CO-EXISTENCE OF CHAOS-BASED AND CONVENTIONAL DIGITAL COMMUNICATION SYSTEMS

Francis C.M. Lau and Chi K. Tse

Department of Electronic and Information Engineering, The Hong Kong Polytechnic University
Hunghom, Kowloon, Hong Kong, China
Email: encmlau@polyu.edu.hk & encktse@polyu.edu.hk

ABSTRACT

This paper studies the performance of selected chaos-based communication systems whose spectrum overlaps with that of conventional narrowband systems. Such a scenario may occur in normal practice when chaos-based systems are introduced while the conventional systems are still in operation. The particular chaos-based systems under study are the coherent chaos-shift-keying (CSK) system and the non-coherent differential CSK system, and the co-existing conventional system employs the binary phase-shift-keying modulation scheme. Analytical expressions for the bit error rates are derived and computer simulations are performed to verify the analytical findings.

Index Terms— Chaos communication, chaos-shift-keying, differential chaos-shift-keying, binary phase-shift-keying.

1. INTRODUCTION

Chaos-based communication systems represent a new category of spread-spectrum communication systems [1]–[3], whose working principle differs significantly from conventional direct-sequence and frequency-hopping spread-spectrum systems. However, like all other kinds of spread-spectrum systems, chaos-based systems are required to provide reasonable bit error performance in the presence of a narrowband signal which can be generated from an intruder or a co-existing conventional communication system.

The basic problem considered in this paper is the co-existence of chaos-based systems and conventional systems. Specifically, we are interested in finding the performance of a chaos-based system and the extent to which it is affected by the presence of a conventional narrowband system whose bandwidth falls within that of the chaos-based system in question. This scenario has practical significance, as can be easily appreciated when one considers the introduction of chaos-based communication systems while conventional systems are still in operation.

In particular, the chaos-based systems under study are the coherent chaos-shift-keying (CSK) system and the non-coherent differential CSK (DCSK) system, whereas the conventional system used in the study employs the standard binary phase-shift-keying (BPSK) scheme. Also, both the chaos-based and conventional systems are assumed to have identical data rates. Analytical expressions for the bit error rates are derived, permitting evaluation of performance for different noise levels, power ratios and spreading factors. Finally, results from computer simulations verify the analytical findings.

This work is supported by a research grant provided by Hong Kong Polytechnic University and also by a competitive earmarked research grant funded by the Hong Kong Research Grants Council (Grant No. PolyU5124/01E).

2. SYSTEM OVERVIEW

We consider a chaos-based communication system and a conventional system whose bandwidths overlap significantly. We refer to the whole system as *combined chaos-based-conventional system,* which can be represented by the block diagram shown in Fig. 1. Our analysis will proceed in a discrete-time fashion. At time k, denote the output of the chaos transmitter by s_k and that of the conventional transmitter by u_k. These two signals are then added, as well as corrupted by noise η_k in the channel, before they arrive at the receiving end. At the receiver, based on the incoming signal r_k, the receivers of the chaos-based system and the conventional system will attempt to recover their respective data streams. Coherent or non-coherent detection schemes may be applied in the receivers, depending upon the modulation methods used in the transmitter. Specifically, we will consider a "combined CSK-BPSK" system and a "combined DCSK-BPSK" system, and will attempt to develop analytical expressions for the bit error rates of the recovered data streams.

3. PERFORMANCE ANALYSIS

3.1. Combined CSK-BPSK Communication System

We first consider a discrete-time baseband equivalent model of a combined CSK-BPSK communication system. We assume that the CSK system and the BPSK system have identical bit rate and that their bit streams are synchronized. Also, the carrier frequencies of the two systems are identical and synchronized. Further, "−1" and "+1" occur with equal probabilities in the bit streams of both systems.

For simplicity, we consider here a CSK system in which one chaos generator is used to produce chaotic signal samples $\{x_k\}$ for $k = 1, 2, \ldots$. Suppose $\alpha_l \in \{-1, +1\}$ is the symbol to be sent during the lth bit period. Define the spreading factor, 2β, as the number of chaotic samples used to transmit one binary symbol. During the lth bit duration, i.e., for $k = 2\beta(l-1)+1, 2\beta(l-1)+2, \ldots, 2\beta l$, the output of the CSK transmitter is $s_k = \alpha_l x_k$.

In the BPSK system, we denote the lth transmitted symbol by $b_l \in \{-1, +1\}$. Moreover, the signal power is P_B. Thus, during the lth bit duration, i.e., for $k = 2\beta(l-1)+1, 2\beta(l-1)+2, \ldots, 2\beta l$, the transmitted signal is constant and is represented by $u_k = \sqrt{P_B} b_l$.

