An Improved Tabu Search for the Global Optimizations of Electromagnetic Devices

S. L. Ho, Shiyou Yang, Guangzheng Ni, and H. C. Wong

***Abstract*—An extended Tabu algorithm with an aspiration factor is proposed. The algorithm is based on the success of techniques such as the memorization of the previously visited subspaces, the systematic diversification as well as the intensification process for neighborhood creations. The numerical results obtained by solving a mathematical test function and the benchmark problem 22 of the TEAM Workshop reported in this paper will demonstrate the usefulness of the proposed method.**

***Index Terms*—Global optimization, inverse problem, optimal design, stochastic algorithm, tabu search method.**

I. Introduction

MORE and more attentions are paid to stochastic methods when one tries to solve the optimal design problems arising from the computation on electromagnetics in recent years. This is because most of the optimal design problems involve objective functions with more than one optima, and by inclusion of the stochastic elements, the stochastic methods would reach the global optimum with certainty under mild conditions. Current stochastic methods popularly used in electromagnetics include Simulated Annealing (SA), Evolution (Genetic), and Tabu Search (TS) algorithms. Unfortunately, the major drawback accompanying this kind of methods is the slow convergence speed or excessive computational burden. In order to alleviate the excessive computational burden and enhance the robustness of the methods, recent research on these methods focuses on the refinements of the methods to enhance their efficiency, or to establish a good trade-off between accuracy, reliability and computational burden [1]–[3]. Compared with SA and Genetic algorithms, the tabu search technique is relatively new, and it is felt that there is a need for further developments. Recently, different modifications on this method are proposed, especially in problems such as combinational optimizations [4]–[9]. Essentially, some of the improvements of the Improved Tabu Search (ITS) presented in this paper are novel, but some are based on methods similar to those used in combinational optimization problems that are rarely used to study electrical engineering problems. Numerical results of the proposed method on a mathematical test function and a benchmark problem are also presented to demonstrate the applicability of the proposed ITS.

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II. Improved Tabu Search

As it is well know, tabu search technique is a metaheuristic procedure that guides the local heuristic search procedure to explore the solution space to avoid local optimality. The structures used in literatures vary from very simple ones that include only certain basic components to those with very complicated strategies such as variable tabu lists, systematic diversification and intensification processes for neighborhood creations as well as short and long term memories, resulting in different performances and complexities in programming. Hence the performance of the TS depends on a proper choice of the neighborhood of a solution, on the number of iterations for which a move is kept as tabu, on the aspiration criterion, on the best combination of short and long term memories, and on the best balance of the intensification and the diversification strategies [9].

This section details the ITS for the global optimization of multimodal objective functions with continuous variables based on the thorough investigation and integration of the recent developments about this method in the related areas.

A. Intensification Phase

To reinforce the moves that incorporate the attributes of good solutions founded in previous search process, an evolution method is used as the intensification phase within the ITS, because through reproduction and crossover, the excellent characteristics of the parent generation is inherited by the new generations, and the fittest survives as the parent generation to reproduce the next generation. The first n_p states with better objective function values thus far searched are selected as the population of the evolution method, and the populations are dynamically updated during the search.

The evolution strategy used in this paper is a simple one with Reproduction and Crossover operators. In view that the intensification phase is only used to drive the search to use the attributes of previously visited good solutions to guide the generation of new states, the Mutation operator is excluded deliberately in the proposed ITS. The iterative procedure of this phase can be described as:

1) Parent Selection: Select the parents randomly from the population according to their fitness;

2) Crossover: For every two selected parents p_1 and p_2, a linear combination of them is used to generate their feasible offsprings (new states) of_1 and of_2 [1], [2], i.e.,

$$of_1 = \alpha p_1 + (1-\alpha) p_2$$
$$of_2 = (1-\alpha) p_1 + \alpha p_2 \quad (1)$$

where α is a random number out the interval of [0, 1].

3) Termination Test: If the test is passed, terminate the intensification phase, and go to diversification strategy; otherwise, go to step 1) for the next iteration cycle.

