PERFORMANCE ANALYSIS OF MULTIPLE ACCESS CHAOTIC-SEQUENCE SPREAD-SPECTRUM COMMUNICATION SYSTEMS EMPLOYING PARALLEL INTERFERENCE CANCELLATION DETECTORS

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ABSTRACT

In this paper, parallel interference cancellation (PIC) detectors are applied to jointly decode symbols in a multiple access chaotic-sequence spread-spectrum communication system. In particular, three types of linear detectors, namely, single-user detector, decorrelating detector and minimum mean-square-error detector, are used to estimate the transmitted symbols for the first stage of the PIC detector. The technique for deriving the approximate bit error rate (BER) is described and computer simulations are performed to verify the analytical BERs.

Index Terms — Chaos-based communications, multiple access, multi-user detection, spread-spectrum communications, parallel interference cancellation.

1. INTRODUCTION

Multi-user detection is an effective technique to reduce mutual interference between users in a multiple access environment. Recently, the parallel and successive interference cancellation detectors have been applied to a chaotic-sequence-based communication system [1]. The performance has been evaluated by computer simulations but no analytical BER has been derived. Also, the combined effects of the linear and nonlinear multi-user detectors have not been examined.

In this paper, we study three types of linear detectors applied in conjunction with parallel interference cancellation (PIC) detector in a multiple access chaotic-sequence spread-spectrum (MA-CSSS) communication system. The linear detectors include single-user detector, decorrelating detector and minimum mean-square-error detector [2, 3]. A PIC detector is a nonlinear multi-user detector that can cancel the total interference for all users simultaneously [4]. The detector requires exact knowledge of the spreading sequences of all users as well as an estimation of the transmitted symbols. Based on the estimation of the transmitted symbols, the mutual interference is computed and subtracted from the incoming signal before decoding is performed. The technique for deriving the approximate BERs for the PIC detectors (i.e., conventional/PIC, DD/PIC and MMSE/PIC detectors) is described. The analytical results are supported by computer simulations.

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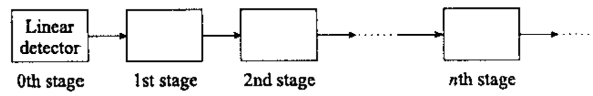

Fig. 2: A multistage parallel interference cancellation (PIC) detector.

2. SYSTEM DESCRIPTION

2.1. Transmitter Structure

Consider an N-user MA-CSSS communication system, as shown in Fig. 1. Define $d_l^{(i)} \in \{-1,+1\}$ as the lth transmitted symbol for the ith user, which assumes the value "+1" or "−1" with equal probability. Denote the chaotic sequence generated by the ith generator by $\{x_k^{(i)}\}$, which is used to spread the binary symbol sequence $\{d_l^{(i)}\}$. Assuming a spreading factor of γ, the transmitted signal for user i at time $k=(l-1)\gamma+1,(l-1)\gamma+2,\ldots,l\gamma$ can be expressed as $s_k^{(i)} = d_l^{(i)} x_k^{(i)}$.

2.2. Receiver Structure

Assuming a simple *additive white Gaussian noise* (AWGN) channel, the received signal is given by $r_k = \sum_{i=1}^{N} s_k^{(i)} + \xi_k$, where ξ_k is an additive white Gaussian noise sample with zero mean and variance $N_0/2$. Suppose the chaotic spreading sequences can be reproduced exactly at the receiver. As shown in Fig. 1, the output of the jth correlator ($j=1,2,\ldots,N$), denoted by $y_l^{(j)}$, equals

$$y_l^{(j)} = \sum_{i=1}^{N} d_l^{(i)} \sum_{k=(l-1)\gamma+1}^{l\gamma} x_k^{(i)} x_k^{(j)} + \sum_{k=(l-1)\gamma+1}^{l\gamma} \xi_k x_k^{(j)}. \tag{1}$$

2.2.1. Conventional Single-user Detector

For the conventional single-user detector, the decoded symbol, denoted by $\hat{d}_l$, can be computed from $\hat{d}_l^{(j)} = \mathrm{sgn}[y_l^{(j)}]$, where sgn[.] represents the sign function. In other words, if $y_l^{(j)} > 0$, "+1" is detected for the jth user. Otherwise, "−1" is decoded.

