SPURIOUS MODULATION ON CURRENT-MODE CONTROLLED DC/DC CONVERTERS: AN EXPLANATION FOR INTERMITTENT CHAOTIC OPERATION

S. C. Wong, C. K. Tse and K. C. Tam

Department of Electronic and Information Engineering
Hong Kong Polytechnic University, Hunghom, Hong Kong
URL: http://chaos.eie.polyu.edu.hk Email: {scwong,cktse,enkctam}@eie.polyu.edu.hk

ABSTRACT

In this paper, we explain the mechanism that causes "intermittent" instability and chaos in a current-mode controlled switching converter. The circuit model used to study the phenomenon incorporates a coupling process through which a spurious signal is coupled to the current sensing and ramp compensation circuitry, resulting in a modulation of the compensation slope which causes the system to become unstable intermittently. We describe a way to find the parameter boundaries where intermittent chaotic operation emerges.

Keywords — Switching power converters, current-mode control, intermittent chaos.

1. INTRODUCTION

Power electronics engineers have frequently reported intermittent instability in switching power converters, especially when the converters are not properly protected against instrusion of spurious signals and noise [1]. The intrusion can take the form of coupling via conducted or radiated paths. Sometimes, the intruders (spurious signals) can live on the same circuit board or be present at a close proximity [2]. In this paper we show how the intermittent chaotic operation in a current-mode controlled switching converter can be properly modelled and explained in terms of intrusion of spurious signals. To facilitate design, we identify the critical parameters that affect intermittent chaotic operation and describe a way to calculate the parameter boundaries where intermittent chaotic operation emerges.

2. REVIEW OF CIRCUIT OPERATION

We first review the operation of the current-mode control boost converter [3, 4]. Refering to Fig. 1, when the switch turns on, the inductor current goes up linearly, and is compared with a reference level $i_{\rm ref}$, which is given by

$$i_{\rm ref} = I_{\rm ref} - m_c \bmod(t, T) \tag{1}$$

where T is the switching period and m_c is the slope of the compensation ramp signal. When the inductor current reaches the reference level, the comparator resets the flip-flop, thereby turning off the switch. When the switch is off, the inductor current falls almost linearly. The cycle repeats when the flip-flop is set again by the clock. It should be noted that the inclusion of the compensation ramp is mandatory for maintaining stable operation [5], and stability may be affected when insufficient compensation is applied.

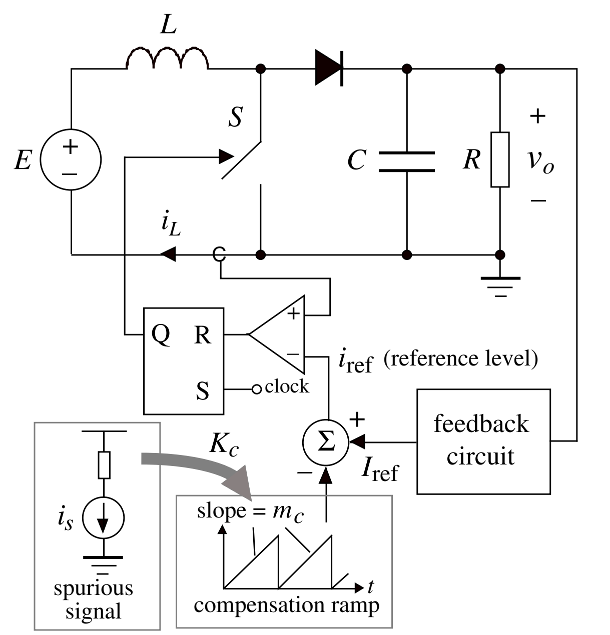


Fig. 1: Boost converter under current-mode control with ramp compensation.

3. INTERMITTENT CHAOTIC OPERATION BY SIMULATION

We consider an additive process which injects the spurious signal directly to the compensation ramp signal of the power supply, as depicted in Fig. 1. Suppose the overall effect of intruding signals is lumped to one spurious source i_s, which

Table 1: Parameter values used in simulations.

