A Family of PWM Based Sliding Mode Voltage Controllers for Basic DC–DC Converters

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Abstract— **This paper discusses a family of fixed-frequency pulse-width-modulation based sliding mode voltage controllers for basic DC–DC converters operating in continuous conduction mode. An experimental comparison is offered between the proposed controller and the PWM peak current mode controller for the case of the boost converter. The performances and properties of the proposed controller, the conventional PWM voltage mode controller, and the PWM current mode controllers are also compared.**

I. Introduction

Requiring only the sensing of the output voltage and a single stage compensation, linear PWM voltage mode control is by far the most convenient method of controlling DC–DC converters. However, the effectiveness of this control methodology is often limited to converters operating in discontinuous conduction mode (DCM) and the buck-type converters. Boost-type and buck-boost-type converters operating in continuous conduction mode (CCM), which inherit the right-half-plane-zero (RPHZ) characteristic in their duty-cycle-to-output-voltage transfer functions, typically restrict designers to choosing low gain bandwidth types of compensation network for their voltage controllers. This makes the performance of the converters sluggish and often unsatisfactory [1]. Moreover, the performance will be further deteriorated in applications requiring wide operating conditions, since the compensation network is only optimized for a fixed operating condition.

A solution to achieving fast dynamical response in RPHZ converter systems is to employ the current mode control [2]. This is a two-loop control methodology that uses an internal current loop in addition to the voltage loop. In a certain sense, this introduces a nonlinear state feedback term which makes the current mode control semi-nonlinear, since the voltage loop of its control is maintained linear. This is unlike the PWM voltage mode control which is entirely linear. Hence, the current mode control supports converter applications with a wider range of operating condition than the linear PWM voltage mode control. Additionally, since the overall system's stability (phase) margin is increased by the adoption of a fast inner-current loop compensator, a higher gain bandwidth compensator over a comparable voltage mode circuit can be achieved. This leads to fast dynamical response in RHPZ systems. However, despite such excellent properties, the current mode control is afterall not fully nonlinear. It does not fully support converter applications with large-signal disturbances.

Thus, in this paper, we introduce a family of fixed-frequency nonlinear controllers, namely, the PWM based sliding mode (SM) voltage controllers to supplement the existing types of PWM controllers for applications in basic DC–DC converters. The idea is to facilitate the control support for the class of converters operating with wide operating ranges. The method of deriving these controllers and their preliminary simulation results have been provided in a previous paper [3]. This paper complements [3] with an experimental validation and a detailed discussion of the practical aspects of the controllers. A comparison of the general aspects of the performances and properties between the proposed controller and various conventional PWM controllers is also provided.

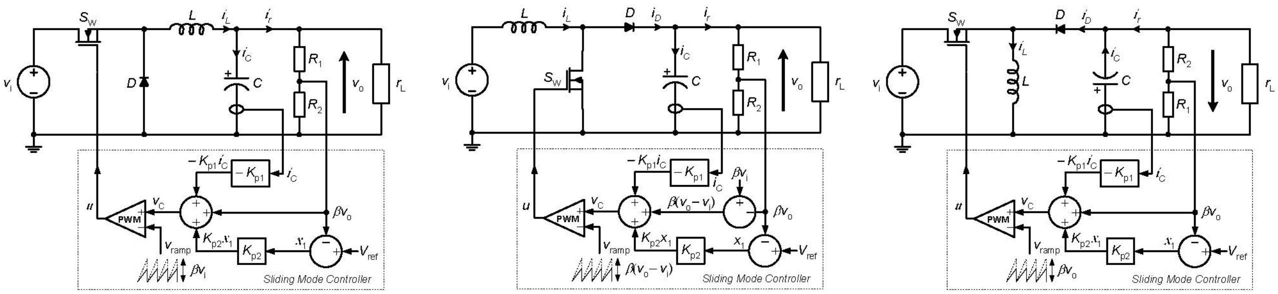
II. A Family of PWM Based SM Controllers

SM controllers are well known for their robustness and stability. Particularly in converter applications requiring a wide operating range, SM controllers are understandably better candidates than conventional PWM controllers due to their ability in handling large-signal perturbations in nonlinear systems [4], [5]. Among the various proposed systems, the fixed-frequency PWM based SM controllers are more suited for practical implementation in power converters [3], [5]–[9]. The main operating mechanism of these controllers is a pulse-width modulator that employs a control signal derived from SM control technique. Detailed discussion of the control strategy can be found in [3], [9].

