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Economic Ordering Policy for VAR Deterioration Model with Non-stationary Two-warehouse Inventory and Demand

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Abstract

This paper adopts the two-warehouse inventory, determination on the first run-time and VAR (Vector Auto Regression) deterioration model. The optimal EOQ in the interval of the finite horizon is determined under critical considerations. The non-stationary two-warehouse inventory, i.e. the inventory and initial inventory are non-stationary at level, but stationary after lag difference similar to demand (demand and initial demand). The output of the proposed model represented the optimal order quantity and optimal first run-time, the optimal total cost as integration of first order with the significant trend and intercept. The optimal demand is decreased during more risk as a deterioration variable to reduce the quantity in the stock. The initial demand is stationary after a first lag and the demand is stationary.

Keywords: **i**nitial inventory; optimal of first run-time; EOQ (Economic Ordering Quantity); total cost function (TC).

Abstrak

Penelitian ini mengadopsi inventori dengan dua gudang penyimpanan, penentuan pada waktu run (*run-time*) awal, dan model deteriorating VAR (Vector Auto Regression). Nilai optimal EOQ dalam interval horizon berhingga ditentukan dengan pertimbangan kritis. Inventori dengan dua gedung yang tidak stasioner, yaitu inventori dan inventori awal tidak stasioner pada level, tetapi stasioner setelah perbedaan lag seperti halnya pada permintaan (permintaan dan permintaan awal). Hasil dari model yang diajukan menunjukkan nilai orde yang optimal dan waktu run awal yang optimal, total biaya optimal sebagai integrasi dari orde pertama dengan tren dan intercept yang signifikan. Permintaan optimal mengalami penurunan ketika lebih banyak risiko sebagai variabel deteroriating untuk mengurangi jumlah dalam stok. Permintaan awal menunjukkan stasioner setelah perbedaan lag pertama dan permintaan juga stasioner.

Kata kunci: inventori awal; optimal run-time awal; EOQ (*Economic Ordering Quantity*); fungsi biaya total.

1. INTRODUCTION

Many types of researchers developed the inventory model, but this paper investigated the inventory model by the new assumptions as of the VAR (Vector Auto Regression) deterioration model and non-stationary inventory and initial inventory model. Maiti [1] developed a fuzzy model for deteriorating multi-outlet by applying triangular fuzzy. Chung et al. [2] investigated the model with permissible delay in payment under providing a discount in cash. Pal et al. [3] deals with the price and dependent demand in stock with permissible in delay in payment. Palanivel and Uthayakumar [4] developed the deteriorating model under new assumptions as price and dependent demand in the advertisement but in the output of the model in numerical analysis, the total cost decreased when the deterioration rate increased that is not supported by the inventory theory as Pal et al. [3]. Pal et al. [5] represented the EPQ under constant production across the year, fuzzy demand, and nonlinear deterioration. Omprakash et al. [6] attempted a model for the policy of the retailer's inventory to minimize the total cost to the retailer's inventory but the movement in the output is slow especially for the total cost. Jolai et al. [7] the framework of optimization is represented to optimize the production under variable deterioration. Neetu and Tomer [8] the inflation is considered as a variable in the deteriorating inventory and demand as the price of selling function. Tiwari et al. [9] developed the model through new consideration to the two-echelon supply chain; demand is non-constant as function in selling price. Tripathi and Kaur [10] investigated the deteriorating items by linear time-dependent deterioration and demand. In this paper, we propose a model to handle the inventory under the VAR (Vector Auto Regression) deterioration model, non-stationary demands to determine the optimal first run-time and total cost, EOQ (Economic Order Quantity) also, reactivation of the output of the assumed model. The model in this paper used when the industry has *stocked* with two sub-stocks for the single items.

2. MATERIAL AND METHOD

2.1. Assumptions and notions

2.1.1. Assumptions

The mathematical model is developed within assumptions

1) The planning horizon is finite.
2) Single item inventory control.
3) The demand is non-stationary.
4) Initial demand is non-stationary at level but stationary at lag=1.
5) Deterioration is estimated by $VAR(2,2)$ model as $\theta = a_1\theta(-1) + a_2\theta(-2) + b_1\theta_0(-1) + b_2\theta_0(-2) + \epsilon$ where a_1, a_2, b_1, b_2 are real constants, ϵ is an error.
6) Initial deterioration is stationary over time.
7) There is no replacement or repair of deterioration items during the supposed period.
8) The shortage is not allowed.
9) The lead time is zero.
10) The inventory level at the end of the planning horizon is zero
11) The cost factors are deterministic.

