Applying Adomian Decomposition Method for solving the Covid-19 epidemic with Vaccine

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Abstract— A pandemic Covid-19 is an epidemic that spreads over a big region, Crosse international borders, and often affects a lot of people. Only a few pandemics result in severe illness in a subset of people or in an entire community. The virus has mainly affects the elderly population that causes Covid-19, has mainly been transmitted through droplet generate once an infected persons exhales, sneezes and coughs. These symptoms are too heavy; to hang in air, and quickly, fall on surface or floor. The COVID-19 model including the Vaccination Campaign is of natural phenomenon which can be represented as a system of differential equations for the first order; the mathematical models include a system of several second order nonlinear equations. We applied the Adomian decomposition methods to the mathematical models of Covid-19. The main advantage of this method is that it can be directly applied to all kinds of linear and nonlinear differential equations, homogeneous or nonhomogeneous, with constant or variable coefficients. The solutions of the model are non-negativity. It indicates that the, infection, will be gradually the epidemic and disappear will, stop. If, R_0>1, the average of each affected individually. More than one person has infected, and the incidence of infection is in wrinkles. That means the epidemic, will not be end, while maintain the existence of the disease, the R_0=1 means that each infected patient results in an average infections.

Keywords—ADM, Covid-19, epidemic, mathematical modeling

1. Introduction

Coronavirus (Covid-19) was a persistent global, epidemic of the Covid-19, which broke out, on December, 2019 in Wuhan city, China. On February 2020, it has been begun to spreading rapidly worldwide. The ASARS-COV-2 virus strain transmitted from infected animal to human, it causes the infection of 10 million people, and the death of hundred thousands of people worldwide. This is according, to statistics that reported, by country authorities, including the local Health Ministry [1-15]. The Covid-19 has 3 types which are also known, as beta, gamma and alpha. There is also, another type that is called, delta SARS- CoV. Humans' coronavirus was first identified in 1960s [16]. The virus has mainly affects the elderly population [1]. The virus, that causes Covid-19, has mainly been transmitted through droplet generate once an infected persons exhales, sneezes and coughs. These symptoms are too heavy; to hang in air, and quickly, fall on surface or floor.

Studies have been discussed the international trade as the driver of virus spreads [17]. Some studies have discussed SARS-CoV-2, and the corresponding diseases [18-23]. Many studies have been discussed the matter of the effective reproduction number [24-25]. The research tries to find the connection between environmental factors and spread of Covid-19 [26]. Also, study the clinical feature of corona virus, and discuss the outcomes in short term of eighteen patients of these patients with the Covid-19 are in intensive, care unit. Instruction has been given on keep the distance, of at least two meters, from other person. The virus was spread by shard of saliva emitted by sneezing, coughing, speech from a person, who infested with the virus. The incubations time were on the average five days. Thus, the health ministry has been decided to the isolation, of at the least 14 days, from moment of the discovery, of the virus. During that time, carrier person can be infecting others. Symptoms are usually shortness of breath, cough and fever. In the elderly, populations, complications might like kidney failure, blood clots accumulation, and pneumonia. There was no specific drag or vaccine against the Covid-19; however, studies were being ducted worldwide [28]. Greater measures may be needed for reduce the airborne transmission of the virus including regular disinfection and room ventilation. It is effectively limit the concentrations of SARS-CoV-2 in aerosol. But, most of the period, social distancing was not respected. Recently, the sanitary protection, that is providing by the universal wearing of mask represent a promising practices for reducing transmission, of Covid-19 infections.

The epidemiological result has been provided at the beginning of pandemics by number of countries, including Singapore, South Korea, Japan, Taiwan, Hong Kong. It shows efficacy of universal, masking as, the sort of a control measures, even in absence, of the severe lockdown over the pandemic. The necessity, of the mask, that against asymptomatic spreads in aerosols, and droplets. The evidence was provided, of fact that no wearing masks maximize exposure, whereas the least, levels of exposure were achieved through a universal mask. The universal use, of face masks helps to reduce the size, of the spread particles from, 100μm to 1–0.1μm, especially in asymptomatic, humans and those with, the mild symptoms. The aim of the study was to estimate mathematical modeling depends on the mathematical language that describes the behavior of a natural phenomenon .

