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An Analog Implementation of Pulse-Width-Modulation Based Sliding Mode Controller for DC–DC Boost Converters

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***Abstract*— This paper addresses the issues concerning the implementation of a pulse-width-modulation based sliding mode controller for boost converters. The methods of modelling the system and translation of the SM control equations for the PWM implementation are illustrated. It is shown that the control technique is easily realized with simple analog circuitries. The proposed system is verified by experimental measurements.**

I. Introduction

The advantages of employing sliding mode (SM) controllers for applications in nonlinear control systems are well discussed and understood [1]. In power converters requiring wide operating range, SM controllers are understandably better candidates than conventional linear PWM controllers due to their excellent robust and stability properties in handling large-signal perturbations [2]. This spurs numerous researches in the area. However, among the various proposed systems, the fixed-frequency SM controllers are particularly suited for practical implementation in power converters [3]–[10].

Study of fixed-frequency SM controllers has been initiated by the practical constraint of power converters in requiring to prevent excessive power losses and EMI noise generation, and also to simplify the design of input and output filters [11]. However, the nature of the SM controller is to ideally operate at infinite, varying, and self-oscillating switching frequency such that the controlled variables can track a certain reference path to achieve steady-state operation [1].

There are numerous methods proposed to constrict the switching frequency of SM controllers [3]–[10]. Those that employ the hysteresis-modulation (HM) (or delta-modulation) as the medium for implementing the control law, will require either constant timer circuits to be incorporated into the hysteretic SM controller to ensure constant switching frequency [3], [4], or the use of an adaptive hysteresis band that varies with parameter changes to control and fix the switching frequency [5], [10]. However, these solutions require additional components and are unattractive for low cost voltage conversion applications. Moreover, some of these converter systems suffer from deteriorated transient response.

Alternatively, the switching frequency of SM controllers can be constricted (fixated) by changing the modulation method from HM to pulse-width-modulation (PWM) [2], [6], [7], [9]. This idea is originated from one of the earliest papers on SM controlled power converters [2], which suggests that under SM control operation, the control signal of *equivalent control approach* u_{eq} in SM control is equivalent to the *duty cycle control signal* d of a PWM controller. The proof was later provided in the papers [12], [13]. It has been shown that at a high switching frequency, the *control action* of a sliding mode controller is equivalent to the *duty cycle control action* of a PWM controller. Hence, the use of PWM techniques in lieu of HM methods in SM control is possible under these principles.

Lately, this idea is revisited and experimentally demonstrated on a sliding mode voltage controlled (SMVC) buck converter [9]. Building on the work of [2], [6], [12], it is demonstrated in [9] how PWM based SM controller can be easily realized with simple analog ICs. However, the discussion does not cover the design methodology of other converters. It was also unknown if the proposed PWM based SM controller can be as easily realized in converters that are more complex.

Therefore, in this paper, we extend the work in [9], by exploring into the possible application of the PWM based SM voltage controller on the boost converter. Specifically, we illustrate the method of modelling the system and the translation of the SM control equations for the PWM implementation. Finally, with an experimental prototype, we validate that the PWM based SM controller under a different circuit architecture, is also applicable for the control of boost converters.

II. The Design Approach

A. System Modelling

A second order PID type of SM voltage controller is adopted. Fig. 1 shows the schematic description of the proposed sliding mode voltage controlled (SMVC) boost converters, where $C, L,$ and r_{L} are the capacitance, inductance, and load resistance of the converters respectively; i_C, i_L, and i_r are the capacitor, inductor, and load currents respectively; V_{ref}, v_{i}, and βv_{o} are the reference, input, and sensed output voltage respectively; and $u = 0$ or 1 is the switching state of power switch S_{W}.

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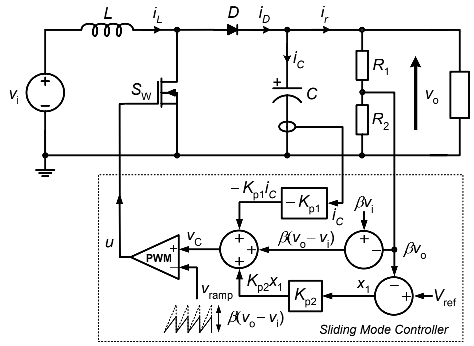

Fig. 1. Schematic diagrams of the PID SMVC boost converter.