Parameter	Value
Inductance L	200 μH
Capacitance C	100 μF
Load resistance R	47 Ω
Switching frequency f_o	25 kHz
Input voltage E	15 V
Reference current $I_{\rm ref}$	8 A
Slope of compensation ramp m_c	75×10^3 A/s
Spurious signal frequency f_s	25.02 kHz

modulates the slope of the compensation ramp in such a way that the actual compensating slope varies in the range $m_c(1 \pm \alpha)$, i.e.,

$$\alpha = \frac{\Delta m_c}{m_c} = \frac{K_c \hat{i}_s}{m_c} \tag{2}$$

with $\hat{i}_s$ being the amplitude of the effective intruding source and K_c being the coupling gain. Note that the overall effect is, at the end, reflected in the magnitude of Δm_c or α. We may therefore study the effect of the intruding source in terms of Δm_c or α.

We assume that the intruding source is sinusoidal of frequency f_s, i.e.,

$$m_c \mapsto m_c(1 + \alpha \sin 2\pi f_s t). \tag{3}$$

It should be noted that the exact form of the intruding source is unimportant as long as the ramp signal is caused to change its slope, thereby destabilizing the inner current loop. Also, f_s is generally different from the switching frequency f_o. In our simulations, we take $f_s = 25020$ Hz and apply varying amplitudes of the spurious signal to the system. Other simulation parameters are shown in Table 1.

In order to reveal the periodicity of the operation and to facilitate investigation of the intermittent behavior, we examine the sampled waveform of the inductor current. Effectively, we sample the inductor current at the switching frequency. When we plot the sampled waveform as a function of time, we can observe how the operation changes from time to time. Such plots are called *time-bifurcation diagrams* [1].

From the simulated time-bifurcation diagrams shown in Fig. 2, the following observations can be made.

1. When the intruding signal strength is very weak (i.e., small α), the converter can still maintain its regular operation, though the steady-state operation point fluctuates. The effect is not significant.

2. As α increases, the converter experiences subharmonic operation intermittently with regular operation. For relatively small α, period-2 operation is observed intermittently with regular operation. Subharmonics of longer periods emerge as α increases further.

3. For a sufficiently large α, chaotic and subharmonic operations are observed between periods of regular operations.

4. The intermittent period is equal to $1/|f_s - f_o|$. Thus, if the intruding signal frequency is very close to the switching frequency of the power converter, the intermittency is long.

4. ANALYSIS

We let $i_{L,n}$ and $i_{L,n+1}$ be the inductor current at $t = nT$ and $t = (n+1)T$, respectively. Also, let the output voltage be v_o. Now, by inspecting the slopes of the inductor current and the compensation ramp (see Fig 3), we get

$$i_{L,n+1} = I_{\rm ref} - f(nT + d_nT) - m_2(1 - d_n)T \tag{4}$$
$$i_{L,n} = I_{\rm ref} - f(nT + d_nT) - m_1 d_n T \tag{5}$$

where d_n is the duty cycle of the nth switching period, $f(t)$ is the displacement of the reference level due to the compensation ramp, and m_1 and m_2 are the rising slope and falling slope, respectively, of the inductor current, i.e.,

$$m_1 = \frac{E}{L} \quad \text{and} \quad m_2 = \frac{v_o - E}{L}. \tag{6}$$

Note that in the steady state, we have $v_o = E/(1-D)$, where D is the operating duty cycle, and hence,

$$\frac{m_2}{m_1} = \frac{D}{1-D}. \tag{7}$$

When analyzing the dynamics at the vicinity of the switching frequency, $I_{\rm ref}$, m_1 and m_2 can be treated as constants. The variations of $i_{L,n}$ and $i_{L,n+1}$ can be calculated as