2.1.2. Notations

Q_1 = The order quantity in stock within $(0, t_1)$.
Q_0 = The order quantity which is in stock within $(0, t_1)$.
Q_S = The difference of order quantity which is in stock within $(0, t_1)$.
TC = The total relevant cost $(0, t_1)$.

2.2. Parameters

The mathematical model is representing the following parameters

A = The fixed ordering cost per replenishment \$/order.
C = The unit purchasing price at time zero \$/order.
D_1 = The demand per unit time for $I_1(t)$.
D_0 = The initial demand per unit time for $I_0(t)$.
B_0 = positive real constant, $B_0 > 0$.
B_1 = positive real constant, $B_1 > 0$.
$I_0(t)$ = The initial inventory level at (0, t_1).
$I_1(t)$ = The inventory level at time during (0, t_1).
I_h = The interest charged per \$per unit by the supplier.
t_1 = The first run time of each replenishment cycle for an emergency order.
θ = Deterioration units/unit time.
θ_0 = Initial deterioration units/unit time which is caused in θ.

3. MATHEMATICAL MODEL

Let $I_1(t)$ is the inventory level at any time t, $0 \leq t \leq t_1$, Depletion due to demand within first component interval $0 \leq t \leq t_1$. The $\frac{dI_1(t)}{dt}$ satisfied the Eq. (1) that describes the instantaneous state of $I_1(t)$ over the open interval $(0, t_1)$ is given by:

$$\frac{dI_1(t)}{dt} = \theta\frac{dI_0(t)}{dt} + B_0D_0 + B_1D_1, B_0D_0 > 0, 0 \leq t \leq t_1, 0 \leq \theta \leq 1, \tag{1}$$

$$I_1(t) - \theta\, I_0(t) = \int_t^{t_1}(B_0D_0 + B_1D_1)du = (B_0D_0 + B_1D_1)(t_1 - t).$$

Let $I_0(t)$ is initial of the inventory level at any time $t, 0 \leq t \leq\ t_1$. Depletion due to initial demand within component interval $t_1 \leq t \leq T$ and there is initial deterioration. The first-order differential equation Eq. (2) that describes the instantaneous state of $I_0(t)$ over the open interval $(0, t_1)$ is given by

$$\frac{dI_0(t)}{dt} + \theta_0 I_0(t) = -D_0, 0 \leq t \leq t_1, \tag{2}$$

where $I_0(t_1) = 0$ for equation of number. $I_0(t) =\ I_{01}(t)\int_t^{t_1} D_0 e^{\theta_0 u^2}du = \frac{D_0}{\theta_0}\left(e^{\theta_0(t_1-t)} - 1\right), I_{01}(t) = e^{-\theta_0 t}$, and $I_0(0) = \frac{D_0}{\theta_0}\left(e^{\theta_0 t_1} - 1\right),\ Q_0 = \frac{D_0}{\theta_0}\left(e^{\theta_0 t_1} - 1\right)$.

According to Eq. (2) the inventory level is given as $I_1(t) = (B_0D_0 + B_1D_1)(t_1 - t) + \frac{\theta D_0}{\theta_0}\left(e^{\theta_0(t_1-t)} - 1\right)$, and $Q_1 = (B_0D_0 + B_1D_1)t_1 + \frac{\theta D_0}{\theta_0}\left(e^{\theta_0 t_1} - 1\right)$.

3.1. Fixed ordering cost

We assumed that the fixed ordering cost over the planning horizon (0, t_1) consideration is: $TC_A = A$.

3.2. Purchasing cost

The purchasing cost of calculated as $TC_P = C\left[(B_0D_0 + B_1D_1)t_1 + \frac{\theta D_0}{\theta_0}\left(e^{\theta_0 t_1} - 1\right)\right]$.

