A Tabu Method to Find the Pareto Solutions of Multiobjective Optimal Design Problems in Electromagnetics

S. L. Ho, Shiyou Yang, Guangzheng Ni, and H. C. Wong

***Abstract*—A tabu search algorithm is proposed for finding the Pareto solutions of multiobjective optimal design problems. In this paper, the contact theorem is used to evaluate the Pareto solutions. The ranking selecting approach and the fitness sharing function are also introduced to identify new current points to begin every iteration cycle. Detailed numerical results are reported in this paper to demonstrate the power of the proposed algorithm in ensuring uniform sampling and obtaining the Pareto optimal front of the multiobjective design problems. The most efficient method of implementing the proposed algorithm is also discussed.**

***Index Terms*—Fitness sharing, multiobjective optimization, ranking, tabu search, vector optimization.**

I. INTRODUCTION

THANKS to the advent in both computer sciences and numerical techniques, researchers in electrical engineering can now give increasing attentions to design optimization problems that could not be solved hitherto. However, almost all engineering design problems involve simultaneous optimizations of multiple and often conflicting objectives. Although the optimal methods for solving single objective problems are well developed, there are very few "truly" suitable algorithms applicable for multiobjective optimizations. Consequently, a common approach for dealing with this kind of problems in electrical engineering is to convert the problem into a scalar one, i.e., to combine multiobjective functions into a single one by adding different weighting factors to different objectives and then solving the weighted objective function. Alternatively, one can select the most dominant one as the objective function and take the others as constraints. However, the designer must give a priority or preference order to each objective function. This means that the optimal results would depend largely on the rules of thumb of the designer, and some important aspects that were unknown prior to the optimization study could well be excluded. Furthermore, the weighting values might also be chosen inaccurately. Thus, a more desirable and natural approach for multiobjective optimizations is to identify a set of solutions known as the Pareto optima from which a single optimal solution corresponding to a particular tradeoff of different objectives is to be chosen by the decision maker. In this paper, a tabu search algorithm for finding the Pareto solutions of multiobjective functions is proposed. The algorithm is validated using numerical examples.

Manuscript received July 5, 2001; revised October 25, 2001. This work was supported in part by the National Science Foundation of China under Grant 59877022.

S. L. Ho is with the Electrical Engineering Department, the Hong Kong Polytechnic University, Hong Kong (e-mail: eeslho@polyu.edu.hk).

S. Y. Yang is with the Electrical Engineering Department, the Hong Kong Polytechnic University, Hong Kong, and with the Electrical Engineering College of Zhejiang University, Hangzhou, China (e-mail: shiyouyang@yahoo.com).

G. Ni is with the Electrical Engineering Department, Zhejiang University, Hangzhou, China (e-mail: nigz@cee.zju.edu.cn).

H. C. Wong is with the Industrial Center, the Hong Kong Polytechnic University, Hong Kong (e-mail: ichcwong@polyu.edu.hk).

Publisher Item Identifier S 0018-9464(02)02361-0.

II. TERMINOLOGY

Although the multiobjective optimization and the corresponding terminologies were frequently quoted, the interpretations are sometimes dependent on the context. Hence, the following concepts/terminologies are first defined in order to ensure an easy understanding of the proposed algorithm.

Multiobjective Optimization Problem: In general, a multiobjective optimization problem, which is also called a vector optimization problem, is to optimize a vector function subject to some constrained conditions. Consider a minimization problem that can be written in a shortened form as

$$\min \overline{f}(\overline{x}) \quad (\overline{x} \in X). \tag{1}$$

$$\overline{f}: X \to F$$
$$X = \left\{\overline{x} \in E^n \left| \overline{g}(\overline{x}) \geq 0, \overline{h}(\overline{x}) = 0\right.\right\} \tag{2}$$
$$F = \left\{f \in E^k \left| \overline{f}(\overline{x}), \overline{x} \in X\right.\right\}$$

where

$$\overline{x} = [x_1 \quad x_2 \quad \cdots \quad x_n]^T$$
$$\overline{f}(\overline{x}) = [f_1(\overline{x}) \quad f_2(\overline{x}) \quad \cdots \quad f_k(\overline{x})]^T$$
$$\overline{g}(\overline{x}) = [\overline{g}_1(\overline{x}) \quad \overline{g}_2(\overline{x}) \quad \cdots \quad \overline{g}_m(\overline{x})]^T$$
$$\overline{h}(\overline{x}) = [\overline{h}_1(\overline{x}) \quad \overline{h}_2(\overline{x}) \quad \cdots \quad \overline{h}_p(\overline{x})]^T.$$

