Synthesis of Impedance Using Switching Converters

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Abstract — A general impedance synthesizer using a minimum number of switching converters is studied in this paper. We begin with showing that any impedance can be synthesized by a circuit consisting of only two simple power converters, one storage element (e.g., capacitor), and one dissipative element (e.g., resistor) or power source. The implementation of such a circuit for synthesizing any desired impedance can be performed by (i) programming the input current given the input voltage such that the desired impedance function is achieved; (ii) controlling the amount of power dissipation (generation) in the dissipative element (source) so as to match the required active power of the impedance to be synthesized. Then, the instantaneous power will automatically be balanced by the storage element. Such impedance synthesizers find a lot of applications in power electronics. For instance, a resistance synthesizer can be used for power factor correction (PFC), a programmable capacitor or inductor synthesizer (comprising of small high-frequency converters) can be used for control applications.

1 INTRODUCTION

Many problems in electrical engineering are reducible to one of *impedance synthesis* or *imitation* [1]. Basically, the impedance observed from the terminals of a given circuit is defined as the ratio of the voltage across the terminals and the current flowing into and out of the terminals, as shown in Fig. 1. In power electronics, for instance, impedance imitation is central to many applications, be it known or clearly recognized by the engineers. Power factor correction (PFC), for example, effectively requires the input impedance to be resistive [2]. Thus, *imitating* a resistor for the input of a converter is the basic requirement for achieving a high power factor for the converter. Many control problems can also be interpreted as a kind of impedance modification, e.g., shaping the load transient of a converter is essentially a process of modifying the output impedance of the converter [3]. In this paper, we consider the problem of synthesizing impedance using a minimal configuration of switching converters. The problem is relevant to power electronics, and may also be applicable to other branches of electrical engineering. For instance, programmable reactive components using some high-frequency converters are useful elements for control applications.

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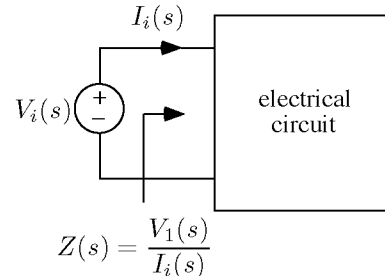
$$Z(s) = \frac{V_1(s)}{I_i(s)}$$

Figure 1: Impedance definition. $Z(s)$ is the impedance observed at the input terminals of the given electrical circuit.
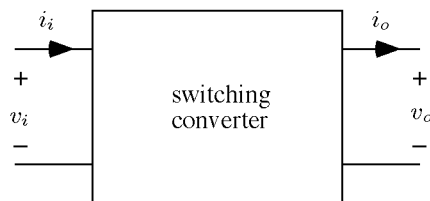

Figure 2: Simple switching converter as two-port.

We will begin with a review of switching converters in Section 2, with particular emphasis on their fundamental functions in terms of terminal voltages and currents. The main task, to be presented in Section 3, is to find the minimal configuration for synthesizing any impedance. Then, in Section 4, we will present a design of such an impedance synthesizer, and provide some experimental results in Section 5.

2 REVIEW OF SWITCHING CONVERTERS

We consider a *simple switching converter* as a two-port circuit, as shown in Fig. 2. Suppose the input voltage, input current, output voltage and output current are v_i, i_i, v_o and i_o, respectively. In practice, a simple switching converter may contain a storage element and a pair of switches, such as the buck, buck-boost and boost converters. The switches are switched periodically at a high frequency (usually hundreds of kHz). The switching frequency can be regarded as being so high that all variables v_i, i_i, v_o and i_o are relatively slowly varying.[1] Precisely, we write

$$f_{\max} \ll f_s \tag{1}$$

where $f_{\max}$ is the maximum frequency of v_i, i_i, v_o and i_o, and f_s is the switching frequency of the converter.

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[1] This assumption holds for all power electronics applications. Ideal switching converters therefore operate at infinite switching frequency.

Specific power processing functions are achieved by adjusting or controlling the relative durations of the "on" and "off" intervals of the switches.

The most fundamental property of an *ideal* switching converter is the conservation of power. Clearly, the average input power over one repetition period (also called switching period) must equal the average output power over the same period, i.e., $\int_T v_i i_i dt = \int_T v_o i_o dt$. Thus, the ideal converter does not store or dissipate any energy over a repetition period.[2] Of course, real converters are never 100% efficient, and here, we ignore this loss to keep our discussion simple. Moreover, in practice, we only consider v_i, i_i, v_o and i_o being *slowly varying*, i.e., varying at a frequency which is much lower than the switching frequency of the converter. Hence, the ideal power conservation equation becomes

$$v_i i_i = v_o i_o, \qquad (2)$$

where v_i, i_i, v_o and i_o can be regarded as instantaneous variables.