2.2.2. Parallel Interference Cancellation (PIC) Detector

A PIC detector with multiple stages is shown in Fig. 2. At the 0th stage of the PIC detector, the transmitted symbols are first es-

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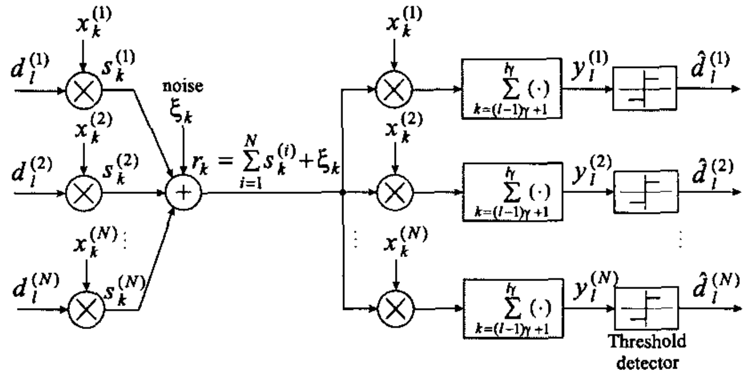


Fig. 1: An N-user multiple access chaotic-sequence spread-spectrum (MA-CSSS) communication system employing single-user detectors.

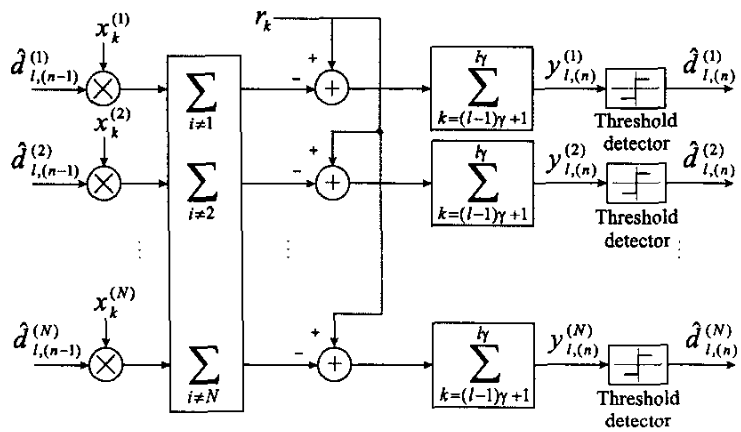


Fig. 3: The nth stage of a PIC detector.

timated using a linear detector such as a conventional single-user detector, decorrelating detector or MMSE detector.

At each of the subsequent stages, the interference is estimated and removed from the decision statistics. The construction of the nth ($n \geq 1$) stage of the PIC detector is shown in Fig. 3. Assuming that the spreading sequences of all users are known at the detector, the inputs from the previous stage are multiplied by the corresponding spreading sequences and the transmitted signals for all users are estimated. Then, the interference is reconstructed and subtracted from the received signal. At the nth stage, the output of the jth correlator is given by

$$\begin{aligned} y_{l,(n)}^{(j)} &= \sum_{k=(l-1)\gamma+1}^{l\gamma} \left(r_k - \sum_{i=1,i\neq j}^{N} \hat{d}_{l,(n-1)}^{(i)} x_k^{(i)} \right) x_k^{(j)} \\ &= \underbrace{d_l^{(j)} \sum_{k=(l-1)\gamma+1}^{l\gamma} \left(x_k^{(j)}\right)^2}_{\text{required signal}} \\ &+ \underbrace{\sum_{k=(l-1)\gamma+1}^{l\gamma} \sum_{i=1,i\neq j}^{N} \left(d_l^{(i)} - \hat{d}_{l,(n-1)}^{(i)} \right) x_k^{(i)} x_k^{(j)}}_{\text{interfering signal}} \\ &+ \underbrace{\sum_{k=(l-1)\gamma+1}^{l\gamma} \xi_k x_k^{(j)}}_{\text{noise}} \end{aligned} \tag{2}$$

and the symbol is decoded according to the sign of $y_{l,(n)}^{(j)}$, i.e., $\hat{d}_{l,(n)}^{(j)} = \text{sgn}[y_{l,(n)}^{(j)}]$.

