Coexistence of Chaos-Based and Conventional Digital Communication Systems of Equal Bit Rate

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***Abstract*—Chaos-based communication systems represent a new category of spread-spectrum communication systems, whose working principle differs significantly from conventional direct-sequence and frequency-hopping spread-spectrum systems. However, like all other kinds of spread-spectrum systems, chaos-based systems are required to provide reasonable bit error performance in the presence of a narrow-band signal which can be generated from an intruder or a coexisting conventional communication system. In particular, the frequency band of this foreign narrow-band signal can fall within the bandwidth of the chaos-based system in question. Such a scenario may occur in normal practice when chaos-based systems are introduced while the conventional systems are still in operation. It is therefore important to examine the coexistence of chaos-based and conventional systems. The objective of this paper is to evaluate the performance of the chaos-based system when its bandwidth overlaps with that of a coexisting conventional system. In particular, the chaos-based systems under study are the coherent chaos shift keying (CSK) system and the noncoherent differential CSK (DCSK) system, whereas the conventional system used in the study employs the standard binary phase shift keying scheme. Also, both the chaos-based and conventional systems are assumed to have identical data rates. Analytical expressions for the bit-error rates are derived, permitting evaluation of performance for different noise levels, power ratios and spreading factors. Finally, results from computer simulations verify the analytical findings.**

***Index Terms*—Chaos communications, chaos shift keying (CSK), coexistence, conventional communications, differential CSK (DCSK).**

I. Introduction

MUCH research effort has recently been devoted to the investigation of chaos-based communication systems. In their analog forms, chaos-based communications systems employing techniques like chaotic masking [1], chaotic modulation [2], and many others, have been proposed. Most of these analog schemes, however, do not perform satisfactorily when the transmission channel is subject to the usual additive noise. On the other hand, digital schemes are shown to be more robust in the presence of noise. Among the many chaos-based digital schemes proposed, the chaos shift keying (CSK) and differential CSK (DCSK) schemes are the most widely studied [3]–[5].

Typically, in a digital chaos-based communication system, digital symbols are mapped to nonperiodic chaotic basis functions. Detection schemes can be categorized into coherent and noncoherent types [6], [7]. In coherent detection, such as in chaotic masking and coherent CSK systems [8], the receiver has to reproduce the same chaotic carrier which has been used to carry the information, often through a process known as *chaos synchronization* [9]–[11] which is very difficult to achieve in practice. Thus, until practical chaos synchronization schemes become available, coherent chaos-based systems remain only of theoretical interest. In noncoherent systems, however, the chaotic carrier does not need to be regenerated at the receiver [12]. Usually, noncoherent detection makes use of some distinguishable properties of the transmitted signals, which can be some inherent deterministic properties (e.g., optimal detection [3], return-map based detection [13] and maximum-likelihood method [14]), or fabricated by a suitable bit arrangement (e.g., DCSK [15], [16]), or some statistical properties (e.g., bit energy detection [4]). Since chaos synchronization is not required, noncoherent systems represent, as yet, a more practical form of chaos-based communication. Moreover, we should stress that coherent systems theoretically outperform their noncoherent counterpart, and the correlator-based coherent detection is the optimal form of detection in terms of bit error performance. Therefore, the study of coherent systems will provide performance indicators which are important for future development of the field.

The basic problem considered in this paper is the coexistence of chaos-based systems and conventional systems. Specifically, we are interested in finding the performance of a chaos-based system and the extent to which it is affected by the presence of a conventional narrow-band system whose bandwidth falls within that of the chaos-based system in question. This scenario has practical significance, as can be easily appreciated when one considers the introduction of chaos-based communication systems while conventional systems are still in operation. When it happens, chaos-based systems and conventional systems are actually interfering with one another. The ability of a chaos-based communication system to coexist with a conventional communication system is therefore an important issue that should be thoroughly investigated. The main questions are whether the interference can be tolerated and under what conditions both kinds of systems will operate with satisfactory performances. To answer these questions, we first present an analytical method for evaluating the performances of the chaos-based communication system and the conventional system when their corresponding bandwidths overlap substantially. Then, based on the analytical bit-error rates (BERs), we evaluate the coexistence performance for a range of noise levels, power ratios and spreading factors. In this paper, we choose the coherent

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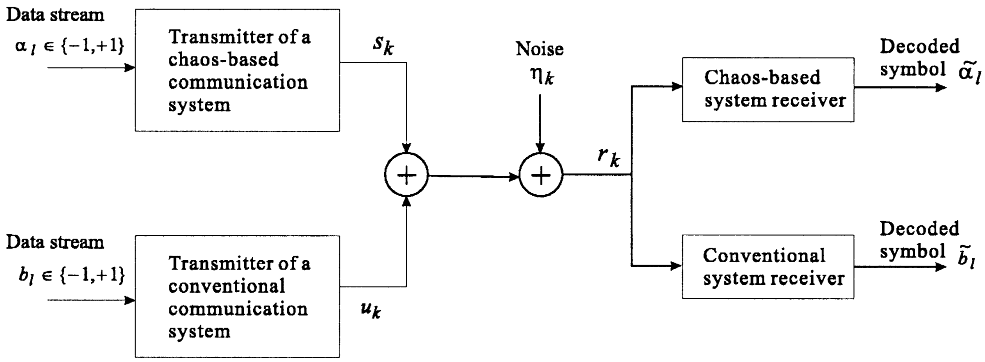

Fig. 1. Block diagram of a combined chaos-based conventional digital communication system.

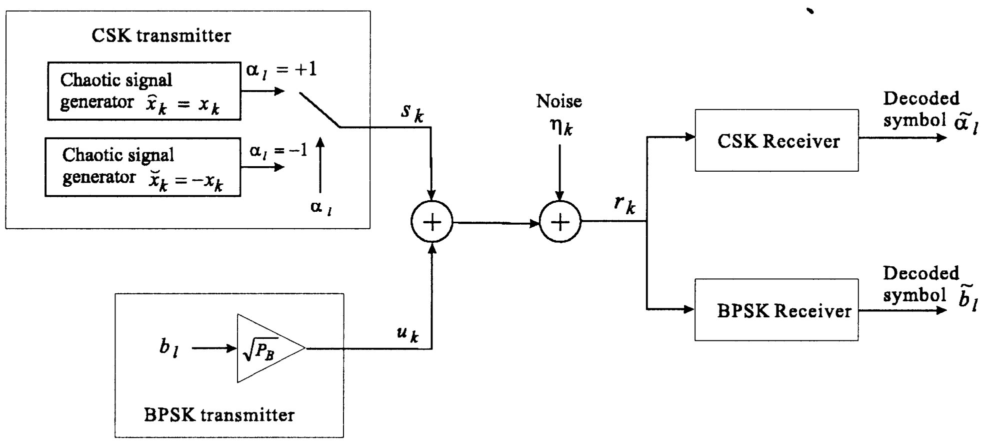

Fig. 2. Block diagram of a combined CSK–BPSK communication system.

binary-phase-shift-keying (BPSK) system as the conventional system, and the coherent CSK and noncoherent DCSK systems as the chaos-based communication systems. Finally, we verify our findings with computer simulations.

II. System Overview

We consider a chaos-based communication system and a conventional system whose bandwidths overlap significantly. We refer to the whole system as *combined chaos-based conventional system*, which can be represented by the block diagram shown in Fig. 1. In this system, two independent data streams are assumed to be sent at the same data rate. Our analysis will proceed in a discrete-time fashion. At time k, denote the output of the chaos transmitter by s_k and that of the conventional transmitter by u_k. These two signals are then added, as well as corrupted by noise η_k in the channel, before they arrive at the receiving end. At the receiver, based on the incoming signal r_k, the receivers of the chaos-based system and the conventional system will attempt to recover their respective data streams. Coherent or noncoherent detection schemes may be applied in the receivers, depending upon the modulation methods used in the transmitter.

Clearly, the signals from the chaotic and the conventional systems will be interfering with each other and thus the performance of each system will be degraded. Specifically, we will consider a "combined CSK–BPSK" system and a "combined DCSK–BPSK" system, and will attempt to develop analytical expressions for the BERs of the recovered data streams.

III. Performance Analysis of Combined CSK–BPSK Communication System

We first consider a discrete-time baseband equivalent model of a combined CSK–BPSK communication system, as shown in Fig. 2. We assume that the CSK system and the BPSK system have identical bit rate and that their bit streams are synchronized. Also, the carrier frequencies of the two systems are identical and synchronized. Further, "-1" and "$+1$" occur with equal probabilities in the bit streams of both systems.

Generally, in the CSK transmitter, a pair of chaotic sequences, denoted by $\{\hat{x}_k\}$ and $\{\check{x}_k\}$, are generated by two chaotic maps. If the symbol "$+1$" is sent, $\{\hat{x}_k\}$ is transmitted during a bit period, and if "-1" is sent, $\{\check{x}_k\}$ is transmitted. For simplicity, we consider here a CSK system in which one chaos generator is used to produce chaotic signal samples $\{x_k\}$ for $k = 1, 2, \ldots$. The two possible transmitted sequences are $\{\hat{x}_k = x_k\}$ and $\{\check{x}_k = -x_k\}$. Suppose $\alpha_l \in \{-1, +1\}$ is the symbol to be sent during the lth bit period. Define the spreading factor, 2β, as the number of chaotic samples used to transmit one binary symbol. During the lth bit duration, i.e., for $k = 2\beta(l-1)+1, 2\beta(l-1)+2, \ldots, 2\beta l$, the output of the CSK transmitter is

$$s_k = \alpha_l x_k. \tag{1}$$

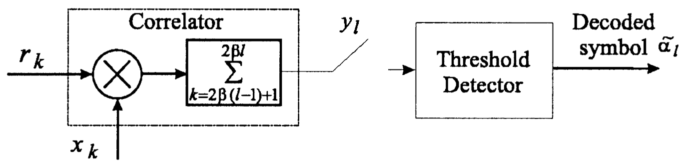

Fig. 3. Block diagram of a coherent CSK receiver.

In the BPSK system, we denote the lth transmitted symbol by $b_l \in \{-1, +1\}$. Moreover, the signal power is P_B. Thus, during the lth bit duration, i.e., for $k = 2\beta(l-1)+1,\ 2\beta(l-1)+2,\ \ldots,\ 2\beta l$, the transmitted signal is constant and is represented by

$$u_k = \sqrt{P_B} b_l. \tag{2}$$

The CSK and BPSK signals are combined and corrupted by an additive white Gaussian noise in the channel, before arriving at the receiving end. Thus, the received signal, denoted by r_k, is given by

$$r_k = s_k + u_k + \eta_k \tag{3}$$

where η_k is a Gaussian noise sample of zero mean and variance (power spectral density) $N_0/2$. For each of the CSK and BPSK receivers, we will consider the lth bit and derive the error probability over all transmitted bits.

A. Performance of the CSK System in Combined CSK–BPSK System

Assume that a correlator-type receiver is employed. Referring to Fig. 3, the correlator output for the lth bit, y_l, is given by

$$\begin{aligned} y_l &= \sum_{k=2\beta(l-1)+1}^{2\beta l} r_k x_k \\ &= \underbrace{\alpha_l \sum_{k=2\beta(l-1)+1}^{2\beta l} x_k^2}_{\text{required signal}} + \underbrace{\sqrt{P_B} b_l \sum_{k=2\beta(l-1)+1}^{2\beta l} x_k}_{\text{interfering BPSK signal}} \\ &\quad + \underbrace{\sum_{k=2\beta(l-1)+1}^{2\beta l} \eta_k x_k}_{\text{noise}}. \end{aligned} \tag{4}$$

Suppose a "+1" is transmitted in both CSK and BPSK systems during the lth symbol duration, i.e., $\alpha_l = +1$ and $b_l = +1$. For simplicity we write $y_l|(\alpha_l = +1, b_l = +1)$ as

$$y_l|(\alpha_l = +1, b_l = +1) = A + B + C \tag{5}$$

where A, B, and C are the required signal, interfering BPSK signal and noise, respectively, and are defined as

$$A = \sum_{k=2\beta(l-1)+1}^{2\beta l} x_k^2 \tag{6}$$

$$B = \sqrt{P_B} \sum_{k=2\beta(l-1)+1}^{2\beta l} x_k \tag{7}$$

$$C = \sum_{k=2\beta(l-1)+1}^{2\beta l} \eta_k x_k. \tag{8}$$

The mean of $y_l|(\alpha_l = +1, b_l = +1)$ is

$$\begin{aligned} &E\left[y_l|(\alpha_l = +1, b_l = +1)\right] \\ &\quad = E[A] + E[B] + E[C] \\ &\quad = E\left[\sum_{k=2\beta(l-1)+1}^{2\beta l} x_k^2\right] + E\left[\sqrt{P_B} \sum_{k=2\beta(l-1)+1}^{2\beta l} x_k\right] \\ &\qquad + E\left[\sum_{k=2\beta(l-1)+1}^{2\beta l} \eta_k x_k\right] \\ &\quad = \sum_{k=2\beta(l-1)+1}^{2\beta l} E\left[x_k^2\right] + \sqrt{P_B} \sum_{k=2\beta(l-1)+1}^{2\beta l} E[x_k] \\ &\qquad + \sum_{k=2\beta(l-1)+1}^{2\beta l} E[\eta_k] E[x_k] \\ &\quad = 2\beta P_s + 2\beta \sqrt{P_B} E[x_k] \end{aligned} \tag{9}$$

where $P_s = E[x_k^2]$ denotes the average power of the chaotic signal. The last equality holds because $E[\eta_k] = 0$. The variance of $y_l|(\alpha_l = +1, b_l = +1)$ is

$$\begin{aligned} \text{var}\left[y_l|(\alpha_l = +1, b_l = +1)\right] &= \text{var}[A] + \text{var}[B] + \text{var}[C] \\ &\quad + 2\text{cov}[A, B] + 2\text{cov}[B, C] + 2\text{cov}[A, C] \end{aligned} \tag{10}$$

where $\text{cov}[X, Y]$ is the covariance between X and Y defined as

$$\text{cov}[X, Y] = E[XY] - E[X]E[Y]. \tag{11}$$

It can be proved that both $\text{cov}[A, C]$ and $\text{cov}[B, C]$ are zero (see Appendix A). Hence, (10) can be simplified to

$$\begin{aligned} &\text{var}\left[y_l|(\alpha_l = +1, b_l = +1)\right] \\ &\quad = \text{var}[A] + \text{var}[B] + \text{var}[C] + 2\text{cov}[A, B]. \end{aligned} \tag{12}$$

The mean value and the average power of the chaotic signal can be computed by numerical simulation. If the invariant probability density function of $\{x_k\}$ is available, in most cases the mean and the average power can be obtained not only by numerical integration, but also in analytical forms. The variance and covariance terms in (12) can also be computed using aforementioned techniques. Hence, $E[y_l|(\alpha_l = +1, b_l = +1)]$ and $\text{var}[y_l|(\alpha_l = +1, b_l = +1)]$ can be evaluated.

