

Calculation and Management of Premium Funds in Sharia Insurance based on Langevin Type Model of Return on Investment

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Abstract

This research discusses the calculation of the premium of term life-insurance based on sharia principles. The difference between the conventional method and the sharia principle is in the concept of interest rates. In this research, the concept of interest in the conventional method is replaced by the Return on Investment (ROI) that changes stochastically following the Langevin type model. The Monte-Carlo simulation is applied to generate the ROI with some initial values. On the mechanism of premium management, we apply the system without a saving element and the *Al-Mudbarabah* relationship where the participants will get a sharing-profit of the operating surplus if they don't make a claim until the end of the agreement period. We assume that the administrative expenses only charged in the first year. Therefore, the operating surplus will be greater after the first year. In addition, we do 20 times of Monte-Carlo simulations to generate the ROI with initial value are 7.5%, 9%, and 10%. The result shows that the annual premiums become smaller when the ROI become greater and vice versa. This is because the company get a smaller return when the initial of ROI is small. So the annual premium will be greater. The annual premium for male participants is greater than women because the rate of death of men is greater than women. The other factors that make the annual premium more expensive are length of the agreement and greater compensation.

Keywords: Langevin type model, stochastic differential equation, system without a saving element, *Al-Mudbarabah* principle, Monte-Carlo simulation.

Abstrak

Penelitian ini membahas mengenai perhitungan dana premi asuransi jiwa berjangka berdasarkan prinsip-prinsip syariah. Perbedaan antara metode konvensional dengan prinsip syariah adalah pada konsep tingkat bunga. Pada penelitian ini, konsep bunga digantikan dengan nilai *Return on Investment* (ROI) yang berubah secara stokastik mengikuti model tipe Langevin. Simulasi Monte-Carlo diterapkan untuk membangkitkan nilai ROI menggunakan beberapa nilai awal. Pada mekanisme pengelolaan dana premi, kami menerapkan sistem tanpa unsur tabungan dan hubungan *Al-Mudbarabah* dimana peserta akan mendapatkan bagi hasil atas surplus operasional jika peserta tersebut tidak melakukan klaim sampai akhir masa perjanjian. Kami mengasumsikan bahwa biaya administrasi hanya dibebankan pada tahun pertama. Sehingga surplus operasional akan menjadi lebih besar setelah tahun pertama. Selain itu, kami melakukan 20 kali simulasi Monte-Carlo untuk membangkitkan ROI dengan nilai awal 7.5%, 9%, dan 10%. Hasil menunjukkan bahwa premi tahunan akan semakin kecil jika nilai awal dari ROI membesar dan sebaliknya. Hal ini disebabkan oleh keuntungan perusahaan yang kecil jika nilai awal ROI semakin kecil sehingga premi tahunan haruslah lebih besar. Premi tahunan untuk peserta laki-laki cenderung lebih besar daripada premi tahunan peserta wanita. Hal ini karena tingkat kematian laki-laki lebih tinggi

daripada wanita. Faktor lain yang membuat premi tahunan lebih besar adalah lamanya masa kontrak asuransi dan kompensasi yang semakin besar.

Kata kunci: Model tipe Langevin, persamaan diferensial stokastik, sistem tanpa unsur tabungan, prinsip *Al-Mudharabah*, simulasi Monte-Carlo.

1. INTRODUCTION

Islam is a majority religion in Indonesia so sharia insurance developed significantly. The fundamental difference between sharia insurance and other insurance is usury [1]. The Sharia Council of the Indonesian Ulema Council says that sharia insurance i.e. ta'min, takaful or tadhmun try to protect and cooperation between Moslems/parties through investments in the form of assets and/or tabarru' which provides a pattern of return to confront the certain risks through contract (agreement) following sharia. The contract in this fatwa referred to a contract that does not contain gharar (fraud), maysir (gambling), usury, zhulm (persecution), risywah (bribe), illicit goods, and immorality. The concepts of development of premiums management in sharia insurance are Mudharabah, Wakala, Hybrid (Mudharabah – Wakala), dan Wakala-Waqf [2]. There are two systems for managing sharia insurance premiums i.e. a system that contains saving elements and a system without saving elements [3]. In the second system, all premiums will be tabarru fund i.e. the fund to pay a claim if the participant experiences a disaster. In this system, there is an al-Mudharabah relationship i.e. agreement to share a profit and loss sharing (PLS) with a certain proportion after the projects are implemented.