Pareto Optimal: A solution $\overline{x}^*$ is a Pareto optimal if no objective function can be improved without worsening at least one other objective functions. Mathematically, a solution $\overline{x}^*$ is a Pareto optimal of (1) if and only if for all $\overline{x} \in X (\overline{x} \neq \overline{x}^*)$

$$f_i(\overline{x}^*) \leq f_i(\overline{x}) \wedge (\forall i \in \{1, 2, \ldots, k\})$$
$$f_i(\overline{x}^*) < f_i(\overline{x}) (\exists i \in \{1, 2, \ldots, k\}). \tag{3}$$

Negative Core: The negative core in E^k is the set

$$C^- = \left\{\overline{f} \in E^K \left| \overline{f} \leq 0\right.\right\}. \tag{4}$$

Contact Theorem: A vector $\overline{f}^*$ is a Pareto optimal solution for a multiobjective optimization problem if and only if

$$\left(C^- + \overline{f}^*\right) \cap F = \left\{\overline{f}^*\right\}. \tag{5}$$

Dominated and Nondominated Solutions: Suppose $\overline{x}^{(i)}$ and $\overline{x}^{(j)}$ are two different feasible points.

1) $\overline{x}^{(i)}$ is said to be dominated by (or inferior to) $\overline{x}^{(j)}$ [which is denoted by $\overline{x}^{(j)} \preceq \overline{x}^{(i)}$] if $f(\overline{x}^{(i)})$ is partially larger than $f(\overline{x}^{(j)})$, i.e.,

$$\overline{f}_i\left(\overline{x}^{(i)}\right) \geq \overline{f}_i\left(\overline{x}^{(j)}\right) \wedge (\forall i \in \{1, 2, \ldots, k\})$$
$$\overline{f}_i\left(\overline{x}^{(i)}\right) > \overline{f}_i\left(\overline{x}^{(j)}\right) (\exists i \in \{1, 2, \ldots, k\}). \tag{6}$$

2) $\overline{x}^{(i)}$ is said to be nondominated if there is no $\overline{x}^{(j)}$ among the specified points.

III. Tabu Method for Pareto Optimal

The inherent solution process of tabu search methods, i.e., the involvement of a number of neighborhood solutions simultaneously in the process, lends itself very conveniently to dealing with multiple objective functions. In general, an efficient solver for multiobjective optimization problems should have the following features: 1) The solutions obtained are Pareto optimal, and 2) the solutions are uniformly sampled from the Pareto optimal set [1]. To achieve these two goals, different improvements are proposed on an improved tabu search method of single objective functions, as reported in [2] and discussed below.

A. Generation of Neighborhoods

A common approach [3] in tabu methods for neighborhood generations is to construct the co-center hyperballs with different radii around the current point and then generate a random point in each hyperball. However, this will result in an inhomogeneous exploration of the variable spaces around the current points. Hence, the concentric "hypercrowns" approach, as reported in [4], is extended and employed in the proposed method for the generation of neighborhood solutions. In this paper, the radii of the hypercrowns are determined on the basis of all the hyperballs having the same volume.

B. Introduction of Pareto Optimal Archive

In order to report the searched Pareto optimal and design the diversity algorithms, a Pareto optimal archive is introduced in the proposed method. The quantities of the archive are dynamically updated and used in the optimization process. The length of this archive is finite.

C. Ranking of New States

It is well known that in the selection of new current points for a tabu search, it is necessary to obtain the objective function values of their neighborhood solutions. As the objective function in a multiobjective optimization problem is a vector, some scalarization techniques must be used. Thus, the ranking method [5] is extended and used for evaluating the "fitness" of a solution of the proposed algorithm. Hence, it is proposed that for a neighborhood solution $\overline{x}^{(i)}$, as shown in Fig. 1, its rank can be given by

$$\mathrm{Rank}\left(\overline{x}^{(i)}\right) = 1 + p_i \tag{7}$$

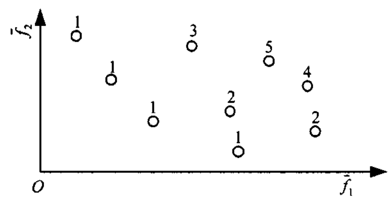

Fig. 1. Ranks for neighborhood solutions for minimizing $\overline{f}_1$ and $\overline{f}_2$.