Fig. 1 shows the schematic diagrams of the family of PID PWM based SM voltage controller to be discussed in this paper. Here, C, L, and r_{L} denote the capacitance, inductance, and instantaneous load resistance of the converters respectively; i_C, i_L, and i_r denote the instantaneous capacitor, inductor, and load currents respectively; V_{ref}, v_{i}, and βv_{o} denote the reference, instantaneous input, and instantaneous output voltages respectively; β denotes the feedback network ratio; and $u = (0, 1)$ is the state of power switch S_{W}.

A. Control Equations

The control equations required for the implementation of the PWM based SM voltage controllers for the respective basic DC–DC converters were derived in [3], and are illustrated in



(a) SMVC buck converter (b) SMVC boost converter (c) SMVC buck-boost converter

Fig. 1. Schematic diagrams of the family of PWM based SM voltage controllers for the buck, boost, and buck-boost converters.

TABLE I
EQUATIONS OF PWM BASED SM VOLTAGE CONTROLLERS

Converter	v_{c}	$\hat{v}_{\mathrm{ramp}}$
Buck	$\beta v_{\mathrm{o}} - K_{\mathrm{p1}} i_C + K_{\mathrm{p2}}\left(V_{\mathrm{ref}} - \beta v_{\mathrm{o}}\right)$	βv_{i}
Boost	$\beta\left(v_{\mathrm{o}} - v_{\mathrm{i}}\right) - K_{\mathrm{p1}} i_C + K_{\mathrm{p2}}\left(V_{\mathrm{ref}} - \beta v_{\mathrm{o}}\right)$	$\beta\left(v_{\mathrm{o}} - v_{\mathrm{i}}\right)$
Buck-boost	$\beta v_{\mathrm{o}} - K_{\mathrm{p1}} i_C + K_{\mathrm{p2}}\left(V_{\mathrm{ref}} - \beta v_{\mathrm{o}}\right)$	βv_{o}

* $K_{\mathrm{p1}} = \beta L\left(\frac{\alpha_1}{\alpha_2} - \frac{1}{r_{\mathrm{L}} C}\right)$ and $K_{\mathrm{p2}} = \frac{\alpha_3}{\alpha_2} LC$ are fixed parameters.

Table I. The control gains K_{p1} and K_{p2} are the constant parameters that can be found in terms of converter's parameters L, C, and r_{L}, and selected values of sliding coefficients α_1, α_2, and α_3. This simplifies the design procedure.

B. Existence Conditions

The compliance of the existence condition ensures the occurrence of SM operation. The method of deriving the existence condition is to inspect the local reachability condition of the controlled system. The existence conditions of the proposed family of controllers were derived in [3] and are shown in Table II. Selection of the sliding coefficients must comply with the respective inequalities.

TABLE II
SLIDING MODE CONTROL EXISTENCE CONDITIONS

Buck	$0 < \beta V_{\mathrm{o}} - K_{\mathrm{p1}} i_C + K_{\mathrm{p2}}\left(V_{\mathrm{ref}} - \beta V_{\mathrm{o}}\right) < \beta V_{\mathrm{i}}$
Boost	$0 < K_{\mathrm{p1}} i_C - K_{\mathrm{p2}}\left(V_{\mathrm{ref}} - \beta V_{\mathrm{o}}\right) < \beta(V_{\mathrm{o}} - V_{\mathrm{i}})$
Buck-boost	$0 < K_{\mathrm{p1}} i_C - K_{\mathrm{p2}}\left(V_{\mathrm{ref}} - \beta V_{\mathrm{o}}\right) < \beta V_{\mathrm{o}}$

C. Sliding Coefficients

The method of selecting sliding coefficients for second-order controllers is illustrated in [9]. Based on the Ackermann's Formula for designing static controllers [10], the adopted approach is to select the sliding coefficients based on the desired dynamic properties. In this way, the *stability condition* of the system is automatically satisfied.