B. Diversification Phase

To drive the search into unexplored regions uniformly and to escape from a local optimum, a diversification strategy is included in the ITS. This is realized by means of:

1) The use of different generation mechanism for new neighborhood—to prevent generations of trivial moves, the new state, denoted by x_{new}, is generated in the h_i neighbor of the current one, identified by x_{old}, by using the following formula, which differs from the most common ones used in the study of computational electromagnetics,

$$x_{new} = x_{old} + r \cdot \delta_i \quad (2)$$

where

$r = \mathrm{diag}(r_1, r_2, \ldots, r_n)$, $r_i (i = 1, 2, \ldots, n)$	is a random integer parameter within the interval $[(N_l)_i, (N_u)_i]$,
n	is the dimension of the independent variables,
$\delta_i \ (i = 1, 2, \ldots, n)$	is the precision parameter of the variable in the ith direction predetermined by the user, and

$$(N_l)_i = \mathrm{integer}\left\{\max\left(\frac{a_i - (x_{old})_i}{\delta_i}, \frac{-h_i}{\delta_i}\right)\right\} \quad (3)$$
$$(N_u)_i = \mathrm{integer}\left\{\min\left(\frac{b_i - (x_{old})_i}{\delta_i}, \frac{h_i}{\delta_i}\right)\right\} \quad (4)$$

where a_i and b_i $(i = 1, 2, \ldots, n)$ are, respectively, the lower and upper bounds of the variable in the ith direction.

2) Use of both short and long term memorizations—by memorizing the most recently visited n_l states, the so called short term memory, and by comparing the newly generated moves with those memorized in both the short term and the population of the intensification phase, the so called long term memory, the procedure will discard unnecessary minimization steps that would otherwise lead to some previously found local optima or searched sub-states. Hence the algorithm is forced to explore un-searched subspaces.

3) Always restart from the last accepted states rather than the best one searched so far in the diversification phase.

Since the proposed method is developed for real value problems, the likelihood of finding solutions that are identical is extremely small, thus a proximity criterion was proposed in order for the procedure to work [10]. Such criterion relaxes the strict comparison of the new solution to those memorized in both the short and long term memories.

C. Memorization of Searched Subspace

The main aim of memorizing the searched subspace for applying tabu search technique for the optimization of multimodal functions with continuous variable is to prevent the algorithm form premature terminations, i.e., to converge to a local optimum, rather than to prevent it from cycling. So besides the memorization of the population of the intensification phase, a short term memory—the memorization of the most recently visited n_l states is also included in the proposed ITS. This short term memory is dynamically updated by adding the last solution to the list and discarding the oldest one from it.

Although the CPU time required for computing the distances between the randomly generated moves with those memorized in both long and short term memories is very small compared with that required for starting new iterative cycles with this kind of points, the accumulation of the overall time is still significant, especially for the long term memory case where the quantities are very large. In order to reduce these computing times, 1) the quantities in the memories are stored in ascending order of the objective function values; 2) the per unit values of the variables are used.

D. Aspiration Level

By dynamically memorizing the searched subspaces and the populations, and comparing the newly generated states with those in the memories, some new states may also be tabu in the ITS. Considering the fact that when a solution to a problem is being searched iteratively, the error reduction would probably become increasingly small [5]. If the moves are still found to lead to better objective function values with respect to the best one so far found, then an aspiration level condition is proposed to override the decision to reject a new move that may lead to a best solution if:

1) this new move happens to be better than the best one which has been found so far, no matter how smaller is the improvement; or

2)

$$f(x_k) - f(x_{k-1}) < 0$$

where x_k is the best solution among the moves generated in the neighborhood of the current point x_{k-1}.

Together with

3) the distances between the new moves to all of the memorized populations are greater than a threshold.

The reason for adding condition 3) to the previous two conditions is that if the newly generated move is very near to one of the populations, then the intensification search in the neighborhood of the prescribed state can be accomplished in the intensification phase, thereby saving the duplication in computational resources.

E. Transition Between Inten- and Diver-sification Phases

The proposed ITS uses a similar criterion as reported in [5] to dynamically determine when to change phases, i.e., when to start from the intensification phase; if the objective function has not improved significantly in the last k iterations, then the ITS will switch to the diversification phase; the algorithm will continue in the diversification phase until the objective begins to improve or if a maximum number of diversification iterations is reached.

F. Termination Criteria

The proposed algorithm has two termination criteria to decide if the global optimum has been found. The first one is: at the end of every iteration cycle of the diversification phase, the search should stop if

$$|f_k^* - f_{k-p}^*| \le \varepsilon \qquad (p = 1, 2, \ldots, n_p)$$
$$f_k^* - f_{opt} \le \varepsilon \tag{5}$$

where

- f_k^* is the best solution of the objective function searched in the kth cycle of the diversification or intensification phases,
- f_{opt} is the best one searched so far,
- n_p is a prescribed integer number, and
- ε is a precision parameter.

The other termination criterion to stop the search is satisfied when the number of consecutive moves with no predefined improvements in the best objective function exceeds a threshold value.