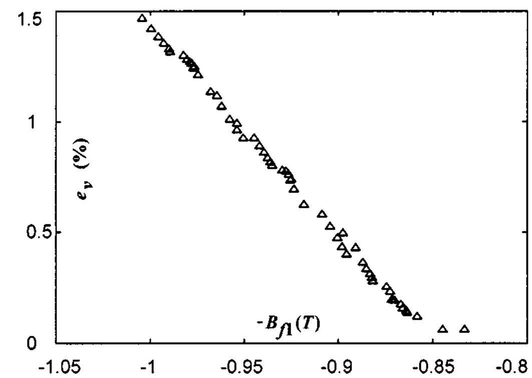

Fig. 2. Searched Pareto optimal of the proposed algorithm for Case 1.

where p_i is the number of solutions in both the neighborhood and the Pareto optimal archive that dominate the point $\overline{x}^{(i)}$.

Contrary to the originally ranking method of [5], the quantities in the Pareto optimal archive are also involved in determining the rank of a neighborhood solution to make the searched Pareto optimal to have uniform distributions in the objective function space.

The rank of a solution, together with its fitness-sharing function value, is then used to decide its "fitness" value as the new current point.

D. Fitness Sharing Function

In order to ensure the searched Pareto optimal to have uniform distributions in the objective space, it is important to maintain the diversity of the algorithm. Besides using the new generation scheme of the neighborhood solutions and to incorporate the quantities of the Pareto optimal archive into the rank computation of a solution, the fitness sharing concept of the evolutionary methods is improved and used in the proposed algorithm. To take into account the density of the searched Pareto optimal around a new point, the following fitness sharing function is proposed. For example, for the neighborhood solution $\overline{x}^{(i)}$, its fitness sharing function is defined by

$$f_{\mathrm{share}}\left(\overline{x}^{(i)}\right) = \frac{1/d\left(\overline{x}^{(i)}\right)}{\sum_{j=1}^{N_h} 1/d\left(\overline{x}^{(j)}\right)} \tag{8}$$

where $d(\overline{x}^{(i)})$ is the density of the Pareto optimal obtained around the specified solution point $\overline{x}^{(i)}$, and N_h is the number of the total neighborhood solutions of $\overline{x}^{(i)}$.

To compute the density of the Pareto optimal for a specified point, a hyperball with the point as the center is constructed, and the number of the Pareto optimal points that lie in this ball are used as a measure for its fitness sharing function. The fitness

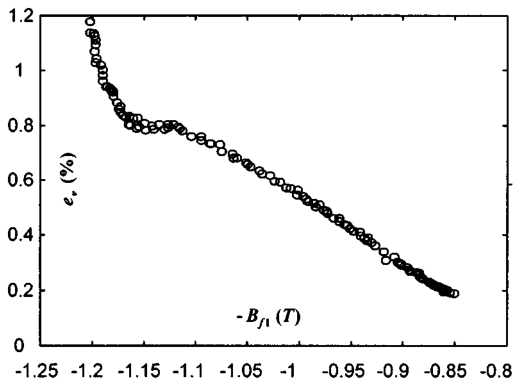

Fig. 3. Computed Pareto optimal of the proposed algorithm for Case 2.

value of a neighborhood solution $\overline{x}^{(i)}$ is the sum of the inverse of its rank and its fitness sharing function value, i.e.,

$$f_{fit}\left(\overline{x}^{(i)}\right) = \frac{1}{1+p_i} + \frac{1/d\left(\overline{x}^{(i)}\right)}{\sum_{j=1}^{N_h} 1/d\left(\overline{x}^{(j)}\right)}. \quad (9)$$

All other things being equal, it can be seen that the sparser the points around the specified point, the more likely it is to be selected as the new current point. This characteristic enables the proposed algorithm to work well to yield an uniform sampling of the Pareto optimal front in a simple way. Moreover, since uniformly sampling the Pareto front is as important as finding them, there are no weighting factors in (9) for ranking the fitness-sharing values.

In addition, there would be some rare situations where more than one neighborhood solutions are sharing the highest fitness value. Under such circumstances, the new current point is selected randomly among those with the highest fitness value.

