GENERALIZED CORRELATION-DELAY-SHIFT-KEYING SCHEME FOR NONCOHERENT CHAOS-BASED COMMUNICATION SYSTEMS

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ABSTRACT

In this paper, we propose a generalized correlation-delay-shift-keying (GCDSK) scheme for noncoherent chaos-based communications. In the proposed scheme, several delayed versions of a chaotic signal are first produced. Some of them will be modulated by the binary data to be transmitted. The delayed signals will then be added to the original chaotic signal and transmitted. At the receiver, a simple correlator-type detector is employed to decode the binary symbols. The approximate bit error rate (BER) of the GCDSK scheme is derived analytically based on a Gaussian approximation. Simulations are performed and compared with the noncoherent correlation-delay-shift-keying (CDSK) and differential chaos-shift-keying (DCSK) modulation schemes. The effects of the spreading factor, length of delay and the number of delay units on the BER are fully studied. It is found that GCDSK can achieve better BER performance than DCSK under reasonable bit-energy-to-noise-power-spectral-density ratios.

1. INTRODUCTION

Noncoherent communication schemes, which do not require the reproduction of the chaotic signals at the receiving end, are more feasible in practice [1]–[3]. The *correlation delay-shift-keying* (CDSK) scheme is one of the noncoherent detection schemes, which is similar to the DCSK scheme in that a reference chaotic signal is embedded in the transmitted signal [3]. Unlike in DCSK [2], however, the reference signal and the information-bearing signal are now added together with a certain time delay in CDSK. As a consequence, each transmitted signal sample includes one reference sample and one information-bearing sample, and the transmitted signal sample is never repeated. Since no individual reference signal is sent, the bandwidth efficiency is improved. Moreover, by eliminating the switch required to perform the switching between the reference chaotic signal and information-bearing signal in the DCSK system, CDSK allows a continuous operation of the transmitter. Also, the transmitted signal is more homogeneous and less prone to interception. However, because the sum of two chaotic signals is sent, more uncertainty (interference) is produced when the received signal correlates with its delayed version at the receiving side. Therefore, the performance of CDSK is worse than that of DCSK.

In this paper, we propose a generalized CDSK (GCDSK) scheme. The transmitted signal is composed of a reference chaotic signal and a number of delayed chaotic signals, some of which are modulated by the data being sent. Such a construction of the transmitted signal allows the transmission of more than one reference signal and more than one information-bearing signal simultaneously. The useful signal component, as well as the interference component, will be enhanced at the receiving side. We show that with an appropriate choice of system parameters, the bit error performance of the proposed system improves over the CDSK scheme. The system also inherits the merits of the CDSK system such as being switchless and allowing continuous operation of the transmitter.

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2. GENERALIZED CORRELATION-DELAY-SHIFT-KEYING (GCDSK) SCHEME

2.1. Transmitter Structure

We propose a generalized CDSK (GCDSK) communication system, as shown in Fig. 1. The transmitter contains a chaotic signal generator and $(M-1)$ delay blocks. We assume that $M>2$ because when $M=2$, the GCDSK system degenerates to the CDSK system. Denote the minimum delay and the lth transmitted symbol by L and $d_l \in \{-1,+1\}$ respectively. Assume that "+1" and "-1" are transmitted with equal probability. First a chaotic signal, denoted by $\{x_k\}$, is generated in the transmitter. The chaotic signals with delays $L, 3L, 5L, \ldots$ are modulated by the data sequence $\{d_l\}$, whereas the signals with delays $2L, 4L, 6L, \ldots$ are unmodulated. Finally, the transmitted signal is formed by adding the original chaotic signal and all the delayed signals. Denote the spreading factor by β, i.e., β chaotic signals s_k, are sent within one bit duration. During the lth bit duration, i.e., for time $k = (l-1)\beta+1, (l-1)\beta+2, \ldots, l\beta$, the transmitted signal is given by

$$s_k = \begin{cases} \displaystyle\sum_{m=0}^{\frac{M-2}{2}} x_{k-2mL} + d_l \sum_{m=0}^{\frac{M-2}{2}} x_{k-(2m+1)L} & \text{if } M \text{ is even} \\ \displaystyle\sum_{m=0}^{\frac{M-1}{2}} x_{k-2mL} + d_l \sum_{m=0}^{\frac{M-3}{2}} x_{k-(2m+1)L} & \text{if } M \text{ is odd} \end{cases} \quad (1)$$

where in each case, the first and second terms represent summation of all the unmodulated and modulated chaotic signals, respectively.

