Integrated RBF Network Based Estimation Strategy of the Output Characteristics of Brushless DC Motors

S. L. Ho, Minrui Fei, W. N. Fu, H. C. Wong, and Edward W. C. Lo

***Abstract*—The circuit-field coupled model is very accurate but it is computationally inefficient in studying the output performance of brushless dc motors. In order to resolve the problem, an estimation strategy based on an integrated radial basis function (RBF) network is proposed in this paper. The strategy introduces new conceptions of the network group that are being realized by three steps, namely: 1) an adaptive RBF network is proposed for modeling the center network; 2) the RBF network group is then used to build the base networks; and 3) an integrated RBF network based on the base network group is used subsequently to predict the non-trained output characteristics of the brushless dc motor.**

***Index Terms*—ANN, brushless dc motor, finite element, nonlinear, radial basis function.**

I. Introduction

THE MATHEMATICAL models of electric machines can be traditionally separated into circuit models and circuit-field coupled models. The circuit model has constant parameters and can be solved quickly, but its accuracy is poor as the electromotive forces and inductances are dependent on the operating conditions of the motor [1], [10]. In the circuit-field coupled model, the circuit equations of the stator windings and the magnetic field are coupled together by the flux-linkages and these equations are then solved simultaneously. When the impressed stator voltages are known, the stator phase currents, the magnetic field distribution, and the output torque can be computed. When the torque balance equation is coupled into the system equations, the dynamic performance of the motor can be computed by using the time stepping method [2]. The circuit-field coupled model is very accurate but it is computationally inefficient. Therefore, the circuit-field coupled model is limited to the application of off-line simulations. Thus, in real-time control, the circuit models are extensively used.

However, it is now possible to perform the simulations and also the real time control of electrical machines by using artificial neural network (ANN) based techniques [3], [4]. Such a system does not require an analytical model and is not restricted by the assumptions in the conventional circuit models. It can also produce results much faster when compared with the complicated numerical methods. Nonetheless, this technique is still in its early development stage. The choice of the ANN structure and its associated training procedures for different types of motors is thus a topical research area for fellow co-researchers.

In this paper, an integrated radial basis function (RBF) network trained by the computational results from the circuit-field coupled model for a multi-pole brushless dc motor is presented. The proposed estimation strategy with a center network and a base network group has a high accuracy and it can produce results very quickly when compared to algorithms using the conventional field computation methods and the back propagation (BP) network-based strategies [5]. Hence, the integrated RBF network can be applied to the real-time control and the optimum design study of brushless dc motors with the same accuracy as that obtained in magnetic field computations using finite-element modeling.

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II. Circuit-Field Coupled Model for Training Network

A circuit-field coupled time stepping finite-element method (FEM) is used to simulate the operation of brushless dc motors [6]. The circuit equations are coupled to the voltage driven FEM of the motor, and hence the stator windings can be fed with the output voltages directly from the inverter. With the FEM model, the effects of high-order harmonics, saturations, and relative movements of tooth-slots can all be directly included into the system equations.

The integrated RBF network is only used to represent the relationship among stator voltages and currents, and to calculate the output torque. When the magnitude of the impressed stator voltage u_m, the normalized stator voltages u_A, u_B, and u_C, the rotor position θ, and the rotor speed ω are known, the algorithm would then compute the stator phase current i_A, i_B, i_C, and the output torque T. The nonlinear relationships of the inputs and the outputs can be expressed as

$$\begin{cases} i_A = f_A(u_A, u_B, u_C, \theta, \omega, u_m) \\ i_B = f_B(u_A, u_B, u_C, \theta, \omega, u_m) \\ i_C = f_C(u_A, u_B, u_C, \theta, \omega, u_m) \end{cases} \tag{1}$$

$$T = f_T(i_A, i_B, i_C, \theta, \omega, u_m). \tag{2}$$

By calling the time-stepping FEM repeatedly for a given set of $(u_A, u_B, u_C, \theta, \omega, u_m)$, a set of profiles of (i_A), i_B, i_C, $T)$ can be obtained. These computed data are used as the training patterns for training the integrated RBF network in the proposed algorithm.