For the lth symbol, an error occurs when $y_l \le 0|(\alpha_l = +1, b_l = +1)$. Since $y_l|(\alpha_l = +1, b_l = +1)$ is the sum of a large number of random variables, we may assume that it follows a normal distribution. The error probability is thus given by

$$\begin{aligned} &\text{Prob}\left(y_l \le 0|(\alpha_l = +1, b_l = +1)\right) \\ &\quad = \frac{1}{2}\text{erfc}\left(\frac{E\left[y_l|(\alpha_l = +1, b_l = +1)\right]}{\sqrt{2\text{var}\left[y_l|(\alpha_l = +1, b_l = +1)\right]}}\right) \end{aligned} \tag{13}$$

where erfc(.) is the complementary error function defined as

$$\text{erfc}(\psi) \equiv \frac{2}{\sqrt{\pi}} \int_{\psi}^{\infty} e^{-\lambda^2} d\lambda. \tag{14}$$

Similarly, when $\alpha_l = +1$ and $b_l = -1$, the output of the correlator can be shown equal to

$$y_l|(\alpha_l = +1, b_l = -1) = A - B + C. \quad (15)$$

Likewise, the mean and variance of $y_l|(\alpha_l = +1, b_l = -1)$, denoted by $E[y_l|(\alpha_l = +1, b_l = -1)]$ and $\text{var}[y_l|(\alpha_l = +1, b_l = -1)]$, can be found as

$$E[y_l|(\alpha_l = +1, b_l = -1)] = 2\beta P_s - 2\beta\sqrt{P_B}E[x_k] \quad (16)$$

$$\text{var}[y_l|(\alpha_l = +1, b_l = -1)] = \text{var}[A] + \text{var}[B] + \text{var}[C] - 2\text{cov}[A, B] \quad (17)$$

where A, B, and C are defined in (6)–(8). The corresponding error probability is

$$\text{Prob}(y_l \leq 0|(\alpha_l = +1, b_l = -1)) = \frac{1}{2}\text{erfc}\left(\frac{E[y_l|(\alpha_l = +1, b_l = -1)]}{\sqrt{2\text{var}[y_l|(\alpha_l = +1, b_l = -1)]}}\right). \quad (18)$$

Hence, for the CSK system, given a "+1" is sent during the lth bit duration, the error probability is given by

$$\begin{aligned}\text{BER}^{(l)}_{\text{CSK-I}} &= \text{Prob}(b_l = +1) \times \text{Prob}(y_l \leq 0|(\alpha_l = +1, b_l = +1)) \\ &\quad + \text{Prob}(b_l = -1) \times \text{Prob}(y_l \leq 0|(\alpha_l = +1, b_l = -1)) \\ &= \frac{1}{4}\left[\text{erfc}\left(\frac{E[y_l|(\alpha_l = +1, b_l = +1)]}{\sqrt{2\text{var}[y_l|(\alpha_l = +1, b_l = +1)]}}\right)\right. \\ &\quad \left. + \text{erfc}\left(\frac{E[y_l|(\alpha_l = +1, b_l = -1)]}{\sqrt{2\text{var}[y_l|(\alpha_l = +1, b_l = -1)]}}\right)\right]. \end{aligned} \quad (19)$$

Also, given "−1" is sent during the lth symbol duration in the CSK system, i.e., $\alpha_l = -1$, it can be shown that

$$E[y_l|(\alpha_l = -1, b_l = +1)] = -2\beta P_s + 2\beta\sqrt{P_B}E[x_k] \quad (20)$$

$$\text{var}[y_l|(\alpha_l = -1, b_l = +1)] = \text{var}[A] + \text{var}[B] + \text{var}[C] - 2\text{cov}[A, B] \quad (21)$$

$$E[y_l|(\alpha_l = -1, b_l = -1)] = -2\beta P_s - 2\beta\sqrt{P_B}E[x_k] \quad (22)$$

$$\text{var}[y_l|(\alpha_l = -1, b_l = -1)] = \text{var}[A] + \text{var}[B] + \text{var}[C] + 2\text{cov}[A, B] \quad (23)$$

where A, B, and C are again defined in (6)–(8). The error probability, given a "−1" is sent, is then equal to

$$\begin{aligned}\text{BER}^{(l)}_{\text{CSK-II}} &= \text{Prob}(b_l = +1) \times \text{Prob}(y_l > 0|(\alpha_l = -1, b_l = +1)) \\ &\quad + \text{Prob}(b_l = -1) \times \text{Prob}(y_l > 0|(\alpha_l = -1, b_l = -1)) \\ &= \frac{1}{4}\left[\text{erfc}\left(\frac{-E[y_l|(\alpha_l = -1, b_l = +1)]}{\sqrt{2\text{var}[y_l|(\alpha_l = -1, b_l = +1)]}}\right)\right. \\ &\quad \left. + \text{erfc}\left(\frac{-E[y_l|(\alpha_l = -1, b_l = -1)]}{\sqrt{2\text{var}[y_l|(\alpha_l = -1, b_l = -1)]}}\right)\right]. \end{aligned} \quad (24)$$

Hence, the overall error probability of the lth transmitted symbol is

$$\begin{aligned}\text{BER}^{(l)}_{\text{CSK}} &= \text{Prob}(\alpha_l = +1) \times \text{BER}^{(l)}_{\text{CSK-I}} \\ &\quad + \text{Prob}(\alpha_l = -1) \times \text{BER}^{(l)}_{\text{CSK-II}} \\ &= \frac{1}{2}\left[\text{BER}^{(l)}_{\text{CSK-I}} + \text{BER}^{(l)}_{\text{CSK-II}}\right]. \end{aligned} \quad (25)$$

It can be seen from (19), (24), and (25) that $\text{BER}^{(l)}_{\text{CSK}}$ is independent of l. Thus, the error probability of the lth transmitted symbol is the same as the BER of the system. In the combined CSK–BPSK system, the BER of the symbols carried by the CSK signal, denoted by BER_{CSK}, is therefore

$$\text{BER}_{\text{CSK}} = \text{BER}^{(l)}_{\text{CSK}} = \frac{1}{2}\left[\text{BER}^{(l)}_{\text{CSK-I}} + \text{BER}^{(l)}_{\text{CSK-II}}\right]. \quad (26)$$

Hence, (19) and (24) can be computed and substituted into (26) to obtain the BER of the system.

At this point, we make a few assumptions in order to further simplify the analysis. These assumptions can be easily justified for the chaotic sequences generated by the logistic map and by all Chebyshev maps of degree larger than one.

1) The mean value of $\{x_k\}$ is zero. The justification for this assumption is that no power should be wasted in sending noninformation-bearing dc component through the channel. The condition also optimizes the performance of the joint CSK/BPSK scheme because it ensures that the chaotic sequences being restricted to the plane orthogonal to the basis vector [1, 1, 1, 1, 1, 1, ...] in use for BPSK. In practice, any dc component generated by the chaos generator can be removed artificially before transmission.
2) The covariance of x_j and x_k vanishes for $j \neq k$.
3) The covariance of x_j^2 and x_k^2 vanishes for $j \neq k$.
4) The correlation of x_j and x_k^2 vanishes for $j - k \neq 1$.

The above assumptions can be translated to

$$E[x_k] = 0 \quad (27)$$

$$\text{cov}[x_j, x_k] = E[x_j x_k] - E[x_j]E[x_k] = 0, \quad \text{for } j \neq k \quad (28)$$

$$\text{cov}\left[x_j^2, x_k^2\right] = E\left[x_j^2 x_k^2\right] - E\left[x_j^2\right]E\left[x_k^2\right] = 0, \quad \text{for } j \neq k \quad (29)$$

$$E\left[x_j x_k^2\right] = 0, \quad \text{for } j - k \neq 1. \quad (30)$$

Thus, (9), (16), (20), and (22) become

$$E[y_l|(\alpha_l = +1, b_l = +1)] = E[y_l|(\alpha_l = +1, b_l = -1)] = 2\beta P_s \quad (31)$$

$$E[y_l|(\alpha_l = -1, b_l = +1)] = E[y_l|(\alpha_l = -1, b_l = -1)] = -2\beta P_s \quad (32)$$

and the variances of the variables A, B, and C, and the covariance between A and B are given by (see Appendix A)

$$\text{var}[A] = 2\beta\Lambda \quad (33)$$

$$\text{var}[B] = 2\beta P_B P_s \quad (34)$$

$$\text{var}[C] = \beta N_0 P_s \quad (35)$$

$$\text{cov}[A, B] = \sqrt{P_B}(2\beta - 1)E\left[x_{k+1}x_k^2\right] \approx 2\beta\sqrt{P_B}\Omega, \quad \text{for large } 2\beta \quad (36)$$

where

$$\Lambda = \mathrm{var}\left[x_k^2\right] \tag{37}$$
$$\Omega = E\left[x_{k+1}x_k^2\right]. \tag{38}$$

Note that Λ denotes the variance of x_k^2 and is different from the average power of the chaotic signal, P_s. Hence, (12), (17), (21), and (23) can be put as

$$\begin{aligned}\mathrm{var}\left[y_l|(\alpha_l=+1,b_l=+1)\right] &= \mathrm{var}\left[y_l|(\alpha_l=-1,b_l=-1)\right]\\ &= 2\beta\Lambda+2\beta P_B P_s+\beta N_0 P_s+2(2\beta-1)\sqrt{P_B}\Omega\\ &\approx \beta(2\Lambda+2P_BP_s+N_0P_s+4\sqrt{P_B}\Omega)\end{aligned} \tag{39}$$

$$\begin{aligned}\mathrm{var}\left[y_l|(\alpha_l=+1,b_l=-1)\right] &= \mathrm{var}\left[y_l|(\alpha_l=-1,b_l=+1)\right]\\ &= 2\beta\Lambda+2\beta P_B P_s+\beta N_0 P_s-2(2\beta-1)\sqrt{P_B}\Omega\\ &\approx \beta(2\Lambda+2P_BP_s+N_0P_s-4\sqrt{P_B}\Omega).\end{aligned} \tag{40}$$

Substituting (31) and (32) and (39) and (40) into (19), (24), and (26), the BER can be found as shown in (41) and (42) at the bottom of page, where $E_b = 2\beta P_s$ denotes the average bit energy of the CSK system. The expression given in (41) or (42) is thus the analytical BER for the noisy coherent CSK system in a combined communication environment. Note that for fixed BPSK signal power P_B and noise power spectral density $N_0/2$, the BER can be improved by making one or more of the following adjustments.

1) Reduce the variance of x_k^2.
2) Reduce the absolute value of $\Omega(E[x_{k+1}x_k^2])$.
3) Increase the spreading factor 2β.
4) Increase the CSK signal power P_s.

In particular, when the BPSK signal power is zero, i.e., $P_B = 0$, it can be readily shown that the BER reduces to [17]

$$\mathrm{BER_{CSK}}|(P_B=0) = \frac{1}{2}\mathrm{erfc}\left(\frac{1}{\sqrt{\frac{\Lambda}{\beta P_s^2}+\frac{N_0}{2\beta P_s}}}\right) \tag{43}$$

$$= \frac{1}{2}\mathrm{erfc}\left(\frac{1}{\sqrt{\left(\frac{E_b^2}{4\beta\Lambda}\right)^{-1}+\left(\frac{E_b}{N_0}\right)^{-1}}}\right). \tag{44}$$

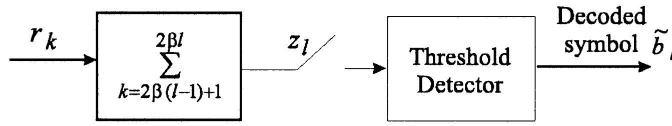

Fig. 4. Block diagram of a BPSK receiver.