2. COVID-19 Mathematical Model

Mathematical modeling depends on the mathematical language that describes the behavior of a natural phenomenon. The COVID-19 pandemic model including the Vaccination Campaign is of natural phenomenon which can be represented as a system of differential equations for the first order, the model is formed (1)[16].

The population of COVID-19 pandemic model is divided into five individuals named compartments, $S(t)$, $E(t)$, $I(t)$, $V(t)$ and $R(t)$, so this model is made ($SEIVR$),[16]. The first stage; Susceptible population is the healthy individuals with the symbol (S), the second stage; Exposed population; the infected individuals in the incubation period who do not show symptoms of disease and are denoted by (E). The third stage; the Infected population is the infected individuals and the given rise to infection after incubation interval and symbolized by (I), the fourth stage; the Vaccinated population is the individuals who take the vaccination and symbolized by (V), the fifth stage; Recovered population are the individuals who die or recover which are coded by (R). The derivatives of all compartments of the coronavirus model are continuous at $t \geq 0$. The solutions of the model are non-negativity. The system of COVID-19 understudy is offered in (1), [16]:

$$\frac{dS(t)}{dt} = \Lambda - (\alpha E + m + \mu)S. \qquad (2.1)$$

$$\frac{dE(t)}{dt} = \alpha SE + pVE - (fI + c + \mu)E. \qquad (2.2)$$

$$\frac{dI(t)}{dt} = fEI - (z + \mu + \sigma)I. \qquad (2.3)$$

$$\frac{dV(t)}{dt} = mS - (pE + \mu)V. \qquad (2.4)$$

$$\frac{dR(t)}{dt} = zI + cE - \mu R. \qquad (2.5)$$

where Table (1) has a description of the parameters of the COVID-19 model.

Table (1) Description parameters of the COVID-19 model with their values

Parameter	**Description**	**Value of Parameters**	**References**
Λ	Recruitment rate of COVID-19 viruses	50	[29]
α	Rate of transition from susceptible individuals to exposed individuals	0.002	estimated
m	Proportion of vaccinated susceptible individuals	0.5	estimated
f	Rate at which exposed people become infected (I)	0.008	estimated
p	Disease exposure rate for vaccinated individuals	0.08	estimated
z	The recovery rate of infected individuals	0.012	estimated
μ	Natural death rate	0.009	estimated
c	Recovery rate of exposed individuals	0.05	estimated
σ	Disease-related mortality	0.25	estimated

3. Solving COVID-19 Model by Analytical Method

A numerical method is a process to find the solutions approximately at specified points, most initial value problems can be solved numerically like the COVID-19 epidemic system in the present work. In this section, the Adomian decomposition method (ADM) can be utilized to solve the COVID-19 epidemic problem in waist in 2020. First, the form of ADM is presented below, the general formula of S_{k+1} $E_{k+1} I_{k+1}$ V_{k+1} and R_{k+1} for the ADM method is:

$$S_{k+1} = l^{-1}(\Lambda - \alpha A_k - mS_k - \mu S_k). k \geq 0 \cdot \tag{3.1}$$

$$E_{k+1} = l^{-1}(\alpha A_k + pB_k - fC_k - cE_k - \mu E_k). k \geq 0 \cdot \tag{3.2}$$

$$I_{k+1} = l^{-1}(fC_k - zI_k - \mu I_k - \sigma I_k). k \geq 0 \cdot \tag{3.3}$$

$$V_{k+1} = l^{-1}(mS_k - pB_k - \mu V_k). k \geq 0 \cdot \tag{3.4}$$

$$R_{k+1} = l^{-1}(zI_k + cE_k - \mu R_k). k \geq 0 \cdot \tag{3.5}$$

The general form of the nonlinear borders A_k, B_k and C_k have to be: by Using the Alternative Method

$$A_k = \left(\sum_{n=0}^{\infty} E_n\right)\left(\sum_{n=0}^{\infty} S_n\right). k = 0.1.2. \ldots$$

