

# *Using The Simplex Method For Optimizing Some Economic Functions Of The Enterprise "MSA Kompani" Dooel-Skopje*

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**Abstract** – In science, in economics, in practice, problems are often encountered in which it is required to find the largest (smallest) value of a numerical function with real values of the type  $f: E \rightarrow \mathbb{R}$ . More precisely, the elements of the set  $E$  are required, where the function  $f$  reaches the optimal value. For this reason, such problems are called optimization problems. Optimization problems are also solved by so-called linear programming methods.

Linear programming is a mathematical programming that deals with the problem of system optimization within given constraints, it is a method for solving production planning problems.

The manufacturer wants to determine how to use the limited quantities of raw materials with maximum profit, the manager how to distribute the assigned work among his employees so that it is done in the shortest possible time, that is, to be as effective as possible. The goal of these problems is optimization, profit maximization or cost minimization [1].

Making profit is the main goal of the company, but many companies still need to learn the maximum profit that can be achieved by optimizing their resources, one of which is the company “MSA KOMPANI”. This research focuses on the products of “MSA KOMPANI” DOOEL-SKOPJE

different according to the price. The purpose of this paper is to optimize the company's profits and costs. In this paper we will use the simplex linear programming method to solve our problem in “MSA KOMPANI”.

Modeling a real-life problem as a linear programming problem requires teamwork of experts from several fields.

**Keywords** – Optimation, Profit, Income, Enterprise, Simplex Method, Linear Programming.

## I. INTRODUCTION

The focus of this research is on the products of the company “MSA KOMPANI” DOOEL-SKOPJE. This study aims to optimize profits, revenues, by determining the composition of the number of products produced. There are many ways to solve this problem, one of which is using linear programming, specifically the simplex method.

A linear program is a mathematical model for determining the optimal combination of products to maximize benefits or minimize costs. A linear program has three essentials: the objective function, the decision variable, and the constraints.

In the beginning, we build the mathematical model from the data of the real problem and then from these data we form the first simplex table. There is often more than one way to describe a particular algorithm. The simplex method is one of the techniques for determining the optimal solution used in linear programming. The simplex method is a general method, which means that it solves any linear programming problem.

The simplex method is an iterative method with a finite number of steps starting from an initial solution and in the last step of the iterations, the optimal solution is obtained. This is the reason why the simplex method is easily applied to the computer as well.

II. FORMULATION OF THE LINEAR PROGRAMMING PROBLEM. MATHEMATICAL MODEL

In science, in economics, in practice, problems are often encountered in which it is required to find the largest (smallest) value of a numerical function with real values of the type  $f: E \rightarrow \mathbb{R}$ . More precisely, the elements of the set  $E$  are required, where the function  $f$  reaches the optimal value. For this reason, such problems are called optimization problems. Such a function is called a goal function. In general it is a function of  $n$  variables  $x_1, x_2, \dots, x_n$ , t.e. its starting set is a subset of Euclidean space  $\mathbb{R}^n$  of vectors  $x = (x_1, x_2, \dots, x_n)$  with  $n$  dimensions, t.e.  $E \subset \mathbb{R}^n$ . Wide practical use represents the case when the objective function  $f$  is a linear function of the variables  $x_1, x_2, \dots, x_n$ :

$$f(x) = c_1 x_1 + c_2 x_2 + \dots + c_n x_n, \tag{1}$$

for  $x = (x_1, x_2, \dots, x_n) \in E$ . The starting set  $E$  is determined by imposing various conditions on the variables  $x_j$  for  $j = 1, 2, \dots, n$ . In particular,  $E$  is the set of solutions of the following  $m$  system of linear equations or inequations of variables  $x_j$ :

$$\begin{cases} \alpha_{11}x_1 + \alpha_{12}x_2 + \dots + \alpha_{1n}x_n \{ \leq, =, \geq \} \beta_1, \\ \alpha_{21}x_1 + \alpha_{22}x_2 + \dots + \alpha_{2n}x_n \{ \leq, =, \geq \} \beta_2, \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots \\ \alpha_{m1}x_1 + \alpha_{m2}x_2 + \dots + \alpha_{mn}x_n \{ \leq, =, \geq \} \beta_m. \end{cases} \tag{2}$$

Usually variables  $x_j$  in practice they take non-negative values, therefore the condition is added to the system (2) of conditions:

$$x_1 \geq 0, x_2 \geq 0, \dots, x_n \geq 0. \tag{3}$$

Any solution of system (2) that satisfies condition (3) is called an *allowed solution* of system (2), while the its allowed solution  $(x_1^0, x_2^0, \dots, x_n^0)$ , where the function  $f$  reaches the largest or smallest value is called the *optimal solution of the problem* (1), (2), (3). The set of allowed solutions of system (2) is denoted  $\Omega$  and is called the *allowed set (area) of problem* (1), (2), (3).