3.3. Holding cost excluding interest cost

The average inventory quantity is used to obtain holding cost

$$\bar{I} = \int_0^{t_1} I_1(t)dt = \int_0^{t_1} (\alpha B_0D_0 + B_1D_1)t_1 + \frac{\theta D_0}{\theta_0}\left(e^{\theta_0 t_1} - 1\right)dt$$
$$= \frac{(B_0D_0+B_1D_1){t_1}^2}{2} + \frac{\theta D_0}{{\theta_0}^2}\left(e^{\theta_0 t_1} - \theta_0 t_1 - 1\right).$$
$$TC_h = I_h\left[\frac{(B_0D_0+B_1D_1){t_1}^2}{2} + \frac{\theta D_0}{\theta_0}\left(e^{\theta_0 t_1} - 1\right)\right],$$
$$TC = TC_A + TC_h + TC_P.$$

Then $TC = \left[A + C[(B_0D_0 + B_1D_1)t_1 + \frac{\theta D_0}{\theta_0}\left(e^{\theta_0 t_1} - 1\right)\right] + I_h\left[\frac{(B_0D_0+B_1D_1){t_1}^2}{2} + \frac{\theta D_0}{\theta_0}\left(e^{\theta_0 t_1} - 1\right)\right]$.

3.4. Economic Order Quantity

To find optimal run-time by minimizing the total cost function we found-out the optimal demand and optimal deterioration as the following

$$TC = \left[\frac{1}{t_1}\right]\left[A + C[(B_0D_0 + B_1D_1)t_1 + \frac{\theta D_0}{\theta_0}\left(e^{\theta_0 t_1} - 1\right)\right] + I_h\left[(B_0D_0 + B_1D_1)t_1 + \frac{\theta D_0}{{\theta_0}^2}\left(e^{\theta_0 t_1} - \theta_0 t_1 - 1\right)\right].$$

$$\frac{dTC}{dt_1} = \left[-\frac{1}{{t_1}^2}\right]\left[A + C[(B_0D_0 + B_1D_1)t_1 + \frac{\theta D_0}{\theta_0}\left(e^{\theta_0 t_1} - 1\right)] + I_h[\frac{(B_0D_0 + B_1D_1){t_1}^2}{2}\right.$$
$$\left. + \frac{\theta D_0}{{\theta_0}^2}\left(e^{\theta_0 t_1} - \theta_0 t_1 - 1\right)]\right] + \left[\frac{1}{t_1}\right]\left[C[(B_0D_0 + B_1D_1) + \theta D_0 e^{\theta_0 t_1}]\right.$$
$$\left. + I_h\left[(B_0D_0 + B_1D_1)t_1 + \frac{\theta D_0}{{\theta_0}^2}\left(e^{\theta_0 t_1} - \theta_0 t_1 - 1\right)\right]\right].$$

Let $\frac{dTC}{dt_1} = 0$ to find the optimum total cost. Then ${t_1}^* = \sqrt{\frac{A}{[I_h\left(\frac{(B_0D_0+B_1D_1)}{2} + \theta D_0\right) + \theta_0 C D_0 \theta]}}$.

To test the ${t_1}^*$ by using the second derivative, we found out that $\frac{d^2TC}{{dt_1}^2}|_{t={t_1}^*} = \frac{2A}{{t_1}^3} > 0$

The total cost has minimum value at $t_1 = t_1^*$.

4. SENSITIVITY ANALYSIS

Because the proposed model assumed the non-stationary demand and initial demand is non-stationary, the optimal order quantity and optimal first run-time, the optimal total cost is integrated at first order rank.

Example 1: We assumed the values of B_0 and B_1 as positive real are arbitrary, whether associated costs are too. We choose $A = 50\$, C = 150\$, I_h = 10\$, C_s = 5\$, B_0 = 0.2\$, B_1 = 0.6$. SPSS 26 and EViews 10 software version are used in our analytics. The estimate $VAR(2,2)$ model of deterioration is $\theta = 0.5579108800025\theta(-1) + 1.08651528812\theta(-2) + 0.269987524\theta_0(-1) - 0.8889578917765\theta_0(-2) + 0.068896924528$.

Table 1. The sensitivity analysis.