0-7803-7761-3/03/$17.00 ©2003 IEEE

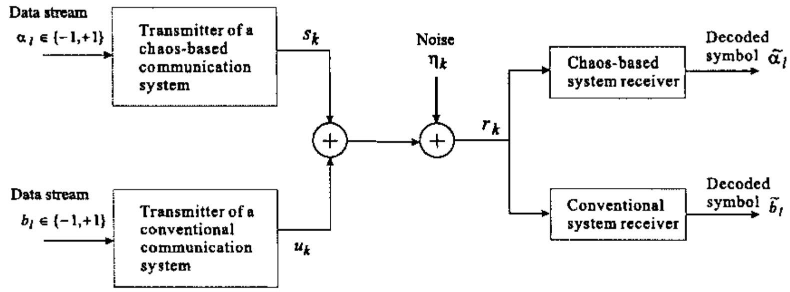

Fig. 1: Block diagram of a combined chaos-based-conventional digital communication system.

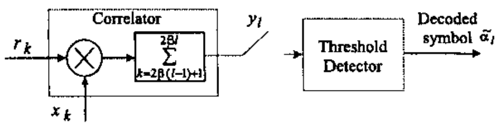

Fig. 2: Block diagram of a coherent CSK receiver.

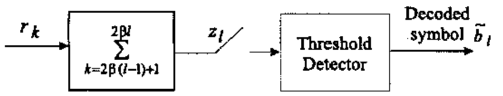

Fig. 3: Block diagram of a BPSK receiver.

The CSK and BPSK signals are combined and corrupted by an additive white Gaussian noise in the channel, before arriving at the receiving end. Thus, the received signal, denoted by r_k, is given by $r_k = s_k + u_k + \eta_k$ where η_k is a Gaussian noise sample of zero mean and variance (power spectral density) $N_0/2$.

3.1.1. *Performance of the CSK System*

Assume that a correlator-type receiver is employed. Referring to Fig. 2, the correlator output for the lth bit, y_l, is given by

$$\begin{aligned} y_l &= \sum_{k=2\beta(l-1)+1}^{2\beta l} r_k x_k \\ &= \underbrace{\alpha_l \sum_{k=2\beta(l-1)+1}^{2\beta l} x_k^2}_{\text{required signal}} + \underbrace{\sqrt{P_B}\, b_l \sum_{k=2\beta(l-1)+1}^{2\beta l} x_k}_{\text{interfering BPSK signal}} \\ &\quad + \underbrace{\sum_{k=2\beta(l-1)+1}^{2\beta l} \eta_k x_k}_{\text{noise}} . \end{aligned} \tag{1}$$

The overall error probability of the lth transmitted symbol can be calculated from

$$\mathrm{BER}_{\text{CSK}}^{(l)} = \frac{1}{2}\left[\mathrm{BER}_{\text{CSK-I}}^{(l)} + \mathrm{BER}_{\text{CSK-II}}^{(l)}\right] \tag{2}$$

where

$$\begin{aligned} &\mathrm{BER}_{\text{CSK-I}}^{(l)} \\ =\ &\mathrm{Prob}(b_l = +1) \times \mathrm{Prob}(y_l \le 0 \mid (\alpha_l = +1, b_l = +1)) \\ &+ \mathrm{Prob}(b_l = -1) \times \mathrm{Prob}(y_l \le 0 \mid (\alpha_l = +1, b_l = -1)) \\ =\ &\frac{1}{4}\mathrm{erfc}\left(\frac{\mathrm{E}[y_l \mid (\alpha_l = +1, b_l = +1)]}{\sqrt{2\mathrm{var}[y_l \mid (\alpha_l = +1, b_l = +1)]}}\right) \\ &+ \frac{1}{4}\mathrm{erfc}\left(\frac{\mathrm{E}[y_l \mid (\alpha_l = +1, b_l = -1)]}{\sqrt{2\mathrm{var}[y_l \mid (\alpha_l = +1, b_l = -1)]}}\right) \end{aligned} \tag{3}$$

$$\begin{aligned} &\mathrm{BER}_{\text{CSK-II}}^{(l)} \\ =\ &\mathrm{Prob}(b_l = +1) \times \mathrm{Prob}(y_l > 0 \mid (\alpha_l = -1, b_l = +1)) \\ &+ \mathrm{Prob}(b_l = -1) \times \mathrm{Prob}(y_l > 0 \mid (\alpha_l = -1, b_l = -1)) \\ =\ &\frac{1}{4}\mathrm{erfc}\left(\frac{-\mathrm{E}[y_l \mid (\alpha_l = -1, b_l = +1)]}{\sqrt{2\mathrm{var}[y_l \mid (\alpha_l = -1, b_l = +1)]}}\right) \\ &+ \frac{1}{4}\mathrm{erfc}\left(\frac{-\mathrm{E}[y_l \mid (\alpha_l = -1, b_l = -1)]}{\sqrt{2\mathrm{var}[y_l \mid (\alpha_l = -1, b_l = -1)]}}\right) . \end{aligned} \tag{4}$$