In this case the problem of optimizing the function, using the symbol  $\Sigma$  of an adder, takes the form of the following mathematical model:





PRODUCE	PRODUCTI ON TIME FOR UNIT	ASSEMBL Y TIME FOR UNIT	NUMBE R OF WORKE RS PER UNIT	REQUES T/ DAY	OFFER / DAY	PROFIT/ UNIT	SELLING PRICE/ UNIT	COST/ UNIT
A) SCS 01 playground	8 hour	4 hour	4	1	5	30000 den	80600.00 den.	50600 den
B)Pencil fence	10 minutes	5 minutes	3	30	10	800 den	2800.00 den	2000 den
C) Swing	3 hour	1 hour	4	1	5	12000 den	45000.den	33000 den
D) Soft blocks	24 hour	0	1	0.5	2	35000 den	150000 den	115000 den
E) seesaw	3 hour	1 hour	3	1	5	5000 den	18000 den	13000 den

The company works in one shift. From the above data we calculated that the company has available 120 hours of work per day, of which Production and Assembly are in the ratio P:M=38,25:6,083. We solve the system  $\begin{cases} P + M = 120 \\ P : M = 38,25 : 6,083 \end{cases}$  and we get that the company has 103,5 hours of daily capacity for production and 16,083 daily capacity hours for assembly.

- *We compile the mathematical model for maximizing the company's daily profit according to the offer presented:*

Decision variables:

$x_1$  = Quantity to be produced of the type A

$x_2$  = Quantity to be produced of the type B

$x_3$  = Quantity to be produced of the type B

$x_4$  = Quantity to be produced of the type B

$x_5$  = Quantity to be produced of the type B

Mathematical modeling of our case

Determine the quantities  $x_1, x_2, x_3, x_4$  and  $x_5$ , with restrictions:

Total units of production:  $x_1 + x_2 + x_3 + x_4 + x_5 \leq 27$

Total cost:  $50600 x_1 + 2000 x_2 + 33000 x_3 + 115000 x_4 + 13000 x_5 \leq 733000$

Total time:  $12 x_1 + 0.25 x_2 + 4 x_3 + 24 x_4 + 4 x_5 \leq 120$

$x_1, x_2, x_3, x_4, x_5 \geq 0$

To maximize the profit function

$F = 30000 x_1 + 800 x_2 + 12000 x_3 + 35000 x_4 + 5000 x_5$

So we have:

To maximize the objective function

$$F(x) = 30000x_1 + 800x_2 + 12000x_3 + 35000x_4 + 5000x_5$$

Under the conditions:

$$\begin{cases} x_1 + x_2 + x_3 + x_4 + x_5 \leq 27 \\ 50600x_1 + 2000x_2 + 33000x_3 + 115000x_4 + 13000x_5 \leq 733000 \\ 12x_1 + 0.25x_2 + 4x_3 + 24x_4 + 4x_5 \leq 120 \end{cases}$$

Where,  $x_1, x_2, x_3, x_4, x_5 \geq 0$ .

We take the additional variables and build the system:

$$\begin{cases} x_1 + x_2 + x_3 + x_4 + x_5 + s_1 = 27 \\ 50600x_1 + 2000x_2 + 33000x_3 + 115000x_4 + 13000x_5 + s_2 = 733000 \\ 12x_1 + 0.25x_2 + 4x_3 + 24x_4 + 4x_5 + s_3 = 120 \\ -30000x_1 - 800x_2 - 12000x_3 - 35000x_4 - 5000x_5 + F(x) = 0 \end{cases}$$

We build the first simplex table:

Base	x1	x2	x3	x4	x5	s1	s2	s3	F	T.L
s1	1	1	1	1	1	1	0	0	0	27
s2	50600	2000	33000	115000	13000	0	1	0	0	733000
s3	12	0.25	4	24	4	0	0	1	0	120
F	-30000	-800	-12000	-35000	-5000	0	0	0	1	0