For any PID SMVC converters type, the control variable $\boldsymbol{x}$ may be expressed in the general form:

$$\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} V_{\mathrm{ref}} - \beta v_{\mathrm{o}} \\ \frac{d(V_{\mathrm{ref}} - \beta v_{\mathrm{o}})}{dt} \\ \int (V_{\mathrm{ref}} - \beta v_{\mathrm{o}}) dt \end{bmatrix} \tag{1}$$

where x_1, x_2, and x_3 represents the *voltage error*, the *voltage error dynamics* (or the rate of change of voltage error), and the *integral of voltage error*, respectively. Substitution of the boost converter's behavioral models under continuous conduction mode (CCM) of operation into (1) produces the following control variable description:

$$\boldsymbol{x}_{\mathrm{boost}} = \begin{bmatrix} V_{\mathrm{ref}} - \beta v_{\mathrm{o}} \\ \frac{\beta v_{\mathrm{o}}}{r_{\mathrm{L}} C} + \int \frac{\beta (v_{\mathrm{o}} - v_{\mathrm{i}}) \bar{u}}{LC} dt \\ \int x_1 dt \end{bmatrix}. \tag{2}$$

where $\bar{u} = 1 - u$ is the inverse logic of u. Next, the time differentiation of equation (2) produces the state space description

$$\dot{\boldsymbol{x}}_{\mathrm{boost}} = \mathbf{A} \boldsymbol{x}_{\mathrm{boost}} + \mathbf{B} \bar{u} \tag{3}$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -\frac{1}{r_{\mathrm{L}} C} & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} 0 \\ \frac{\beta v_{\mathrm{o}}}{LC} - \frac{\beta v_{\mathrm{i}}}{LC} \\ 0 \end{bmatrix}. \tag{4}$$

B. Controller Design

For this system, it is appropriate to have a general SM control law that adopts a switching function such as

$$u = \begin{cases} 1 & \text{when } S > 0 \\ 0 & \text{when } S < 0 \end{cases} \tag{5}$$

where S is the instantaneous state variable's trajectory, and is described as

$$S = \alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3 = \boldsymbol{J}^{\mathrm{T}} \boldsymbol{x}, \tag{6}$$

with $\boldsymbol{J}^{\mathrm{T}} = [\alpha_1 \ \ \alpha_2 \ \ \alpha_3]$ and α_1, α_2, and α_3 representing the control parameters termed as sliding coefficients.

1) Derivation of Existence Conditions: To ensure the existence[1] of SM operation, the local reachability condition

$$\lim_{S \to 0} S \cdot \dot{S} < 0, \tag{7}$$

must be satisfied. For the proposed converter, this can be expressed as

$$\begin{cases} \dot{S}_{S \to 0^+} = \boldsymbol{J}^{\mathrm{T}} \mathbf{A} \boldsymbol{x}_{\mathrm{boost}} + \boldsymbol{J}^{\mathrm{T}} \mathbf{B} u_{S \to 0^+} < 0 \\ \dot{S}_{S \to 0^-} = \boldsymbol{J}^{\mathrm{T}} \mathbf{A} \boldsymbol{x}_{\mathrm{boost}} + \boldsymbol{J}^{\mathrm{T}} \mathbf{B} u_{S \to 0^-} > 0 \end{cases}. \tag{8}$$

The specific conditions for the existence of SM control operation for the boost converters are

- Case 1: $S \to 0^+$, $\dot{S} < 0$ – substitution of $u_{S \to 0^+} = \bar{u} = 0$, and the matrices in (2) and (4) into (8) gives $-\alpha_1 \frac{\beta i_C}{C} + \alpha_2 \frac{\beta i_C}{r_L C^2} + \alpha_3 (V_{\mathrm{ref}} - \beta v_{\mathrm{o}}) < 0$.
- Case 2: $S \to 0^-$, $\dot{S} > 0$ – substitution of $u_{S \to 0^-} = \bar{u} = 1$, and the matrices in (2) and (4) into (8) gives $-\alpha_1 \frac{\beta i_C}{C} + \alpha_2 \frac{\beta i_C}{r_L C^2} + \alpha_3 (V_{\mathrm{ref}} - \beta v_{\mathrm{o}}) - \alpha_2 \frac{\beta v_{\mathrm{i}}}{LC} + \alpha_2 \frac{\beta v_{\mathrm{o}}}{LC} > 0$.