D. Evaluation

The controller circuits (refer to Fig. 1 and Table I) basically adopts the same structure as the PWM PD linear control, but with additional components consisting of the instantaneous input voltage βv_{i} and/or the instantaneous output voltage βv_{o}. These are the components contributing to the nonlinearity of the feedback control, and are therefore the key properties that keep the controller robust to load and line regulation under wide operating ranges. In case of large disturbance, the component v_{o} is highly influential in the control. However, at steady state, v_{o} becomes a fixed point. The equation then reduces to the PWM PD linear controller form.

On the other hand, these controllers can be viewed as a type of nonlinear state-feedback controllers designed from nonlinear averaged models of the converters. The main difference between these two approaches is that the SM approach, which averages the model during the implementation of the controller, retains much of the power converter dynamics. This results in a set of design restrictions: the existence conditions, which are evolved from the instantaneous dynamics of the converter. Such design restrictions are absent from the nonlinear PWM controller design approach [11]. Since the abidance of the existence condition implies that the system's trajectory will strictly follow the desired sliding surface, the system's dynamic response will strictly obey the designed dynamics. Such stringency is not present in the nonlinear PWM controller design approach.

Table III shows a comparison of the general aspects of the performances and properties between the proposed controller and various conventional PWM controllers.

III. EXPERIMENTAL RESULTS AND DISCUSSIONS

The proposed PWM based SM voltage controllers have been experimentally verified on the buck converter [3]. This section presents the experimental validation of the controller on a 100 W boost converter with the specification shown in Table IV. Here, the PWM based SM voltage controller is designed to give a critically damped response at a bandwidth

TABLE III

A COMPARISON OF THE VARIOUS FIXED-FREQUENCY PWM CONTROL SCHEMES

Category of Comparison	PWM Voltage Mode	PWM Current Mode/Average Current Mode	PWM Based SM Voltage Mode
type of control	linear	some nonlinearity	nonlinear
compensation design	difficult	moderate/difficult	easy
steady-state line and load regulation	average	good	average
large-signal dynamic response consistency	poor	average	good
gain bandwidth in RHPZ converter	low	relatively higher	low
requirement for current sensing	no	yes	yes
noise-to-signal ratio	low	high/moderate	moderate
current protection	external circuit	inherent	external circuit
audiosusceptibility protection	external scheme	inherent	existing

TABLE IV

SPECIFICATION OF BOOST CONVERTER

Description	Parameter	Nominal Value
Input voltage	$v_{\rm i}$	24 V
Capacitance	C	2000 μF
Capacitor ESR	c_r	69 mΩ
Inductance	L	300 μH
Inductor resistance	l_r	0.14 Ω
Switching frequency	$f_{\rm S}$	200 kHz
Minimum load resistance (full load)	$r_{\rm L(min)}$	24 Ω
Maximum load resistance (10 % load)	$r_{\rm L(max)}$	240 Ω
Desired output voltage	$V_{\rm od}$	48 V

of $\omega_n = 1.5$ krad/s, i.e., $\tau = 0.66$ ms. The sliding coefficients are $\frac{\alpha_1}{\alpha_2} = 3000$ and $\frac{\alpha_3}{\alpha_2} = 2250000$ (full-load condition design) according to the design equations in [9]; the reference voltage is $V_{\rm ref} = 8$ V, $\beta = \frac{1}{6}$; and control parameters $K_{\rm p1} = \beta L\left(\frac{\alpha_1}{\alpha_2} - \frac{1}{r_{\rm L}C}\right) = 0.149$ and $K_{\rm p2} = \frac{\alpha_3}{\alpha_2}LC = 1.35$.