2.2. Receiver Structure

We assume an additional white Gaussian noise (AWGN) channel. The received signal, denoted by r_k, is given by $r_k = s_k + \xi_k$ where ξ_k denotes the AWGN signal with zero mean and variance

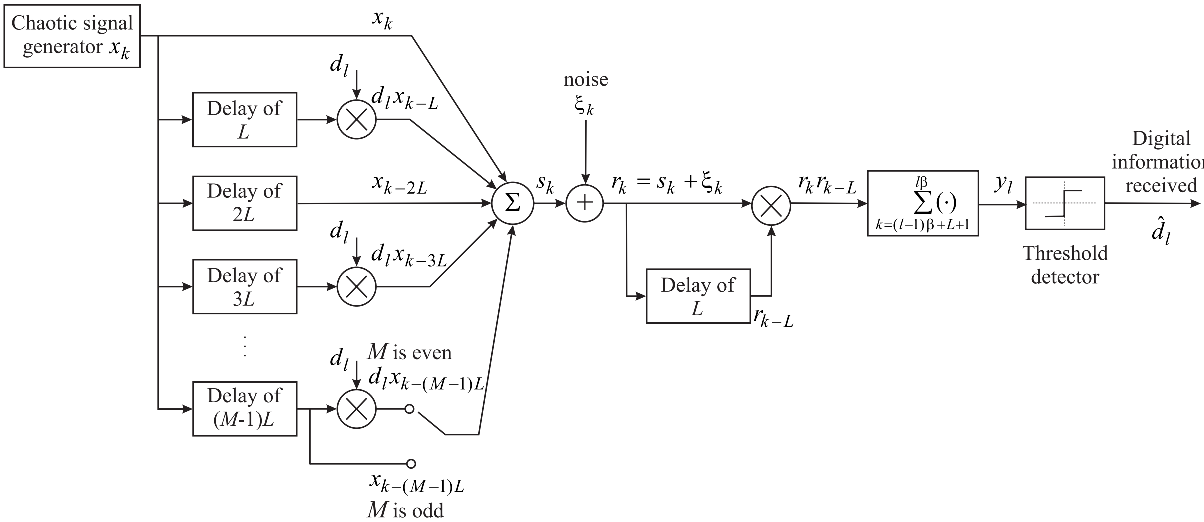

Fig. 1. Block diagram of a generalized correlation-delay-shift-keying communication system.

$N_0/2$. Similar to CDSK, a correlator-type detector is used. The only difference in the correlator is that only $(\beta - L)$ terms (assuming $\beta > L$) will be added in the summation block. Although part of the useful signal component will be lost by summing only $(\beta - L)$ terms, the intra-signal interference component will also be reduced because the appearance of the bit value d_{l-1} will be avoided in the received signal for d_l. For the lth symbol, the corresponding output of the correlator equals