θ	θ_0	D_0	D_1	t_1^*	EOQ_1*	EOQ_0*	Q_S*	TC*
0.00007	0.000002	250	500	0.169022	59.1608	42.25561	16.90519	53094.23
0.0007	0.0001	300	520	0.163864	60.99181	49.15923	11.83258	56441.42
0.07	0.01	350	580	0.145195	64.25121	50.85503	13.39618	67025.49
0.1	0.015	400	610	0.135582	65.89815	54.2878	11.61035	73577.24
0.2	0.15	450	660	0.096629	55.72135	43.79948	11.92187	87251.55
0.3	0.25	500	690	0.071814	47.78199	36.23143	11.55056	100687.4
0.4	0.35	550	715	0.05514	41.9692	30.62162	11.34757	115233.4
0.5	0.45	600	725	0.043832	37.60676	26.56018	11.04658	129972.5
0.6	0.55	650	750	0.035817	34.8812	23.51202	11.36918	147601.1
0.7	0.65	700	790	0.029949	33.20744	21.16967	12.03777	168112.7
0.8	0.75	750	840	0.025521	32.15019	19.32473	12.82546	191052.8
0.9	0.85	800	880	0.022093	31.25692	17.84121	13.41571	214617.4
0.99	0.95	850	900	0.019471	30.36239	16.70473	13.65766	236611.1

Table 2. The output of VAR model

R-squared	0.998513	0.999119
Adj. R-squared	0.997522	0.998532
Sum sq. Resids	0.001595	0.000903
S.E. equation	0.016305	0.012266
F-statistic	1007.323	1701.074
Log likelihood	33.00468	36.13542
Akaike AIC	-5.091759	-5.660986
Schwarz SC	-4.910898	-5.480124

Table 3. The stationary test of EOQ1 at lag = 1.

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-12.54827	0.0001
Test critical values:	1% level	-5.521860	
	5% level	-4.107833	
	10% level	-3.515047	

Variable	Coefficient	Std. Error	t-Statistic	Prob.
D(Q1*(-1))	-1.295156	0.103214	-12.54827	0.0002
D(Q1*(-1),2)	0.215836	0.079825	2.703876	0.0539
D(Q1*(-2),2)	0.128770	0.080352	1.602569	0.1843
C	-13.85027	1.107274	-12.50844	0.0002
@TREND("1")	1.120089	0.121347	9.230464	0.0008

Table 4. The stationary test of TC* at lag = 1.

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-4.247838	0.0429
Test critical values:	1% level	-5.521860	
	5% level	-4.107833	
	10% level	-3.515047	

Variable	Coefficient	Std. Error	t-Statistic	Prob.
D(TC_(-1))	-2.571953	0.605473	-4.247838	0.0132
D(TC_(-1),2)	1.428007	0.480548	2.971622	0.0411
D(TC_(-2),2)	0.745408	0.248256	3.002583	0.0398
C	6757.713	1347.356	5.015538	0.0074
@TREND("1")	4181.679	1079.222	3.874715	0.0179

Table 5. The stationary test of t_1* at lag = 1

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-8.343314	0.0007
Test critical values:	1% level	-5.521860	
	5% level	-4.107833	
	10% level	-3.515047	

Variable	Coefficient	Std. Error	t-Statistic	Prob.
D(t_1*(-1))	-1.738569	0.208379	-8.343314	0.0011
D(t_1*(-1),2)	0.543628	0.165157	3.291575	0.0302
D(t_1*(-2),2)	0.291296	0.117724	2.474387	0.0686
C	-0.067110	0.008077	-8.308422	0.0011
@TREND("1")	0.005395	0.000697	7.744471	0.0015

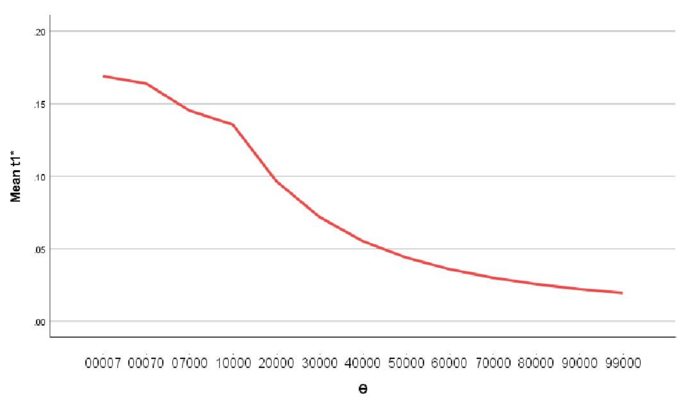

Figure 1. The deterioration in the stock versus optimal first-run time.