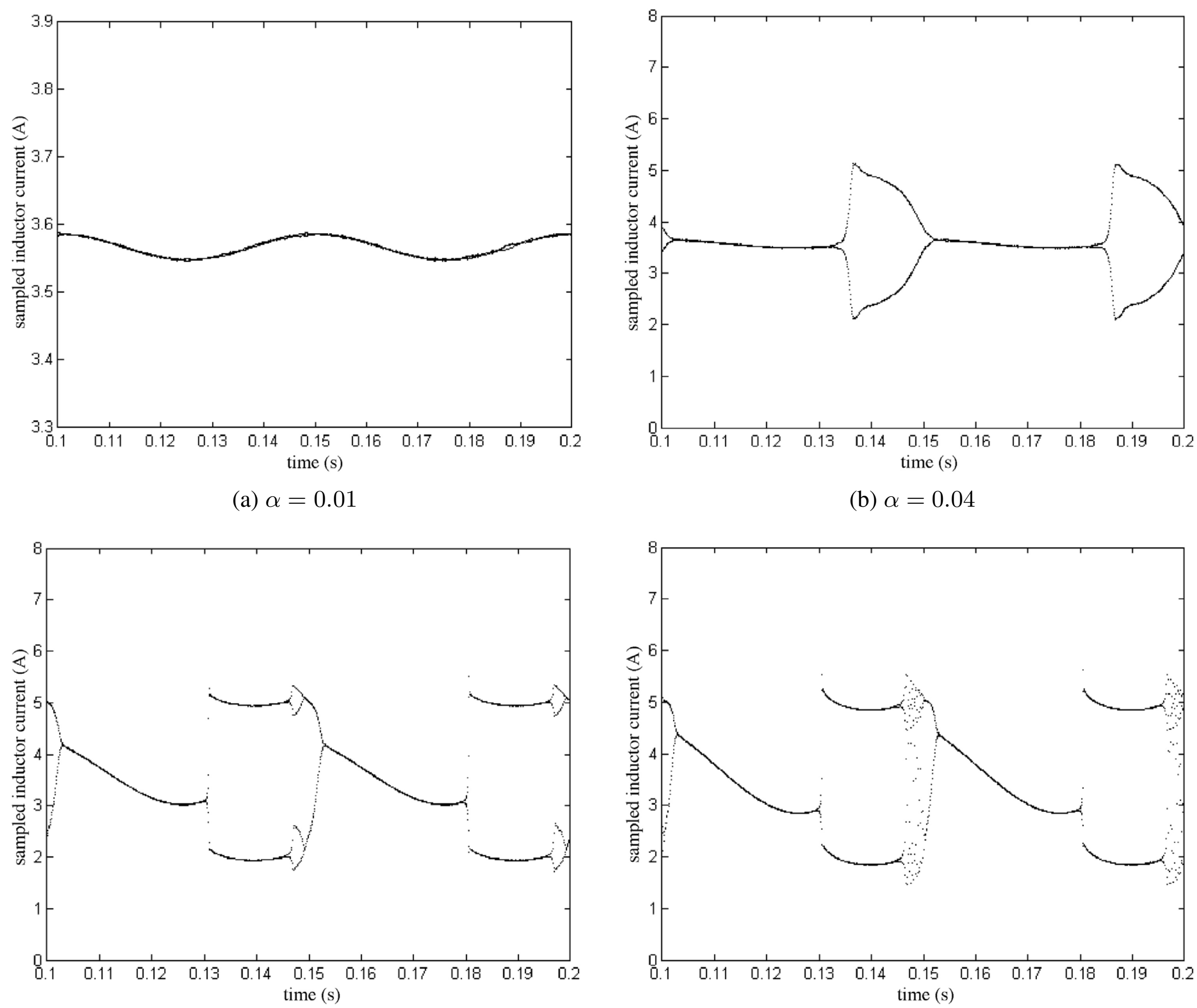
$$\delta i_{L,n+1} = -\frac{\partial f(nT + d_nT)}{\partial d_n}\delta d_n + m_2 T \delta d_n \tag{8}$$
$$\delta i_{L,n} = -\frac{\partial f(nT + d_nT)}{\partial d_n}\delta d_n - m_1 T \delta d_n. \tag{9}$$

