personal and social experiences accumulated so far are overused and the particle might be driven away from the local optimum. For the latter case, both the personal and social experiences are not used fully, and the convergence performance of the algorithm is undermined. However, in human social activities such as in hunting, most people have the abstract reasoning ability to make the best use of his knowledge and that of the group's in determining the most promising regions to go searching. In other words, the two random weighting parameters reflecting the experiences of his own and that of his companions are not completely independent (i.e., if one parameter is large, the other should be small or vice versa). By modeling this reasoning ability into an updating formula and noting the sum of the two inter-related weighting parameters can be set to 1, one single random parameter that includes the cognitive and social experiences of the particle for updating its velocity is proposed.

Finally, to control the balance of exploration and exploitation, another random parameter r_2 in (5) is introduced, and the velocity is updated by using

$$\begin{aligned} v_d^i(k+1) = {} & r_2 v_d^i(k) + (1-r_2)c_1 r_1 \left(p_d^i - x_d^i(k)\right) \\ & + (1-r_2)c_2(1-r_1)\left(g_d^i - x_d^i(k)\right) \end{aligned} \quad (5)$$

$$\begin{aligned} v_d^i(k+1) &= \frac{v_d^i(k+1)\cdot v_d^{\max}}{|v_d^i(k+1)}, \qquad \text{if } \left|v_d^i(k+1)\right| > v_d^{\max} \\ v_d^i(k+1) &= \frac{v_d^i(k+1)\cdot v_d^{\min}}{\left|v_d^i(k+1)\right|}, \qquad \text{if } \left|v_d^i(k+1)\right| < v_d^{\min} \end{aligned} \quad (6)$$

where r_1 and r_2 are two random parameters uniformly chosen within the interval [0, 1], $v_d^{\min}$ is a parameter that controls the inappreciable velocities in the dth coordinate direction.

It should be noted that the communication between different particles in the proposed search procedure is set up in a heterarchical rather than a hierarchical manner.

C. Exceeding Boundary Control

In updating the position of particles using (3)–(6), it is very common to find the coordinates of the new particles lying outside the boundaries of the parameter space. In such cases, the popular approaches used in available PSO algorithms are either to take the boundaries as the coordinates of the new particles, or to keep the coordinates of the particle unchanged but to assign the particle with an extremely poor objective function value. However, either treatment will reduce the diversity of the particles in the searching process and reduce the global search ability of the algorithm correspondingly. Therefore, in the improved PSO algorithm, a different approach is proposed in that if a new particle moves outside the boundaries, the current velocity of the particle in question is modified using

$$v_d^i(k)_{\text{new}} = \text{sign}(r_3)\cdot\alpha\cdot v_d^i(k) \quad (7)$$

and a new particle is then generated again using (3)–(6) with the newly modified velocity vector. This process is repeated until a feasible particle is generated.

In (7), $\alpha < 1, r_3$ is a random parameter uniformly selected within [0, 1], and $\mathrm{sign}(\cdot)$ is a sign function defined as

$$\mathrm{sign}(r) = \begin{cases} 1, & r \geq 0.5 \\ -1, & r < 0.5. \end{cases} \quad (8)$$

D. Intensification Searches

To enhance the refined pinpointing search ability of the PSO algorithm to locate the exact optimal solution, an intensification search phase is incorporated into the search procedure as proposed below. In essence, once a new g_{best} is founded, the algorithm will activate automatically an intensification search in the small neighborhood around this point using only its speed vector with the cognitive and social influences being deliberately excluded in the velocity updating formula. In this iterative process, if a search is successful, the algorithm will keep the velocity vector unchanged while continuing its exploitation using this speed vector; otherwise, the algorithm will generate randomly a new speed vector to begin the next refinement search. The intensification search process will be repeated until the number of consecutive unsuccessful explorations around a new g_{best} reaches 10.

III. Numerical Examples

To test and validate the proposed algorithm, experiments on different mathematical test functions and practical design problems of electromagnetic (EM) devices are conducted. Only typical results are, however, reported due to space limitations. In these experiments, the parameters used for the PSO algorithms, including the proposed one, are set as $c_1 = c_2 = 2, \alpha = 0.9, N_{\mathrm{popsize}} = 10, v_d^{\max} = (u_d - l_d)/4, v_d^{\min} = 10^{-5} \cdot (u_d - l_d)$ (u_d and l_d are, respectively, the upper and lower bounds of the dth variable). The iterative process of an algorithm will stop once the successive iterations without improvements in the best objective function value searched so far reaches 200.