$$A_k = (E_0 + E_1 + E_2 + \cdots)(S_0 + S_1 + S_2 + \cdots).$$

$$= E_0S_0 + E_0S_1 + E_0S_2 + \cdots + E_1S_0 + E_1S_1 + E_1S_2 + \cdots + E_2S_0 + E_2S_1 + E_2S_2 + \cdots$$

$$B_k = \left(\sum_{n=0}^{\infty} E_n\right)\left(\sum_{n=0}^{\infty} V_n\right). k = 0.1.2. \ldots$$

$$B_k = (E_0 + E_1 + E_2 + \cdots)(V_0 + V_1 + V_2 + \cdots).$$

$$= E_0V_0 + E_0V_1 + E_0V_2 + \cdots + E_1V_0 + E_1V_1 + E_1V_2 + \cdots + E_2V_0 + E_2V_1 + E_2V_2 + \cdots$$

$$C_k = \left(\sum_{n=0}^{\infty} E_n\right)\left(\sum_{n=0}^{\infty} I_n\right). k = 0.1.2. \ldots$$

$$C_k = (E_0 + E_1 + E_2 + \cdots)(I_0 + I_1 + I_2 + \cdots).$$

$$= E_0I_0 + E_0I_1 + E_0I_2 + \cdots + E_1I_0 + E_1I_1 + E_1I_2 + \cdots + E_2I_0 + E_2I_1 + E_2I_2 + \cdots$$

Substituting for all A_0 and S_0 by Eq. (3.1), A_0 B_0 C_0 and E_0 by Eq. (3.2), C_0 and I_0 by Eq. (3.3), S_0 B_0 and V_0 by Eq. (3.4), I_0 E_0 and R_0 Eq. (3.5), to obtain :

$$S_1 = l^{-1}(\Lambda - \alpha A_0 - mS_0 - \mu S_0). k \geq 0 \cdot$$

$$E_1 = l^{-1}(\alpha A_0 + pB_0 - fC_0 - cE_0 - \mu E_0). k \geq 0 \cdot$$

$I_1 = l^{-1}(fC_0 - zI_0 - \mu I_0 - \sigma I_0).\, k \geq 0 \cdot$

$V_1 = l^{-1}(mS_0 - pB_0 - \mu V_0).\, k \ \ \geq 0 \cdot$

$R_1 = l^{-1}(zI_0 + cE_0 - \mu R_0).\, k \geq 0 \cdot$

Now we find $S_2. E_2. I_2. V_2$ and R_2.

Substituting for all A_1 and S_1 by Eq. (3.1), $A_1. B_1. C_1$ and E_1 by Eq. (3.2), C_1 and I_1 by Eq. (3.3), $S_1. B_1$ and V_1 by Eq. (3.4), $I_1. E_1.$ and R_1 Eq. (3.5), to find :

$S_2 = l^{-1}(\Lambda - \alpha A_1 - mS_1 - \mu S_1).\, k \geq 0 \cdot$

$E_2 = l^{-1}(\alpha A_1 + pB_1 - fC_1 - cE_1 - \mu E_1).\, k \geq 0 \cdot$

$I_2 = l^{-1}(fC_1 - zI_1 - \mu I_1 - \sigma I_1).\, k \geq 0 \cdot$

$V_2 = l^{-1}(mS_1 - pB_1 - \mu V_1).\, k \ \ \geq 0 \cdot$

$R_2 = l^{-1}(zI_1 + cE_1 - \mu R_1).\, k \geq 0 \cdot$

Now we find $S_3. E_3. I_3. V_3$ and R_3.

Substituting for all A_2 and S_2 by Eq. (3.1), $A_2. B_2. C_2$ and E_2 by Eq. (3.2), C_2 and I_2 by Eq. (3.3), $S_2. B_2$ and V_2 by Eq. (3.4), $I_2. E_2.$ and R_2 Eq. (3.5), to get :