Many researchers developed sharia insurance, i.e. Kurniandi [4] simulated the calculation of premiums by changing the interest rate to become the rate of investment (ROI) that follows the stochastic process. ROI is a performance measure used to evaluate the efficiency of an investment or compare the efficiency of some different investments [5]. Rasyidah [3] made a formulation of premium management compensation that following the premium system with the savings element in Family Takaful Insurance by applying the concept of the Profit Analysis Method. This method can only be applied to profit-sharing problems for short-term insurance, but for long-term insurance, this problem resolved by separating the part of the premium for savings and the part of the tabarru funds to pay the claim. Purnomo [6] models the calculation of premium rates for savings and non-saving products in sharia insurance. This model calculates the premium rates based on the table of mortality, interest/discount, and cost, and there is surplus underwriting for the customer who holds the joint non-saving policies.

In this paper, we will develop a mechanism to calculate the annual premium for the term life insurance where the interest rates changed to ROI that follows the Langevin type of stochastic model [7] in the non-saving system. We will apply the Monte-Carlo simulation to obtain the annual premium and use some initial values of ROI i.e. 7.5%, 9%, and 10%.

2. METHOD

In this study, we use the system without saving elements. All premium paid by the participant will be put into tabarru fund, this system is generally intended for group insurance with a minimum of 30 participants. This collection of funds is used for payment of claims to the heirs if the participant dies and the agreement ends (if there is a surplus of funds). Figure 1 describes the system without saving elements. In this system, we assume that the administrative expenses are only charged in the first year.

So the investment fund in the first year equal to the annual premium minus the administrative expenses. The reinsurance ratio is 10% of total premiums.

In this research, we assume the ROI parameter, $r(t)$, follows the Langevin type of stochastic model [8]:

$$dr(t) = (\theta - \alpha r(t))dt + \sigma dW(t), \tag{1}$$

where θ, α, σ are constants, and $W(t)$ is a Wiener process.

The discrete form for equation (1) is

$$r(t + 1) = r(t) + (\theta - \alpha r(t))\Delta t + \sigma \varepsilon \sqrt{\Delta t}. \tag{2}$$

where ε is a random variable with the normal standard distribution. If $\Delta t = 1$ then equation (2) becomes

$$r(t + 1) = r(t) + (\theta - \alpha r(t)) + \sigma \varepsilon. \tag{3}$$

The discount factor for ROI when a policy is issued until the death benefit is paid defined as [9]:

$$v_t = \prod_{s=1}^t \frac{1}{(1 + r(s))}, r(t) > -1, \forall s, \tag{4}$$

assumes that $v_0 = 1$.

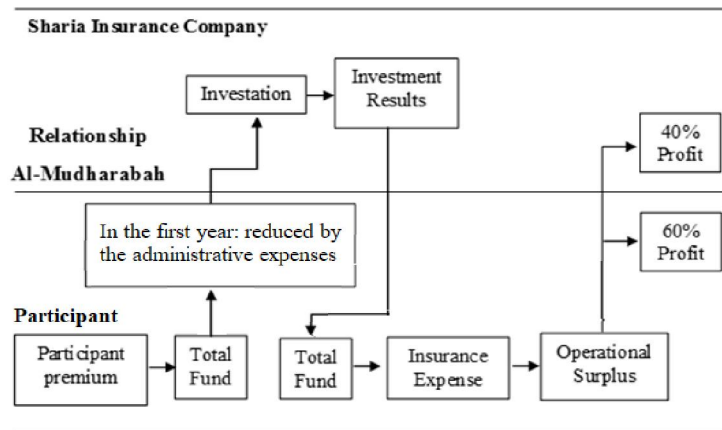


Figure 1. The premium management scheme without savings elements.