**Step 1:**

In the fourth row and the columns of  $x$ , we look for the largest negative number in absolute value, which is  $|-35000|=35000$  and the  $x_4$  column is the key column. We divide the free terms by the corresponding coefficients of the  $x_4$  column and get:

$$\frac{120}{24} = 5; \frac{733000}{115000} = 6.374, \frac{27}{1} = 27$$

We look for the smallest factor which is 5 and fix the row of  $s_3$ , we see that the fixed element is 24. Since the key must always be 1, then we divide the third row (the  $s_3$ ) with 24 and we get:

Base	x1	x2	x3	↓x4	x5	s1	s2	s3	T.L
s1	1	1	1	1	1	1	0	0	27
s2	50600	2000	33000	115000	13000	0	1	0	733000
←s3	0.5	1/96	1/6	[1]	1/6	0	0	1/24	5
F	-30000	-800	-12000	-35000	-5000	0	0	0	0

Get out of the base s3, enter on the base x4. We perform the transformations:  $R_1 - R_3$ ,  $R_2 - 115000R_3$  and  $R_4 + 35000R_3$  and we get:

Base	x1	x2	x3	x4	x5	s1	s2	s3	T.L
s1	0.5	0.99	0.83	0	0.83	1	0	-0.04	22
s2	-6900	802.08	13833.33	0	-6166.67	0	1	-4791.67	158000
x4	0.5	0.01	0.17	1	0.17	0	0	0.04	5
F	-12500	-435.42	-6166.67	0	833.33	0	0	1458.33	175000

Thus the first step of the simplex algorithm is completed.

**Step 2:** We act the same as in the step 1 and fix the element 0.5. To make it 1, we multiply the third row (the x4) with 2.

Base	↓x1	x2	x3	x4	x5	s1	s2	s3	T.L
s1	0.5	0.99	0.83	0	0.83	1	0	-0.04	22
s2	-6900	802.08	13833.33	0	-6166.67	0	1	-4791.67	158000
←x4	[1]	0.02	0.34	2	0.34	0	0	0.08	10
F	-12500	-435.42	-6166.67	0	833.33	0	0	1458.33	175000

Get out of the base x4, enter on the base x1. We perform the transformations:  $R_1 - 1/2R_3$ ,  $R_2 + 6900R_3$  and  $R_4 - 12500R_3$  and we get:

Base	x1	x2	x3	x4	x5	s1	s2	s3	T.L
s1	0	0.98	0.67	-1	0.67	1	0	-0.08	17
s2	0	945.83	16133.33	13800	-3866.67	0	1	-4216.67	227000
x1	1	0.02	0.33	2	0.33	0	0	0.08	10
F	0	-175	-2000	25000	5000	0	0	2500	300000

Thus the second step of the simplex algorithm is completed.

**Step 3.** We act the same as in the step 1 and fix the element 16133.33. To make the key 1, we divide the second row (the s<sub>2</sub>) with 16133.33 and we get:

Base	x1	x2	↓x3	x4	x5	s1	s2	s3	T.L
s1	0	0.98	0.67	-1	0.67	1	0	-0.08	17
←s <sub>2</sub>	0	0.06	[1]	0.86	-0.24	0	0	-0.26	14.07
x1	1	0.02	0.33	2	0.33	0	0	0.08	10
F	0	-175	-2000	25000	5000	0	0	2500	300000

Get out of the base s<sub>2</sub>, enter on the base x<sub>3</sub>. We perform the transformations:  $R_1 - 0.67R_2$ ,  $R_3 - 0.33R_2$  and  $R_4 + 2000R_2$  and we get:

Base	x1	x2	x3	x4	x5	s1	s2	s3	T.L
s1	0	<b>0.94</b>	0	-1.57	0.83	1	0	0.09	7.62
x <sub>3</sub>	0	0.06	1	0.86	-0.24	0	0	-0.26	14.07
x1	1	0	0	1.71	0.41	0	0	0.17	5.31
F	0	-57.75	0	26710.74	4520.66	0	0.1 2	1977.27	328140.5

Thus the third step of the simplex algorithm is completed.