$$
\begin{aligned}
y_l &= \sum_{k=(l-1)\beta+L+1}^{l\beta} r_k r_{k-L} \\
&= \begin{cases}
\sum_{k=(l-1)\beta+L+1}^{l\beta} \left[\left(\sum_{m=0}^{\frac{M-2}{2}} \left(x_{k-2mL} + d_l x_{k-(2m+1)L}\right) + \xi_k \right) \times \left(\sum_{m=0}^{\frac{M-2}{2}} \left(x_{k-(2m+1)L} + d_l x_{k-(2m+2)L}\right) + \xi_{k-L} \right) \right] & \text{if } M \text{ is even} \\
\sum_{k=(l-1)\beta+L+1}^{l\beta} \left[\left(\sum_{m=0}^{\frac{M-1}{2}} x_{k-2mL} + d_l \sum_{m=0}^{\frac{M-3}{2}} x_{k-(2m+1)L} + \xi_k \right) \times \left(\sum_{m=0}^{\frac{M-1}{2}} x_{k-(2m+1)L} + d_l \sum_{m=0}^{\frac{M-3}{2}} x_{k-(2m+2)L} + \xi_{k-L} \right) \right] & \text{if } M \text{ is odd}
\end{cases} \\
&= \begin{cases}
\lambda_{\text{even}} + \mu_{\text{even}} + \nu_{\text{even}} & \text{if } M \text{ is even} \\
\lambda_{\text{odd}} + \mu_{\text{odd}} + \nu_{\text{odd}} & \text{if } M \text{ is odd}
\end{cases}
\end{aligned}
\quad (2)
$$

where

$$
\lambda_{\text{even}} = \sum_{k=(l-1)\beta+L+1}^{l\beta} d_l \left(\sum_{m=1}^{\frac{M-2}{2}} x_{k-2mL}^2 + \sum_{m=0}^{\frac{M-2}{2}} x_{k-(2m+1)L}^2 \right) \quad (3)
$$

$$
\mu_{\text{even}} = \sum_{k=(l-1)\beta+L+1}^{l\beta} \left[\left(\sum_{m=0}^{\frac{M-2}{2}} \left(x_{k-2mL} + d_l x_{k-(2m+1)L}\right) \right) \times \left(\sum_{m=0}^{\frac{M-2}{2}} \left(x_{k-(2m+1)L} + d_l x_{k-(2m+2)L}\right) \right) \right] - \lambda_{\text{even}} \quad (4)
$$

$$
\nu_{\text{even}} = \sum_{k=(l-1)\beta+L+1}^{l\beta} \left[\sum_{m=0}^{\frac{M-2}{2}} \left(x_{k-2mL} + d_l x_{k-(2m+1)L}\right) \xi_{k-L} + \sum_{m=0}^{\frac{M-2}{2}} \left(x_{k-(2m+1)L} + d_l x_{k-(2m+2)L}\right) \xi_k + \xi_k \xi_{k-L} \right] \quad (5)
$$

$$
\lambda_{\text{odd}} = \sum_{k=(l-1)\beta+L+1}^{l\beta} d_l \left(\sum_{m=1}^{\frac{M-1}{2}} x_{k-2mL}^2 + \sum_{m=0}^{\frac{M-3}{2}} x_{k-(2m+1)L}^2 \right) \quad (6)
$$

$$
\mu_{\text{odd}} = \sum_{k=(l-1)\beta+L+1}^{l\beta} \left[\left(\sum_{m=0}^{\frac{M-1}{2}} x_{k-2mL} + d_l \sum_{m=0}^{\frac{M-3}{2}} x_{k-(2m+1)L} \right) \times \left(\sum_{m=0}^{\frac{M-1}{2}} x_{k-(2m+1)L} + d_l \sum_{m=0}^{\frac{M-3}{2}} x_{k-(2m+2)L} \right) \right] - \lambda_{\text{odd}} \quad (7)
$$

$$
\nu_{\text{odd}} = \sum_{k=(l-1)\beta+L+1}^{l\beta} \left[\left(\sum_{m=0}^{\frac{M-1}{2}} x_{k-2mL} + d_l \sum_{m=0}^{\frac{M-3}{2}} x_{k-(2m+1)L} \right) \xi_{k-L} + \left(\sum_{m=0}^{\frac{M-1}{2}} x_{k-(2m+1)L} + d_l \sum_{m=0}^{\frac{M-3}{2}} x_{k-(2m+2)L} \right) \xi_k + \xi_k \xi_{k-L} \right] \quad (8)
$$