The following steps are to calculate the annual premium in the system without saving elements:

1. Determine the age of the participants, the term of the agreement, and the amount of compensation.
2. Generate the random number $\varepsilon \sim N(0,1)$ in equation (3).
3. Calculate the $r(t)$ in equation (3) and v_t in equation (4) with $r(0)$ are 7.5%, 9%, and 10.
4. Calculate a single premium $A_{1:\overline{n}|}$ using [9]

$$A_{1 \overline{x:n}|} = \sum_{t=0}^{n-1} \sum_{s=0}^t v_{s+1} {}_t p_x q_{x+t},$$

where ${}_t p_x$ is the probability of a person x years old lives for another t years to exact age $x + t$, and q_{x+t} is the probability of a person $x + t$ years old dying in next year. The value of ${}_t p_x$ will be taken from The 2011 Indonesian Mortality Table [10] with ${}_t p_x = \frac{l_{x+t}}{l_x}$.

5. Calculate the annual premium $P(A_{1 \overline{x:n}|})$ using [11]

$$P(A_{1 \overline{x:n}|}) = \frac{A_{1 \overline{x:n}|}}{\ddot{a}_{x:n|}} \times b_t,$$

where $\ddot{a}_{x:n|} = \sum_{t=0}^{n-1} v_t {}_t p_x$ is the life annuity [12] and $b_{t+1} = \begin{cases} 1, & t = 0, 1, \dots, n - 1 \\ 0, & t = n, n + 1, \dots \end{cases}$.

3. RESULT AND DISCUSSION

In this section, we will calculate the annual premium for sharia insurance for n -year life types insurance and illustrate the system without savings. We use a case study for the participants are men and women aged 25 – 45 years old with the compensation are IDR 100 million, IDR 200 million, and IDR 500 million and term of the agreement are 10 years, 15 years and 20 years. Based on Kurniandi [4], the parameters in equation (3) are assumed that $\theta = 0.06$, $\alpha = 0.5$, $\sigma = 0.15$, and the initial ROI are 7.5%, 9%, 10%, and the number of the simulation is 20 times. The profit-sharing ratio between the company and the participant is 60%:40%. We use Monte-Carlo simulation to generate the random numbers of ε and we calculate the annual premium in each simulation. Table 1 shows the average ROI from 20 Monte-Carlo simulations with a term of the agreement is 10 years. Table 2 shows the average annual premium from 20 simulations for men and women aged 24 – 40 years old with the initial values are 7.5%, 9%, and 10%.

Table 1. The average ROI from 20 Monte-Carlo simulations with a term of the agreement is 10 years.

Year	$r(t)$
1 st	10.94%
2 nd	11.32%
3 rd	9.87%
4 th	8.56%
5 th	8.63%
6 th	10.52%
7 th	10.39%
8 th	13.24%
9 th	14.28%
10 th	12.32%

The illustration of the premium management in the system without saving will be shown in Table 3. In this illustration, we take the compensation is IDR 100 million, the term is 10 years and the number of participants is 100.000 of men aged 30 years old. In the first year, the annual premium is IDR. 582.203. Therefore, the total premiums are IDR. 58.220.300.000. Based on Table 1, the ROI in the first year is 10.94%. If we assume that the administrative and the reinsurance ratio are 35% and 10%, respectively, then the total funds to invest are total premium minus administrative expenses and reinsurance. Therefore, the total funds plus the investment result will become IDR. 41.983.240.533. If the probability of death is 0.076% then the total claims are IDR 7.600.000.000. Therefore, at the end of the first year, the operating surplus is IDR. 28.561.210.533. This surplus will be distributed to the company and the participants. In the next years, the funds invested are total premiums minus reinsurance so that the operating surplus becomes greater.

Table 2. The average annual premium from 20 simulations of ROI with the compensation is IDR 100.000.000, the term is 10 years, and initial values, $r(0)$, are 7.5%, 9%, and 10%.