2) Derivation of Control Equations for PWM Based Controller: The conventional SM controller implementation based on HM [8] requires only control equations (5) and (6). However, if the PWM based SM voltage controller is to be adopted, an indirect translation of the SM control law is required so that pulse-width modulation can be used in lieu of hysteresis modulation [9]. The procedure for the PWM design can be summarized in two steps. Firstly, the equivalent control signal u_{eq}, which is a smooth function of the discrete input function u, is formulated using the *invariance conditions* by setting the time differentiation of (6) as $\dot{S} = 0$ [1]. Secondly, the equivalent control function is mapped onto the duty cycle function of the pulse-width modulator [9]. For the PWM based SMVC boost converter, the derivations are as illustrated.

- Equating $\dot{S} = \boldsymbol{J}^{\mathrm{T}} \mathbf{A} \boldsymbol{x} + \boldsymbol{J}^{\mathrm{T}} \mathbf{B} \bar{u}_{\mathrm{eq}} = 0$ yields the equivalent control function

$$\bar{u}_{\mathrm{eq}} = \frac{\beta L}{\beta (v_{\mathrm{o}} - v_{\mathrm{i}})} \left(\frac{\alpha_1}{\alpha_2} - \frac{1}{r_{\mathrm{L}} C} \right) i_C - \frac{\alpha_3 LC}{\alpha_2 \beta (v_{\mathrm{o}} - v_{\mathrm{i}})} (V_{\mathrm{ref}} - \beta v_{\mathrm{o}})$$

  where $\bar{u}_{\mathrm{eq}}$ is continuous and $0 < \bar{u}_{\mathrm{eq}} < 1$. Since $u = 1 - \bar{u}$, which also implies $u_{\mathrm{eq}} = 1 - \bar{u}_{\mathrm{eq}}$, the substitution of (9) into the inequality and a multiplication by $\beta(v_{\mathrm{o}} - v_{\mathrm{i}})$ gives

$$0 < {u_{\mathrm{eq}}}^{*} = -\beta L \left(\frac{\alpha_1}{\alpha_2} - \frac{1}{r_{\mathrm{L}} C} \right) i_C + LC \frac{\alpha_3}{\alpha_2} (V_{\mathrm{ref}} - \beta v_{\mathrm{o}}) + \beta (v_{\mathrm{o}} - v_{\mathrm{i}}) < \beta (v_{\mathrm{o}} - v_{\mathrm{i}}). \tag{9}$$

- Finally, the mapping of the equivalent control function (9) onto the duty ratio control d, where $0 < d = \frac{v_{\mathrm{c}}}{\hat{v}_{\mathrm{ramp}}} < 1$,

[1] Satisfaction of the existence condition is one of the three necessary conditions for SM control operation to occur. It ensures that the state trajectory at locations near the sliding surface will always be directed towards the sliding surface. The other two necessary conditions are the hitting condition, which is satisfied by the control law in eqn. (5), and the stability condition, which is satisfied through the assignment of sliding coefficients [14].

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gives the following relationships for the control signal v_{c} and ramp signal $\hat{v}_{\mathrm{ramp}}$ where

$$v_{\mathrm{c}} = u_{\mathrm{eq}}{}^{*} = -\beta L\left(\frac{\alpha_1}{\alpha_2} - \frac{1}{r_{\mathrm{L}}C}\right) i_C + LC\frac{\alpha_3}{\alpha_2}\left(V_{\mathrm{ref}} - \beta v_{\mathrm{o}}\right) + \beta\left(v_{\mathrm{o}} - v_{\mathrm{i}}\right) \qquad (10)$$

and

$$\hat{v}_{\mathrm{ramp}} = \beta\left(v_{\mathrm{o}} - v_{\mathrm{i}}\right) \qquad (11)$$

for the practical implementation of the PWM based SM controller.

III. Implementation of the PWM based SM Controller

A. Conversion of Control Equations to Circuit Form

1) Control Signal Computation: The computation of the control signal v_{c} in (10) can be performed using simple gain amplification and summing functions. In our prototype, we realize the equation using only three analog gain amplifiers and a summer circuit (LM318). The parameters of these circuitries can be easily calculated using known values of L, C, r_{L}, and β, and proper choices of α_1, α_2, and α_3.