with $\{\lambda_{\text{even}}, \mu_{\text{even}}, \nu_{\text{even}}\}$ and $\{\lambda_{\text{odd}}, \mu_{\text{odd}}, \nu_{\text{odd}}\}$ denoting the sets of required signal, the intra-signal interference and the noise component when M is even and odd, respectively. The intra-signal interference, similar to the inter-user interference in a multiple access system [4], originates from the correlation between the chaotic samples and may contribute positively or negatively to the required signal. Based on the value of y_l, the symbol is decoded according to the following rule:

$$
\hat{d}_l = \begin{cases} +1 & \text{if } \ y_l > 0 \\ -1 & \text{if } \ y_l \leq 0. \end{cases} \quad (9)
$$

It can be observed that when L is small compared to β, the useful signal component in the GCDSK receiver is approximately $(M-1)$ times larger than that of the CDSK case. Although both the intra-signal interference and the noise component increase compared with the CDSK case, their effect can be compensated by the increase in signal component. Therefore, with appropriate values of M and L, GCDSK can be designed to outperform CDSK.

2.2.1. *Gaussian-Approximated BERs*

In our study, we make use of the Chebyshev map of degree 2 to generate the chaotic signal. The map is given by

$$x_{k+1} = 2x_k^2 - 1 \tag{10}$$

and its correlation properties have been reported previously [5, 6]. Assuming that the chaotic signal $\{x_k\}$ is stationary, it is readily shown that for a given transmitted symbol d_l, the mean value of the correlator output equals

$$\mathrm{E}[y_l \mid d_l] = d_l(\beta - L)(M-1)P_s \tag{11}$$

where E[.] represents the expectation operator and $P_s = \mathrm{E}[x_k^2]$. If the conditional correlator output follows a Gaussian distribution, the approximate bit error rate for the GCDSK system can then be derived analytically and is given by

$$\begin{aligned}\mathrm{BER_{GCDSK}} &= \frac{1}{2}\mathrm{Prob}(y_l \le 0 \mid (d_l = +1)) \\ &\qquad + \frac{1}{2}\mathrm{Prob}(y_l > 0 \mid (d_l = -1)) \\ &= \begin{cases} \frac{1}{4}\mathrm{erfc}\left(\frac{(\beta-L)(M-1)P_s}{\sqrt{2\mathrm{var}[y_l \mid (d_l=+1)]}}\right) & \\ \quad + \frac{1}{4}\mathrm{erfc}\left(\frac{(\beta-L)(M-1)P_s}{\sqrt{2\mathrm{var}[y_l \mid (d_l=-1)]}}\right) & \text{when } L = 1 \\ \frac{1}{2}\mathrm{erfc}\left(\frac{(\beta-L)(M-1)P_s}{\sqrt{2\mathrm{var}[y_l \mid (d_l=+1)]}}\right) & \text{when } 2 \le L < \beta \end{cases}\end{aligned} \tag{12}$$

where erfc(·) represents the complementary error function [5]. Due to the limitation of space, the derivation of the $\mathrm{var}[y_l \mid d_l]$ is not described in this paper.

3. RESULTS AND DISCUSSIONS

In this section, we present our findings on the bit error performance of the GCDSK system. We denote the average bit energy by E_b which can be readily shown equal to

$$E_b = \beta M P_s. \tag{13}$$

For various average-bit-energy-to-noise-psd (E_b/N_0) ratios, we simulate the GCDSK system and record the BERs. Also, we compute the approximate BERs using (12). We then compare our results with those derived from the CDSK and DCSK systems whenever appropriate.