A. Validation

A well-designed mathematical function having 10^5 local optima, which is categorized as hard functions for an optimizer to find the global solution, is first solved using the proposed and the original PSO algorithms for performance comparison. Mathematically, the function is defined as

$$\begin{aligned} \text{minimize} f(x) = \frac{\pi}{n} & 10 \sin^2(\pi x_1) + (x_n - 1)^2 \\ & + \textstyle\sum_{i=1}^{n-1} \left[(x_i - 1)^2 (1 + 10 \sin^2(\pi x_{i+1})) \right] \\ \text{subject to} \quad & -10 \leq x_i \leq 10, \qquad i = 1, 2, \ldots, 5. \end{aligned} \quad (9)$$

For this function, each of the two algorithms is run independently 100 times, and the averaged performance comparison results are given in Table I. Here, a success run or "to find the exact global solution" means that the tolerance between the searched and the exact global solutions is 10^{-7} in absolute value. To demonstrate the refined pinpointing search ability of the proposed algorithm, a simple two-dimensional (2-D) mathematical function as defined below is then solved using the proposed algorithm with and without the intensification phase. Moreover

TABLE I
Performances Comparison of the Proposed and Original PSOs on the First Mathematical Test Function With 100 Independent Runs

Algorithms	No. of averaged iterations	Success rate
Original PSO	798	9/100
Proposed PSO	1143	100/100

TABLE II
Ten Successive Iterations Immediately After the Finding of a Point Close to the Exact Minimum of Zero for the Proposed Algorithm With and Without Intensification Search for the 2D Test Function

Iteration	Proposed without Intensification			Proposed with Intensification		
No.	*x*	*y*	*f(x,y)*	*x*	*y*	*f(x,y)*
N*	6.76×10^{-5}	-9.18×10^{-4}	8.47×10^{-7}	-3.87×10^{-4}	-3.19×10^{-4}	2.51×10^{-7}
N+1	6.76×10^{-5}	7.81×10^{-4}	6.14×10^{-7}	-3.87×10^{-4}	-3.13×10^{-4}	1.50×10^{-7}
N+2	6.76×10^{-5}	-7.70×10^{-3}	5.93×10^{-5}	-3.87×10^{-4}	3.35×10^{-4}	2.61×10^{-7}
N+3	6.76×10^{-5}	-2.40×10^{-3}	5.76×10^{-6}	-1.42×10^{-4}	2.92×10^{-4}	1.05×10^{-7}
N+4	-0.390	7.81×10^{-4}	0.152	-1.42×10^{-4}	2.95×10^{-5}	2.10×10^{-8}
N+5	-0.857	7.81×10^{-4}	0.734	-1.42×10^{-4}	2.95×10^{-7}	2.02×10^{-8}
N+6	-1.19	7.81×10^{-4}	3.54	-3.08×10^{-5}	2.95×10^{-7}	9.50×10^{-10}
N+7	-0.742	7.81×10^{-4}	0.550	2.03×10^{-4}	3.24×10^{-7}	4.12×10^{-8}
N+8	-0.139	7.81×10^{-4}	1.92×10^{-2}	-2.04×10^{-4}	2.95×10^{-7}	4.17×10^{-8}
N+9	1.36×10^{-3}	-2.760	7.617	-1.42×10^{-4}	3.81×10^{-5}	2.17×10^{-8}
N+10	1.36×10^{-3}	2.72	7.418	-1.21×10^{-4}	3.81×10^{-5}	1.61×10^{-8}

*N is the iterative number for the optimizer to find a 'close approximation' of the exact minimum of zero.

$$\text{minimize} \quad f(x, y) = x^2 + y^2 (-5 \leq x, y \leq 5). \quad (10)$$

After an optimizer finds a "close approximation" of the exact minimum of zero in this study, it will report the search trajectories of the next ten successive iterations, and the corresponding results are presented in Table II. Here, a "close approximation" means that the distance between its position and that of the exact minimum of zero is less than 10^{-3} in absolute value.

From these numerical results, it is obvious that (1) for a hard optimal problem with 10^5 local optima, the 100 independent runs of the proposed algorithm are all successful whereas only nine out 100 runs of the original PSO algorithm satisfy the stipulated solution precisions (2) after finding a "close approximation" of the exact minimum of zero of the 2-D function, the proposed algorithm can reach the exact global solution in six iterations. On the other hand, if one uses the same algorithm with the intensification search being deliberately excluded, the search will wander around the "close solution" in the first three iterations before diverging away in the next seven iterations without any improvement in the objective function.