$S_3 = l^{-1}(\Lambda - \alpha A_2 - mS_2 - \mu S_2).\, k \geq 0 \cdot$

$E_3 = l^{-1}(\alpha A_2 + pB_2 - fC_2 - cE_2 - \mu E_2).\, k \geq 0 \cdot$

$I_3 = l^{-1}(fC_2 - zI_2 - \mu I_2 - \sigma I_2).\, k \geq 0 \cdot$

$V_3 = l^{-1}(mS_2 - pB_2 - \mu V_2).\, k \ \ \geq 0 \cdot$

$R_3 = l^{-1}(zI_2 + cE_2 - \mu R_2).\, k \geq 0 \cdot$

The Adomian decomposition method assumes that the unknown functions $S(t). E(t). I(t). V(t)$ and $R(t)$that can be written by series as follows:

$$S(t) = \sum_{k=0}^{\infty} S_k. \quad E(t) = \sum_{k=0}^{\infty} E_k. \quad I(t) = \sum_{k=0}^{\infty} I_k. \quad V(t) = \sum_{k=0}^{\infty} V_k. \quad R(t) = \sum_{k=0}^{\infty} R_k.$$

$$S(t) = \sum_{k=0}^{\infty} S_k = S_0 + S_1 + S_2 + S_3 + \cdots \qquad (3.6)$$

$$E(t) = \sum_{k=0}^{\infty} E_t = E_0 + E_1 + E_2 + E_3 + \cdots \qquad (3.7)$$

$$I(t) = \sum_{k=0}^{\infty} I_t = I_0 + I_1 + I_2 + I_3 + \cdots \quad (3.8)$$

$$V(t) = \sum_{k=0}^{\infty} V_t = V_0 + V_1 + V_2 + V_3 + \cdots \quad (3.9)$$

$$R(t) = \sum_{k=0}^{\infty} R_t = R_0 + R_1 + R_2 + R_3 + \cdots \quad (3.10)$$

$S(t).E(t).I(t).V(t)$ and $R(t)$ of ADM results are unsettled terms.

4. Results and Discussion

The initial conditions are taken which is as follows; S_0= 500 , E_0=0.622 I_0=0.151, V_0=0 ,R_0=0.189 The problem of the Corona epidemic is discussed in 2020. It is analyzed in this section. A model is given for COVID-19 [16] because an exact solution is not available for the current model, in Figure 4.1 the expected values were used to compare the current approximate solutions for ADM and 〖RK〗_4. We note the convergence of the numerical curve with the approximate curve, which indicates that the ADM method is an effective, convergent and reliable method for solving polynomials. Note that the population decreased in period 0.02. Then it stabilizes at one level in the period 0.04 until the epidemic will continue.

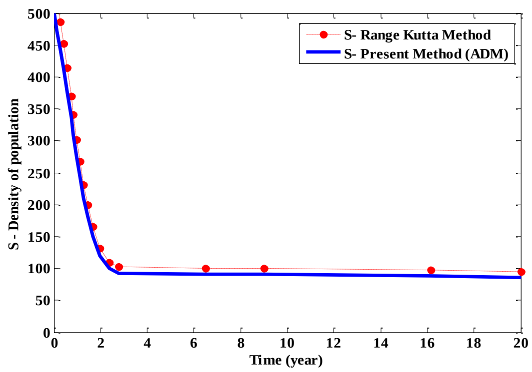

Figure 4.1: Comparison Susceptible (S) between RK4 Method and present method (ADM) for Covied-19 model in [16]

Figures 4.2 describe the behavior of each phase for the COVID19 epidemic. The behavior of S(t), the population has decreased gradually since the virus began, then will be stable at the end of 0.02 to the following years. On the other hand, it is noticeable that the E(t) has increased since the beginning of the spread of the virus, reaching its highest possible in 0.04. Then it starts declining with 0.06 to the end. After that, it settles to the same level. It is clear that the behavior of I(t) population has been increasing since the beginning of the epidemic and begins to decrease gradually after 0.04, it will maintain the same level after 0.06. At the beginning of the spread of the virus, the vaccine was not available until the end of 0.02, therefore, the behavior of the vaccine curve before the middle of 2021 should be ignored. With the vaccination campaign began, we note that the curve of V (t) gradually rised to 0.04 and remained in a noticeable rise until 0.06, then began to decline and stabilize at a certain level in 0.04 for the coming years. Finally, the curve R(t) began to increase and rise gradually until 0.06. The increase would be very little in the coming years for the curve R(t) . All the present study results agree with the previous studies [29] for all stages of the Coronavirus epidemic in the results.