In addition to the power conservation property, a defining objective of a switching converter is that *given one of the terminal variables (normally the input voltage v_i), a switching converter is able to regulate or program* **any one** *of the remaining three terminal variables by controlling some parameter.* For example, for dc power supplies, the input voltage is given, and the switching converter aims to regulate the output voltage by controlling the duty cycle. Moreover, for power factor correction, the input voltage is given (normally a rectified sinewave), and the switching converter aims to program the input current such that it varies at the same frequency and in phase with the input sinewave voltage.

Note that the above definition implicitly assumes that all terminal voltages and currents are slowly varying compared to the switching frequency. Thus, theoretically, if the switching frequency approaches infinity, the switching converter can process voltage or current of arbitrarily high frequencies.

3 MINIMAL CONFIGURATION OF IMPEDANCE SYNTHESIZER BASED ON SWITCHING CONVERTERS

In finding the basic configuration of an impedance synthesizer, we first observe that an impedance can be realized by programming the current if the voltage is given, and vice versa. This fits the definition of the switching converter described in Section 2. Thus, a switching converter can be used to program the input current i_i given its input voltage v_i in order to create the desired impedance seen from the converter's input.[3] Here, both v_i and i_i are slowly varying in the sense of (1). Moreover, the switching converter is subject to the constraint of (2), i.e., power conservation. Clearly, no impedance

can be synthesized with one converter terminated with a fixed load impedance because (2) can never be satisfied.[4] Thus, *one switching converter is insufficient.* Clearly, we need to balance the power by ensuring that the output of the converter "emits" the right amount of instantaneous power.

If two switching converters are available, one of them can be used to program the input current (given the input voltage) so as to achieve the desired impedance. The other can then be used to match the power conservation requirement by controlling its output to dissipate or generate the correct amount of real power since $v_i i_i = v_o i_o$ must be satisfied by both converters. This is possible according to the definition of the switching converter described in Section 2 that, given the load (a resistor or a dc current source), the converter can adjust/control its output voltage v_o so as to emit or absorb a desired amount of dc power to or from the output port. Clearly, there must then exist a storage element connecting between the two converters in order to absorb the right amount of instantaneous power to meet the power balance for both converters. Thus, the minimum configuration consists of

- two switching converters,
- one storage element,
- one dissipative element or power source.

Obviously the dissipative element can be realized by a negative power source (e.g., current load). Furthermore, because power flow can be in either directions, the converters must be bi-directional.

4 IMPLEMENTATION

Some suitable circuit configurations satisfying the above conditions can be found in Liu *et al.* [4], which describes a family of two-converter configurations for achieving two control functions. A minimal configuration is shown in Fig. 3. Note that the choice of the exact types of Converters 1 and 2 remains a design issue.

To synthesize a desired impedance $Z(s)$, we have to ensure the correct magnitude and phase relationships between v_i and i_i. Essentially we want i_i to follow a reference template i_{ref} which is related to v_i as follows, in the complex frequency domain:

$$I_{\mathrm{ref}}(s) = V_i(s)G(s) \qquad (3)$$

where $G(s)$ is the transfer function from the input voltage to the reference template. Assume that the transfer function from I_{ref} to I_i is

$$K(s) = \frac{I_i(s)}{I_{\mathrm{ref}}(s)}. \qquad (4)$$

Suppose $Z(s)$ is to be synthesized. Then, $G(s)$ becomes

$$G(s) = \frac{1}{K(s)Z(s)}. \qquad (5)$$

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[2] For any of the simple converters, viz. buck, boost and buck-boost converters, the inductor stores no energy over one switching period.

[3] This is exactly what a PFC converter does.

[4] A single converter having unity power factor cannot provide perfect dc output regulation because $v_i i_i \neq v_o i_o$, where $v_i i_i$ is square of sine and $v_o i_o$ is constant.
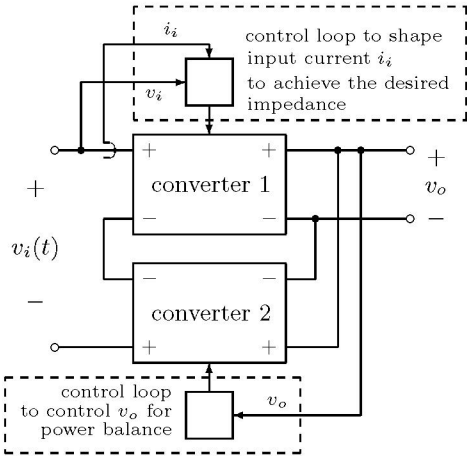
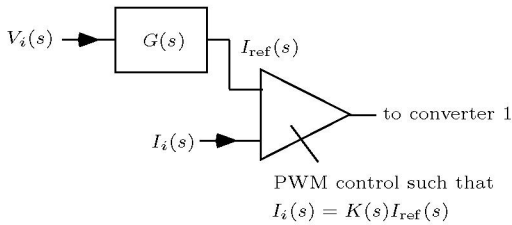Figure 3: A minimal configuration.