B. Application

In order to evaluate the performance of the proposed algorithm upon practical engineering design problems, the proposed PSO algorithm is used to study the Team Workshop problem 22 of a superconducting magnetic energy storage (SMES) configuration with eight free parameters, as shown in Fig. 1. This problem can be expressed mathematically as

$$\begin{aligned} \text{minimize} \quad & f = \frac{B_{\mathrm{stary}}^2}{B_{\mathrm{norm}}^2} + \frac{|\mathrm{Energy} - E_{\mathrm{ref}}|}{E_{\mathrm{ref}}} \\ \text{subject to} \quad & J_i \leq (-6.4|(B_{\max})_i| + 56)(\mathrm{A/mm_2}) \end{aligned} \quad (11)$$

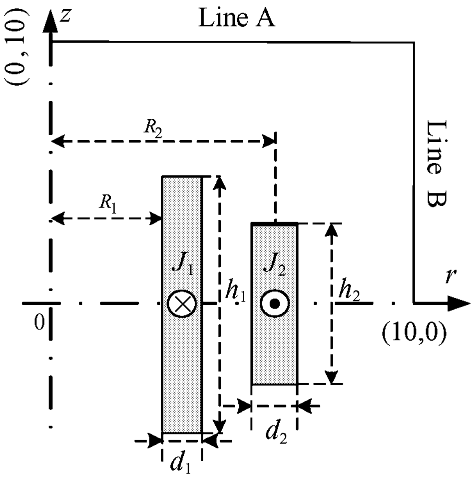

Fig. 1. Schematic diagram of an SMES.

TABLE III
FINAL OPTIMAL RESULTS SEARCHED USING THE PROPOSED METHOD

Results	R_1(m)	R_2(m)	$h_1/2$(m)	$h_2/2$(m)	d_1(m)	d_2(m)
Proposed	1.5704	2.1020	0.7850	1.4206	0.6003	0.2581
By IGTE	1.5703	2.0999	0.7846	1.4184	0.5943	0.2562

TABLE IV
FINAL OPTIMAL RESULTS SEARCHED USING THE PROPOSED METHOD

Results	J_1(MA/m^2)	J_2(MA/m^2)	B^2_{stary} (T^2)	*Energy*(MJ)	f_{obj}
Proposed	17.2469	-12.9653	2.2205×10^{-10}	180.2301	6.8295×10^{-3}
By IGTE	17.3367	-12.5738	2.1913×10^{-10}	179.9924	5.5203×10^{-3}

where Energy is the stored energy in the SMES device, $E_{\rm ref} = 180MJ; B_{\rm norm} = 2 \times 10^{-4}$ T; J_i and $(B_{\rm max})_i (i = 1, 2)$ are, respectively, the current density and the maximum field in the ith coil, $B^2_{\rm stary}$ is a measure of the stray fields which is evaluated along 22 equidistant points of line A and line B of Fig. 1 and is formulated as

$$B^2_{\rm stary} = \sum_{i=1}^{22} (B_{\rm stary})^2_i / 22. \tag{12}$$

For this design problem, the parameters to be optimized are the geometric parameters and current densities of coils 1 and 2, as shown in Fig. 1, and the details about this problem are given in the case reported by Institut für Grundlagen und Theorie der Elektrotechnik (IGTE) [8]. In the numerical study, the performance parameters as required by (11) and (12) are determined based on the 2-D finite-element analysis although more accurate and efficient semianalytical formulae are applicable for this simple field problem. The parameters of the proposed algorithm used for this application are almost the same as that given previously except that a different stop criterion is used (i.e., the proposed algorithm will stop the iterative process when the number of successive iterations without improvements in the best objective function so far searched reaches 100). For comparative purposes, the proposed algorithm is run independently for five times, and the averaged iterative number for the proposed algorithm to converge to a solution is 3465. Tables III and IV give the computed results of a typical run as well as the best ones searched so far by IGTE. From these numerical results, it can be seen that the optimal values of the decision parameters found by the proposed algorithm are nearly identical to those of the most recent best ones contributed by IGTE. This therefore is a solid demonstration of the robustness and effectiveness of the proposed algorithm in solving complex EM design problems. However, the optimized objective function value using the proposed PSO algorithm is worse than that of the best ones searched so far by IGTE.

IV. CONCLUSION

The PSO algorithm is a new entrant to the family of EA, and it has shown great potential for solving difficult design problems in different engineering disciplines. Indeed, there are conferences devoted solely on this topic. Notwithstanding its recent popularity, the PSO algorithm has a number of shortcomings as pointed out by fellow researchers. This paper focuses on enhancing the global search ability and the refined pinpointing search ability of available PSOs, and some approaches are proposed to address these two issues. The numerical results, as reported in this paper, suggest that the proposed algorithm is successful in realizing the goals as stipulated earlier. As for the future work along this line, the authors will strive to develop an adaptive algorithm to use the information of the design problem acquired during the course of the search process for tuning the parameters automatically.

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