Combining the above equations, we get the characteristic multiplier or eigenvalue, λ, as

$$\lambda = \frac{\delta i_{L,n+1}}{\delta i_{L,n}} = \frac{-\dfrac{\partial f(nT + d_nT)}{\partial d_n} + m_2 T}{-\dfrac{\partial f(nT + d_nT)}{\partial d_n} - m_1 T}. \tag{10}$$

For the sinusoidal intruding source under consideration, we have

$$\frac{\partial f(nT + d_nT)}{\partial d_n} = m_c T\left[1 + \alpha \sin(\omega_s d_n T + \theta)\right. \tag{11}$$
$$\left. + \alpha\omega_s d_n T \cos(\omega_s d_n T + \theta)\right] \tag{12}$$



(a) $\alpha = 0.01$ (b) $\alpha = 0.04$ (c) $\alpha = 0.3$ (d) $\alpha = 0.4$

Fig. 2: Sampled inductor current waveforms (time-bifurcation diagrams) for the current-mode controlled boost converter with unintended coupling of sinusoidal source to the compensation ramp signal for different spurious signal strengths. (a) Regular operation with fluctuation of average value; (b)–(c) intermittent subharmonic operation; (d) intermittent chaotic operation.

where $\theta \in [0, 2\pi]$ can be regarded as a random variable for any particular switching period. Thus, we get

$$\lambda = -\frac{m_2 - g(\alpha)m_c}{m_1 + g(\alpha)m_c} = -\frac{\left(\dfrac{D}{1-D}\right) - g(\alpha)M_c}{1 + g(\alpha)M_c} \quad (13)$$

where $M_c = \frac{m_c L}{E}$ and $g(\alpha) = 1 + \alpha \sin(\omega_s DT + \theta) + \alpha\omega_s DT \cos(\omega_s DT + \theta)$, with D being the operating duty cycle. Here, we note that M_c is the normalized slope of the compensation ramp and $g(\alpha)$ acts as an adjustment factor to the compensation slope due to the presence of the spurious signal. Thus, we may define an effective compensation slope M_{eff} as

$$M_{\text{eff}} = g(\alpha)M_c \quad (14)$$

which is less than M_c if $g(\alpha) < 1$. In particular, $g(0) = 1$ corresponds to the case where the spurious signal is absent and $M_{\text{eff}} = M_c$.

Now, since subharmonics and chaos occur when $\lambda \leq -1$, we can find, from (13), the condition for maintaining regular operation as

$$M_{\text{eff}} > -\frac{1-2D}{2(1-D)}. \quad (15)$$

Also, the extreme values of $g(\alpha)$ are given by

$$\sup_{\alpha} g(\alpha) = 1 \pm \alpha\sqrt{1 + \omega_s^2 D^2 T^2}. \quad (16)$$

Here, we consider only the case where $g(\alpha) < 1$ since the effect of the spurious signal being considered is to reduce the effective compensation slope. Thus, the minus sign should be taken for the extreme value of $g(\alpha)$ given in

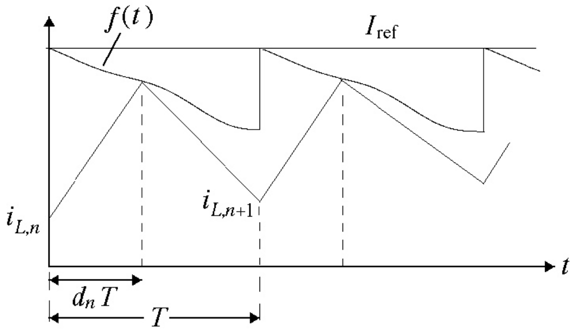


Fig. 3: Inductor current waveform and compensation ramp signal in the presence of intruding source.