Figure 4: Current shaping control for achieving the required magnitude and phase relationships between input voltage and input current. $K(s)$ is nearly constant if all variables are slowly varying relative to the switching frequency.

Thus, in the circuit implementation, we have to realize this transfer function in order to synthesize the required impedance. A block diagram showing the control requirement is shown in Fig. 4.

Clearly, $K(s)$ in general depends on the converter response. However, if the bandwidth of the converter response is much higher than that of the applied voltage v_i, we may assume that $K(s)$ is nearly a constant, i.e., i_i is proportional to i_{ref}. Thus, we can find the control transfer function $G(s)$ from (5).

Example1: Inductive Impedance Synthesis

The impedance of a series connection of an inductor L and a resistor R_L is

$$Z_{LR}(s) = sL + R_L \qquad (6)$$

and $G(s)$ can be calculated according to (5) as

$$G(s) = \frac{1}{KR_L \left(1 + \frac{sL}{R_L}\right)}. \qquad (7)$$

The transfer function $G(s)$ is a single pole system and can be easily implemented [5].
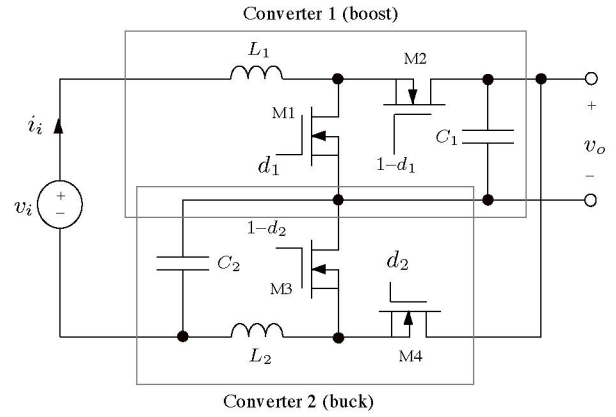

Figure 5: Experimental impedance synthesizer based on the configuration of Fig. 3, with converter 1 and converter 2 realized by a boost converter and a buck converter, respectively. Control circuits omitted.

Example 2: Capacitive Impedance Synthesis

The impedance of a parallel connection of a capacitor C and a resistor R_C is

$$Z_{CR}(s) = \frac{R_C}{1 + sCR_C} \qquad (8)$$

and hence $G(s)$ can be calculated according to (5) as

$$G(s) = \frac{1}{K\left(\frac{R_C}{1+sCR_C}\right)} = \frac{1}{K}\frac{1 + sCR_C}{R_C}. \qquad (9)$$

The transfer function $G(s)$ is a one zero system and can also be readily implemented [5].

5 EXPERIMENTAL RESULTS

A particular choice of practical converter types for the implementation of the impedance synthesizer is shown in Fig. 5. Here, the input v_i is a sinusoidal voltage source, and the output v_o is connected to a current load. The switching frequency is 50 kHz. The two examples described in the previous section are evaluated.

The first impedance to be synthesized is $L = 0.3$ H in series with $R_L = 50\ \Omega$. We have realized $G(s)$ using op-amps and discrete components. The transfer function K was found to be 1.18 A/V. Figure 6 (a) shows the measured waveforms with V_i (CH1) = 80 V(rms) 50 Hz AC. CH4 shows the current I_i with inductive characteristics of lagging phase. The measured phase lag of I_i is 54.72° and the amplitude is 0.8436 A(rms). The measured synthesized impedance is equivalent to $L = 0.246$ H and $R_L = 54.772\ \Omega$ in series and is close to the designed values of $L = 0.28$ H and $R_L = 45.107\ \Omega$. The error is expected to be caused by the phase difference shown in Fig. 5 (b) of I_{ref} (CH3) and I_i, which has been ignored in the calculation of $G(s)$.