**Step 4.** We act the same as in the step 1 dhe fiksojmë elementin **0.94**. To make the key 1, we divide the first row (the s<sub>1</sub>) with 0.94 and we get:



Base	x <sub>1</sub>	↓x <sub>2</sub>	x <sub>3</sub>	x <sub>4</sub>	x <sub>5</sub>	s <sub>1</sub>	s <sub>2</sub>	s <sub>3</sub>	T.L
←s <sub>1</sub>	0	[1]	0	-1.67	0.88	1.06	0	0.1	8.11
x <sub>3</sub>	0	0.06	1	0.86	-0.24	0	0	-0.26	14.07
x <sub>1</sub>	1	0	0	1.71	0.41	0	0	0.17	5.31
F	0	-57.75	0	26710.74	4520.66	0	0.12	1977.27	328140.5

Get out of the base s<sub>1</sub>, enter on the base x<sub>2</sub>. We perform the transformations:  $R_2 - 0.06R_1$ ,  $R_4 + 57.75R_1$  and we get:

Base	x <sub>1</sub>	x <sub>2</sub>	x <sub>3</sub>	x <sub>4</sub>	x <sub>5</sub>	s <sub>1</sub>	s <sub>2</sub>	s <sub>3</sub>	T.L
x <sub>2</sub>	0	1	0	-1.67	0.88	1.06	0	0.1	8.11
x <sub>3</sub>	0	0	1	0.95	-0.29	-0.06	0	-0.27	13.6
x <sub>1</sub>	1	0	0	1.72	0.41	0	0	0.17	5.3
F	0	0	0	26614.29	4571.43	61.43	0.12	1982.86	328608.57

Since the row of F has no negative values, then the algorithm ends.

We conclude that the optimal solution is reached for  $x_1=5.3$ ,  $x_2=8.11$ ,  $x_3=13.6$ ,  $x_4=0$  and  $x_5=0$  as well as the maximum profit per day that can be achieved is  $\max F=328608.57$  den.

V. CONCLUSIONS:

Based on the calculation using the simplex linear programming method and verifying the results also with the help of the online software <https://linprog.com/>, the following conclusions can be drawn:

1. To maximize profits, the company should produce 5.3 units of type A) SCS01 playground, 8.11 units of type B) Pencil fence and 13.6 units of type C) Swing, while from two other products from 0 units
2. The maximum profit received in one day in this case will be 328608.57 den. from 178000den that it was at the beginning according to the offer made, representing an enormous increase in profits of 185%.
3. Under optimal conditions, the total production cost increased to 733200 den (initially 733000 den) and the required production time was the same as before the study, which represents a negligible cost increase of 0.03%.

• We compile the mathematical model for minimizing the cost of production, so that the daily profit according to the company's request is preserved:

Mathematical modeling:

Determine the quantities  $x_1$ ,  $x_2$ ,  $x_3$ ,  $x_4$  and  $x_5$ , with restrictions:

Total units of production:  $x_1 + x_2 + x_3+x_4+x_5 \leq 27$

Profit:  $30000 x_1 + 800 x_2 + 12000 x_3 + 35000 x_4 + 5000 x_5 = 88500$

Total time:  $12 x_1 + 0.25 x_2 + 4 x_3 + 24 x_4 + 4 x_5 \leq 120$

$x_1, x_2, x_3, x_4, x_5 \geq 0$

So we have:

To minimize the cost function

$$C = 50600x_1 + 2000x_2 + 33000x_3 + 115000x_4 + 13000x_5$$

Under the conditions:

$$\begin{cases} x_1 + x_2 + x_3 + x_4 + x_5 \leq 27 \\ 30000x_1 + 800x_2 + 12000x_3 + 35000x_4 + 5000x_5 = 88500 \\ 12x_1 + 0.25x_2 + 4x_3 + 24x_4 + 4x_5 \leq 120 \end{cases}$$

Where,  $x_1, x_2, x_3, x_4, x_5 \geq 0$ .