Age	The Average of annual Premium (IDR)					
	$r(0) = 7.5\%$		$r(0) = 9\%$		$r(0) = 10\%$	
	Men	Women	Men	Women	Men	Women
25	453999.30	315388.25	453554.65	315050.76	453256.59	314824.68
26	463694.93	329875.30	463229.24	329523.19	462917.14	329287.31
27	480659.43	348519.28	480163.55	348145.80	479831.27	347895.60
⋮	⋮	⋮	⋮	⋮	⋮	⋮
42	2340662.99	1520348.31	2337670.21	1518532.72	2335667.67	1517317.39
43	2663445.33	1700213.91	2660028.23	1698175.29	2657741.80	1696810.70
44	3019876.28	1862907.49	3015995.84	1860773.49	3013399.43	1859344.7
45	3406643.18	2145431.10	3402265.82	2142829.85	3399336.90	2141088.77

Table 3. The illustration of premium fund management mechanism without savings elements with compensation IDR 100 million.

Year	1 st	2 nd	3 rd	...	8 th	9 th	10 th
Amount of Participants	100,000	99,924	99,844	...	99,401	99,292	99,172
Total Premium from Participants (IDR × 10 ⁹)	58.22	58.18	58.13	...	57.87	57.81	57.74
Administration expenses (= 35% from Total Premium, IDR × 10 ⁹)	20.38	0	0	...	0	0	0
Total of fund (IDR × 10 ⁹)	37.84	58.18	58.13	...	57.87	57.81	57.74
Investment Percentage	10,94%	11,32%	9,87%	...	13,24%	14,28%	12,32%
Investment Results (IDR × 10 ⁹)	4.14	6.58	5.74	...	7.66	8.25	7.11
Total of fund + Investment Results (IDR × 10 ⁹)	41.98	64.76	63.87	...	65.53	66.06	64.85
Probability of death	0,00076	0,0008	0,00083	...	0,000109	0,00012	0,000135
Total of Claim (IDR × 10 ⁹)	7.60	8.00	8.30	...	1.09	1.20	1.35

Reinsurance Premium (10% from Total Premium, IDR × 10 ⁹)	5.82	5.82	5.81	...	5.79	5.78	5.77
Claim + Reinsurance Premium (IDR × 10 ⁹)	13.42	13.82	14.11	...	6.88	6.98	7.12
Operating Surplus (Total of fund + Investment result, IDR × 10 ⁹)	28.56	50.94	49.75	...	58.66	59.08	57.73
Profit-sharing for participants (60% from operating surplus, IDR × 10 ⁹)	17.14	30.57	29.85	...	35.19	35.45	34.64
Profit-sharing for Company (40% from operating surplus, IDR × 10 ⁹)	11.42	20.38	19.90	...	23.46	23.63	23.09

Figures 2 – 5 show the comparison of annual premiums based on several categories. In Figure 2, we can see that the greater the ROI, the smaller the annual premiums. From Figure 3 we can see that the annual premium for men is greater than women's. This is caused by the probability of death for men is greater than women's. However, following the theory in insurance generally, the longer the contract or the more compensation was given at the claim, the annual premium will be greater. These can be seen in Figure 4 – 5.

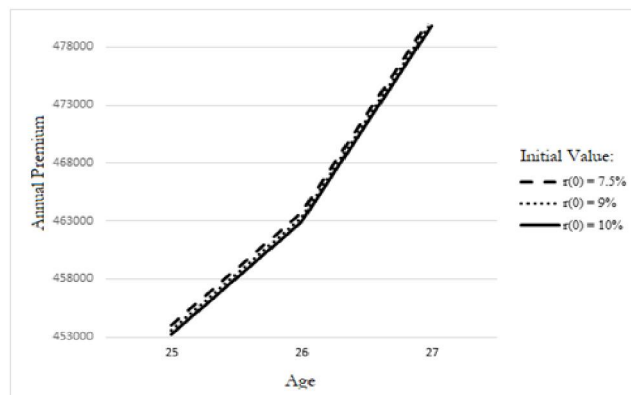


Figure 2. The comparison of the annual premium for men aged 25 – 27 years old based on the initial values with compensation IDR. 100 million and term 10 years.