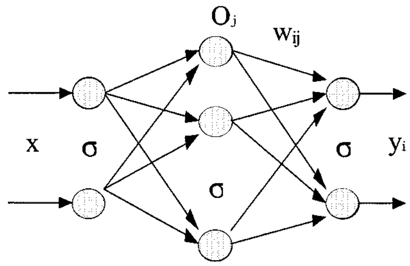

Fig. 1. Model of the radial basis function (RBF) network.

III. RBF NETWORK PRINCIPLE

The RBF network has a linear output layer and a simple structure with a nonlinear hidden layer that constructs the local approximations which correspond to the nonlinear input–output mapping as given in (1) and (2) and Fig. 1.

The base function of the hidden layer is defined as the Gaussian base function

$$O_j(x) = e^{-\|x-C_j\|^2/\sigma_j^2}. \tag{3}$$

where c_j is the center point of the j base function and σ_j is the selectable parameter controlling the width of the base function around the center point. For the input $x \in R^m$, only a few nodes near the center point c_j are active. The nonlinear mapping from x to $O_j(x)$ is thus realized in the hidden layer.

The linear output layer realizes a linear input–output mapping as follows:

$$y_i = \sum_{j=1}^{n} w_{ij} O_j(x) \tag{4}$$

$$w_{ij}(k+1) = \left(w_{ij}(k) + \lambda \left[y_i^d - y_i(k) \right] O_j(x) \right) \Big/ \sum_{j=1}^{n} O_j^2(x) \tag{5}$$

where

- w_{ij} linking weight between the $O_j(x)$ node in the hidden layer and the y_i node in the output layer;
- y_i^d desired network output;
- λ learning rate.

When $0 < \lambda < 2$, the convergence of the recurrence equation (5) is assured as described below.

First, let

$$e_i(k) \triangleq y_i^d - y_i(k). \tag{6}$$

From (4), one has

$$e_i(k) = y_i^d - \sum_{j=1}^{n} w_{ij}(k) O_j(x) \tag{7}$$

Substituting (6) into (5) and then into (8) yields

$$\Delta e_i(k) = -\lambda e_i(k). \tag{9}$$

Then one obtains

$$\Delta e_i(k) = -\sum_{j=1}^{n} \Delta w_{ij}(k) O_j(x) \tag{8}$$

that is

$$e_i(k+1) = (1-\lambda) e_i(k). \tag{10}$$

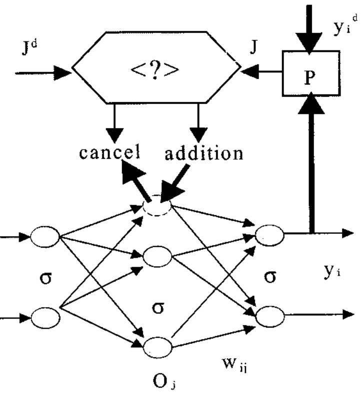

Fig. 2. Model of the adaptive RBF network.

If the iterative process is to be stabilized, one sets $\lim_{m\to\infty} e_i(k) = 0$. Moreover, the stability condition is satisfied when $|1-\lambda| < 1$, i.e., $0 < \lambda < 2$. In order to meet the monotonic attenuation condition of the changes in $e_i(k)$, one has $0 < \lambda < 1$.

IV. ADAPTIVE RBF NETWORK PRINCIPLE

Indeed, the approximating characteristics of the RBF network have a higher accuracy and an acceptable speed of computation when compared to that of the BP network. However, the relatively small number of nodes in the RBF hidden layer normally has a low accuracy, and hence one may not be able to minimize the root-mean-square error function. Consequently, the adaptive RBF network as proposed is used.

In Fig. 2, J is the root-mean-square error function and J^d is the allowable maximum value of J. The block P refers to the process of getting the function J. The block <?> is the process for comparing J with J^d to make the following decisions.

If $J > J^d$, then $n = n+1$ (the number of nodes in the hidden layer is increased);

If $J \leq J^d$ and $T = 0$, then $T = T+1$ and $n = n+1$ (the initial state of the counter T is 0 and the number of nodes in the hidden layer is increased).