B. Performance of the BPSK System in Combined CSK–BPSK System

In the BPSK receiver shown in Fig. 4, the incoming signal samples within a symbol period are summed to give z_l, i.e.,

$$\begin{aligned}z_l &= \sum_{k=2\beta(l-1)+1}^{2\beta l} r_k\\ &= \underbrace{\alpha_l\sum_{k=2\beta(l-1)+1}^{2\beta l} x_k}_{\text{interfering CSK signal}} + \underbrace{2\beta\sqrt{P_B}b_l}_{\text{required signal}}\\ &\quad+\underbrace{\sum_{k=2\beta(l-1)+1}^{2\beta l}\eta_k}_{\text{noise}}.\end{aligned} \tag{45}$$

Using similar procedures as in the Section III-A, it can be shown that the mean and variance of z_l, denoted by $E[z_l]$ and $\mathrm{var}[z_l]$, respectively, are given by

$$E[z_l] = 2\beta\alpha_l E[x_k] + 2\beta\sqrt{P_B}b_l \tag{46}$$
$$\mathrm{var}[z_l] = \mathrm{var}\left[\sum_{k=2\beta(l-1)+1}^{2\beta l} x_k\right] + \beta N_0. \tag{47}$$

Assuming that: i) the mean value of $\{x_k\}$ is zero, and ii) the covariance of x_j and x_k vanishes for $j \neq k$, putting (27) and (28) in (46) and (47) gives

$$E[z_l] = 2\beta\sqrt{P_B}b_l \tag{48}$$
$$\mathrm{var}[z_l] = 2\beta P_s + \beta N_0. \tag{49}$$

Suppose $b_l = +1$. As $z_l|(b_l=+1)$ is the sum of a large number of random variables, we may assume that it follows a normal

$$\begin{aligned}\mathrm{BER_{CSK}} &\approx \frac{1}{4}\mathrm{erfc}\left(\frac{2\beta P_s}{\sqrt{2\beta(2\Lambda+2P_BP_s+N_0P_s+4\sqrt{P_B}\Omega)}}\right)+\frac{1}{4}\mathrm{erfc}\left(\frac{2\beta P_s}{\sqrt{2\beta(2\Lambda+2P_BP_s+N_0P_s-4\sqrt{P_B}\Omega)}}\right)\\ &= \frac{1}{4}\mathrm{erfc}\left(\frac{1}{\sqrt{\frac{\Lambda}{\beta P_s^2}+\frac{P_B}{\beta P_s}+\frac{N_0}{2\beta P_s}+\frac{2\sqrt{P_B}\Omega}{\beta P_s^2}}}\right)+\frac{1}{4}\mathrm{erfc}\left(\frac{1}{\sqrt{\frac{\Lambda}{\beta P_s^2}+\frac{P_B}{\beta P_s}+\frac{N_0}{2\beta P_s}-\frac{2\sqrt{P_B}\Omega}{\beta P_s^2}}}\right)\end{aligned} \tag{41}$$

$$\begin{aligned}&= \frac{1}{4}\mathrm{erfc}\left(\frac{1}{\sqrt{\left(\frac{E_b^2}{4\beta\Lambda}\right)^{-1}+\left(\frac{E_b}{2P_B}\right)^{-1}+\left(\frac{E_b}{N_0}\right)^{-1}+\left(\frac{E_b^2}{8\beta\sqrt{P_B}\Omega}\right)^{-1}}}\right)\\ &\quad+\frac{1}{4}\mathrm{erfc}\left(\frac{1}{\sqrt{\left(\frac{E_b^2}{4\beta\Lambda}\right)^{-1}+\left(\frac{E_b}{2P_B}\right)^{-1}+\left(\frac{E_b}{N_0}\right)^{-1}-\left(\frac{E_b^2}{8\beta\sqrt{P_B}\Omega}\right)^{-1}}}\right)\end{aligned} \tag{42}$$

distribution. An error occurs when $z_l \le 0|(b_l = +1)$, and the corresponding error probability is given by

$$\text{Prob}\left(z_l \le 0|(b_l = +1)\right) = \frac{1}{2}\text{erfc}\left(\frac{E\left[z_l|(b_l = +1)\right]}{\sqrt{2\text{var}\left[z_l|(b_l = +1)\right]}}\right). \tag{50}$$

Likewise, given $b_l = -1$, the error probability is

$$\text{Prob}\left(z_l > 0|(b_l = -1)\right) = \frac{1}{2}\text{erfc}\left(\frac{-E\left[z_l|(b_l = -1)\right]}{\sqrt{2\text{var}\left[z_l|(b_l = -1)\right]}}\right). \tag{51}$$

Putting (48) and (49) to (50) and (51), the error probability for the lth transmitted BPSK symbol can be found as

$$\begin{aligned}\text{BER}^{(l)}_{\text{BPSK-CSK}} &= \text{Prob}(b_l = +1) \times \text{Prob}\left(z_l \le 0|(b_l = +1)\right) \\ &\quad + \text{Prob}(b_l = -1) \times \text{Prob}(z_l > 0|(b_l = -1)) \\ &= \frac{1}{4}\text{erfc}\left(\frac{E\left[z_l|(b_l = +1)\right]}{\sqrt{2\text{var}\left[z_l|(b_l = +1)\right]}}\right) \\ &\quad + \frac{1}{4}\text{erfc}\left(\frac{-E\left[z_l|(b_l = -1)\right]}{\sqrt{2\text{var}\left[z_l|(b_l = -1)\right]}}\right) \\ &= \frac{1}{2}\text{erfc}\left(\frac{2\beta\sqrt{P_B}}{\sqrt{4\beta P_s + 2\beta N_0}}\right). \end{aligned} \tag{52}$$

Since $\text{BER}^{(l)}_{\text{BPSK-CSK}}$ is independent of l, the error probability of the lth transmitted symbol is the same as the BER of the system. Therefore, the BER of the BPSK system, denoted by $\text{BER}_{\text{BPSK-CSK}}$, is

$$\begin{aligned}\text{BER}_{\text{BPSK-CSK}} &= \text{BER}^{(l)}_{\text{BPSK-CSK}} \\ &= \frac{1}{2}\text{erfc}\left(\frac{2\beta\sqrt{P_B}}{\sqrt{4\beta P_s + 2\beta N_0}}\right) \\ &= \frac{1}{2}\text{erfc}\left(\frac{1}{\sqrt{\frac{P_s}{\beta P_B} + \frac{N_0}{2\beta P_B}}}\right) \qquad (53) \\ &= \frac{1}{2}\text{erfc}\left(\sqrt{\frac{E_B}{\tilde{N}_0}}\right) \qquad (54)\end{aligned}$$

where

$$E_B = 2\beta P_B \tag{55}$$

denotes the bit energy of the BPSK signal and

$$\frac{\tilde{N}_0}{2} = \frac{N_0}{2} + P_s \tag{56}$$

represents the equivalent noise power spectral density when the interfering CSK signal is taken into consideration. Thus, the interfering CSK signal simply raises the noise level of the BPSK signal. The expression given in (53) or (54) is the analytical BER for the noisy coherent BPSK system in a combined communication environment. For a fixed chaotic signal power P_s, the BER can be improved by increasing the spreading factor 2β and/or increasing the BPSK signal power P_B.

C. Example

Consider the case where a logistic map is used for chaos generation. The form of the map is

$$x_{k+1} = g(x_k) = 1 - 2x_k^2. \tag{57}$$

The invariant probability density function of x_k, denoted by $\rho(x)$, is [18]

$$\rho(x) = \begin{cases} \frac{1}{\pi\sqrt{1-x^2}}, & \text{if } |x| < 1 \\ 0, & \text{otherwise.} \end{cases} \tag{58}$$

Since $\rho(x)$ is an even function, the mean value of x_k is

$$E[x_k] = \int_{-\infty}^{\infty} x\rho(x)dx = \int_{-1}^{1} x\rho(x)dx = 0. \tag{59}$$

Define

$$\begin{aligned} g^{(1)}(x) &= g(x) \\ g^{(2)}(x) &= g\left(g^{(1)}(x)\right) \\ \vdots &= \vdots \\ g^{(n)}(x) &= g\left(g^{(n-1)}(x)\right). \end{aligned}$$

Since $g(x)$ is an even function, $g^{(n)}(x)$ is also even. Further, since $xg^{(n)}(x)\rho(x)$ is the product of one odd function and two even functions, it is also an odd function, and we have, for $n \ne 0$

$$E[x_k x_{k+n}] = \int_{-1}^{1} xg^{(n)}(x)\rho(x)dx = 0. \tag{60}$$

Thus, from (59) and (60), we clearly see that the assumptions corresponding to (27) and (28) made earlier in Section III-A and the two assumptions made in Section III-B are all well justified. In Appendix B, it is also shown that (29) and (30) are valid for the chaotic sequence generated by the logistic map.[1] Moreover, we have

$$P_s = E\left[x_k^2\right] = \int_{-\infty}^{\infty} x^2\rho(x)dx = \int_{-1}^{1} x^2\rho(x)dx = \frac{1}{2} \tag{61}$$

$$\begin{aligned}\Lambda &= \text{var}\left[x_k^2\right] = E\left[x_k^4\right] - E^2\left[x_k^2\right] \\ &= \int_{-1}^{1} x^4\rho(x)dx - \frac{1}{4} = \frac{1}{8}\end{aligned} \tag{62}$$

$$\Omega = E\left[x_{k+1}x_k^2\right] = -\frac{1}{4} \tag{63}$$

from which we can write

$$E[x_k x_m] = \begin{cases} \frac{1}{2}, & \text{for } k = m \\ 0, & \text{for } k \ne m. \end{cases} \tag{64}$$

[1]In Appendix C, it is illustrated that (27) to (30) are satisfied by the chaotic sequences generated by the class of Chebyshev maps of degree larger than one.

For the case where the logistic map is used to generate the chaotic samples, we substitute (61) to (63) into (41) to obtain the BER of the CSK system, i.e.,

$$\mathrm{BER}_{\mathrm{CSK}} \approx \frac{1}{4}\mathrm{erfc}\left(\sqrt{\frac{2\beta}{1+4P_B+2N_0-4\sqrt{P_B}}}\right) + \frac{1}{4}\mathrm{erfc}\left(\sqrt{\frac{2\beta}{1+4P_B+2N_0+4\sqrt{P_B}}}\right). \tag{65}$$

Moreover, for the BPSK system, we put (61) into (53) to obtain

$$\mathrm{BER}_{\mathrm{BPSK-CSK}} = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{2\beta P_B}{1+N_0}}\right). \tag{66}$$

IV. Performance Analysis of Combined DCSK–BPSK Communication System

In this section, we move on to a combined DCSK–BPSK system. In a DCSK system, the basic modulation process involves dividing the bit period into two equal slots. The first slot carries a reference chaotic signal, and the second slot bears the information. For a binary system, the second slot is the same copy or an inverted copy of the first slot depending upon the symbol sent being "+1" or "−1." This structural arrangement allows the detection to be done in a noncoherent manner requiring no reproduction of the same chaotic carrying signals at the receiver. Essentially, the detection of a DCSK signal can be accomplished by correlating the first and the second slots of the same symbol and comparing the correlator output with a threshold. Fig. 5 shows the block diagram of a DCSK transmitter and receiver pair.

Making the same assumptions as in Section III, we obtain the transmitted DCSK signal in the lth bit duration as

$$s_k = \begin{cases} x_k, & \text{for } k = 2\beta(l-1)+1, 2\beta(l-1)+2, \\ & \quad \ldots, 2\beta(l-1)+\beta \\ \alpha_l x_{k-\beta}, & \text{for } k = 2\beta(l-1)+\beta+1, \\ & \quad 2\beta(l-1)+\beta+2, \ldots, 2\beta l \end{cases} \tag{67}$$

and the BPSK signal as

$$u_k = \sqrt{P_B}\, b_l, \qquad \text{for } k = 2\beta(l-1)+1, 2\beta(l-1)+2, \ldots, 2\beta l. \tag{68}$$

All symbols and notations are as defined in the previous section. The noisy received signal r_k is given by

$$r_k = s_k + u_k + \eta_k. \tag{69}$$

A. Performance of the DCSK System in Combined DCSK–BPSK System

At the DCSK receiver, the detector essentially computes the correlation of the corrupted reference and data slots of the same symbol. We consider the output of the correlator for the lth received bit, y_l, which is given by

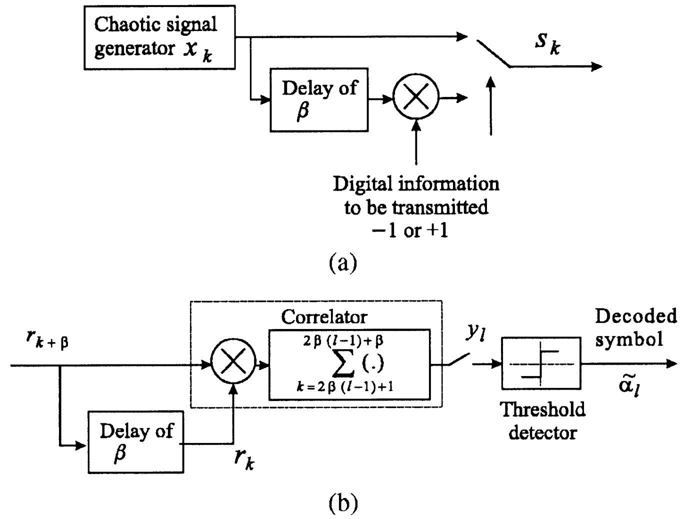

Fig. 5. Block diagram of a noncoherent DCSK system. (a) Transmitter. (b) Receiver.