We also verified the results of this linear programming minimization problem in the online software <https://linprog.com/>. Take:

We add the additional variables and get:

$$C = 50600x_1 + 2000x_2 + 33000x_3 + 115000x_4 + 13000x_5 + 0s_1 + 0s_2 + Ms_3$$

Under the conditions:

$$\begin{cases} x_1 + x_2 + x_3 + x_4 + x_5 + s_1 = 27 \\ 30000x_1 + 800x_2 + 12000x_3 + 35000x_4 + 5000x_5 + s_3 = 88500 \\ 12x_1 + 0.25x_2 + 4x_3 + 24x_4 + 4x_5 + s_2 = 120 \end{cases}$$

The initial table is:

<b>Basis</b>	<b>x1</b>	<b>x2</b>	<b>x3</b>	<b>x4</b>	<b>x5</b>	<b>s1</b>	<b>s2</b>	<b>s3</b>	<b>T.L</b>
<b>s1</b>	1	1	1	1	1	1	0	0	27
<b>s3</b>	30000	800	12000	35000	5000	0	0	1	88500
<b>s2</b>	12	0.25	4	24	4	0	1	0	120
<b>Min C</b>	30000M- -50600	800M- 2000	12000M- 33000	35000M- 115000	5000M- 13000	0	0	0	88500M

**Step 1.** We set the key which is 35000, we divide the corresponding row by 35000.

<b>Basis</b>	<b>x<sub>1</sub></b>	<b>x<sub>2</sub></b>	<b>x<sub>3</sub></b>	<b>↓x<sub>4</sub></b>	<b>x<sub>5</sub></b>	<b>s<sub>1</sub></b>	<b>s<sub>2</sub></b>	<b>s<sub>3</sub></b>	<b>T.L</b>
<b>s<sub>1</sub></b>	1	1	1	1	1	1	0	0	27
<b>←s<sub>3</sub></b>	0.86	0.02	0.34	<b>[1]</b>	0.14	0	0	0	2.53
<b>s<sub>2</sub></b>	12	0.25	4	24	4	0	1	0	120
<b>Min C</b>	30000M-50600	800M-2000	12000M-33000	35000M-115000	5000M-13000	0	0	0	88500M

We perform the necessary transformations. At the base enters x<sub>4</sub>, while s<sub>3</sub> exits.

After the appropriate calculations are done, we move on

**Step 2.** We set the key which is **0.86**, we divide the corresponding row by 0.86.

<b>Basis</b>	<b>↓x<sub>1</sub></b>	<b>x<sub>2</sub></b>	<b>x<sub>3</sub></b>	<b>x<sub>4</sub></b>	<b>x<sub>5</sub></b>	<b>s<sub>1</sub></b>	<b>s<sub>2</sub></b>	<b>s<sub>3</sub></b>	<b>T.L</b>
<b>s<sub>1</sub></b>	0.14	0.98	0.66	0	0.86	1	0	0	24.47
<b>←x<sub>4</sub></b>	<b>[1]</b>	0.03	0.4	1.17	0.17	0	0	0	2.95
<b>s<sub>2</sub></b>	-8.57	-0.3	-4.23	0	0.57	0	1	0	59.31
<b>Min C</b>	47971.43	628.57	6428.57	0	3428.57	0	0	-M+3.29	290785.71

We perform the necessary transformations. At the base enters x<sub>1</sub>, while x<sub>4</sub> exits.

After the appropriate calculations are done, we move on

**Step 3:**

<b>Basis</b>	<b>x<sub>1</sub></b>	<b>x<sub>2</sub></b>	<b>x<sub>3</sub></b>	<b>x<sub>4</sub></b>	<b>x<sub>5</sub></b>	<b>s<sub>1</sub></b>	<b>s<sub>2</sub></b>	<b>s<sub>3</sub></b>	<b>T.L</b>
<b>s<sub>1</sub></b>	0	0.97	0.6	-0.17	0.83	1	0	0	24.05
<b>x<sub>1</sub></b>	1	0.03	0.4	1.17	0.17	0	0	0	<b>2.95</b>
<b>s<sub>2</sub></b>	0	-0.07	-0.8	10	2	0	1	0	84.6
<b>Min C</b>	0	-650.67	-12760	-55966.67	-4566.67	0	0	-M+1.69	<b>149270</b>

The algorithm is completed and the results obtained are: Min C=149270 which is achieved for x<sub>1</sub>=2.95 and others 0.

From which we get that the cost can be reduced to 149270 den for the same profit of 88500 den from 214100 den that was at the beginning, if we manage to sell 2.95 units of production SCS01 playground. So,

in this case, we will have a cost reduction of 64830 den, which would be transferred to the coffers of the enterprise as profit, and the total amount of profit will be 153330 den.

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