The operators E[.] and var[.] denote the mean and variance, respectively, of the argument. Also, in the derivation of (3) and (4), it has been assumed that the conditional correlator outputs, i.e., $y_l \mid (\alpha_l = \pm 1, b_l = \pm 1)$, are normally distributed.

It can be seen from (2) to (4) that $\mathrm{BER}_{\text{CSK}}^{(l)}$ is independent of l. Thus, the error probability of the lth transmitted symbol is the same as the BER of the system.

3.1.2. *Performance of the BPSK System*

In the BPSK receiver shown in Fig. 3, the incoming signal samples within a symbol period are summed to give z_l, i.e.,

$$\begin{aligned} z_l &= \sum_{k=2\beta(l-1)+1}^{2\beta l} r_k \\ &= \underbrace{\alpha_l \sum_{k=2\beta(l-1)+1}^{2\beta l} x_k}_{\text{interfering CSK signal}} + \underbrace{2\beta\sqrt{P_B}\, b_l}_{\text{required signal}} + \underbrace{\sum_{k=2\beta(l-1)+1}^{2\beta l} \eta_k}_{\text{noise}} \end{aligned} \tag{5}$$

Asssuming that z_l is normal, the error probability for the lth transmitted BPSK symbol can be found as

$$\begin{aligned} &\mathrm{BER}_{\text{BPSK-CSK}}^{(l)} \\ =\ &\mathrm{Prob}(b_l = +1) \times \mathrm{Prob}(z_l \le 0 \mid (b_l = +1)) \\ &+ \mathrm{Prob}(b_l = -1) \times \mathrm{Prob}(z_l > 0 \mid (b_l = -1)) \\ =\ &\frac{1}{2}\mathrm{erfc}\left(\frac{2\beta\sqrt{P_B}}{\sqrt{4\beta P_s + 2\beta N_0}}\right) \end{aligned} \tag{6}$$

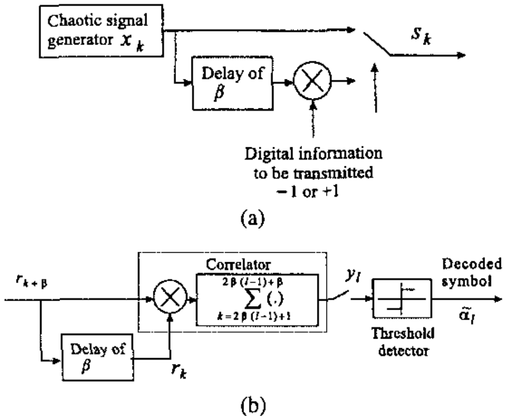

Fig. 4: Block diagram of a non-coherent DCSK system. (a) Transmitter; (b) receiver.

where $P_s = \mathrm{E}[x_k^2]$ denotes the average chaotic signal power. Since $\mathrm{BER}^{(l)}_{\text{BPSK-CSK}}$ is independent of l, it also represents the BER of the system. For a fixed chaotic signal power P_s, the BER can be improved by increasing the spreading factor 2β and/or increasing the BPSK signal power P_B.

3.2. Combined DCSK-BPSK Communication System

In this section, we move on to a combined DCSK-BPSK system. Figure 4 shows the block diagram of a DCSK transmitter and receiver pair [1]. Making the same assumptions as in Section 3.1, we obtain the transmitted DCSK signal in the lth bit duration as

$$s_k = \begin{cases} x_k & \text{for } k = 2\beta(l-1)+1, \ldots, 2\beta(l-1)+\beta \\ \alpha_l x_{k-\beta} & \text{for } k = 2\beta(l-1)+\beta+1, \ldots, 2\beta l. \end{cases} \tag{7}$$

The noisy received signal, r_k, is given by $r_k = s_k + u_k + \eta_k$. At the receiver, the detector essentially computes the correlation of the corrupted reference and data slots of the same symbol, as shown in Fig. 4. The output of the correlator for the lth received bit equals $y_l = \sum_{k=2\beta(l-1)+1}^{2\beta(l-1)+\beta} r_k r_{k+\beta}$. Using a likewise procedure as in Section 3.1, the bit error rate of the DCSK system under a combined communication environment can be derived.