$$\begin{aligned} y_l &= \sum_{k=2\beta(l-1)+1}^{2\beta(l-1)+\beta} r_k r_{k+\beta} \\ &= \sum_{k=2\beta(l-1)+1}^{2\beta(l-1)+\beta} [x_k + u_k + \eta_k][\alpha_l x_k + u_{k+\beta} + \eta_{k+\beta}] \\ &= \alpha_l \sum_{k=2\beta(l-1)+1}^{2\beta(l-1)+\beta} x_k^2 + \sqrt{P_B}\, b_l (1+\alpha_l) \sum_{k=2\beta(l-1)+1}^{2\beta(l-1)+\beta} x_k \\ &\quad + \beta P_B + \sqrt{P_B}\, b_l \sum_{k=2\beta(l-1)+1}^{2\beta(l-1)+\beta} [\eta_{k+\beta} + \eta_k] \\ &\quad + \sum_{k=2\beta(l-1)+1}^{2\beta(l-1)+\beta} x_k(\eta_{k+\beta} + \alpha_l \eta_k) + \sum_{k=2\beta(l-1)+1}^{2\beta(l-1)+\beta} \eta_k \eta_{k+\beta} \\ &= \underbrace{\alpha_l D}_{\text{required signal}} + \underbrace{\sqrt{P_B}\, b_l (1+\alpha_l) F + \beta P_B + \sqrt{P_B}\, b_l G}_{\text{interfering BPSK signal}} \\ &\quad + \underbrace{H + \alpha_l J + K}_{\text{noise}} \end{aligned} \tag{70}$$

where

$$D = \sum_{k=2\beta(l-1)+1}^{2\beta(l-1)+\beta} x_k^2 \tag{71}$$

$$F = \sum_{k=2\beta(l-1)+1}^{2\beta(l-1)+\beta} x_k \tag{72}$$

$$G = \sum_{k=2\beta(l-1)+1}^{2\beta(l-1)+\beta} [\eta_{k+\beta} + \eta_k] \tag{73}$$

$$H = \sum_{k=2\beta(l-1)+1}^{2\beta(l-1)+\beta} x_k \eta_{k+\beta} \tag{74}$$

$$J = \sum_{k=2\beta(l-1)+1}^{2\beta(l-1)+\beta} x_k \eta_k \tag{75}$$

$$K = \sum_{k=2\beta(l-1)+1}^{2\beta(l-1)+\beta} \eta_k \eta_{k+\beta}. \tag{76}$$

Suppose a "+1" is transmitted in both DCSK and BPSK systems during the lth symbol duration, i.e., $\alpha_l = +1$ and $b_l = +1$. Then, (70) can be rewritten as

$$y_l|(\alpha_l = +1, b_l = +1) = D + 2\sqrt{P_B}F + \beta P_B + \sqrt{P_B}G + H + J + K. \tag{77}$$

Similar to the combined CSK–BPSK environment (Section III-A), the mean and variance of $y_l|(\alpha_l = +1, b_l = +1)$ for the DCSK system can be evaluated by numerical simulations. Denote the respective mean and variance by $E[y_l|(\alpha_l = +1, b_l = +1)]$ and $\mathrm{var}[y_l|(\alpha_l = +1, b_l = +1)]$. As $y_l|(\alpha_l = +1, b_l = +1)$ is the sum of a large number of random variables, we may assume that it is normally distributed. An error occurs when $y_l \le 0|(\alpha_l = +1, b_l = +1)$, and the corresponding error probability is given by

$$\mathrm{Prob}\left(y_l \le 0|(\alpha_l = +1, b_l = +1)\right) = \frac{1}{2}\mathrm{erfc}\left(\frac{E\left[y_l|(\alpha_l = +1, b_l = +1)\right]}{\sqrt{2\mathrm{var}\left[y_l|(\alpha_l = +1, b_l = +1)\right]}}\right). \tag{78}$$

Likewise, for the case $\alpha_l = +1$ and $b_l = -1$, (70) becomes

$$y_l|(\alpha_l = +1, b_l = -1) = D - 2\sqrt{P_B}F + \beta P_B - \sqrt{P_B}G + H + J + K. \tag{79}$$

The corresponding error probability is

$$\mathrm{Prob}\left(y_l \le 0|(\alpha_l = +1, b_l = -1)\right) = \frac{1}{2}\mathrm{erfc}\left(\frac{E\left[y_l|(\alpha_l = +1, b_l = -1)\right]}{\sqrt{2\mathrm{var}\left[y_l|(\alpha_l = +1, b_l = -1)\right]}}\right). \tag{80}$$

Given a "+1" is sent by the DCSK signal in the lth symbol duration, the probability that an error occurs is equal to

$$\begin{aligned}\mathrm{BER}_{\mathrm{DCSK-I}}^{(l)} &= \mathrm{Prob}(b_l = +1) \times \mathrm{Prob}\left(y_l \le 0|(\alpha_l = +1, b_l = +1)\right) \\ &\quad + \mathrm{Prob}(b_l = -1) \times \mathrm{Prob}\left(y_l \le 0|(\alpha_l = +1, b_l = -1)\right) \\ &= \frac{1}{4}\left[\mathrm{erfc}\left(\frac{E\left[y_l|(\alpha_l = +1, b_l = +1)\right]}{\sqrt{2\mathrm{var}\left[y_l|(\alpha_l = +1, b_l = +1)\right]}}\right)\right. \\ &\quad \left. + \mathrm{erfc}\left(\frac{E\left[y_l|(\alpha_l = +1, b_l = -1)\right]}{\sqrt{2\mathrm{var}\left[y_l|(\alpha_l = +1, b_l = -1)\right]}}\right)\right]. \end{aligned} \tag{81}$$

Similarly, given "−1" is sent during the lth symbol duration in the DCSK system, i.e., $\alpha_l = -1$, it can be shown that

$$y_l|(\alpha_l = -1, b_l = +1) = -D + \beta P_B + \sqrt{P_B}G + H - J + K \tag{82}$$

$$y_l|(\alpha_l = -1, b_l = -1) = -D + \beta P_B - \sqrt{P_B}G + H - J + K \tag{83}$$

where D, G, H, J, and K are defined in (71) to (76). Denote the respective means and variances of $y_l|(\alpha_l = -1, b_l = +1)$ and $y_l|(\alpha_l = -1, b_l = -1)$ by $E[y_l|(\alpha_l = -1, b_l = +1)]$, $E[y_l|(\alpha_l = -1, b_l = -1)]$, $\mathrm{var}[y_l|(\alpha_l = -1, b_l = +1)]$ and $\mathrm{var}[y_l|(\alpha_l = -1, b_l = -1)]$. The error probability, given a "−1" is sent, is then equal to

$$\begin{aligned}\mathrm{BER}_{\mathrm{DCSK-II}}^{(l)} &= \mathrm{Prob}(b_l = +1) \times \mathrm{Prob}\left(y_l > 0|(\alpha_l = -1, b_l = +1)\right) \\ &\quad + \mathrm{Prob}(b_l = -1) \times \mathrm{Prob}\left(y_l > 0|(\alpha_l = -1, b_l = -1)\right) \\ &= \frac{1}{4}\left[\mathrm{erfc}\left(\frac{-E\left[y_l|(\alpha_l = -1, b_l = +1)\right]}{\sqrt{2\mathrm{var}\left[y_l|(\alpha_l = -1, b_l = +1)\right]}}\right)\right. \\ &\quad \left. + \mathrm{erfc}\left(\frac{-E\left[y_l|(\alpha_l = -1, b_l = -1)\right]}{\sqrt{2\mathrm{var}\left[y_l|(\alpha_l = -1, b_l = -1)\right]}}\right)\right]. \end{aligned} \tag{84}$$

Since both (81) and (84) are independent of l, the BER of the DCSK system under a combined communication environment, denoted by $\mathrm{BER}_{\mathrm{DCSK}}$, equals the overall error probability of the lth transmitted symbol ($\mathrm{BER}_{\mathrm{DCSK}}^{(l)}$), i.e.,

$$\begin{aligned}\mathrm{BER}_{\mathrm{DCSK}} &= \mathrm{BER}_{\mathrm{DCSK}}^{(l)} \\ &= \mathrm{Prob}(\alpha_l = +1) \times \mathrm{BER}_{\mathrm{DCSK-I}}^{(l)} \\ &\quad + \mathrm{Prob}(\alpha_l = -1) \times \mathrm{BER}_{\mathrm{DCSK-II}}^{(l)} \\ &= \frac{1}{2}\left[\mathrm{BER}_{\mathrm{DCSK-I}}^{(l)} + \mathrm{BER}_{\mathrm{DCSK-II}}^{(l)}\right]. \end{aligned} \tag{85}$$

To simplify the analysis, we make similar assumptions as in Section III-A. With these assumptions, we apply (27)–(30) to (71)–(76) and obtain the relevant means, variances and covariances, i.e.,

$$\begin{array}{ll} E[D] = \beta E\left[x_k^2\right] \equiv \beta P_s & \mathrm{var}[D] = \beta \mathrm{var}\left[x_k^2\right] \equiv \beta\Lambda \\ E[F] = 0 & \mathrm{var}[F] = \beta P_s \\ E[G] = 0 & \mathrm{var}[G] = \beta N_0 \\ E[H] = 0 & \mathrm{var}[H] = \frac{\beta P_s N_0}{2} \\ E[J] = 0 & \mathrm{var}[J] = \frac{\beta P_s N_0}{2} \\ E[K] = 0 & \mathrm{var}[K] = \frac{\beta N_0^2}{4} \\ \mathrm{cov}[D,F] = (\beta-1)E\left[x_{k+1}x_k^2\right] \approx \beta\Omega & \mathrm{cov}[\chi,\gamma] = 0 \end{array} \tag{86}$$

where $\chi, \gamma \in \{D, F, G, H, J, K\}$, and $(\chi, \gamma) \ne (D, F)$ or (F, D). Furthermore, it can be readily shown that

$$E\left[y_l|(\alpha_l = +1, b_l = +1)\right] = E[D] + 2\sqrt{P_B}E[F] + \beta P_B + \sqrt{P_B}E[G] + E[H] + E[J] + E[K] \tag{87}$$

$$E\left[y_l|(\alpha_l = +1, b_l = -1)\right] = E[D] - 2\sqrt{P_B}E[F] + \beta P_B - \sqrt{P_B}E[G] + E[H] + E[J] + E[K] \tag{88}$$

$$E\left[y_l|(\alpha_l = -1, b_l = +1)\right] = -E[D] + \beta P_B + \sqrt{P_B}E[G] + E[H] - E[J] + E[K] \tag{89}$$

$$E\left[y_l|(\alpha_l = -1, b_l = -1)\right] = -E[D] + \beta P_B - \sqrt{P_B}E[G] + E[H] - E[J] + E[K] \tag{90}$$

$$\begin{aligned}\mathrm{var}\left[y_l|(\alpha_l=+1,b_l=+1)\right] &= \mathrm{var}[D]+4P_B\mathrm{var}[F]\\ &\quad +P_B\mathrm{var}[G]+\mathrm{var}[H]\\ &\quad +\mathrm{var}[J]+\mathrm{var}[K]\\ &\quad +4\sqrt{P_B}\mathrm{cov}[D,F] \qquad (91)\\ \mathrm{var}\left[y_l|(\alpha_l=+1,b_l=-1)\right] &= \mathrm{var}[D]+4P_B\mathrm{var}[F]\\ &\quad +P_B\mathrm{var}[G]+\mathrm{var}[H]\\ &\quad +\mathrm{var}[J]+\mathrm{var}[K]\\ &\quad -4\sqrt{P_B}\mathrm{cov}[D,F] \qquad (92)\\ \mathrm{var}\left[y_l|(\alpha_l=-1,b_l=+1)\right] &= \mathrm{var}\left[y_l|(\alpha_l=-1,b_l=-1)\right]\\ &= \mathrm{var}[D]+P_B\mathrm{var}[G]\\ &\quad +\mathrm{var}[H]+\mathrm{var}[J]\\ &\quad +\mathrm{var}[K]. \qquad (93)\end{aligned}$$

Putting (86) into (87) through (93), we obtain

$$\begin{aligned}E\left[y_l|(\alpha_l=+1,b_l=+1)\right] &= E\left[y_l|(\alpha_l=+1,b_l=-1)\right]\\ &= \beta P_s+\beta P_B \qquad (94)\\ E\left[y_l|(\alpha_l=-1,b_l=+1)\right] &= E\left[y_l|(\alpha_l=-1,b_l=-1)\right]\\ &= -\beta P_s+\beta P_B \qquad (95)\\ \mathrm{var}\left[y_l|(\alpha_l=+1,b_l=+1)\right] &\approx \beta\Lambda+4\beta P_BP_s+\beta P_BN_0\\ &\quad +\beta P_sN_0+\frac{\beta N_0^2}{4}\\ &\quad +4\beta\Omega\sqrt{P_B} \qquad (96)\\ \mathrm{var}\left[y_l|(\alpha_l=+1,b_l=-1)\right] &\approx \beta\Lambda+4\beta P_BP_s+\beta P_BN_0\\ &\quad +\beta P_sN_0+\frac{\beta N_0^2}{4}\\ &\quad -4\beta\Omega\sqrt{P_B} \qquad (97)\\ \mathrm{var}\left[y_l|(\alpha_l=-1,b_l=+1)\right] &= \mathrm{var}\left[y_l|(\alpha_l=-1,b_l=-1)\right]\\ &= \beta\Lambda+\beta P_BN_0\\ &\quad +\beta P_sN_0+\frac{\beta N_0^2}{4}. \qquad (98)\end{aligned}$$

Also, putting (94)–(98) into (81), (84) and (85), we get the BER of the DCSK system, as shown in (99) at bottom of page. The expression given in (99) is then the analytical BER for the noisy DCSK signal in a combined communication environment. Note that for fixed BPSK signal power P_B and noise power spectral density $N_0/2$, the BER can be reduced by making one or a combination of the following adjustments:

1) reduce the variance of x_k^2;
2) reduce the absolute value of $\Omega(E[x_{k+1}x_k^2])$;
3) increase the spreading factor 2β;
4) increase the DCSK signal power P_s.