This paper adopted the vector autoregressive inventory model and applied the permissible range for considering the deterioration with the initial determination as $VAR(2,2)$ model. The optimal first-run time, the optimal total cost across the period. The sensitivity analysis considered variable deterioration. The optimal first run-time decreased when the initial and deterioration, increased to satisfy the real situation as figure 1 – 2.

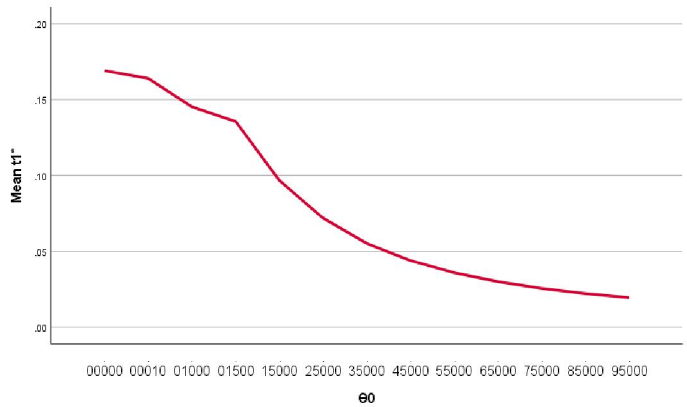

Figure 2. The initial deterioration in the stock versus the optimal first-run time.

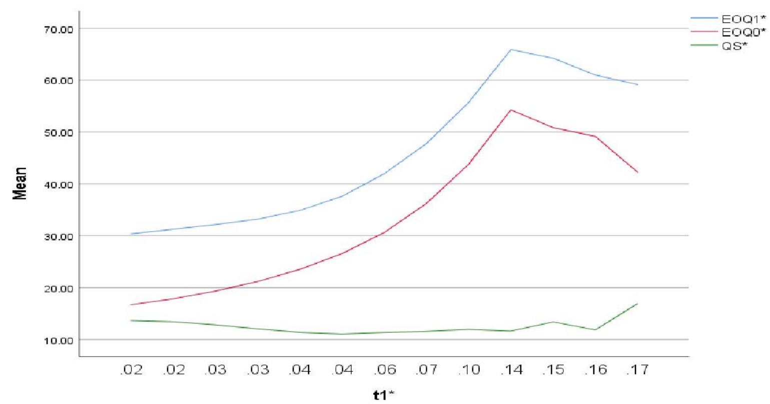

Figure 3. The optimal order quantity in the stock versus the optimal first-run time.

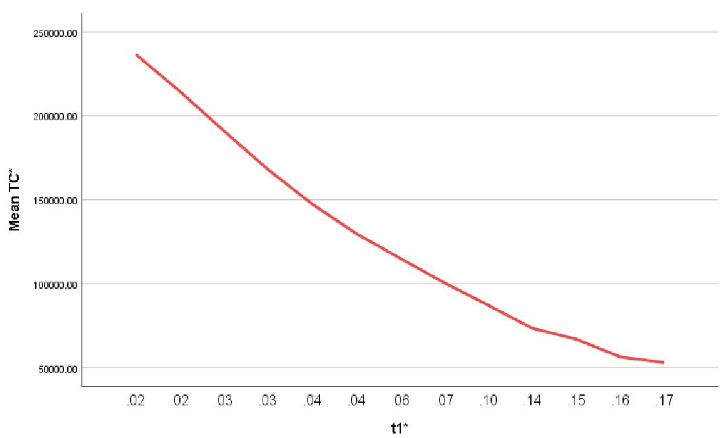

Figure 4. The optimal total cost versus optimal first-run time.

The optimal initial EOQ and EOQ decreased when the initial and deterioration, increased to satisfy the realistic situation, especially when initial deterioration equal to 0.015 or more that, deterioration equal to 0.1 or more than that as figure 3. The optimal total cost increased when the first run-time increased and that expressed a positive relationship based on reality. The optimal of total cost increased when optimal of the first run-time was decreased as figure 4. Because the deterioration risk is increasing. The output of the proposed model analyzed by Eviews 10 to check the stationary of EOQ as a table 3, the optimal total cost was stationary at lag=1 with significance trend and intercept as the table 4. Optimal first run-time was stationary at lag=1 with a significant trend and intercept as a table 5.

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