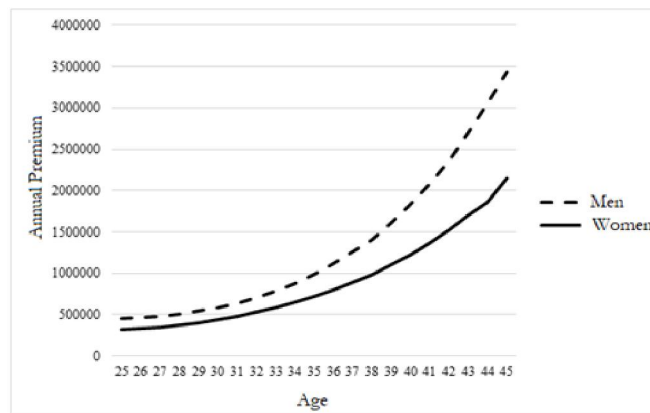


Figure 3. The comparison of annual premium between men and women aged 25 – 45 years old with $r(0) = 9\%$ and compensation IDR. 100 million.

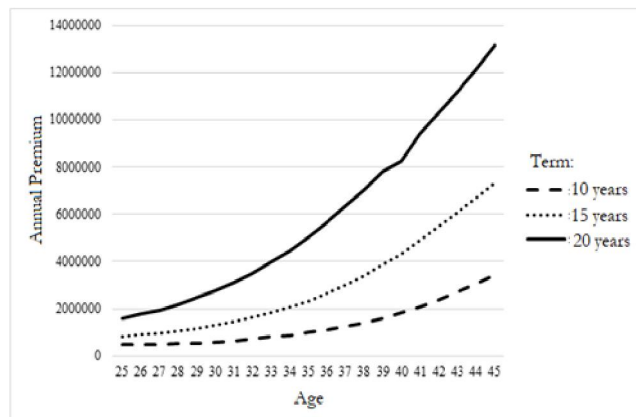


Figure 4. The comparison of the annual premium for men aged 25 – 27 years old based on the term of the agreement with compensation IDR. 100 million and $r(0) = 9\%$.

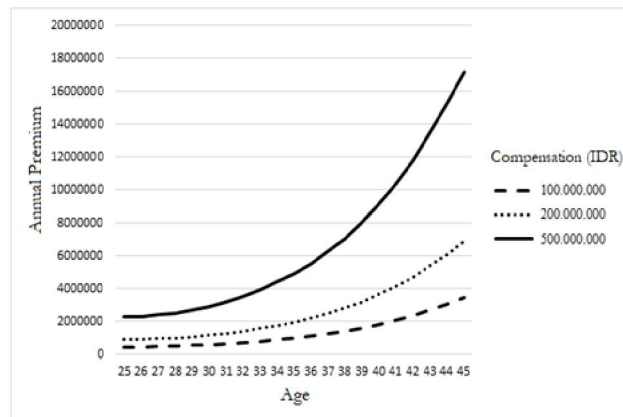


Figure 5. The comparison of the annual premium for men aged 25 – 45 years old based on the compensations with term 10 years and $r(0) = 10\%$.

4. CONCLUSION

In this paper, we discuss the mechanism of premium management without savings elements. The annual premium is calculated based on Return on Investment (ROI) that changes stochastically following the Langevin type model. The Illustration of management of premium funds in the system without the saving element, we assume that the profit-sharing ratio between the participants and company is 60%: 40% and the administration fee only charged in the first year so after the first year, the funds to be invested will be a greater and the operating surplus becomes greater too. Besides, the Monte-Carlo simulation is applied to get ROI with the initial values are 7.5%, 9%, and 10%. For our cases, we can conclude that the greater the ROI, the smaller annual premiums and vice versa. This is because when the initial ROI is small, the company only gets a little return or return on investment. So the annual premium will be even greater. The annual premium for male participants is greater than

for women because the rate of death or risk of men is greater than for women. The length of the agreement and the greater the value of compensation, the annual premium will greater too.

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