2) Ramp Signal Generation: The peak magnitude of the variable ramp signal $\hat{v}_{\mathrm{ramp}}$ is to follow description (11). In our prototype, a transistor configuration of multiple current mirror circuitries (9015 and 9016) and a charging capacitor are employed to realize the ramp generation. Since the desired output voltage is normally constant with little deviation, only the input voltage change is considered in the design. As for the frequency of the ramp signal, it is controlled by an impulse generator (LMC555 and CD4049).

3) Duty Cycle Protection: The incorporation of the control and ramp signal circuitries into the pulse-width-modulator (LM311) forms the basic architecture of the PWM based SM controller. However, recalling that the boost type converter cannot operate with a switching signal u that has a duty cycle $d = 1$, a small protective circuitry is required to ensure that the duty cycle of the controller's output is always $d < 1$. In our prototype, this is satisfied by multiplying the logic state u_{PWM} of the pulse-width-modulator with the logic state u_{CLK} of the impulse generator using a logic AND IC chip (CD4081). By doing so, the maximum duty cycle of the controller is clamped by the duty cycle of the impulse generator.

B. Experimental Prototype

The derived controller is verified through an experimental prototype developed with the specification shown in Table I.

Figure 2 shows the schematic diagram of the proposed PWM based SMVC boost converter. The enumerated points on the diagram represent different test locations in the controller where waveforms are captured and analyzed. The theoretical description of the signals are derived and shown in Table I.

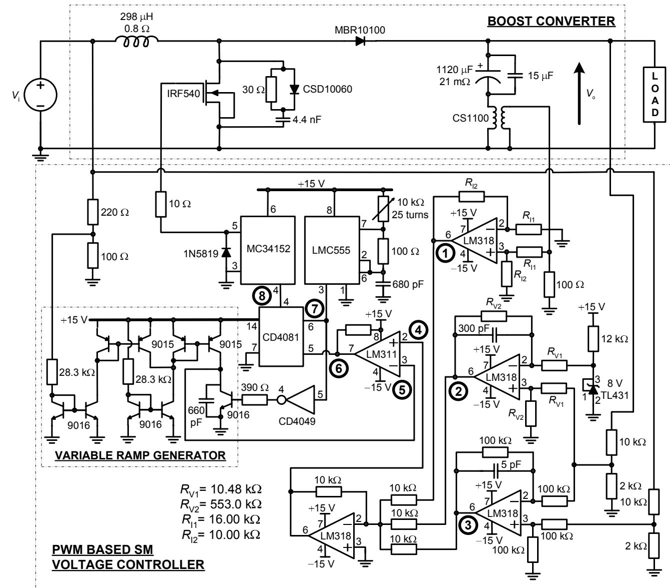

Fig. 2. Schematic diagram of the proposed PWM based SMVC boost converter.

TABLE I
THEORETICAL DESCRIPTION OF SIGNALS

Test Location	Description
1	$\beta L\left(\frac{\alpha_1}{\alpha_2} - \frac{1}{r_{\mathrm{L}}C}\right) i_C$
2	$-LC\frac{\alpha_3}{\alpha_2}\left(V_{\mathrm{ref}} - \beta v_{\mathrm{o}}\right)$
3	$-\beta\left(v_{\mathrm{o}} - v_{\mathrm{i}}\right)$
4	v_{c}
5	$\hat{v}_{\mathrm{ramp}}$
6	u_{PWM}
7	u_{CLK}
8	$u = u_{\mathrm{PWM}} \bullet u_{\mathrm{CLK}}$

IV. Experimental Results and Discussions

A. Measured Signal of Test Locations

Fig. 3 shows the steady-state waveforms of the test locations when the converter is operating at full-load condition. The captured results are consistent with the theoretical expectation.