3. PERFORMANCE ANALYSIS

In this section, we describe briefly the steps taken to derive the bit error rate of the PIC detector with n stages. Without loss of generality, we consider the jth user in an N-user system and we derive the probability of error for the lth symbol. For brevity, we omitted the subscript l in the following analysis. Define

$$D_{(n-1)}^{(j)} = [D_{(n-1)}^{(1)} \cdots D_{(n-1)}^{(j-1)} \; D_{(n-1)}^{(j+1)} \cdots D_{(n-1)}^{(N)}] \tag{3}$$

where

$$D_{(n-1)}^{(i)} = d^{(i)} - \hat{d}_{(n-1)}^{(i)} \quad (4)$$

$$\hat{y}_{(n)}^{(j)} = y_{(n)}^{(j)} | (d^{(j)} = +1, \boldsymbol{D}_{(n-1)}^{(j)}) \quad (5)$$

$$\check{y}_{(n)}^{(j)} = y_{(n)}^{(j)} | (d^{(j)} = -1, \boldsymbol{D}_{(n-1)}^{(j)}). \quad (6)$$

For a given $\boldsymbol{D}_{(n-1)}^{(j)}$ and assuming that the transmitted symbol is "+1" for the jth user, the mean and variance of $y_{(n)}^{(j)} | \ (d^{(j)} = +1, \boldsymbol{D}_{(n-1)}^{(j)})$ (or simply $\hat{y}_{(n)}^{(j)}$) can be shown equal to, respectively,

$$\mathrm{E}[\hat{y}_{(n)}^{(j)}] = \gamma \mathrm{E}[(x_k^{(j)})^2] \quad (7)$$

and

$$\begin{aligned} \mathrm{var}[\hat{y}_{(n)}^{(j)}] = & \ \gamma \mathrm{var}[(x_k^{(j)})^2] \\ & + \sum_{k=1}^{\gamma} \sum_{m=1, m \neq k}^{\gamma} \mathrm{cov}[(x_k^{(j)})^2, (x_m^{(j)})^2] \\ & + \gamma \mathrm{E}[(x_k^{(j)})^2] \sum_{i=1, i \neq j}^{N} \left(D_{(n-1)}^{(i)} \right)^2 \mathrm{E}[(x_k^{(i)})^2] \\ & + \sum_{i=1, i \neq j}^{N} \sum_{k=1}^{\gamma} \sum_{m=1, m \neq k}^{\gamma} \\ & \left[\mathrm{E}[x_k^{(i)} x_m^{(i)}] \mathrm{E}[x_k^{(j)} x_m^{(j)}] + \frac{\gamma N_0 \mathrm{E}[(x_k^{(j)})^2]}{2} \right] \end{aligned} \quad (8)$$

where E[.], var[.] and cov[.] denote the mean, variance and covariance operators, respectively [5]. Similarly, we can derive $\mathrm{E}[\check{y}_{(n)}^{(j)}]$ and $\mathrm{var}[\check{y}_{(n)}^{(j)}]$. Assuming that both $\hat{y}_{(n)}^{(j)}$ and $\check{y}_{(n)}^{(j)}$ are normal when γ is large, it can be readily shown that

$$\begin{aligned} \mathrm{Prob}\left(\hat{y}_{(n)}^{(j)} \leq 0 \right) & = \mathrm{Prob}\left(\check{y}_{(n)}^{(j)} > 0 \right) \\ & = \frac{1}{2} \mathrm{erfc} \left(\frac{\mathrm{E}[\hat{y}_{(n)}^{(j)}]}{\sqrt{2 \mathrm{var}[\hat{y}_{(n)}^{(j)}]}} \right). \end{aligned} \quad (9)$$

At the $(n-1)$th stage, denote the number of users (excluding the jth user) that make a wrong decision by $M_{(n-1)}^{(j)}$. Then, the BER at the nth stage can be computed from

$$\begin{aligned} \mathrm{BER}_{(n)}^{(j)} & = \sum_{\boldsymbol{D}_{(n-1)}^{(j)}} \mathrm{Prob}\left(\hat{y}_{(n)}^{(j)} \leq 0 \right) \times \mathrm{Prob}(\boldsymbol{D}_{(n-1)}^{(j)}) \\ & = \sum_{c=0}^{N-1} \mathrm{Prob}\left(y_{(n)}^{(j)} \leq 0 | (d^{(j)} = +1, M_{(n-1)}^{(j)} = c) \right) \\ & \qquad \times \ \mathrm{Prob}\left(M_{(n-1)}^{(j)} = c \right). \end{aligned} \quad (10)$$