E. Evaluation and Reporting of Pareto Solutions

In the optimization process, the Pareto optimal archive is automatically updated. To test if a solution is a Pareto optimal, the contact theorem is used [6]. To simplify the description, let $\overline{x}^{(l)}$ be a new solution to be considered. In the set of the Pareto optimal archive, if there is a solution $\overline{x}_j^p$ such that

1) if $(C^- + \overline{f}(\overline{x}^{(l)})) \subset (C^- + \overline{f}(\overline{x}_j^p))$, then $\overline{x}_j^p$ is substituted by $\overline{x}^{(l)}$;
2) if $(C^- + \overline{f}(\overline{x}^{(l)})) \supset (C^- + \overline{f}(\overline{x}_j^p))$, then $\overline{x}^{(l)}$ is discarded;
3) if none in the Pareto optimal archive satisfies (1) or (2), then $\overline{x}^{(l)}$ becomes a new Pareto solution, and in this case
   i) if the Pareto optimal archive is not full, add $\overline{x}^{(l)}$ to it;
   ii) if the Pareto solution is full and $\overline{x}^{(l)}$ is in a sparser region than some members of the Pareto optimal archive, replace the solution with the highest point density by $\overline{x}^{(l)}$;
   iii) if neither of i) or ii) is satisfied, then discard $\overline{x}^{(l)}$.

Since the proposed method is developed for real-value problems, the likelihood of finding identical solutions is extremely small, and hence, a proximity criterion was proposed in order for the procedure to work [7]. Such a criterion relaxes the strict comparison of the new solution to those stored in the Pareto solution archive, thereby tending to give rise to uniformly distributed Pareto optimal solutions in the feasible variable spaces.

F. Stop Criterion

The proposed algorithm has two termination criteria to determine when to stop the iterative process.

Criterion One: Once the number of iterations exceeds a prescribed threshold value, the algorithm will stop the iterative process.

Criterion Two: Once the Pareto optimal archive is full and the point density for every member exceeds a threshold value, the algorithm will also stop the iterative process.

The analysis and numerical experiences indicate that the second criterion is more robust in producing an uniform Pareto optimal, but it is computationally rather inefficient.

IV. Numerical Examples

The performances of the proposed algorithm have been investigated extensively on the multiobjective optimization of a practical engineering design problem for determining the optimal geometry parameters of the multisectional pole arc of large hydro-generators with the following goals:

$$\begin{aligned} &\max \quad B_{f1}(X) \\ &\min \quad e_v \\ &\text{s.t} \quad SCR - SCR_0 \geq 0 \\ &\qquad X_d' - X_{d0}' \leq 0 \\ &\qquad THF - THF_0 \leq 0 \end{aligned} \quad (10)$$

where B_{f1} is the amplitude of the fundamental component of the flux density in the air gap, e_v is the distortion factor of a sinusoidal voltage of the machine on no-load condition, THF is the telephone harmonic factor, X_d' is the direct axis transient reactance of the motor, and SCR is the short circuit ratio.

The details about this problem are referred to [8]. In order to explore the performance of the proposed method, it can be studied in the following four different cases.

Case 1: The fitness-sharing function is excluded from the computation of the fitness values of the new points, and termination criterion one is used to decide when to stop the iterative process of the algorithm.

Case 2: The fitness-sharing function is incorporated into the evaluation of the fitness value of a newly generated point, and termination criterion two is used to stop the iterative procedure.

Case 3: The only difference of this case from Case 2 is that criterion one is used.

Case 4: The only difference from Case 2 is that the threshold value of the point densities is set to half of that of Case 2. Figs. 2–5, respectively, show the computed Pareto optimal front of the proposed algorithm under the aforementioned four different running conditions for a 300-MW, 20-pole hydrogenerator. The corresponding performance comparisons are given in Table I. For each case, the iteration number given in this table is the average value of five runs from different starting points. From these results, it can be seen that with the fitness-sharing

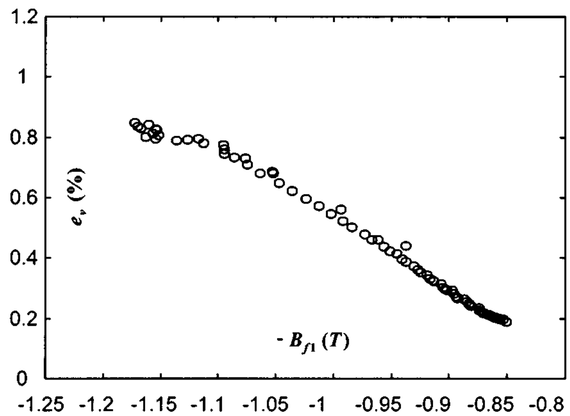

Fig. 4. Searched Pareto optimal of the proposed algorithm for Case 3.