If $J \leq J^d$ and $T > 0$, then $n = n-1$ (the number of nodes in the hidden layer is reduced, and the adaptive process of RBF is complete).

Consequently, the number of nodes in the hidden layer is dependent on the error function J and it will be varied from a specific minimum to a maximum. As the network with an adaptive structure is always designed to satisfy a specific accuracy, such algorithms are thus commonly employed to model a special point network.

V. RBF NETWORK GROUP PRINCIPLE

In (1) and (2), the common mappings is based on the relationship of the normalized stator input voltage u_A, u_B, u_C, the rotor position θ, and the outputs of stator phase current i_A, i_B, i_C, as well as the motor torque T at an impressed stator voltage with a magnitude of $u_m = u_{mc}$ and a rotor speed $\omega = \omega_c$.

According to experimental results and the experiences of the authors, the outputs i_A, i_B, i_C, and T are dependent on the inputs u_A, u_B, u_C, θ, u_m, and ω in modes which are highly

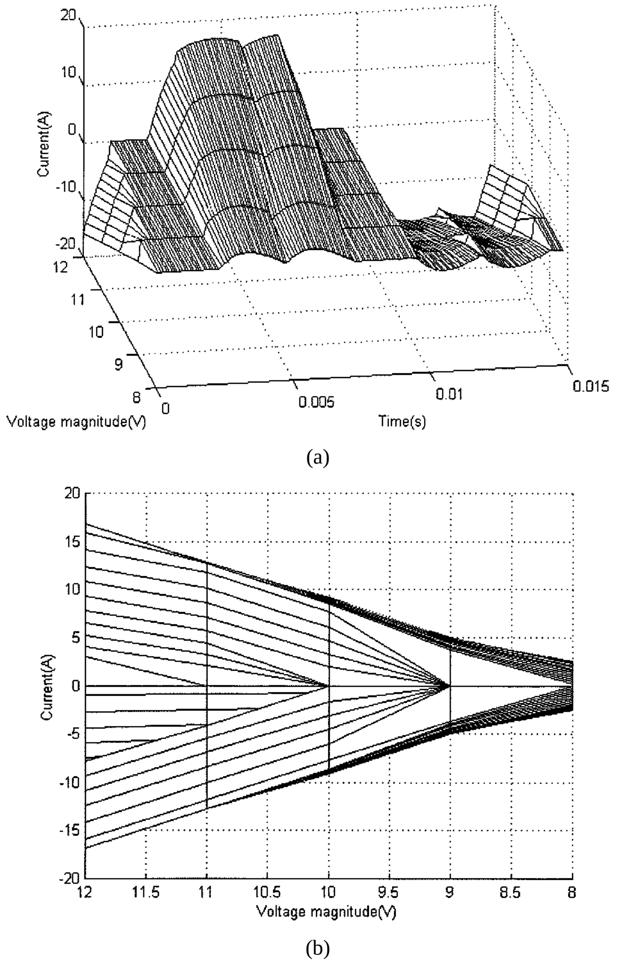

Fig. 3. Dynamic relationship of i_A and u_m in one period. (a) $i_A - u_m - time$ 3-D chart. (b) $i_A - u_m$ 2-D chart.