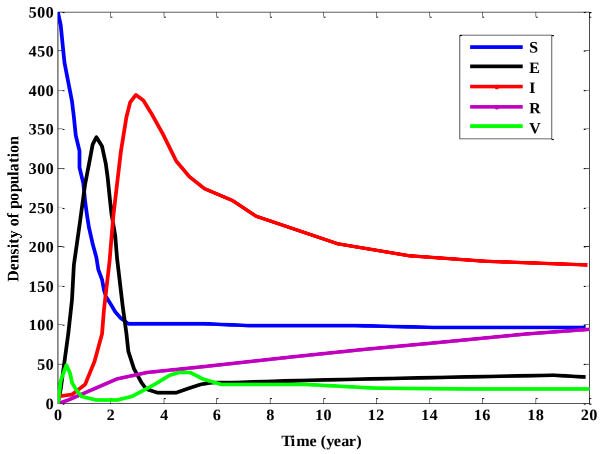

Figure 4.2: Population densities of the COVID-19 model for the estimated values in [16]

5. Conclusion

The study has conclude that the population has decreased gradually since the virus began, then will be stable at the end of 0.02 to the following years. On the other hand, it is noticeable that the E(t) has increased since the beginning of the spread of the virus, reaching its highest possible in 0.04. Then it starts declining with 0.06 to the end. Population has been increasing since the beginning of the epidemic and begins to decrease gradually after 0.04, it will maintain the same level after 0.06. At the beginning of the spread of the virus, the vaccine was not available until the end of 0.02.

References

[1] Nave, O., Shemesh, U., & HarTuv, I. (2021). Applying Laplace Adomian decomposition method (LADM) for solving a model of Covid-19. Computer Methods in Biomechanics and Biomedical Engineering, 24(14), 1618-1628.

[2] Ndaïrou, F., Area, I., Nieto, J. J., & Torres, D. F. (2020). Mathematical modeling of COVID-19 transmission dynamics with a case study of Wuhan. Chaos, Solitons & Fractals, 135, 109846.

[3] Samui, P., Mondal, J., & Khajanchi, S. (2020). A mathematical model for COVID-19 transmission dynamics with a case study of India. Chaos, Solitons & Fractals, 140, 110173.

[4] Hernandez-Vargas, E. A., & Velasco-Hernandez, J. X. (2020). In-host mathematical modelling of COVID-19 in humans. Annual reviews in control, 50, 448-456.

[5] Giordano, G., Blanchini, F., Bruno, R., Colaneri, P., Di Filippo, A., Di Matteo, A., & Colaneri, M. (2020). Modelling the COVID-19 epidemic and implementation of population-wide interventions in Italy. Nature medicine, 26(6), 855-860.

[6] Abdulkadhim, M. M., & Al-Husseiny, H. F. (2020). Global stability and bifurcation of a COVID-19 virus modeling with possible loss of the immunity. In AIP conference Proceedings (Vol. 2292, No. 1, p. 030003). AIP Publishing LLC.

[7] Zumla, A., & Niederman, M. S. (2020). The explosive epidemic outbreak of novel coronavirus disease 2019 (COVID-19) and the persistent threat of respiratory tract infectious diseases to global health security. Current opinion in pulmonary medicine.

[8] Yao, S. Y., Lei, C. Q., Liao, X., Liu, R. X., Chang, X., & Liu, Z. M. (2021). Integrated Chinese and Western medicine in treatment of critical coronavirus disease (COVID-19) patient with endotracheal intubation: a case report. Chinese journal of integrative medicine, 27(4), 300.

[9] Mandal, M., Jana, S., Nandi, S. K., Khatua, A., Adak, S., & Kar, T. K. (2020). A model based study on the dynamics of COVID-19: Prediction and control. Chaos, Solitons & Fractals, 136, 109889.

[10] Kaur, S. P., & Gupta, V. (2020). COVID-19 Vaccine: A comprehensive status report. Virus research, 288, 198114.

[11] Zhang, R., Gao, Y., Xie, D., Lian, R., & Tian, J. (2021). Characteristics of systematic reviews evaluating treatments for COVID-19 registered in Prospero. Epidemiology & Infection, 149.