Figs. 4(a) and 4(b) show respectively the DC output voltage versus the input voltage and operating load resistance. For line variation test, it can be observed that v_{o} decreases with increasing v_{i}. Specifically, the output voltage deviation is -0.15 V (i.e. -0.31 % of V_{od}) for the entire input range 18 V $\leq v_{\mathrm{i}} \leq$ 30 V , i.e., line regulation $\frac{dv_{\mathrm{o}}}{dv_{\mathrm{i}}}$ averages at -12.5 mV/V. For load variation test, it can be concluded that voltage regulation of the converter is robust to load changes, with only a 0.08 V deviation (i.e. 0.17 % of V_{od}) in v_{o} for the entire load range 48 Ω $\leq r_{\mathrm{L}} \leq$ 900 Ω, i.e., load regulation $\frac{dv_{\mathrm{o}}}{dr_{\mathrm{L}}}$ averages at 0.09 mV/Ω.

B. Transient Performance

The dynamic behavior of the controller is studied using a load resistance that alternates between resistances of 240 Ω and 480 Ω, and an input voltage that alternates between

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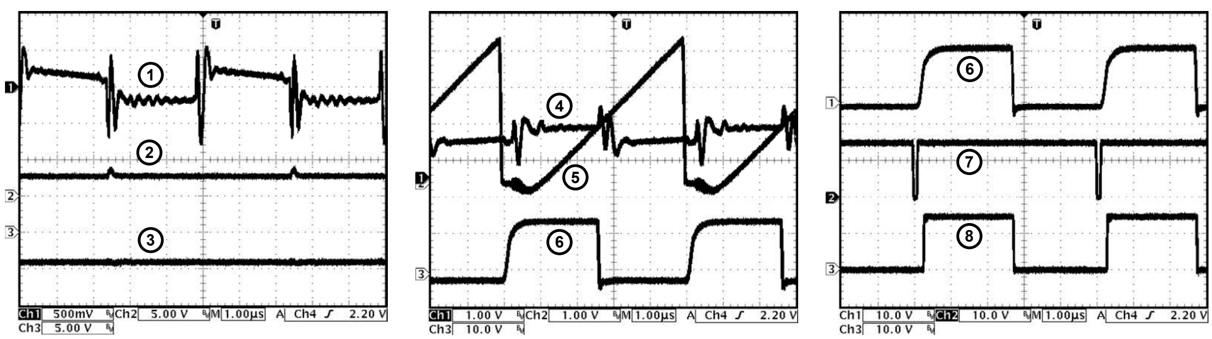
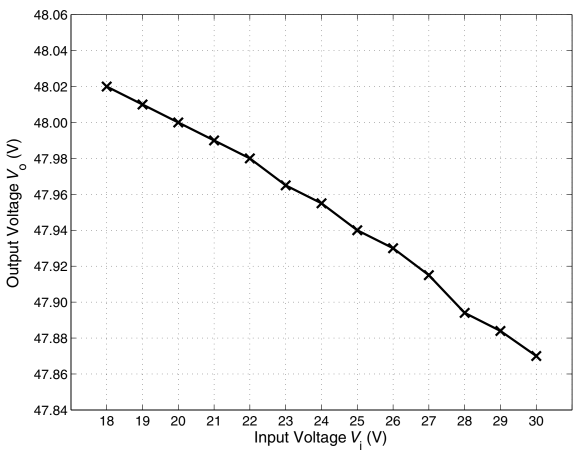
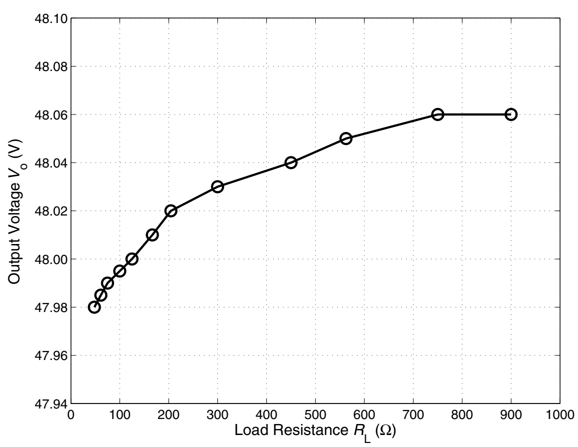


Fig. 3. Experimental waveforms of the test locations under full-load operation.