For large c, $\mathrm{Prob}\left(M_{(n-1)}^{(j)} = c \right)$ is small. In our analysis, we assume that $\mathrm{Prob}\left(M_{(n-1)}^{(j)} = c \right)$ is negligible when $M_{(n-1)}^{(j)}$ is greater than three for conventional/PIC and two for DD/PIC and MMSE/PIC, and hence those terms are ignored. Using the analytical BERs for the linear detectors (e.g., the conventional single-user detector, decorrelating detector and MMSE detector [3, 6]) as the BER values of the 0th stage (denoted as $\mathrm{BER}_{(0)}^{(j)}$), and by applying (10) repeatedly, the BER at the nth stage can be found.

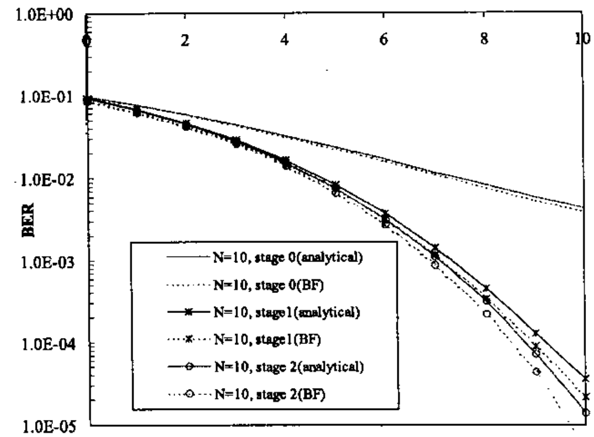


Fig. 4: Analytical and brute-force simulation (BF) BERs versus E_b/N_0 for the conventional/PIC detector. $\gamma = 100$ and $N = 10$.

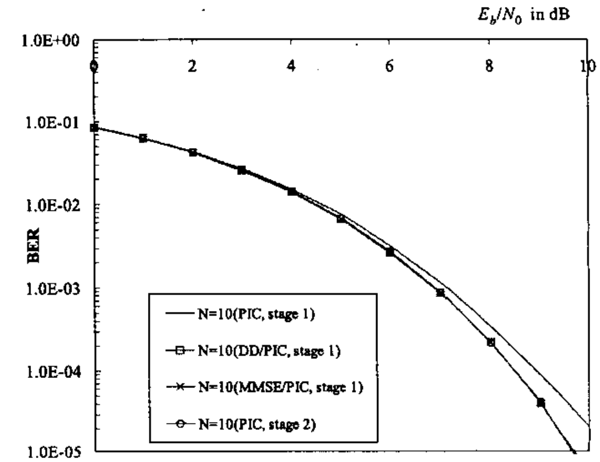


Fig. 5: Comparison of the conventional/PIC, DD/PIC and MMSE/PIC detectors. $\gamma = 100$ and $N = 10$.