[12] Babaei, A., Ahmadi, M., Jafari, H., & Liya, A. (2021). A mathematical model to examine the effect of quarantine on the spread of coronavirus. Chaos, Solitons & Fractals, 142, 110418.

[13] Mandal, S., Bhatnagar, T., Arinaminpathy, N., Agarwal, A., Chowdhury, A., Murhekar, M., & Sarkar, S. (2020). Prudent public health intervention strategies to control the coronavirus disease 2019 transmission in India: A mathematical model-based approach. The Indian journal of medical research, 151(2-3), 190.

[14] Liu, L., Hong, X., Su, X., Chen, H., Zhang, D., Tang, S., & Shi, Y. (2020). Optimizing screening strategies for coronavirus disease 2019: A study from Middle China. Journal of Infection and Public Health, 13(6), 868-872.

[15] Kiran, R., Roy, M., Abbas, S., & Taraphder, A. (2020). Effect of population migration and punctuated lockdown on the spread of SARS-CoV-2. arXiv preprint.

[16] Yavuz, M., Coşar, F. Ö., Günay, F., & Özdemir, F. N. (2021). A new mathematical modeling of the COVID-19 pandemic including the vaccination campaign. Open Journal of Modelling and Simulation, 9(3), 299-321.

[17] Bontempi E, Coccia M, Vergalli S. (2021) Can commercial trade represent the main indicator of the COVID-19 diffusion due to human-to-human interactions? A comparative analysis between Italy, France, and Spain. Environ. Res. 201: 111529

[18] Al-Huraimel K, Alhosani M, Kunhabdulla S. (2020). SARS-CoV-2 in the environment: Modes of transmission, early detection and potential role of pollutions. Sci Total Environ 744: 140946.

[19] Cao, J., Hu, X., Cheng, W., Yu, L., Tu, W. J., & Liu, Q. (2020). Clinical features and short-term outcomes of 18 patients with corona virus disease 2019 in intensive care unit. Intensive care medicine, 46(5), 851-853.

[20] Pengpeng, S., Shengli, C., & Peihua, F. (2020). SEIR Transmission dynamics model of 2019 Covid coronavirus with considering the weak infectious ability and changes in latency duration. MedRxiv.

[21] Huang, C., Wang, Y., Li, X., Ren, L., Zhao, J., Hu, Y. & Cao, B. (2020). Clinical features of patients infected with 2019 novel coronavirus in Wuhan, China. The lancet, 395(10223), 497-506.

[22] Xu, R., Cui, B., Duan, X., Zhang, P., Zhou, X., & Yuan, Q. (2020). Saliva: potential diagnostic value and transmission of 2019-nCoV. International journal of oral science, 12(1), 1-6.

[23] Galanakis, C. M. (2020). The food systems in the era of the coronavirus (COVID-19) pandemic crisis. Foods, 9(4), 523.

[24] Liu Y, Gayle A, Wilder-Smith A. (2020). The reproductive number of COVID-19 is higher compared to SARS coronavirus. J Travel Med 27: taaa021.

[25] Abolmaali, S., & Shirzaei, S. (2021). A comparative study of SIR Model, Linear Regression, Logistic Function and ARIMA Model for forecasting COVID-19 cases. AIMS Public Health, 8(4), 598.

[26] Rosario D, Mutz Y, Bernardes P, et al. (2020) Relationship between COVID-19 and weather: Case study in a tropical country. Int. J. Hyg Environ. Health, 229: 113587.

[27] Bhatnagar, M. R. (2020). COVID-19: Mathematical modeling and predictions. ResearchGate, 10.

[28] Mandal, M., Jana, S., Nandi, S.K., Khatua, A., Adak, S. and Kar, T.K. (2020) A Model Based Study on the Dynamics of COVID-19: Prediction and Control. Chaos, Solitons and Fractals, 136, Article ID: 109889.https://doi.org/10.1016/j.chaos.2020.109889

[29] Higueras, I. (2006). Strong stability for additive Runge Kutta methods. SIAM journal on numerical analysis, 44(4), 1735-1758.

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