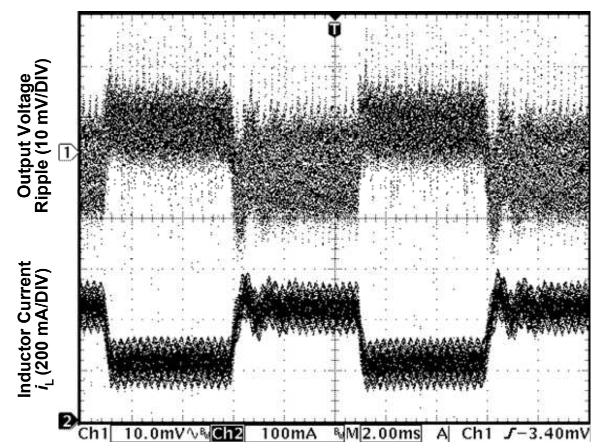


(a) Line variation

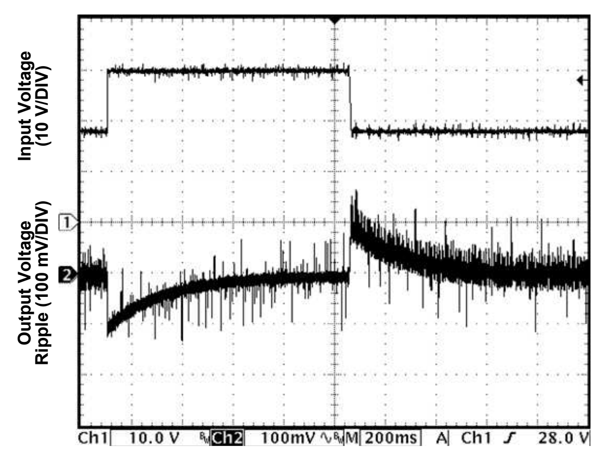


(b) Load variation

Fig. 4. Graphs of DC output voltage v_{o} against (a) input voltage v_{i} and (b) load resistance r_{L} for the PWM based SMVC boost converter.



(a) $v_{\mathrm{i}} = 24$ V; r_{L} = 240 Ω ⇔ 480 Ω



(b) $r_{\mathrm{L}} = 96$ Ω; v_{i} = 18 V ⇔ 30 V

Fig. 5. Waveforms of (a) output voltage v_{o} and inductor current i_L; and (b) input voltage v_{i} and output voltage v_{o} of the SMVC boost converter operating in dynamic line change and dynamic load change conditions.

voltages of 18 V and 30 V. Figs. 5(a) and 5(b) show respectively the output voltage ripple waveforms for both the dynamic load change and dynamic line change operations. The dynamic performances of the converter are adequate for common voltage conversion applications.

C. Operating in Discontinuous Conduction Mode

The PWM based SMVC boost converter, which is designed for operation in CCM, is also tested in discontinuous conduction mode (DCM). Fig. 6 shows the variation of the DC output voltage against different operating load resistances in the DCM. The voltage regulation of the converter has a 0.12 V

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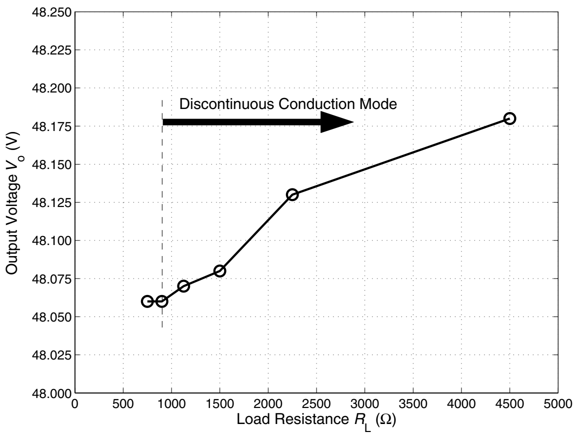

Fig. 6. Graphs of DC output voltage v_o against load resistance r_L for the PWM based SMVC boost converter in light load conditions.

deviation (i.e. 0.25 % of V_{od}) in v_o for the DCM load range $900\ \Omega \leq r_L \leq 4500\ \Omega$, i.e., load regulation $\frac{dv_o}{dr_L}$ averages at 0.033 mV/Ω.

V. Conclusion

A fixed-frequency PWM based SM voltage controller for boost converter is presented. The methods of modelling the system and translation of the SM control equations for the PWM implementation are illustrated. It is shown that the control technique is easily realized with simple analog circuitries. Different static and dynamic tests with line and load changes are also performed. It can be concluded from the results that the derived controller/converter system is feasible for common step-up conversion purposes.

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