G. Algorithm Description

Based on the previous description, one should now be in a position to give a schematic explanation of the ITS below:

1) Initialization—randomly generate the population for the intensification phase;
2) Starting the Intensification phase—if the objective function has not improved significantly in the last k iterations, switch to the diversification phase;
3) Starting diversification phase—the algorithm continues in this phase until the solution begins to improve or a maximum number of iterations is reached. The population is also dynamically updated in this phase to include recently visited moves with better solutions;
4) Termination test—if the test is passed, stop; otherwise go to 2) to continue the next cycle iteration.

III. Numerical Examples

A. Mathematical Test Function

A complex mathematical function with many local minima is used to test the proposed algorithm. The details of the function are:

$$\text{minimize} \quad f(x) = -\left[\sum_{i=1}^{6} \frac{1}{6} \sin\left(2\pi \left(x_i + \frac{i}{5}\right)\right)\right]^2 \tag{6}$$

$$\text{subject to} \quad 0 \le x_i \le 1 \qquad (i = 1, 2, \ldots, 6). \tag{7}$$

TABLE I
Global Optima of the Mathematical Function

Optima (x_{opt})	Object function (f_{opt})
(0.05,0.85,0.65,0.45,0.25,0.05)	-1.0
(0.55,0.35,0.15,0.95,0.75,0.55)	-1.0

TABLE II
Comparison for the Mathematical Function

Algorithm	Optimal solution	Function value	No iteration
ITS	(0.5500170, 0.3500574, 0.1500490, 0.9500023, 0.7500157, 0.5499760)	-1.0000000	1260
SA	(0.5500832, 0.3503048, 0.1497125, 0.9501942, 0.7501021, 0.5502390)	-0.9999981	328669

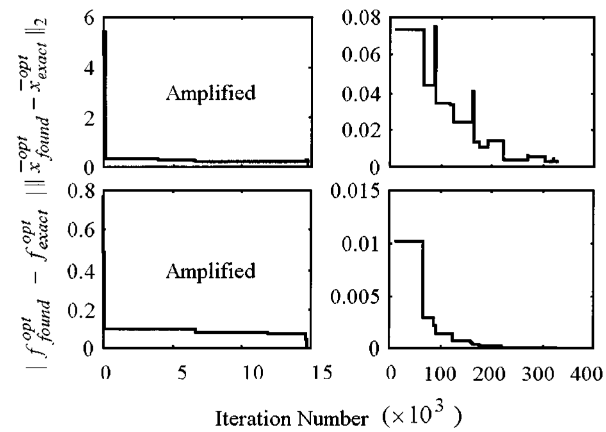

Fig. 1. The convergence trajectories of the optimal solutions for SA.

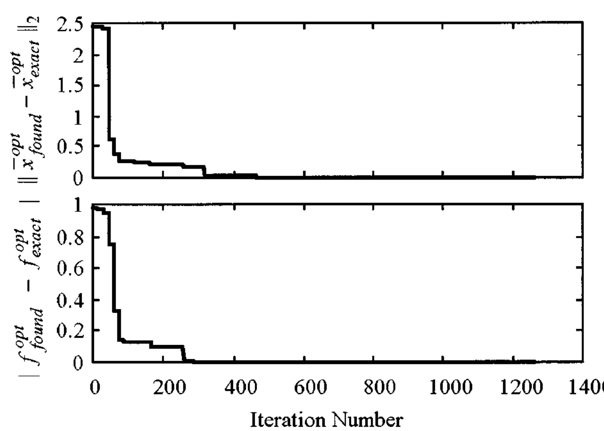

Fig. 2. The convergence trajectories of the optimal solutions for ITS.

The two global optima of the function are given in Table I. The performance comparison of the proposed ITS with SA algorithm is shown in Table II. These results are the average values of 10 runs from different initial states. In the numerical calculations, the initial value of the temperature for SA is set to 0.2, and the n_p of ITS to 4. The convergence trajectories of the optimal solutions of some runs for the two methods are given, respectively, in Figs. 1 and 2. The percentage values are used in these two figures to show the same effect for variables with different base values. In addition, the numerical calculations show that the termination criterion (5) works well for the proposed ITS.

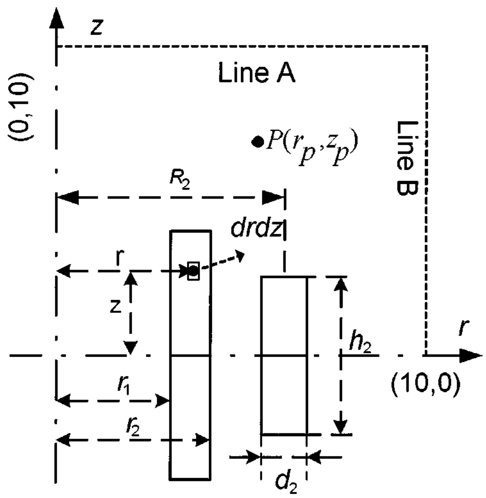

Fig. 3. The schematic diagram of the SMES.