Next, we synthesize a parallel connection of $C = 22\ \mu$F and $R_C = 300\ \Omega$. Figure 7 (a) shows the measured waveforms with V_i (CH1) = 80 V(rms) 50 Hz AC. CH4 shows the current I_i with capacitive characteristics

9

(a)

(a)

(b)
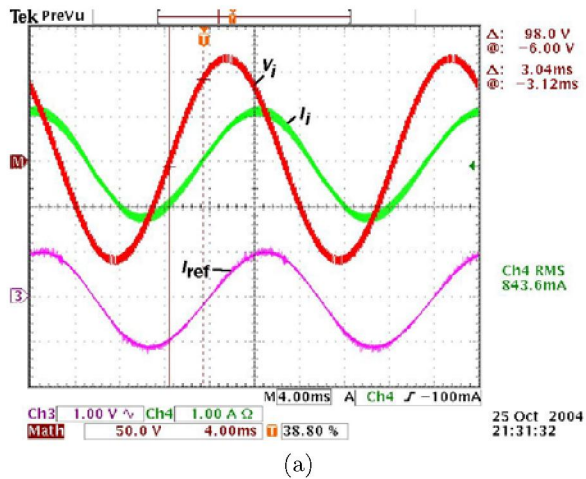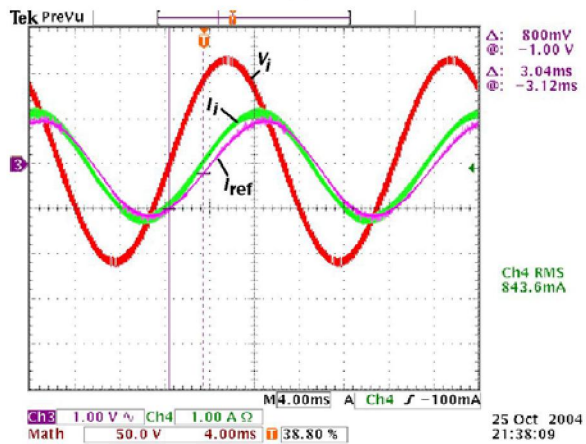
(b)

Figure 6: Measured input current and voltage from inductive impedance synthesizer showing (a) phase lead of voltage; (b) small phase difference between I_{ref} and input current. Impedance being synthesized is 50 Ω in series with 0.3 H.

Figure 7: Measured input current and voltage from capacitive impedance synthesizer showing (a) phase lead of current; (b) small phase difference between I_{ref} and input current. Impedance being synthesized is 300 Ω in parallel with 22 μF.
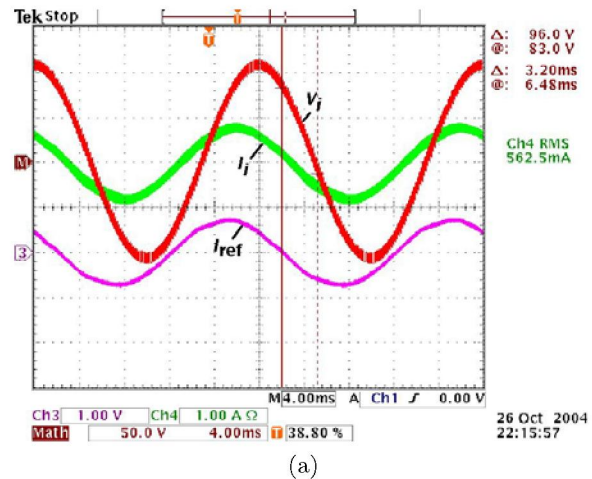
of leading phase. The measured synthesized impedance is equivalent to $C = 18.1$ μF and $R = 242.6$ Ω in parallel connection and is close to the designed values of $C = 21.24$ μF and $R = 291.902$ Ω. The error is also expected to be caused by the phase difference of I_{ref} and I_i shown in Fig. 7 (b).
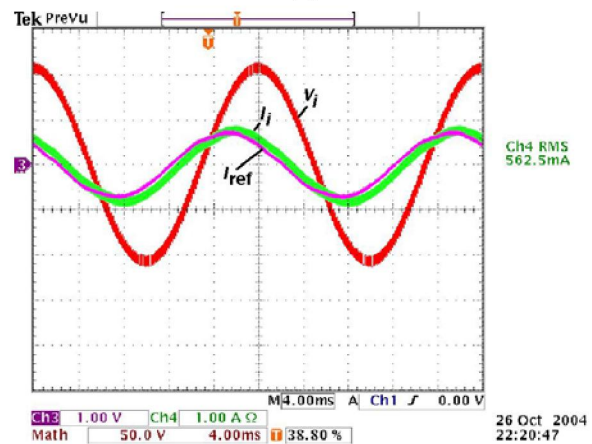
6 CONCLUSIONS

A general impedance synthesis scheme using high frequency switching converters is proposed. With this method, we can imitate any impedance for control purpose, such as in power factor correction. It is also possible to synthesize impedances such as negative inductor and negative capacitor which are not generally available as circuit components.

ACKNOWLEDGMENTS

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10