3.1. Effect of delay L

First, we investigate the effect of the delay L on the bit error performance. Figure 2 plots the BER of the CDSK system together with that of the GCDSK system with $M = 4$ and 6. A spreading factor β of 100 is used. It is observed that the bit error performance for the GCDSK system degrades as the delay L increases. As stated in Section 2.2, the correlator in the GDCSK receiver computes the sum of $(\beta - L)$ terms before deciding upon whether the received symbol is a "+1" or "−1". Hence, the correlator output becomes more unreliable when the number of terms reduces due to an increase in L, thereby increasing the BER. For the CDSK system, the number of the terms used in the correlator block is fixed at β and is independent of the delay L. Therefore, the bit error performance of the CDSK system is found to be unaffected by the

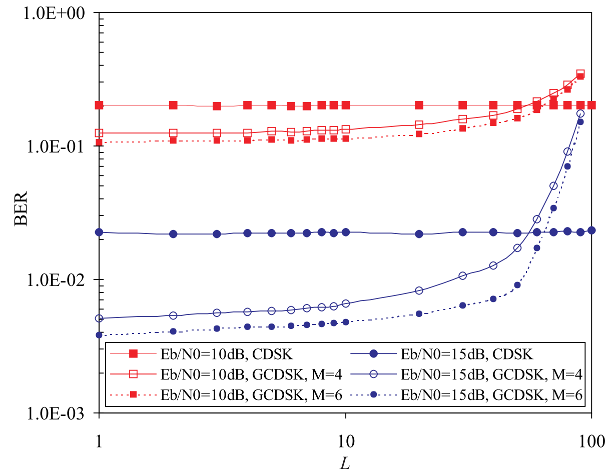

Fig. 2. Simulated BER versus delay L for CDSK and GCDSK systems. $\beta = 100$.

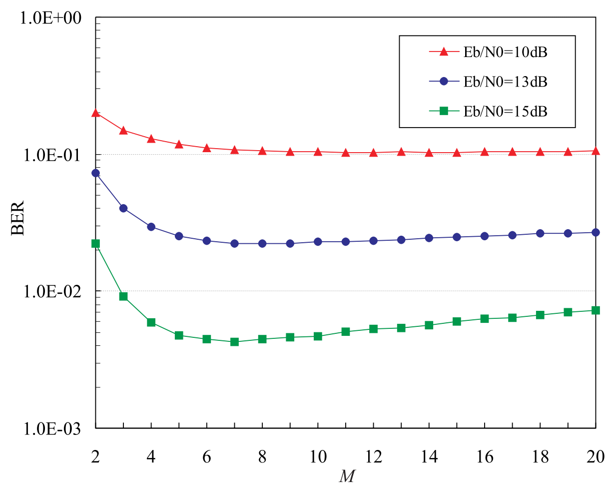

Fig. 3. Simulated BER versus M for the CDSK ($M = 2$) and GCDSK systems. $\beta = 100$ and $L = 5$.

delay L. Comparing the CDSK and GCDSK systems, it can be observed that for the same E_b/N_0, the GCDSK system outperforms the CDSK system with delay values up to 50.

3.2. Effect of the number of delay blocks

Note that the number of delay blocks in the transmitter side equals $(M - 1)$. In Fig. 3, we plot the simulated BER against M. (Note that the CDSK system corresponds to the case where $M = 2$.) A spreading factor β of 100 is used. It is shown that for a fixed E_b/N_0 value, the BER reaches an optimal value at a certain value of M. Specifically, when M increases, the average bit energy and the detected signal component given by (13) and (11), respectively, increase initially. Although the intra-signal interference and the noise power also go up for a given E_b/N_0, there is a net improvement in the signal quality initially, thereby improving the BER. As

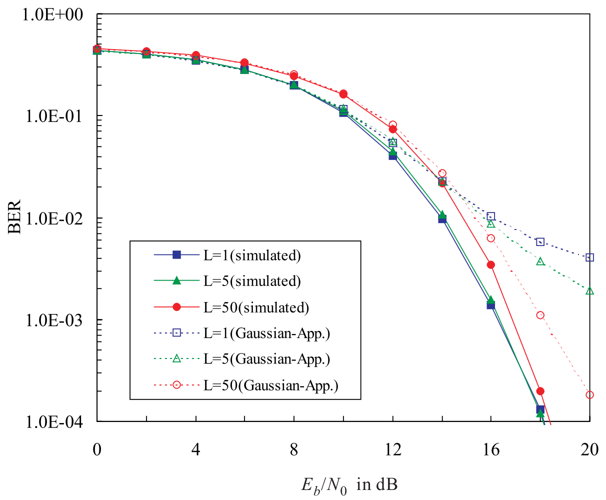

Fig. 4. Simulated and analytical BERs versus E_b/N_0 for the GCDSK system with $M = 6$ and $\beta = 100$.