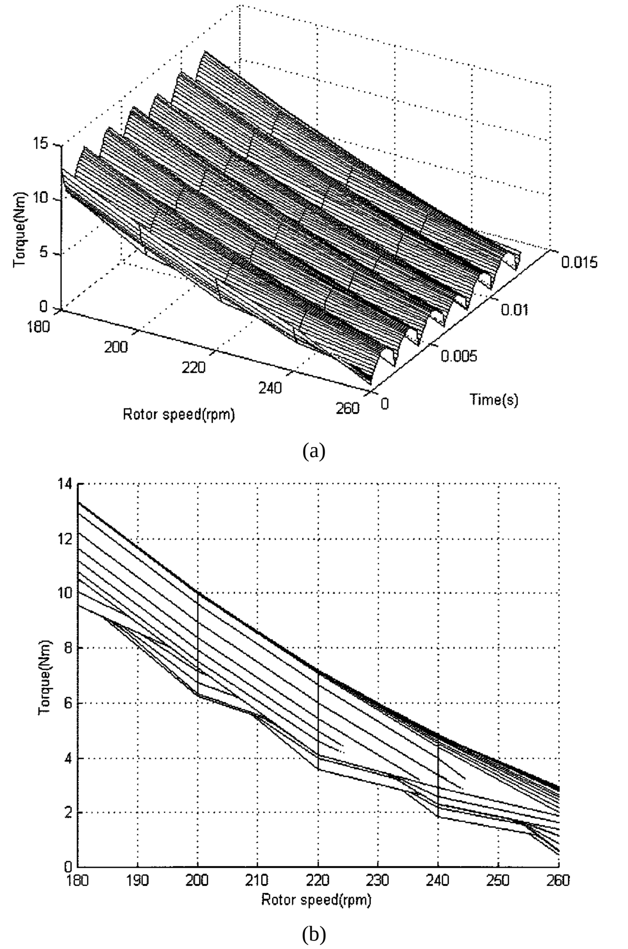

Fig. 4. Dynamic relationship of T and ω in one period. (a) $T - u_m - time$ 3-D chart. (b) $T - u_m$ 2-D chart.

nonlinear. Within a limited range, the changes of the current and torque with u_m or ω are, to a first order of approximation, mostly linear, as can be seen in Figs. 3 and 4, in which the data are generated from FEM computations.

In order to increase the learning speed of the RBF network and improve the generalization ability of the integrated RBF networks, the concept of the RBF network group is proposed. The key of this new concept is that, based on the approximate piecewise-linear assumption within a certain range, one should model the center point ($u_m = u_{mc}$ and $\omega = \omega_c$) by organizing the RBF network adaptively, and then one could model the adjacent and discrete points ($u_m = u_{mb1}, u_{mb2}, \cdots$, and $\omega_m = \omega_{mb1}, \omega_{mb2}, \cdots$) by the approximate RBF networks which have the same number of hidden layer nodes and the same parameters as the Gaussian base functions. The only difference between the proposed algorithm with the RBF network is in the linear layer. Hence, the proposed RBF network group would also have the fast learning power of the approximate RBF networks and a good convergence characteristic.

VI. Integrated RBF Network Principle

Both the adaptive RBF network for modeling the center point and the RBF network group for modeling the adjacent points are dependent on the training data from the circuit-field coupled time-stepping FEM computations. With the same hidden layer, the integrated RBF network does not require any training data. It only needs the weighting matrices of the linear layer of the RBF network group that are used as the integrated network bases.

The integrated algorithm of the linear weighting matrices based on the network bases can be expressed as

$$w_{ij}^{(I)} = \sum_{k=1}^{N_b} \mu_k w_{ij}^{(k)} \tag{11}$$

$$\mu_k = \left[\left(\frac{\omega_{bk} - \omega_c}{d_1} \right)^2 + \left(\frac{u_{mbk} - u_{mc}}{d_2} \right)^2 \right] \cdot \exp\left(- \left[\left(\frac{\omega - \omega_{bk}}{\sigma_\omega} \right)^2 + \left(\frac{u_m - u_{mbk}}{\sigma_u} \right)^2 \right] \right) \tag{12}$$

where the superscript (I) refers to the network of current output or torque output, k is the number of networks in the RBF network group, and c is the center point network, i.e., the adaptive RBF network. d_1, d_2 and σ_ω, σ_u are undetermined coefficients; the exponential function determines the weighting of the mutual influence between the voltage magnitude and the rotor speed. The larger the values of d_1 or d_2, the smaller are their influences in the integrated RBF network. On the other hand, the larger the values of σ_ω or σ_u, the bigger are their influences in the integrated RBF network.