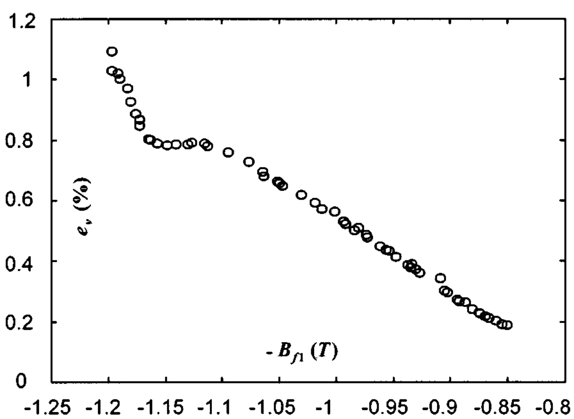

Fig. 5. Computed Pareto optimal of the proposed algorithm for Case 4.

function being excluded from the decision of the fitness value of a solution and a simple termination criterion is being used, the algorithm produces "deceptive" Pareto optimal fronts as shown in Fig. 2. We also see that although the fitness function is incorporated in the computation of the fitness value of a new solution, the use of a simple terminative criterion makes the algorithm unable to find some parts of the Pareto optimal front, as shown in Fig. 4. Moreover, even if the Pareto optimal front is found, the point densities are denser in some sections and sparser in others. Hence, a simple termination criterion cannot guarantee to yield the total Pareto optimal front or even parts of it in a uniform manner. In addition, the algorithm using a simple termination criterion (criterion one) is generally computationally efficient, as can be seen by the iterative number of Case 1 in Table I. We also find that once the fitness-sharing function is included and a delicate termination criterion is used as in Case 2, the proposed algorithm produces nearly perfect results for the Pareto

TABLE I
PERFORMANCE COMPARISON OF THE PROPOSED ALGORITHM UNDER FOUR DIFFERENT RUNNING CONDITIONS

	Case1	Case 2	Case 3	case 4
No. of Iterations	6500	8946	6500	5841

optimal front of the test problem. Moreover, when one halves the threshold value of the point density as in Case 4, the algorithm can still produce uniform distributions of the exact Pareto optimal front. In addition, the iteration number used in Case 4 is significantly smaller than that in Case 3. Finally, the iterative numbers, ranging from 5841 to 8946 in the proposed algorithm under all cases, are acceptable for the complex multiobjective optimal design problems being studied.

V. CONCLUSION

By introducing the ranking concept and incorporating the fitness sharing function into the neighborhood selection of tabu search methods and by proposing the new termination criteria as well as making other novel improvements as discussed, this paper presents a vector optimal design technique (the tabu search algorithm) for finding the Pareto solutions of multiobjective optimal design problems. The detailed numerical results fully demonstrate the robustness of the proposed algorithm in obtaining and ensuring uniform sampling of the Pareto optimal front of the multiobjective design problems. Moreover, the numerical results also suggest that if one has *a priori* knowledge about the distributions of the objective functions, one can use a simple termination criterion in the proposed algorithm in order to be more computationally efficient.

REFERENCES

[1] D. A. V. Veldhuizen and G. B. Lamont, "Multiobjective evolutionary algorithms: Analyzing the state-of-the-art," *Evol. Comput.*, vol. 8, pp. 125–147, 2000.
[2] Y. Shiyou and G. Ni *et al.*, "An universal tabu search algorithm for global optimization of multimodal functions with continuous variables in electromagnetics," *IEEE Trans. Magn.*, vol. 34, pp. 2901–2904, Sept. 1998.
[3] N. Hu, "Tabu search method with random moves for globally optimal design," *Int. J. Num. Methods, Eng.*, vol. 35, pp. 1055–1070, 1992.
[4] P. Siarry, "Fitting of tabu search to optimize functions of continuous variables," *Int. J. Num. Methods, Eng.*, vol. 40, pp. 2449–2457, 1997.
[5] D. E. Goldberg, *Genetic Algorithms in Search, Optimization, and Machine Learning*. Reading, MA: Addision-Wesley, 1989.
[6] A. Osyczka, *Multi-Criterion Optimization in Engineering With FORTRAN Program*. London, U.K.: Ellis Horwood, 1984.
[7] S. L. Ho, S. Yang, P. Zhou, H. C. Wong, and M. A. Rahman, "A combined finite element-domain elimination method for minimizing torque ripples in inverter-fed motor drive systems," *IEEE Trans. Magn.*, vol. 36, pp. 1817–1821, July 2000.
[8] R. Y. Tang and S. Yang *et al.*, "Combined strategy of improved simulated annealing and genetic algorithm for inverse problems," *IEEE Trans. Magn.*, vol. 32, pp. 1326–1329, May 1996.