In particular, when the BPSK signal power is zero, i.e., $P_B = 0$, it can be readily shown that the BER reduces to [17]

$$\begin{aligned}&\mathrm{BER_{DCSK}}|(P_B=0)\\ &\quad=\frac{1}{2}\mathrm{erfc}\left(\frac{1}{\sqrt{\frac{2\Lambda}{\beta P_s^2}+\frac{2N_0}{\beta P_s}+\frac{N_0^2}{2\beta P_s^2}}}\right) \qquad (100)\\ &\quad=\frac{1}{2}\mathrm{erfc}\left(\frac{1}{\sqrt{\left(\frac{E_b^2}{8\beta\Lambda}\right)^{-1}+4\left(\frac{E_b}{N_0}\right)^{-1}+2\beta\left(\frac{E_b}{N_0}\right)^{-2}}}\right) \qquad (101)\end{aligned}$$

where $E_b = 2\beta P_s$ denotes the average bit energy.

B. Performance of the BPSK System in Combined DCSK–BPSK System

The same BPSK receiver shown in Fig. 4 is used to demodulate the BPSK signal in the combined DCSK–BPSK communication system. The output of the summer at the end of the lth symbol duration is

$$\begin{aligned}z_l &= \sum_{k=2\beta(l-1)+1}^{2\beta l} r_k\\ &= \underbrace{(1+\alpha_l)\sum_{k=2\beta(l-1)+1}^{2\beta(l-1)+\beta} x_k}_{\text{interfering DCSK signal}} + \underbrace{2\beta\sqrt{P_B}b_l}_{\text{required signal}}\\ &\quad + \underbrace{\sum_{k=2\beta(l-1)+1}^{2\beta l}\eta_k}_{\text{noise}}. \qquad (102)\end{aligned}$$

When the transmitted symbol for the DCSK system is "-1," i.e., $\alpha_l = -1$, (102) becomes

$$z_l|(\alpha_l=-1) = 2\beta\sqrt{P_B}b_l + \sum_{k=2\beta(l-1)+1}^{2\beta l}\eta_k. \qquad (103)$$

Clearly, from (103), the interference from the DCSK signal vanishes. This is because the interfering DCSK signals coming from

$$\begin{aligned}\mathrm{BER_{DCSK}} \approx\ & \frac{1}{8}\mathrm{erfc}\left(\frac{\beta P_s+\beta P_B}{\sqrt{2\beta\Lambda+8\beta P_BP_s+2\beta P_BN_0+2\beta P_sN_0+\frac{\beta N_0^2}{2}+8\beta\Omega\sqrt{P_B}}}\right)\\ &+\frac{1}{8}\mathrm{erfc}\left(\frac{\beta P_s+\beta P_B}{\sqrt{2\beta\Lambda+8\beta P_BP_s+2\beta P_BN_0+2\beta P_sN_0+\frac{\beta N_0^2}{2}-8\beta\Omega\sqrt{P_B}}}\right)\\ &+\frac{1}{4}\mathrm{erfc}\left(\frac{\beta P_s-\beta P_B}{\sqrt{2\beta\Lambda+2\beta P_BN_0+2\beta P_sN_0+\frac{\beta N_0^2}{2}}}\right). \qquad (99)\end{aligned}$$

the first half and second half of the symbol duration exactly cancel each other. When $\alpha_l = +1$, (102) becomes

$$z_l|(\alpha_l=+1)=2\sum_{k=2\beta(l-1)+1}^{2\beta(l-1)+\beta} x_k+2\beta\sqrt{P_B}b_l+\sum_{k=2\beta(l-1)+1}^{2\beta l}\eta_k. \tag{104}$$

Using a likewise procedure as in Section III-B, it can be shown that the means and variances of $z_l|(\alpha_l = -1)$ and $z_l|(\alpha_l = +1)$, denoted by $E[z_l|(\alpha_l = -1)]$, $E[z_l|(\alpha_l = +1)]$, $\mathrm{var}[z_l|(\alpha_l = -1)]$ and $\mathrm{var}[z_l|(\alpha_l = +1)]$, respectively, are given by

$$E[z_l|(\alpha_l=-1)] = 2\beta\sqrt{P_B}b_l \tag{105}$$

$$E[z_l|(\alpha_l=+1)] = 2E\left[\sum_{k=2\beta(l-1)+1}^{2\beta(l-1)+\beta} x_k\right] + 2\beta\sqrt{P_B}b_l \tag{106}$$

$$\mathrm{var}[z_l|(\alpha_l=-1)] = \beta N_0 \tag{107}$$

$$\mathrm{var}[z_l|(\alpha_l=+1)] = 4\mathrm{var}\left[\sum_{k=2\beta(l-1)+1}^{2\beta(l-1)+\beta} x_k\right] + \beta N_0. \tag{108}$$

Assuming that: i) the mean value of x_k is zero, and ii) the chaotic samples x_j are uncorrelated with other samples x_k for $j \neq k$, we combine (27) and (28) with (106) and (108) to get

$$E[z_l|(\alpha_l=+1)] = 2\beta\sqrt{P_B}b_l \tag{109}$$

$$\mathrm{var}[z_l|(\alpha_l=+1)] = 4\beta P_s + \beta N_0. \tag{110}$$

Suppose $b_l = +1$ and $\alpha_l = +1$. As $z_l|(b_l = +1, \alpha_l = +1)$ is the sum of a large number of random variables, we assume that it follows a normal distribution. An error occurs when $z_l \leq 0|(b_l = +1, \alpha_l = +1)$, and the corresponding error probability is given by

$$\begin{aligned}\mathrm{Prob}\,(z_l \leq 0|(b_l=+1,\alpha_l=+1)) &= \frac{1}{2}\mathrm{erfc}\left(\frac{E[z_l|(b_l=+1,\alpha_l=+1)]}{\sqrt{2\mathrm{var}[z_l|(b_l=+1,\alpha_l=+1)]}}\right)\\ &= \frac{1}{2}\mathrm{erfc}\left(\frac{2\beta\sqrt{P_B}}{\sqrt{8\beta P_s+2\beta N_0}}\right).\end{aligned} \tag{111}$$

Likewise, it can be shown that

$$\mathrm{Prob}\,(z_l \leq 0|(b_l=+1,\alpha_l=-1)) = \frac{1}{2}\mathrm{erfc}\left(\frac{2\beta\sqrt{P_B}}{\sqrt{2\beta N_0}}\right) \tag{112}$$

$$\mathrm{Prob}\,(z_l > 0|(b_l=-1,\alpha_l=+1)) = \frac{1}{2}\mathrm{erfc}\left(\frac{2\beta\sqrt{P_B}}{\sqrt{8\beta P_s+2\beta N_0}}\right) \tag{113}$$

$$\mathrm{Prob}\,(z_l > 0|(b_l=-1,\alpha_l=-1)) = \frac{1}{2}\mathrm{erfc}\left(\frac{2\beta\sqrt{P_B}}{\sqrt{2\beta N_0}}\right). \tag{114}$$

The error probability for the lth transmitted BPSK symbol is given by

$$\begin{aligned}\mathrm{BER}^{(l)}_{\mathrm{BPSK-DCSK}} &= \mathrm{Prob}(b_l=+1)\times\mathrm{Prob}(\alpha_l=+1)\\ &\quad\times\mathrm{Prob}\,(z_l\leq 0|(b_l=+1,\alpha_l=+1))\\ &\quad+\mathrm{Prob}(b_l=+1)\times\mathrm{Prob}(\alpha_l=-1)\\ &\quad\times\mathrm{Prob}\,(z_l\leq 0|(b_l=+1,\alpha_l=-1))\\ &\quad+\mathrm{Prob}(b_l=-1)\times\mathrm{Prob}(\alpha_l=+1)\\ &\quad\times\mathrm{Prob}\,(z_l> 0|(b_l=-1,\alpha_l=+1))\\ &\quad+\mathrm{Prob}(b_l=-1)\times\mathrm{Prob}(\alpha_l=-1)\\ &\quad\times\mathrm{Prob}\,(z_l> 0|(b_l=-1,\alpha_l=-1))\\ &= \frac{1}{4}\mathrm{erfc}\left(\frac{2\beta\sqrt{P_B}}{\sqrt{8\beta P_s+2\beta N_0}}\right)\\ &\quad+\frac{1}{4}\mathrm{erfc}\left(\frac{2\beta\sqrt{P_B}}{\sqrt{2\beta N_0}}\right).\end{aligned} \tag{115}$$

Since $\mathrm{BER}^{(l)}_{\mathrm{BPSK-DCSK}}$ is independent of l, the error probability of the lth transmitted symbol is the same as the BER of the system. Therefore, the BER of the BPSK system, denoted by $\mathrm{BER}_{\mathrm{BPSK-DCSK}}$, is

$$\begin{aligned}\mathrm{BER}_{\mathrm{BPSK-DCSK}} &= \mathrm{BER}^{(l)}_{\mathrm{BPSK-DCSK}}\\ &= \frac{1}{4}\mathrm{erfc}\left(\frac{2\beta\sqrt{P_B}}{\sqrt{8\beta P_s+2\beta N_0}}\right)+\frac{1}{4}\mathrm{erfc}\left(\frac{2\beta\sqrt{P_B}}{\sqrt{2\beta N_0}}\right) \qquad (116)\\ &= \frac{1}{4}\mathrm{erfc}\left(\sqrt{\frac{E_B}{\breve{N}_0}}\right)+\frac{1}{4}\mathrm{erfc}\left(\sqrt{\frac{E_B}{N_0}}\right) \qquad (117)\end{aligned}$$

where E_B is as defined in (55) and

$$\frac{\breve{N}_0}{2} = \frac{N_0}{2} + 2P_s \tag{118}$$

represents the equivalent noise power spectral density when the interfering DCSK signal is taken into consideration. It can be seen that the BPSK signal remains unaffected by the DCSK signal for half of the time and the noise power affecting the BPSK signal increases by $2P_s$ for another half of the time. The expression given in (116) or (117) is the analytical BER for the noisy coherent BPSK system in a combined communiction environment. For a fixed chaotic signal power P_s, the BER can be improved by increasing the spreading factor 2β and/or increasing the BPSK signal power P_B.

C. Example

Consider the case where the logistic map described in Section III-C is used for generating the chaotic sequences. We substitute (61) and (62) into (99) to obtain the BER of the DCSK system, i.e., we obtain (119) shown at the bottom the next page. For the BPSK system, we combine (61) with (116) to obtain

$$\mathrm{BER}_{\mathrm{BPSK-DCSK}} = \frac{1}{4}\mathrm{erfc}\left(\sqrt{\frac{2\beta P_B}{2+N_0}}\right)+\frac{1}{4}\mathrm{erfc}\left(\sqrt{\frac{2\beta P_B}{N_0}}\right). \tag{120}$$

V. Computer Simulations and Discussions

In this section, we study the performances of the chaos-based and conventional digital communication systems under a combined environment by computer simulations. The logistic map described in Section III-C has been used to generate the chaotic sequences. In particular, the BER performance of each of the chaos-based and conventional communication systems will be investigated under variation of the following parameters:

- average bit-energy-to-noise-spectral-density ratio;
- conventional-to-chaotic-signal-power ratio;
- spreading factor.

For comparison, we also plot in each case, the analytical BERs obtained from the expressions derived in Sections III and IV.[2] Results are shown in Figs. 6 and 7 for the combined CSK–BPSK system, and in Figs. 8 and 9 for the combined DCSK–BPSK system. In general, computer simulations and analytical results are in good agreement. Also, as would be expected, the coherent CSK system generally performs better than the noncoherent DCSK system. Further observations are summarized as follows.

1) Except for the DCSK system, the BERs of the combined chaos-based and conventional systems generally decreases (improves) as the spreading factor or the bit-energy-to-noise-power-spectral-density (E_B/N_0 or E_b/N_0) increases.
2) The BER of the chaos-based system in the combined environment generally deteriorates (increases) as P_B/P_s increases for any given E_b/N_0. This is apparently due to the increasing power of the BPSK signal which causes more interference to the chaos-based system, thus giving a higher BER.
3) At a fixed E_B/N_0, the BER of the BPSK system in the combined environment improves as P_B/P_s increases. This result comes with no surprise because as P_B/P_s increases, the power of the chaotic signal becomes weaker compared to the BPSK signal power. Thus, the interference due to the chaotic signal diminishes, resulting in an improved BER for the BPSK system.
4) Comparing the two types of chaos-based communication systems, the performance of the DCSK system is degraded to a larger extent under the influence of a BPSK signal. For example, from Fig. 8, for a spreading factor of 200 and $E_b/N_0 = 20$ dB, we observe that the BER of the DCSK system increases from 2×10^{-4} to 0.5 when P_B/P_s increases from -5 dB to 5 dB. For the CSK system employing the same spreading factor, at $E_b/N_0 = 8$ dB, the BER only increases from around 2×10^{-4} to 7×10^{-4} when P_B/P_s increases from -5 dB to 5 dB.

[2]It has been verified by computer simulations that the conditional receiver outputs are "sufficiently Gaussian" for large spreading factors, e.g., 100 or higher.