(16). Furthermore, we limit ourselves to the practical situation where the amplitude of the spurious signal is relatively small such that the modulation effect does not completely cancel the slope compensation, i.e., $1-\alpha\sqrt{1+\omega_s^2 D^2 T^2} > 0$. Clearly, this condition is equivalent to

$$\alpha < \frac{1}{\sqrt{1+\omega_s^2 D^2 T^2}}, \tag{17}$$

which can also be translated into $D > 0.5$ for all $M_c > 0$. This is consistent with our usual understanding that the use of slope compensation is only needed for $D > 0.5$. We henceforth omit the discussion of the impractical case where (17) is not satisfied.

Now, from (15) and (16), we can find the critical spurious signal strength, $\alpha_{\rm crit}$, at which the first period-doubling occurs and regular operation fails intermittently, i.e.,

$$\alpha_{\rm crit} = \frac{(1+M_c)(1-D)-0.5}{M_c(1-D)\sqrt{1+\omega_s^2 T^2 D^2}}. \tag{18}$$

Thus, from (18), we can compute the boundary of regular operation for any given set of steady-state operating parameters. Note that since (17) has to be satisfied, we restrict the plotting range within $\alpha > 0$ and $0.5 < D < 1$. For ease of visualization, we show in Fig. 4 a few specific boundary curves for some selected values of the compensating slope. We have also plotted the simulation data alongside the curves and found perfect agreement with the analysis.

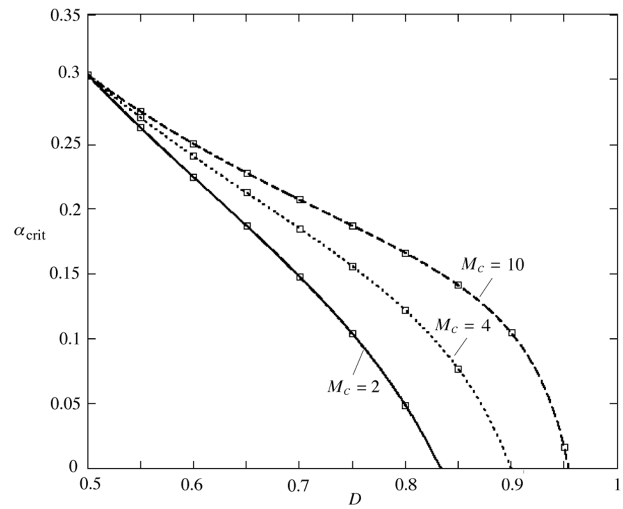


Fig. 4: Boundaries of regular operation. Analytical curves are plotted for $M_c = 2$ (solid curve), $M_c = 4$ (dotted curve) and $M_c = 10$ (dashed curve). Data obtained from cycle-by-cycle simulations are plotted as boxes. Region below the curve corresponds to stable regular operation, and region above the curve corresponds to intermittent subharmonic and/or chaotic operation.

5. CONCLUSION

It should be clear that intermittent operation occurs at a frequency which is simply given by the difference between the spurious signal and an integer multiple of the switching frequency, i.e., $|f_s - nf_o|$ for $n = 0, 1, 2, ...$ Also, intermittent operation can be observed only when the transient is sufficiently fast (or the frequency of intermittency sufficiently low) such that regular, subharmonic or chaotic operation can show up successively in time and be orchestrated as an intermittent operation. In summary, when the spurious signal frequency is sufficiently close to an integer multiple of the switching frequency and the spurious signal is strong enough, intermittent operation occurs. The question of how close the two frequencies should be is therefore dependent upon the transient response of the converter.

Acknowledgments

This work was supported by Hong Kong Research Grants Council under Grant PolyU5241/03E.

6. REFERENCES

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