From these results one can observe clearly the advantages of the proposed ITS.

B. Team Workshop Problem

The TEAM Workshop problem 22 of a Super-conducting Magnetic Energy Storage (SMES) configuration with three free parameters, as shown in Fig. 3, is selected to validate the proposed ITS [12]. The objective function to be optimized is

$$\begin{aligned} \text{minimize} \quad & f = w_1 \frac{B_{stary}^2}{B_{norm}^2} + w_2 \frac{|Energy - E_{ref}|}{E_{ref}} \\ \text{subject to} \quad & J_i \le (-6.4|(B_{\max})_i| + 56)(A/mm_2) \\ & (i = 1, 2) \end{aligned} \quad (8)$$

where

$Energy$	is the stored energy in the SMES device,
E_{ref}	$= 180$ MJ,
B_{norm}	$= 3 \times 10^{-3}$ T,
w_1 and w_2	are weighting factors,
J_i and $(B_{\max})_i$ $(i = 1, 2)$	are, respectively, the current density and the maximum field in the ith coil, and
B_{stary}^2	is a measure of the stray fields which is evaluated along 22 equidistant points of line A and line B of Fig. 3 by

$$B_{stary}^2 = \sum_{i=1}^{22} (B_{stary})_i^2 \Big/ 22. \quad (9)$$

The Biot–Savart law is used to evaluate the fields and the corresponding stored energy because 1) this approach is computationally inexpensive; 2) the computed stray fields are very sensitive to the meshes of finite element type methods. For an arbitrary point P, as given in Fig. 3, the field produced by coil 1 can be determined by

$$(B_p)_r = \int_{-h_1/2}^{h_1/2} \int_{r_1}^{r_2} \frac{u_0 J_1}{2\pi} \frac{z_p - z}{r_p \sqrt{(r + r_p)^2 + (z_p - z)}} \cdot \left[\frac{r^2 + r_p^2 + (z_p - z)^2}{(r_p - r)^2 + (z_p - z)^2} E - K \right] dr\, dz \quad (10)$$

TABLE III
VALIDATION OF THE PROPOSED ITS USING TEAM WORKSHOP PROBLEM 22

Optimal method	R_2	$h_2/2$	d_2	No. iteration
ITS	3.10	0.240	0.388	1842
SA	3.09	0.241	0.389	9846
Best ones [12]	3.08	0.239	0.394	/

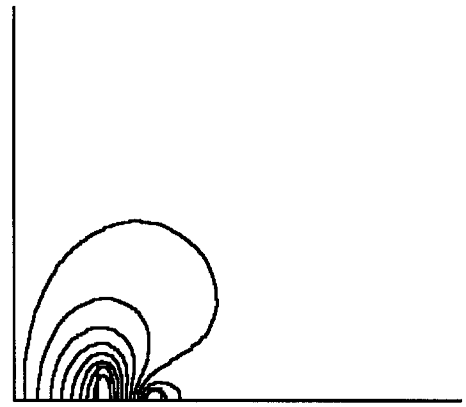

Fig. 4. The field contours under optimized geometry.

$$(B_p)_z = \int_{-h_1/2}^{h_1/2} \int_{r_1}^{r_2} \frac{u_0 J_1}{2\pi} \frac{1}{r_p \sqrt{(r + r_p)^2 + (z_p - z)}} \cdot \left[\frac{r^2 - r_p^2 - (z_p - z)^2}{(r_p - r)^2 + (z_p - z)^2} E + K \right] dr\, dz \quad (11)$$

where K and E are, respectively, the complete elliptic integrals of the first and second kinds, i.e.,

$$K(k) = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - k^2 \sin^2 \theta}} \quad (12)$$

$$E(k) = \int_0^{\pi/2} \sqrt{1 - k^2 \sin^2 \theta}\, d\theta \quad (13)$$

where

$$k^2 = \frac{4 r r_p}{(r + r_p)^2 + (z_p - z)^2}. \quad (14)$$

Table III gives the comparison of the computed results using the proposed ITS with those obtained by using simulated annealing (SA) as well as the best one so far reported [12]. The field contours under optimized geometry is given in Fig. 4. From these results it can be seen that the proposed method uses less than one fifth of the iteration numbers required by SA to reach almost the same optimal solutions.

IV. CONCLUSION

This paper proposed an improved tabu search algorithm for a practical application in finding the optimal designs of electromagnetic devices. The numerical results on both a mathematical test function and the team workshop problem reveal that the iteration number of the proposed method is very small compared with those algorithms such as simulated annealing.

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