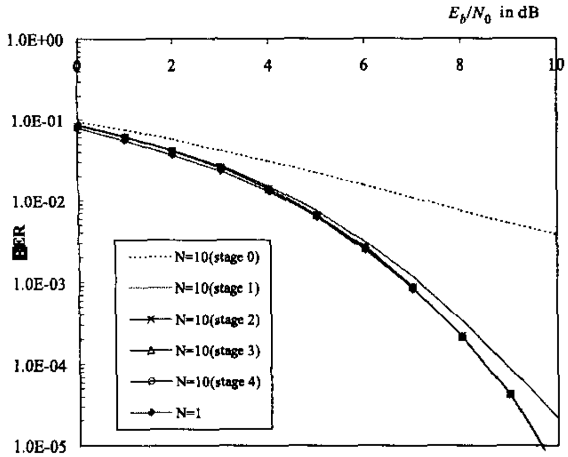
4. COMPUTER SIMULATIONS AND DISCUSSIONS

We assume that the cubic map given by

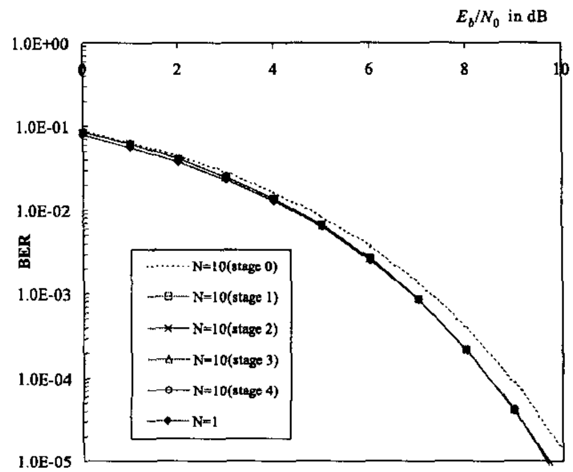
$$x_{k+1} = 4x_k^3 - 3x_k \quad (11)$$

is used by all users, and each uses a different initial condition. Also, the spreading factor (γ) is 100 and the number of users (N) is 10.

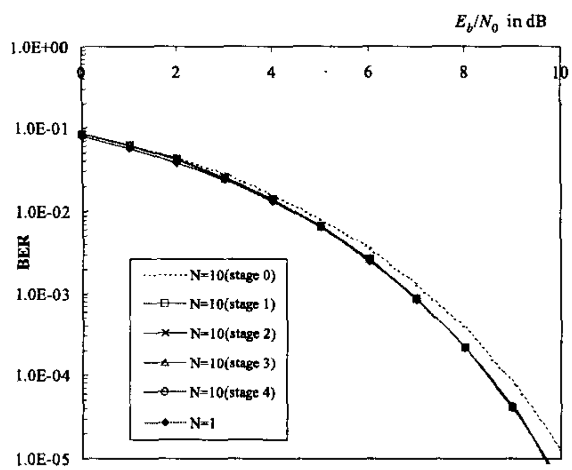
Figure 4 shows the analytical and brute-force (BF) simulation BERs when the combined conventional/PIC detector is applied. Stage 0 corresponds to the case when a conventional MA-CSSS system with single-user detector is used. When PIC detector is used for the MA-CSSS systems (Stages 1 and 2), the performance is enhanced. Moreover, it can be seen that the analytical and the simulated results agree for all stages of the conventional/PIC detector.



(a)



(b)



(c)

Fig. 6: Simulated BER versus E_b/N_0 for (a) conventional/PIC detector; (b) DD/PIC detector; (c) MMSE/PIC detector. $\gamma = 100$ and $N = 10$.

In Figure 5, the simulated BERs for the conventional/PIC, DD/PIC and MMSE/PIC detectors are given. Results show that the BERs for the DD/PIC and MMSE/PIC detectors are almost the same. When $n = 1$, the performance for the DD/PIC and MMSE/PIC detectors is better than that for the conventional/PIC detector. However, the conventional/PIC with 3 stages (stages 0, 1 and 2) can achieve the same performance as the DD/PIC and MMSE/PIC detectors with only 2 stages.

Figures 6(a)–(c) show the simulated BERs for the conventional/PIC, DD/PIC and MMSE/PIC detectors at different stages. The BER for a single-user system (interference-free) which is equivalent to the case when orthogonal spreading codes are used for all users, is also given as a reference. For all the detectors, the BERs at stage 1 are lower than those at stage 0. For the conventional/PIC detector, the BER at stage 2 is further reduced compared to that at stage 1. The BER results for the conventional/PIC with 3 stages, and for the DD/PIC and the MMSE/PIC detectors with 2 stages are close to that of the single-user system. When the number of stages is further increased, no improvement is observed. Overall, the results indicate that the parallel interference cancellation technique can improve the performance of linear multi-user detectors.

5. CONCLUSION

In this paper, we have applied conventional/PIC, DD/PIC and MMSE/PIC detectors to multiple access chaotic-sequence spread-spectrum communication systems. The technique for deriving the approximated BERs has been described. It is found that the analytical BERs agree with the simulation results. Also, it is shown that the PIC detectors can improve the performance further compared to that of linear multi-user detectors. On the other hand, the complexity of the PIC detectors increases as the number of users or the number stages increases. As a result, there is a trade off between the system complexity and the performance.

6. REFERENCES

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