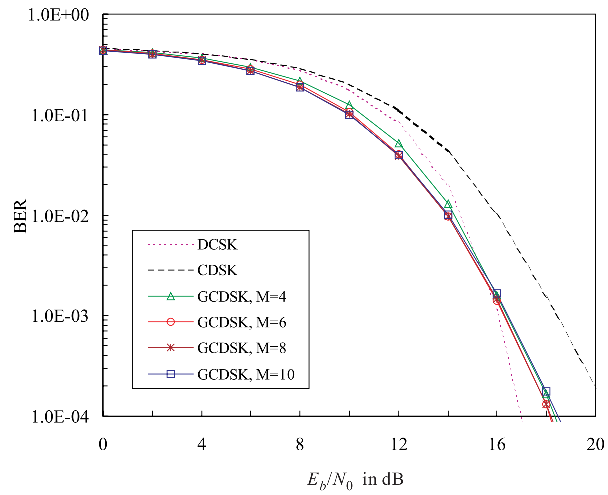

Fig. 5. Simulated BER versus E_b/N_0 for the CDSK, GCDSK and DCSK systems. $\beta = 100$ for all systems, and $L = 1$ for the CDSK and GCDSK systems.

the value of M is further increased beyond the optimal point, the percentage increase in the detected signal component is overshadowed by the degradation due to intra-signal interference and noise. Therefore, the BER starts to degrade for large values of M.

3.3. Comparison of the simulated and Gaussian-approximated BERs

Figure 4 compares the simulated and Gaussian-approximated BERs for the GCDSK system. A spreading factor of 100 is used. For the GCDSK system with $M = 6$, the discrepancy between the simulated and Gaussian-approximated BERs is quite significant when E_b/N_0 is large. When we analyse the statistics of the correlator output from the simulations, it is found that the conditional means and variances match with those derived in this paper. However, the distribution of the conditional correlator output does not follow a Gaussian distribution.

3.4. Comparison with DCSK system

In Fig. 5, the simulated BERs are plotted again for the CDSK, GCDSK and DCSK systems. $\beta = 100$ is used for all systems and a delay of $L = 1$ is employed for the CDSK and GCDSK systems. It is found that the CDSK system gives the worst BER and is approximately 2 dB worse than the DCSK system. For the GCDSK system, the BERs are about the same for $M = 4, 6, 8$ and 10. Its performance is similar to that of the DCSK system and is better for E_b/N_0 values below 16 dB.

4. CONCLUSIONS

In this paper, we develop and study in detail a generalized correlation-delay-shift-keying (GCDSK) scheme for noncoherent chaos-based digital communications. We also compare the bit error rate (BER) of the proposed system with two previously studied noncoherent chaos-based communication schemes, namely, the correlation-delay-shift-keying (CDSK) scheme and the differential chaos-shift-keying (DCSK) scheme. Results show that the CDSK scheme gives the worst BER and is approximately 2 dB worse than the DCSK scheme. For the GCDSK scheme, the BERs are about the same as the DCSK scheme and are better for E_b/N_0 values below 16 dB. Furthermore, we find that a simple Gaussian approximation is not sufficient to model the conditional correlator output at the receiver end. Hence, a more accurate model should be further investigated. Finally, compared with the CDSK and DCSK systems, the GCDSK system has the highest hardware requirement because of the additional delays and multipliers.

5. REFERENCES

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