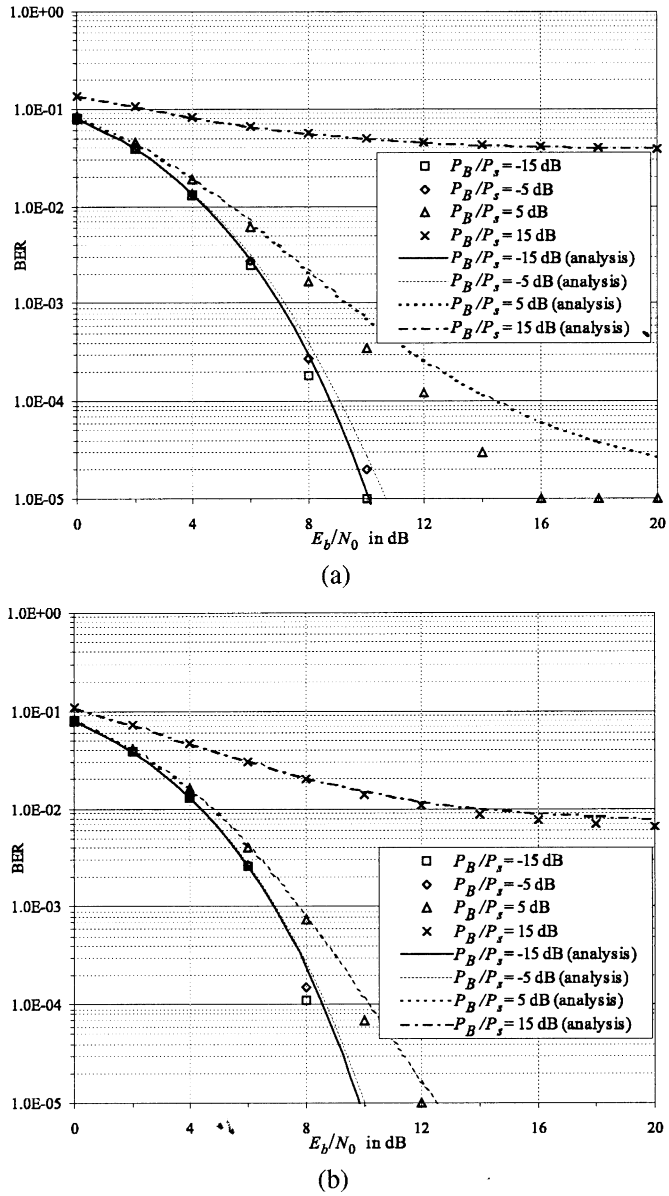

Fig. 6. BERs versus E_b/N_0 of the coherent CSK system in a combined CSK–BPSK environment. Simulated BERs are plotted as points and analytical BERs plotted as lines. (a) Spreading factor is 100. (b) Spreading factor is 200.

$$\begin{aligned}\mathrm{BER}_{\mathrm{DCSK}} \approx\ & \frac{1}{8}\mathrm{erfc}\left(\frac{0.5\beta+\beta P_B}{\sqrt{0.25\beta+4\beta P_B+2\beta P_B N_0+\beta N_0+\frac{\beta N_0^2}{2}-2\beta\sqrt{P_B}}}\right)\\ &+\frac{1}{8}\mathrm{erfc}\left(\frac{0.5\beta+\beta P_B}{\sqrt{0.25\beta+4\beta P_B+2\beta P_B N_0+\beta N_0+\frac{\beta N_0^2}{2}+2\beta\sqrt{P_B}}}\right)\\ &+\frac{1}{4}\mathrm{erfc}\left(\frac{0.5\beta-\beta P_B}{\sqrt{0.25\beta+2\beta P_B N_0+\beta N_0+\frac{\beta N_0^2}{2}}}\right). \qquad (119)\end{aligned}$$

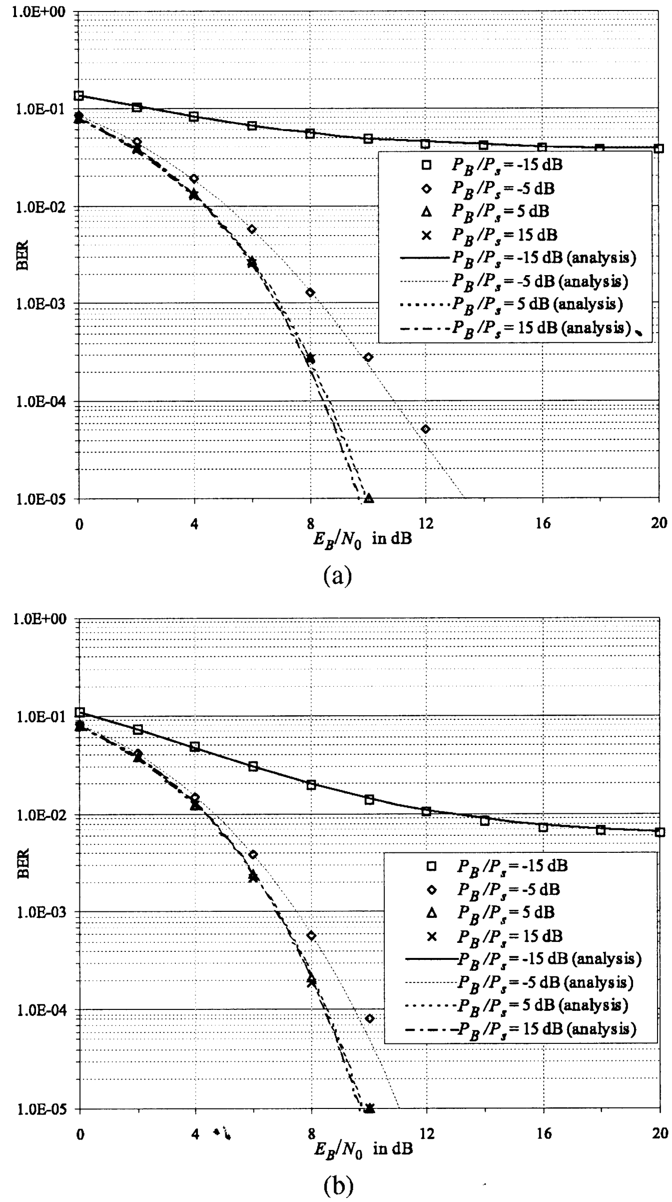

Fig. 7. BERs versus E_B/N_0 of the BPSK system in a combined CSK–BPSK environment. Simulated BERs are plotted as points and analytical BERs plotted as lines. (a) Spreading factor is 100. (b) Spreading factor is 200.

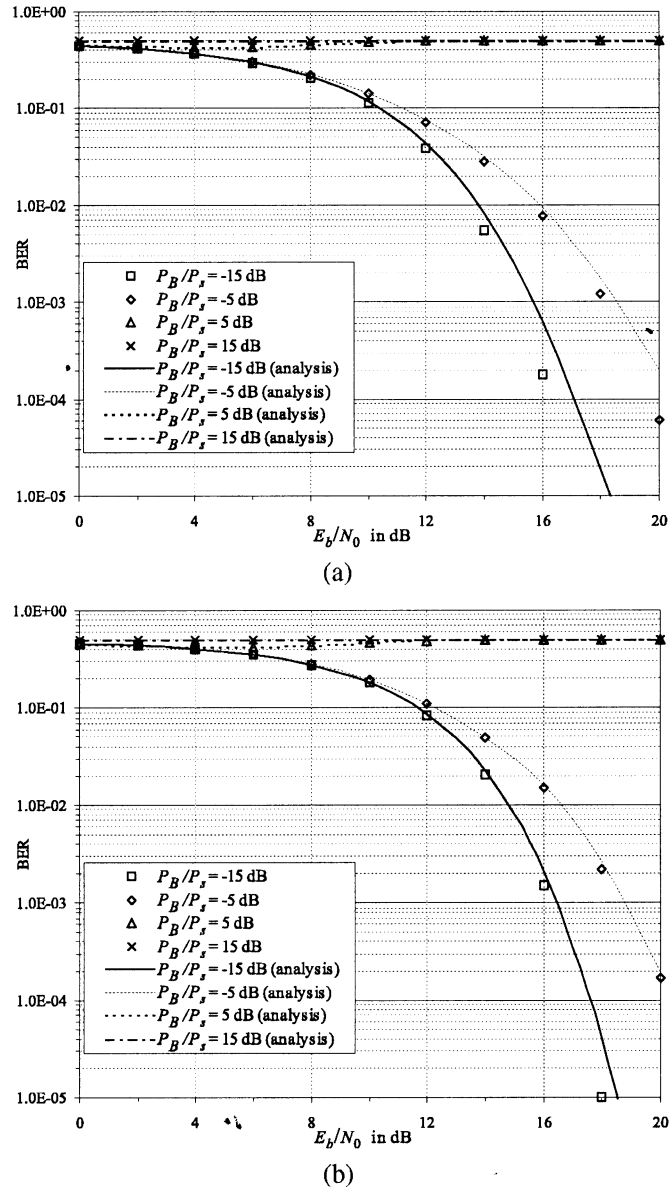

Fig. 8. BERs versus E_b/N_0 of the noncoherent DCSK system in a combined DCSK–BPSK environment. Simulated BERs are plotted as points and analytical BERs plotted as lines. (a) Spreading factor is 100. (b) spreading factor is 200.

5) From Figs. 6 and 7, we observe that for a spreading factor of 100 and $P_B/P_s = 5$ dB, both the CSK and BPSK systems can achieve a BER of 10^{-4} if they can operate at around $E_b/N_0 = 12$ dB and $E_B/N_0 = 9$ dB respectively. In other words, both the BPSK and CSK systems can perform reasonably well under a combined environment.
6) From Figs. 8 and 9, for both the DCSK and BPSK systems to operate with BERs near 10^{-4}, a possible set of operating parameters is $P_B/P_s = -5$ dB, $E_b/N_0 = 20$ dB, $E_B/N_0 = 16$ dB and spreading factor $= 100$. Compared to the combined CSK–BPSK system, the DCSK–BPSK system requires more restrictive operating conditions in order to maintain performance.

Finally, we investigate the channel capacity for a given total-bit-energy-to-noise-power-spectral-density ratio, defined as E_t/N_0 where $E_t = E_b + E_B$. The total capacity of the combined system is the sum of the capacity of the chaos-based system and that of the BPSK system. The capacity of each individual system is further evaluated using the capacity formula for a binary symmetric channel. Hence, the total capacity, denoted by C, for the CSK–BPSK system and the DCSK–BPSK system is given by $C = (1-H(\mathrm{BER}_{\mathrm{CSK}}))+(1-H(\mathrm{BER}_{\mathrm{BPSK-CSK}}))$ and $C = (1-H(\mathrm{BER}_{\mathrm{DCSK}}))+(1-H(\mathrm{BER}_{\mathrm{BPSK-DCSK}}))$, respectively, where $H(.)$ represents the entropy function [19]. The results are plotted in Fig. 10. The observations on the figures are summarized as follows.

1) Under the same condition, the capacity of the combined CSK–BPSK system is higher than that of the combined DCSK–BPSK system. It is because the bit error performances of the former system are better than those of the latter one.

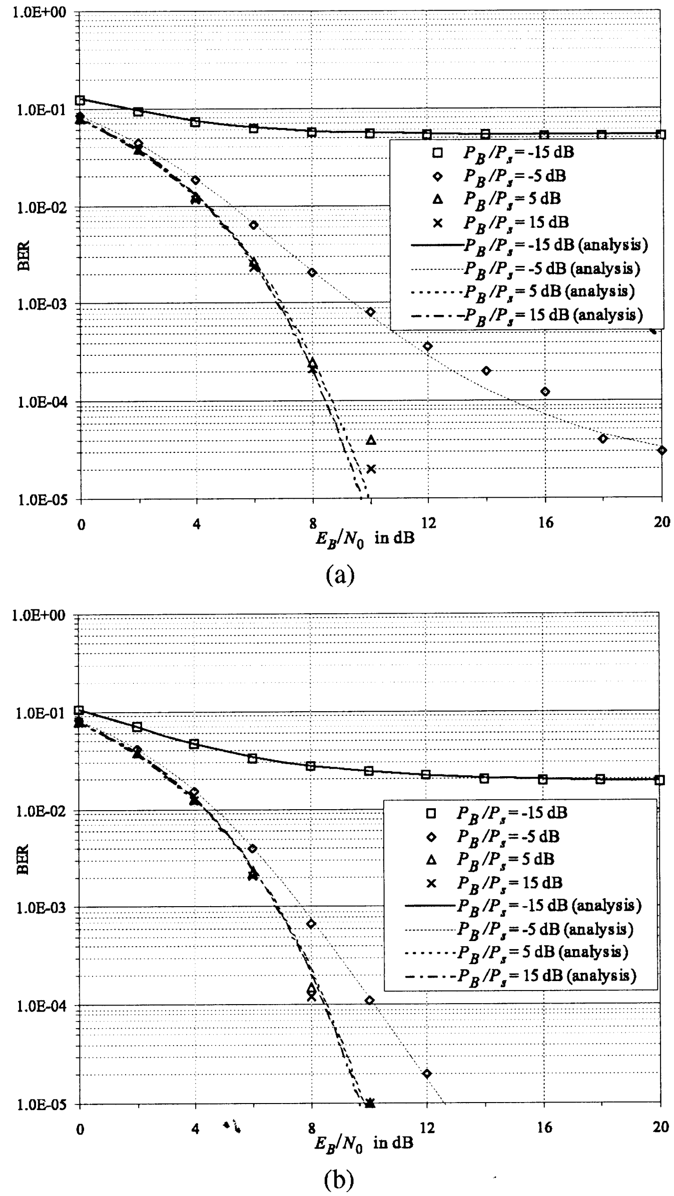

Fig. 9. BERs versus E_B/N_0 of the BPSK system in a combined DCSK–BPSK environment. Simulated BERs are plotted as points and analytical BERs plotted as lines. (a) Spreading factor is 100. (b) Spreading factor is 200.