A. Regulation Performance

A tabulation of the data in terms of the load and line regulation properties is also given in Tables V and VI respectively. According to Table V, the maximum load-regulation error occurs at $v_{\rm i} = 20$ V, with a deviation of 1.75 % from $v_{\rm o(nominal\ condition)}$. Similarly, it can be found from Table VI that the maximum line-regulation error occurs at minimum load $r_{\rm L} = 240\ \Omega$, with a deviation of 1.42 % from $v_{\rm o(nominal\ condition)}$.

B. Proposed Controller Versus Peak Current Mode Controller

The dynamic behavior of the proposed PWM controller is compared to that of a UC3843 PWM peak current mode controller that is optimally tuned to operate a boost converter at step load change from $R_{\rm L} = 24\ \Omega$ to $R_{\rm L} = 240\ \Omega$ for the input condition $V_{\rm i}$ = 24 V. Figs. 2(a)–2(f) show the experimental waveforms of the boost converter under these control schemes.

It can be seen that with the PWM peak current mode controller, the dynamic behavior of the system differs for different operating conditions. Specifically, the response becomes more oscillatory at higher input voltages. Moreover, the dynamic behavior and transient settling time are also very much different between the case when the load changes from $R_{\rm L} = 24\ \Omega$ and $R_{\rm L} = 240\ \Omega$ and the case when it changes from $R_{\rm L} = 240\ \Omega$ and $R_{\rm L} = 24\ \Omega$. This is expected since the peak current mode controller is designed under a linearized small signal model that is only optimal for a specific operating condition. Thus, when a different operating condition is engaged, the responses varies.

On the other hand, with the PWM based SM voltage controller, the dynamic behavior of the output voltage ripple is basically similar (i.e. critically damped) for all operating input and load conditions. Moreover, the transient setting time, which is around 3.4 ms, is also independent of the direction of the step load change. This coincides with our design, which being a 1.5 krad/s bandwidth controller, is expected to have a settling time of $5\tau = \frac{5}{1.5} = 3.33$ ms. This demonstrates the strength of the SM controller in terms of robustness in the dynamic behavior under different operating conditions and uncertainties. Additionally, the example also illustrates a major difference between a large-signal controlled system (SM) and a small-signal controlled system (PWM), that is, the former complies to the design with a similar response for all operating conditions, while the response of the latter will only comply to the design at a specific operating condition.

IV. CONCLUSION

We provide a detailed discussion on a newly proposed family of fixed-frequency pulse-width-modulation based sliding mode voltage controllers for basic DC–DC converters. An experimental comparison between the proposed controller and the PWM peak current mode controller is performed on the boost converter. The general differences in terms of control performances and properties between the proposed controller and the conventional PWM controllers are highlighted. It can be concluded that the PWM based sliding mode voltage controllers possible control alternatives for DC–DC conversion applications requiring wide operating conditions.

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TABLE V

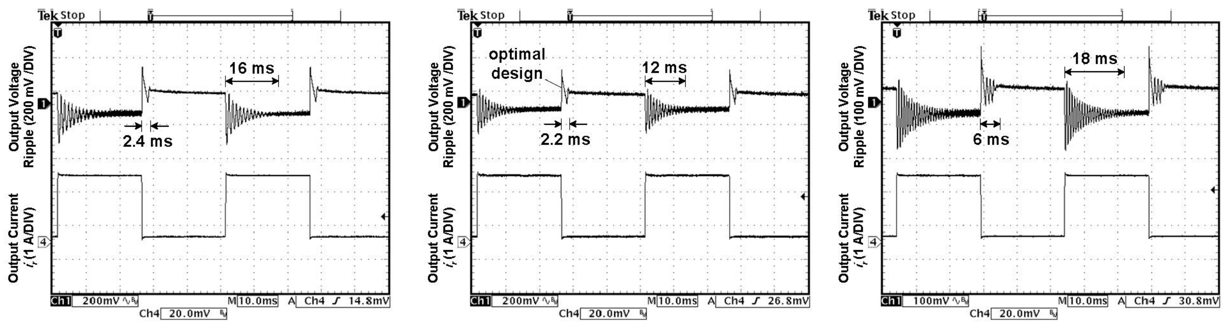
LOAD REGULATION PROPERTY: OUTPUT VOLTAGE AT NOMINAL OPERATING CONDITION $v_{\rm i} = 24$ V AND $r_{\rm L} = 24\ \Omega$ IS $v_{\rm o(nominal\ condition)} = 47.95$ V.