Similarly, the output of the BPSK correlator can be computed and hence the BER is found.

3.3. Example

Consider the case where a logistic map is used for chaos generation. The form of the map is $x_{k+1} = g(x_k) = 1 - 2x_k^2$ and the invariant probability density function of $\{x_k\}$ equals [4]

$$\rho(x) = \begin{cases} \dfrac{1}{\pi\sqrt{1-x^2}} & \text{if } |x| < 1 \\ 0 & \text{otherwise.} \end{cases} \tag{8}$$

For this case, the numerical values of the means and variances of $y_l \mid (\alpha_l = \pm 1, b_l = \pm 1)$ are obtained and substituted into (2) to obtain the BER of the CSK system, i.e.,

$$\mathrm{BER}_{\text{CSK}} \approx \frac{1}{4}\mathrm{erfc}\left(\sqrt{\frac{2\beta}{1+4P_B+2N_0-4\sqrt{P_B}}}\right) + \frac{1}{4}\mathrm{erfc}\left(\sqrt{\frac{2\beta}{1+4P_B+2N_0+4\sqrt{P_B}}}\right). \tag{9}$$

Moreover, for the BPSK system under a combined CSK-BPSK environment, we obtain

$$\mathrm{BER}_{\text{BPSK-CSK}} = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{2\beta P_B}{1+N_0}}\right). \tag{10}$$

For the combined DCSK-BPSK system, it can also be shown that the BERs for the DCSK system and BPSK system are, respectively,

$$\begin{aligned} \mathrm{BER}_{\text{DCSK}} = {} & \frac{1}{8}\mathrm{erfc}\left(\frac{0.5\beta+\beta P_B}{\sqrt{\zeta-2\beta\sqrt{P_B}}}\right) \\ & + \frac{1}{8}\mathrm{erfc}\left(\frac{0.5\beta+\beta P_B}{\sqrt{\zeta+2\beta\sqrt{P_B}}}\right) \\ & + \frac{1}{4}\mathrm{erfc}\left(\frac{0.5\beta-\beta P_B}{\sqrt{0.25\beta+2\beta P_B N_0+\beta N_0+\frac{\beta N_0^2}{2}}}\right) \end{aligned} \tag{11}$$

and

$$\mathrm{BER}_{\text{BPSK-DCSK}} = \frac{1}{4}\mathrm{erfc}\left(\sqrt{\frac{2\beta P_B}{2+N_0}}\right) + \frac{1}{4}\mathrm{erfc}\left(\sqrt{\frac{2\beta P_B}{N_0}}\right) \tag{12}$$

where

$$\zeta = 0.25\beta + 4\beta P_B + 2\beta P_B N_0 + \beta N_0 + \frac{\beta N_0^2}{2}. \tag{13}$$

4. COMPUTER SIMULATIONS AND DISCUSSIONS

In this section we study the performances of the chaos-based and conventional digital communication systems under a combined environment by computer simulations. The logistic map described in Section 3.3 has been used to generate the chaotic sequences. For comparison, we also plot in each case the analytical BERs obtained in Section 3.3. Results are shown in Fig. 5 for the combined CSK-BPSK system, and in Fig. 6 for the combined DCSK-BPSK system. In general, computer simulations and analytical results are in good agreement. Also, as would be expected, the coherent CSK system generally performs better than the non-coherent DCSK system. Further observations are summarized as follows.

1. The BER of the chaos-based system in the combined environment generally deteriorates (increases) as P_B/P_s increases for any given E_b/N_0. This is apparently due to the increasing power of the BPSK signal which causes more interference to the chaos-based system, thus giving a higher BER.
2. At a fixed E_B/N_0, the BER of the BPSK system in the combined environment improves as P_B/P_s increases.
3. Comparing the two types of chaos-based communication systems, the performance of the DCSK system is degraded to a larger extent under the influence of a BPSK signal.