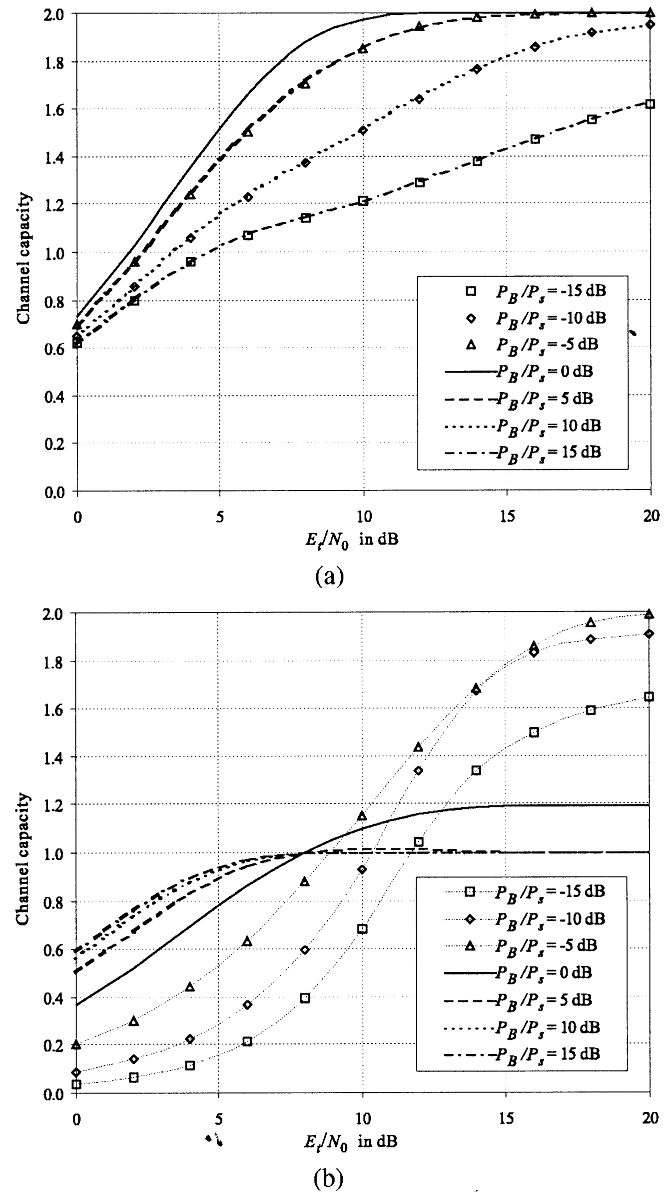

Fig. 10. Channel capacity of the combined chaos-based conventional system. Spreading factor is 100. (a) Combined CSK–BPSK system. (b) Combined DCSK–BPSK system.

2) For the combined CSK–BPSK system, under the same E_t/N_0, the capacity is highest when $P_B/P_s = 0$ dB, i.e., both the CSK and BPSK signals have the same (average) power, the reason being that both the CSK and the BPSK systems have similar BER performance when their powers are equal (due to the assumptions made on the statistics of the chaotic signal). For a fixed power ratio between the chaos and conventional signals, the capacity is the same regardless of CSK or BPSK signal having a higher power. In other words, the capacity is the same for the same absolute value of P_B/P_s in decibels. Moreover, when the power ratio increases, the capacity decreases.
3) For the combined DCSK–BPSK system, the capacity is very low when E_t/N_0 is less than 8 dB. Under the same E_t/N_0 with value above 8 dB, the highest capacity is achieved when $P_B/P_s = -5$ dB.

VI. Conclusion

In this paper, the problem of coexistence of chaos-based communication systems and conventional communication systems is studied in terms of two specific sample systems, namely, a combined CSK–BPSK system and a combined DCSK–BPSK system. This problem is important technically since spread-spectrum communications should be designed to resist interference and the proposed combined systems represent practical future scenarios. To the authors' knowledge, no previous work has been reported in the study of the aforementioned coexistence problem, despite its potential significance. In particular this paper has shown that chaos-based systems can indeed coexist with narrow-band conventional systems whose frequency bands fall within those of the chaos-based systems. Note that for the combined CSK–BPSK system, coherent correlation CSK receiver has been assumed. Since robust chaos synchronization techniques are still not available, the corresponding results represent the *benchmark* performance that a combined CSK–BPSK system can achieve. If a noncoherent CSK receiver, such as the one based on optimal detection [3], is used instead, the performance is bound to degrade.

In this study, it has been assumed that the chaos-based (CSK or DCSK) communication system and the conventional (BPSK) communication system are synchronized. Also, the bit rates are taken to be identical. In general, the systems may not be synchronized and they may operate at different data rates. Under such conditions, the performances of the systems may deviate from the reported results significantly. In addition, the study of the coexistence problem is being extended to the case of wide-band conventional systems. All the aforementioned scenarios are being investigated by the authors and the results will be reported in future publications.

Appendix A
Derivation of Covariances and Variances Relevant to the Analysis of Combined CSK–BPSK System

All symbols are as defined as in Section III.

$$\begin{aligned}\mathrm{cov}[A,C] &= E[AC]-E[A]E[C]\\ &= E\left[\left(\sum_{k=2\beta(l-1)+1}^{2\beta l} x_k^2\right)\left(\sum_{k=2\beta(l-1)+1}^{2\beta l} \eta_k x_k\right)\right] - E\left[\sum_{k=2\beta(l-1)+1}^{2\beta l} x_k^2\right] E\left[\sum_{k=2\beta(l-1)+1}^{2\beta l} \eta_k x_k\right]\\ &= E\left[\sum_{k=2\beta(l-1)+1}^{2\beta l} x_k^3\eta_k + \sum_{\substack{k=2\beta(l-1)+1\\ k\neq j}}^{2\beta l}\sum_{j=2\beta(l-1)+1}^{2\beta l} \eta_k x_k x_j^2\right] - \left(\sum_{k=2\beta(l-1)+1}^{2\beta l} E\left[x_k^2\right]\right)\times\left(\sum_{k=2\beta(l-1)+1}^{2\beta l} E[\eta_k]E[x_k]\right)\\ &= \sum_{k=2\beta(l-1)+1}^{2\beta l} E\left[x_k^3\right]E[\eta_k] + \sum_{\substack{k=2\beta(l-1)+1\\ k\neq j}}^{2\beta l}\sum_{j=2\beta(l-1)+1}^{2\beta l} E[\eta_k]E\left[x_k x_j^2\right]\\ &= 0 \end{aligned} \tag{121}$$

$$\begin{aligned}\mathrm{cov}[B,C] &= E[BC]-E[B]E[C]\\ &= E\left[\left(\sqrt{P_B}\sum_{k=2\beta(l-1)+1}^{2\beta l} x_k\right)\times\left(\sum_{k=2\beta(l-1)+1}^{2\beta l} \eta_k x_k\right)\right] - E\left[\sqrt{P_B}\sum_{k=2\beta(l-1)+1}^{2\beta l} x_k\right]\times E\left[\sum_{k=2\beta(l-1)+1}^{2\beta l} \eta_k x_k\right]\\ &= \sqrt{P_B}E\left[\sum_{k=2\beta(l-1)+1}^{2\beta l} x_k^2\eta_k + \sum_{\substack{k=2\beta(l-1)+1\\ k\neq j}}^{2\beta l}\sum_{j=2\beta(l-1)+1}^{2\beta l} \eta_k x_k x_j\right] - \sqrt{P_B}\left(\sum_{k=2\beta(l-1)+1}^{2\beta l} E[x_k]\right)\times\left(\sum_{k=2\beta(l-1)+1}^{2\beta l} E[\eta_k]E[x_k]\right)\\ &= \sqrt{P_B}\left[\sum_{k=2\beta(l-1)+1}^{2\beta l} E\left[x_k^2\right]E[\eta_k] + \sum_{\substack{k=2\beta(l-1)+1\\ k\neq j}}^{2\beta l}\sum_{j=2\beta(l-1)+1}^{2\beta l} E[\eta_k]E[x_k x_j]\right]\\ &= 0 \end{aligned} \tag{122}$$

$$\begin{aligned}\mathrm{var}[A] &= \mathrm{var}\left[\sum_{k=2\beta(l-1)+1}^{2\beta l} x_k^2\right]\\ &= \sum_{k=2\beta(l-1)+1}^{2\beta l}\mathrm{var}\left[x_k^2\right] + \sum_{\substack{j=2\beta(l-1)+1\\ j\neq k}}^{2\beta l}\sum_{k=2\beta(l-1)+1}^{2\beta l}\mathrm{cov}\left[x_j^2, x_k^2\right]\\ &= 2\beta\mathrm{var}\left[x_k^2\right] \quad \text{apply} \quad (29)\\ &= 2\beta\Lambda \quad \text{where} \quad \Lambda = \mathrm{var}\left[x_k^2\right] \end{aligned} \tag{123}$$

$$\begin{aligned}\mathrm{var}[B] &= \mathrm{var}\left[\sqrt{P_B}\sum_{k=2\beta(l-1)+1}^{2\beta l} x_k\right]\\ &= P_B\sum_{k=2\beta(l-1)+1}^{2\beta l}\mathrm{var}[x_k] + P_B\sum_{\substack{j=2\beta(l-1)+1\\ j\neq k}}^{2\beta l}\sum_{k=2\beta(l-1)+1}^{2\beta l}\mathrm{cov}[x_j, x_k]\\ &= 2\beta P_B P_s \quad \text{apply} \quad (28) \end{aligned} \tag{124}$$

$$\mathrm{var}[C] = \mathrm{var}\left[\sum_{k=2\beta(l-1)+1}^{2\beta l} \eta_k x_k\right]$$

$$
\begin{aligned}
&= \sum_{k=2\beta(l-1)+1}^{2\beta l} \mathrm{var}[\eta_k x_k] \\
&\quad + \sum_{\substack{j=2\beta(l-1)+1\\ j\neq k}}^{2\beta l} \sum_{k=2\beta(l-1)+1}^{2\beta l} \mathrm{cov}[\eta_j x_j, \eta_k x_k] \\
&= \sum_{k=2\beta(l-1)+1}^{2\beta l} \left\{E\left[\eta_k^2 x_k^2\right] - E^2[\eta_k x_k]\right\} \\
&\quad + \sum_{\substack{j=2\beta(l-1)+1\\ j\neq k}}^{2\beta l} \sum_{k=2\beta(l-1)+1}^{2\beta l} \\
&\quad \times \left\{E[\eta_j x_j \eta_k x_k] - E[\eta_j x_j]E[\eta_k x_k]\right\} \\
&= \sum_{k=2\beta(l-1)+1}^{2\beta l} \left\{E\left[\eta_k^2\right]E\left[x_k^2\right] - E^2[\eta_k]E^2[x_k]\right\} \\
&\quad + \sum_{\substack{j=2\beta(l-1)+1\\ j\neq k}}^{2\beta l} \sum_{k=2\beta(l-1)+1}^{2\beta l} \\
&\quad \times \left\{E[x_j x_k]E[\eta_j \eta_k] - E[x_j]E[x_k]E[\eta_j]E[\eta_k]\right\} \\
&= 2\beta\left(\frac{N_0}{2}\right)P_s \\
&= \beta N_0 P_s
\end{aligned} \tag{125}
$$

$$
\begin{aligned}
\mathrm{cov}[A,B] &= \mathrm{cov}\left[\sum_{k=2\beta(l-1)+1}^{2\beta l} x_k^2, \sqrt{P_B}\sum_{k=2\beta(l-1)+1}^{2\beta l} x_k\right] \\
&= \sqrt{P_B}E\left[\left(\sum_{k=2\beta(l-1)+1}^{2\beta l} x_k^2\right) \times \left(\sum_{k=2\beta(l-1)+1}^{2\beta l} x_k\right)\right] \\
&\quad - \sqrt{P_B}E\left[\sum_{k=2\beta(l-1)+1}^{2\beta l} x_k^2\right]E\left[\sum_{k=2\beta(l-1)+1}^{2\beta l} x_k\right] \\
&= \sqrt{P_B}E\left[\sum_{j=2\beta(l-1)+1}^{2\beta l}\sum_{k=2\beta(l-1)+1}^{2\beta l} x_j x_k^2\right] \\
&\quad - \sqrt{P_B}\left(\sum_{k=2\beta(l-1)+1}^{2\beta l} E\left[x_k^2\right]\right) \times \left(\sum_{k=2\beta(l-1)+1}^{2\beta l} E[x_k]\right) \\
&= \sqrt{P_B}\sum_{j=2\beta(l-1)+1}^{2\beta l}\sum_{k=2\beta(l-1)+1}^{2\beta l} E\left[x_j x_k^2\right] \\
&\quad - \sqrt{P_B}\left(\sum_{k=2\beta(l-1)+1}^{2\beta l} E\left[x_k^2\right]\right) \times \left(\sum_{k=2\beta(l-1)+1}^{2\beta l} E[x_k]\right) \\
&= \sqrt{P_B}\sum_{k=2\beta(l-1)+1}^{2\beta l-1} E\left[x_{k+1}x_k^2\right] \\
&\qquad \text{apply (27) and (30)} \\
&= \sqrt{P_B}(2\beta-1)E\left[x_{k+1}x_k^2\right].
\end{aligned} \tag{126}
$$

APPENDIX B
DERIVATION OF $\mathrm{cov}[x_j^2, x_k^2]$ AND $E[x_j x_k^2]$ FOR THE CHAOTIC SEQUENCE GENERATED BY THE LOGISTIC MAP

All symbols are as defined as in Section III.