Input Voltage	Voltage Deviation: $\Delta v_{\rm o} = v_{\rm o(240\ \Omega)} - v_{\rm o(24\ \Omega)}$	Percentage Change: $\frac{\Delta v_{\rm o}}{v_{\rm o(nominal\ condition)}}$ x 100 %
$v_{\rm i}$= 20 V	0.84 V	1.75 % of $v_{\rm o(nominal\ condition)}$
$v_{\rm i}$= 24 V	0.61 V	1.27 % of $v_{\rm o(nominal\ condition)}$
$v_{\rm i}$= 28 V	0.56 V	1.16 % of $v_{\rm o(nominal\ condition)}$

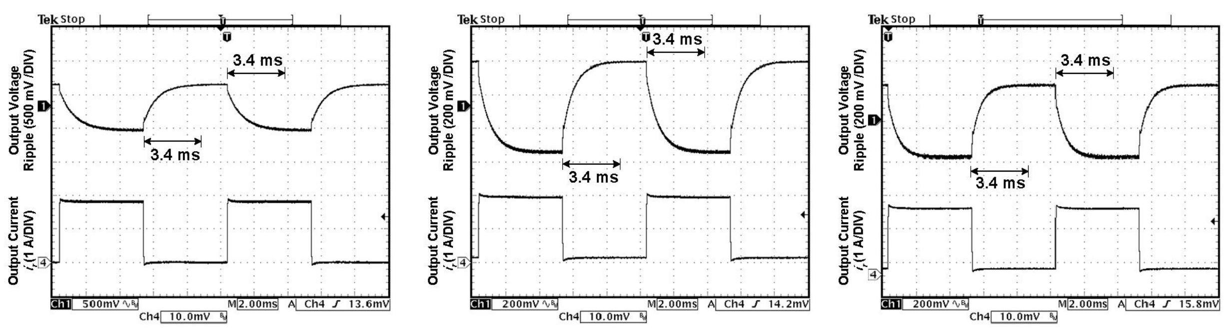
TABLE VI

LINE REGULATION PROPERTY: OUTPUT VOLTAGE AT NOMINAL OPERATING CONDITION $v_{\rm i} = 24$ V AND $r_{\rm L} = 24\ \Omega$ IS $v_{\rm o(nominal\ condition)} = 47.95$ V.

Loading Condition	Voltage Deviation: $\Delta v_{\rm o} = v_{\rm o(v_i=20\ V)} - v_{\rm o(v_i=28\ V)}$	Percentage Change: $\frac{\Delta v_{\rm o}}{v_{\rm o(nominal\ condition)}}$ x 100 %
Minimum load (240 Ω)	0.68 V	1.42 % of $v_{\rm o(nominal\ condition)}$
Half load (48 Ω)	0.58 V	1.21 % of $v_{\rm o(nominal\ condition)}$
Full load (24 Ω)	0.40 V	0.83 % of $v_{\rm o(nominal\ condition)}$



(a) $v_{\rm i}$ = 20 V (peak current mode controller) (b) $v_{\rm i}$ = 24 V (peak current mode controller) (c) $v_{\rm i}$ = 28 V (peak current mode controller)



(d) $v_{\rm i}$ = 20 V (SM controller) (e) $v_{\rm i}$ = 24 V (SM controller) (f) $v_{\rm i}$ = 28 V (SM controller)

Fig. 2. Experimental waveforms of output voltage ripple $\tilde{v_{\rm o}}$ and output current i_r of the boost converter, with the peak current mode controller (a)–(c) and the 1.5 krad/s bandwidth PWM based SM controller (d)–(f), operating at load resistance alternating at 24 Ω (minimum) and 240 Ω (maximum).

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