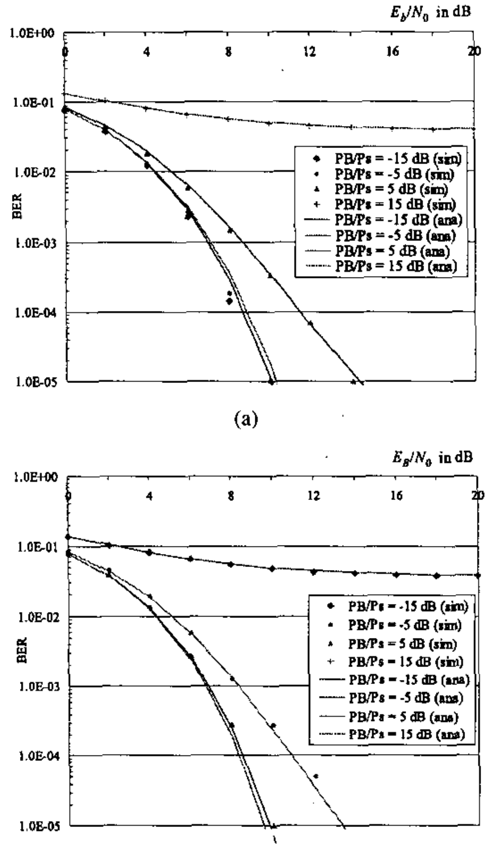

Fig. 5: BERs versus E_b/N_0 of the individual systems in a combined CSK-BPSK environment. Simulated BERs are plotted as points and analytical BERs plotted as lines. Spreading factor is 100. (a) Coherent CSK system; (b) BPSK system.

4. From Fig. 5, it can be observed that both CSK and BPSK systems can perform reasonably well for a P_B/P_s value between -5 dB and 5 dB. It fact, we find that the system capacity is maximum at a P_B/P_s value of 0 dB (not shown here because of space limitation).

5. In the case of the combined DCSK-BPSK system, satisfactory BER performance can be maintained for power ratio around $P_B/P_s = -5$ dB. The system capacity is found to be maximized under such a condition.

5. CONCLUSION

In this paper, the problem of co-existence of chaos-based communication systems and conventional narrowband communication systems is studied in terms of two specific sample systems, namely, a combined CSK-BPSK system and a combined DCSK-BPSK system. A parallel study of the co-existence problem of chaos-based conventional spread-spectrum systems can be found in [5].

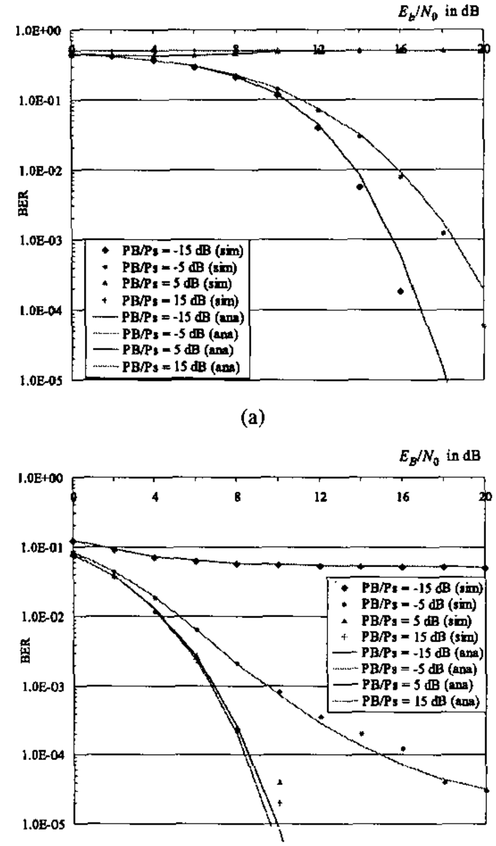

Fig. 6: BERs versus E_b/N_0 of the individual systems in a combined DCSK-BPSK environment. Simulated BERs are plotted as points and analytical BERs plotted as lines. Spreading factor is 100. (a) Non-coherent DCSK system; (b) BPSK system.

6. REFERENCES

[1] G. Kolumbán, M.P. Kennedy and L.O. Chua, "The role of synchronization in digital communications using chaos - Part II: Chaotic modulation and chaotic synchronization," *IEEE Trans. on Circ. Syst. I*, **45**(11), pp. 1129–1140, 1998.

[2] M. Hasler and T. Schimming, "Chaos communication over noisy channels," *Int. J. Bifurc. Chaos*, **10**, pp. 719–735, 2000.

[3] G. Kolumbán and M.P. Kennedy, "The role of synchronization in digital communications using chaos - Part III: Performance bounds for correlation receivers," *IEEE Trans. on Circ. Syst. I*, **47**(12), pp. 1673–1683, 2000.

[4] T. Kohda and A. Tsuneda, "Even- and odd-correlation functions of chaotic Chebychev bit sequences for CDMA," *Proc. IEEE Int. Symp. Spread Spectrum Tech. & Appl.*, Oulu, Finland, pp. 391–395, 1994.

[5] F.C.M. Lau and C.K. Tse, "Co-existence of chaos-based communication systems and conventional spread-spectrum communication systems," *Int. Symp. Nonl. Theory & Appl.*, Xian, China, 2002.