Derivation of $\mathrm{cov}[x_j^2, x_k^2]$

The autovariance of $\{x_k^2\}$ is given by

$$\mathrm{cov}\left[x_j^2, x_k^2\right] = E\left[x_j^2 x_k^2\right] - E\left[x_j^2\right]E\left[x_k^2\right]. \tag{127}$$

We consider the case where $j \neq k$. Without loss of generality, assume $k = j + n$ for some positive integer n.

$$
\begin{aligned}
E\left[x_j^2 x_k^2\right] &= \int_{-\infty}^{\infty} \rho(x)x^2\left(g^{(n)}(x)\right)^2 dx \\
&= \int_{-1}^{1} \frac{1}{\pi\sqrt{1-x^2}} x^2\left(g^{(n)}(x)\right)^2 dx
\end{aligned} \tag{128}
$$

where $\rho(x)$ denotes the invariant probability density function of x_j; $g^{(2)}(x) = g(g(x)), g^{(3)}(x) = g(g^{(2)}(x)), \cdots, g^{(n)}(x) = g(g^{(n-1)}(x))$ and $g(x) = 1 - 2x^2$ for the logistic map under study. Making the substitution $x = \cos\phi$, (128) becomes

$$
\begin{aligned}
E\left[x_j^2 x_k^2\right] &= \int_{\pi}^{0} \frac{1}{\pi\sin\phi}\cos^2\phi\left(g^{(n)}(\cos\phi)\right)^2(-\sin\phi)d\phi \\
&= \frac{1}{\pi}\int_{0}^{\pi} \cos^2\phi\left(g^{(n)}(\cos\phi)\right)^2 d\phi.
\end{aligned} \tag{129}
$$

Applying the formula $1 - 2\cos^2\upsilon = -\cos 2\upsilon$ to $g^{(i)}(\cos\phi)$ $(i = 1, 2, \ldots, n-1)$ repeatedly, we have

$$
\begin{aligned}
g(\cos\phi) &= 1 - 2\cos^2\phi = -\cos 2\phi \\
g^{(2)}(\cos\phi) &= g\left(g(\cos\phi)\right) = g(-\cos 2\phi) \\
&= 1 - 2\cos^2(2\phi) = -\cos(2^2\phi) \\
g^{(3)}(\cos\phi) &= g\left(g^{(2)}(\cos\phi)\right) = g\left(-\cos(2^2\phi)\right) \\
&= 1 - 2\cos^2(2^2\phi) = -\cos(2^3\phi) \\
\vdots &= \vdots \\
g^{(n)}(\cos\phi) &= g\left(g^{(n-1)}(\cos\phi)\right) = g\left(-\cos(2^{n-1}\phi)\right) \\
&= 1 - 2\cos^2(2^{n-1}\phi) = -\cos(2^n\phi).
\end{aligned} \tag{130}
$$

Substituting (130) into (129), we obtain

$$\begin{aligned} E\left[x_j^2 x_k^2\right] &= \frac{1}{\pi}\int_0^\pi \cos^2\phi\cos^2(2^n\phi)d\phi \\ &= \frac{1}{\pi}\int_0^\pi \left(\frac{1+\cos 2\phi}{2}\right)\left(\frac{1+\cos(2^{n+1}\phi)}{2}\right)d\phi \\ &= \frac{1}{4\pi}\int_0^\pi \Big(1+\cos 2\phi+\cos(2^{n+1}\phi) \\ &\qquad +\frac{1}{2}\cos\left((2^{n+1}+2)\phi\right) \\ &\qquad +\frac{1}{2}\cos\left((2^{n+1}-2)\phi\right)\Big)d\phi \\ &= \frac{1}{4\pi}\Big[\phi+\frac{1}{2}\sin 2\phi+\frac{1}{2^{n+1}}\sin(2^{n+1}\phi) \\ &\qquad +\frac{1}{2^{n+2}+4}\sin\left((2^{n+1}+2)\phi\right) \\ &\qquad +\frac{1}{2^{n+2}-4}\sin\left((2^{n+1}-2)\phi\right)\Big]_0^\pi \\ &= \frac{1}{4}. \end{aligned} \tag{131}$$

Simliarly it is readily shown that

$$E\left[x_j^2\right] = E\left[x_k^2\right] = \int_{-\infty}^{\infty}\rho(x)x^2dx = \int_{-1}^{1}\frac{1}{\pi\sqrt{1-x^2}}x^2dx = \frac{1}{2}. \tag{132}$$

Putting (131) and (132) into (127), it is proved that the autovariance for $\{x_k^2\}$ is vanishing for the logistic map.

Derivation of $E[x_j x_k^2]$

When $j = k$,

$$E\left[x_j x_k^2\right] = E\left[x_k^3\right] = \int_{-\infty}^{\infty} x^3\rho(x)dx = \int_{-1}^{1} x^3\rho(x)dx = 0 \tag{133}$$

because $\rho(x)$ is an even function whereas x^3 is odd. Next, we consider the case where $j < k$. Assume $k = j + n$ for some positive integer n and we obtain

$$E\left[x_j x_k^2\right] = \int_{-\infty}^{\infty}\rho(x)x\left(g^{(n)}(x)\right)^2dx. \tag{134}$$

Within the integral, both $\rho(x)$ and $(g^{(n)}(x))^2$ are even while x is odd. Thus, it can be concluded that $E[x_j x_k^2] = 0$ for $j < k$. Finally, for $j > k$, we assume $j = k + m$ for some positive integer m. Making the substitution $x = \cos\phi$ and applying (130), we have

$$\begin{aligned} E\left[x_{k+m}x_k^2\right] &= \int_{-\infty}^{\infty}\rho(x)x^2g^{(m)}(x)dx \\ &= \frac{1}{\pi}\int_0^\pi \cos^2\phi\left(-\cos(2^m\phi)\right)d\phi \\ &= \begin{cases} 0, & \text{for } m \ge 2 \\ -\frac{1}{4}, & \text{for } m = 1. \end{cases} \end{aligned} \tag{135}$$

Thus, we conclude that

$$E\left[x_j x_k^2\right] = \begin{cases} 0, & \text{for } j-k \ne 1 \\ -\frac{1}{4}, & \text{for } j-k = 1. \end{cases} \tag{136}$$

Appendix C
Derivation of the Statistical Properties for the Chaotic Sequences Generated by Chebyshev Maps of Degree Larger Than One

In this appendix, we show that the chaotic sequences generated by Chebyshev maps of degree larger than one satisfy the assumptions (27) to (30) mentioned in Section III-A. A Chebyshev map of degree μ is defined as [20]

$$x_{k+1} = h(x_k) = \cos(\mu\cos^{-1}x_k) \tag{137}$$

where μ is an integer. We consider the case where $\mu > 1$. The invariant probability density function of $\{x_k\}$, denoted by $\rho(x)$, is known to be [20]

$$\rho(x) = \begin{cases} \frac{1}{\pi\sqrt{1-x^2}}, & \text{if } |x| < 1 \\ 0, & \text{otherwise.} \end{cases} \tag{138}$$

Derivation of $E[x_k]$

Since $\rho(x)$ is an even function, the mean value of $\{x_k\}$ is

$$E[x_k] = \int_{-\infty}^{\infty} x\rho(x)dx = \int_{-1}^{1} x\rho(x)dx = 0. \tag{139}$$

Derivation of $\mathrm{cov}[x_j, x_k]$

The autovariance of $\{x_k\}$ is given by

$$\mathrm{cov}[x_j, x_k] = E[x_j x_k] - E[x_j]E[x_k] = E[x_j x_k]. \tag{140}$$

We consider the case where $j \ne k$. Without loss of generality, assume $k = j + n$ for some positive integer n. Define

$$\begin{aligned} h^{(1)}(x) &= h(x) \\ h^{(2)}(x) &= h\left(h^{(1)}(x)\right) \\ &\vdots = \vdots \\ h^{(n)}(x) &= h\left(h^{(n-1)}(x)\right). \end{aligned} \tag{141}$$

The autovariance of $\{x_k\}$ can be rewritten as

$$\operatorname{cov}[x_j, x_k] = \int_{-\infty}^{\infty} \rho(x) x h^{(n)}(x) dx = \int_{-1}^{1} \frac{1}{\pi\sqrt{1-x^2}} x h^{(n)}(x) dx. \tag{142}$$

Making the substitution $x = \cos\phi$, (142) becomes

$$\begin{aligned}\operatorname{cov}[x_j, x_k] &= \int_{\pi}^{0} \frac{1}{\pi \sin\phi} \cos\phi h^{(n)}(\cos\phi)(-\sin\phi) d\phi \\ &= \frac{1}{\pi}\int_0^{\pi} \cos\phi h^{(n)}(\cos\phi) d\phi \\ &= \frac{1}{\pi}\int_0^{\pi} \cos\phi \cos(\mu^n \phi) d\phi \\ &= \frac{1}{2\pi}\int_0^{\pi} \cos\left((\mu^n+1)\phi\right) + \cos\left((\mu^n-1)\phi\right) d\phi \\ &= \frac{1}{2\pi}\left[\frac{\sin\left((\mu^n+1)\phi\right)}{\mu^n+1} + \frac{\sin\left((\mu^n-1)\phi\right)}{\mu^n-1}\right]_0^{\pi} \\ &= 0. \end{aligned} \tag{143}$$

Derivation of $\operatorname{cov}[x_j^2, x_k^2]$

The autovariance of $\{x_k^2\}$ is given by

$$\operatorname{cov}\left[x_j^2, x_k^2\right] = E\left[x_j^2 x_k^2\right] - E\left[x_j^2\right] E\left[x_k^2\right]. \tag{144}$$

We consider the case where $j \neq k$. Without loss of generality, assume $k = j + n$ for some positive integer n.

$$E\left[x_j^2 x_k^2\right] = \int_{-\infty}^{\infty} \rho(x) x^2 \left(h^{(n)}(x)\right)^2 dx = \int_{-1}^{1} \frac{1}{\pi\sqrt{1-x^2}} x^2 \left(h^{(n)}(x)\right)^2 dx. \tag{145}$$

Making the substitution $x = \cos\phi$, (145) becomes

$$\begin{aligned}E\left[x_j^2 x_k^2\right] &= \int_{\pi}^{0} \frac{1}{\pi\sin\phi} \cos^2\phi \left(h^{(n)}(\cos\phi)\right)^2 (-\sin\phi) d\phi \\ &= \frac{1}{\pi}\int_0^{\pi} \cos^2\phi \left(h^{(n)}(\cos\phi)\right)^2 d\phi \\ &= \frac{1}{\pi}\int_0^{\pi} \cos^2\phi \cos^2(\mu^n\phi) d\phi \\ &= \frac{1}{\pi}\int_0^{\pi} \left(\frac{1+\cos 2\phi}{2}\right)\left(\frac{1+\cos(2\mu^n\phi)}{2}\right) d\phi \\ &= \frac{1}{4\pi}\int_0^{\pi} \Big(1 + \cos 2\phi + \cos(2\mu^n\phi) \\ &\quad + \frac{1}{2}\cos\left(2\phi(\mu^n+1)\right) \\ &\quad + \frac{1}{2}\cos\left(2\phi(\mu^n-1)\right)\Big) d\phi \\ &= \frac{1}{4\pi}\Big[\phi + \frac{1}{2}\sin 2\phi + \frac{1}{2\mu^n}\sin(2\mu^n\phi) \\ &\quad + \frac{1}{4(\mu^n+1)}\sin\left(2\phi(\mu^n+1)\right) \\ &\quad + \frac{1}{4(\mu^n-1)}\sin\left(2\phi(\mu^n-1)\right)\Big]_0^{\pi} \\ &= \frac{1}{4}. \end{aligned} \tag{146}$$

Simliarly it is readily shown that

$$E\left[x_j^2\right] = E\left[x_k^2\right] = \int_{-\infty}^{\infty} \rho(x) x^2 dx = \int_{-1}^{1} \frac{1}{\pi\sqrt{1-x^2}} x^2 dx = \frac{1}{2}. \tag{147}$$

Putting (146) and (147) into (144), it is proved that the autovariance for $\{x_k^2\}$ is vanishing for the Chebyshev map of degree larger than one.

Derivation of $E[x_j x_k^2]$

When $j = k$

$$E\left[x_j x_k^2\right] = E\left[x_k^3\right] = \int_{-\infty}^{\infty} x^3 \rho(x) dx = \int_{-1}^{1} x^3 \rho(x) dx = 0 \tag{148}$$

because $\rho(x)$ is an even function whereas x^3 is odd. Next, we consider the case where $j < k$. Assume $k = j + n$ for some positive integer n and we obtain

$$\begin{aligned}E\left[x_j x_k^2\right] &= \int_{-\infty}^{\infty} \rho(x) x \left(h^{(n)}(x)\right)^2 dx \\ &= \int_{-1}^{1} \frac{1}{\pi\sqrt{1-x^2}} x \left(h^{(n)}(x)\right)^2 dx \\ &= \frac{1}{\pi}\int_0^{\pi} \cos\phi \left(h^{(n)}(\cos\phi)\right)^2 d\phi \\ &= \frac{1}{\pi}\int_0^{\pi} \cos\phi \cos^2(\mu^n\phi) d\phi \\ &= 0 \end{aligned} \tag{149}$$

in which the substitution $x = \cos\phi$ has been made. Thus, it can be concluded that $E[x_j x_k^2] = 0$ for $j < k$. Finally, for $j > k$, we assume $j = k + m$ for some positive integer m. Making the substitution $x = \cos\phi$ and applying (141), we have

$$\begin{aligned}E\left[x_{k+m} x_k^2\right] &= \int_{-\infty}^{\infty} \rho(x) x^2 h^{(m)}(x) dx \\ &= \frac{1}{\pi}\int_0^{\pi} \cos^2\phi \cos(\mu^m\phi) d\phi \end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{2\pi}\int_0^{\pi}\left(\cos(\mu^m\phi) + \frac{\cos(\mu^m+2)\phi}{2} + \frac{\cos(\mu^m-2)\phi}{2}\right)d\phi \\
&= \frac{1}{4\pi}\int_0^{\pi}\cos(\mu^m-2)\phi d\phi \\
&= \begin{cases} 0, & \text{for } m \geq 2 \text{ and all } \mu \geq 2 \\ 0, & \text{for } m = 1 \text{ and } \mu > 2 \\ \frac{1}{4}, & \text{for } m = 1 \text{ and } \mu = 2. \end{cases}
\end{aligned} \tag{150}
$$

Thus, we conclude that for the Chebyshev map of degree larger than one

$$E\left[x_j x_k^2\right] = 0, \qquad \text{for} \quad j - k \neq 1. \tag{151}$$

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