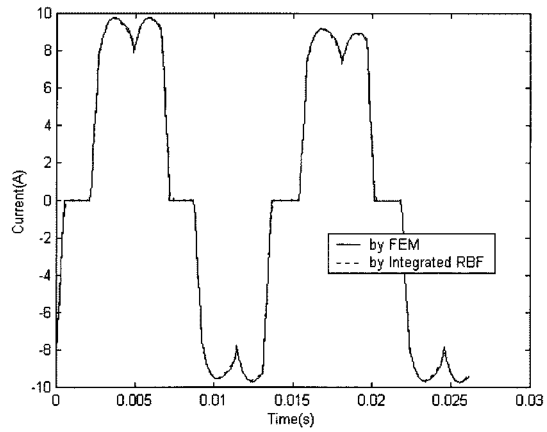

Fig. 5. Stator phase current obtained from the FEM computation and the integrated RBF network.

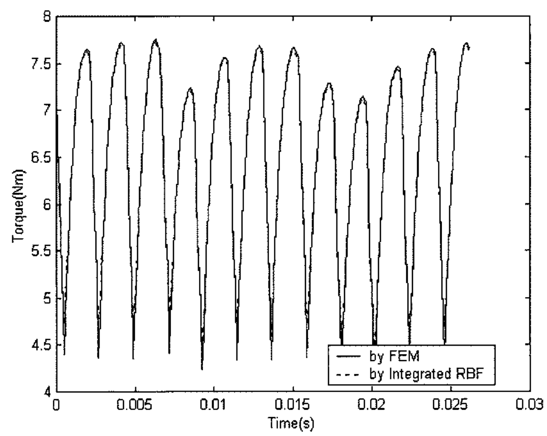

Fig. 6. Output torque using the FEM computation and the integrated RBF network.

Because of the convergence of the integrated network bases, the nontrained integrated RBF network must be convergent by virtue of the Superposition Theorem. The analysis is given as follows.

With the assumption as mentioned previously, one has the same hidden layer in the RBF network group, that is

$$O_j^{(I)}(x) = O_j^{(k)}(x). \tag{13}$$

According to (4), (11), and (13), one has

$$\begin{aligned} y_i^{(I)} &= \sum_{j=1}^{n} w_{ij}^{(I)} O_j^{(I)}(x) = \sum_{j=1}^{n} \sum_{k=1}^{N_b} \mu_k w_{ij}^{(k)} O_j^{(k)}(x) \\ &= \sum_{k=1}^{N_b} \mu_k (\sum_{j=1}^{n} w_{ij}^{(k)} O_j^{(k)}(x). \end{aligned} \tag{14}$$

This will obviously satisfy the superposition and convergence requirements as $\sum_{j=1}^{n} w_{ij}^{(k)} O_j^{(k)}(x)$ $(k = 1, 2, ..., N_b)$ is convergent in the training when $0 < \lambda < 1$ in (5).

VII. Examples

The proposed strategy has been used to establish an integrated RBF network model of a multi-pole PM brushless dc motor (200 W, 12 V, 22 pole, 24 stator slots, with Nd–Fe–B magnets). In the FEM computation, the FEM mesh has about 6000 nodes. According to the learning data from the circuit-field coupled time-stepping FEM computation, three types of experiments are designed. It can be shown that for $u_{mc} = 10$ V and $\omega_c = 220$ rpm, any computation accuracy of currents and torques can be realized by using the adaptive RBF network. Moreover, if the root-mean-square error of the adaptive RBF network is specified to be less than 0.01 and if the node number n is fixed at 159 and 239 for, respectively, the currents and the torques, the error of the RBF network group can be trained to be less than 0.03. Lastly, if the integrated RBF network is synthesized from the network bases from the RBF network group, it can be shown that the target error of 0.05 can be reached within the range of $\omega \in (200-\varepsilon, 240+\varepsilon)$ rpm and $u_m \in (9-\varepsilon, 11+\varepsilon)$ V. Figs. 5 and 6 are only the illustrations in the range.

VIII. Conclusion

In order to resolve the computational inefficiency of the circuit-field coupled model, an estimation strategy based on an integrated RBF network is proposed. The experimental results show that the integrated RBF network is capable of fast learning, e.g., the training time of the center network is about 92∼225 s and the training time of each base network of RBF network group is about 0.43 s. The real estimation or mapping takes about 1.8 ms on a PC 330-MHz computer. Consequently, the proposed estimation strategy should be very useful for the precision real-time control or optimal design of brushless dc motors and many other applications.

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