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# Identification of New Keynesian Phillips Curves from a Global Perspective*

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### Abstract

This paper is concerned with the estimation of New Keynesian Phillips Curves (NKPC) and focusses on two issues: the weak instrument problem and the characterisation of the steady states. It proposes some solutions from a global perspective. Using a global vector autoregressive model (GVAR) steady states are estimated as long-horizon expectations and valid instruments are constructed from the global variables as weighted averages. The proposed estimation strategy is illustrated using estimates of the NKPC for 8 developed industrial countries. The GVAR generates global factors that are valid instruments and help alleviate the weak instrument problem. The steady states also reflect global influences and any long-run theoretical relationships that might prevail within and across countries in the global economy. The GVAR measure of the steady state performed better than the HP measure, and the use of foreign instruments substantially increased the precision of the estimates of the output coefficient.

**Keywords**: Steady States, Long Horizon Expectations, Global VAR, identification, New Keynesian Phillips Curve, Trend-Cycle Decomposition.

**JEL Classification**: C32, E17, F37, F42.

# 1  Introduction

New Keynesian Phillips Curves (NKPC) have been widely used in the macroeconomic literature. Yet their empirical implementation raises a number of issues that continue to be of some concern. In this paper we shall focus on two of these issues - the weak instrument problem and the characterisation of the steady states - and propose some solutions from a global perspective.

The first issue relates to the quality of the instruments used to estimate the NKPC model. The problem arises since in a closed economy setting it turns out that there are not many variables that can be used to produce inflation forecasts that improve significantly over a first order autoregressive model of inflation. Under the NKPC lagged observations are admissible as instruments, in the sense that they are not correlated with the error terms. However, in order to be valid instruments the variables also need to be sufficiently correlated with the endogenous explanatory variables so that the necessary rank condition is satisfied. The solution of the rational expectations (RE) model indicates that this rank condition will not be satisfied unless the lag order of the equation determining the driving variable is greater than that of the NKPC and the extra lags significantly improve the prediction of the driving variable. See, for example, Mavroeidis (2005) and Nason and Smith (2008). In this paper, by taking a global perspective, we suggest some possible routes to resolving the weak instrument problem at least for NKPC models of small open economies.

In addition, the NKPC is typically derived from the first order optimisation conditions of a representative firm, subject to staggered pricing behaviour, in the context of a dynamic stochastic general equilibrium (DSGE) framework. Since these first order conditions are complicated non-linear stochastic equations, usually they are log-linearised around a steady state. Such an approximation procedure is appropriate if a unique steady state exists, the log-linearisation is carried out around the correct steady state, and the approximation errors are relatively small. In cases where the steady states exist and are not time-varying the analysis of the DSGE equations as deviations from the steady states does not pose any new difficulties. Inclusion of intercepts in the log-linearised version of the first order conditions will suffice. Similarly, when the steady state values follow deterministic trends, residuals from regressions on such trend components can be used in the log-linearised DSGE model. The problem arises if the first order conditions contain variables with stochastic trends that could be cointegrated. In such cases any misspecification of the steady states can seriously bias the estimates of the DSGE equations. In practice, the stochastic trends, for example in the case of output, are often approximated by statistical methods such as the Hodrick-Prescott (HP) filter or a variety of the band pass filters as discussed in Christiano and Fitzgerald (2003). These procedures are purely *statistical,* in the sense that they are not derived from the assumed DSGE model and need not be consistent with it. In this paper we present an alternative approach where the derivation of steady states is made consistent with the underlying DSGE model. We propose to measure the steady states by the long horizon expectations, where expectations are taken consistently with respective to the underlying DSGE model. This is in line with the idea of the model consistent expectations that underpin the NKPC and simply extends it

to the long run.

In the empirical section of the paper the steady states are estimated using a global model for 33 countries estimated over the period 1979Q1-2006Q4. Using these estimates NKPC equations are estimated for eight developed economies where it is shown that using global instruments and economic measures of the steady states provide better determined estimates of the NKPC not only for the US but also for a number of European economies, notably UK, France and Spain.

The rest of the paper is set out as follows: Section 2 discusses the identification of the NKPC. Section 3 describes the solution of a multi-country RE DSGE model and shows how the use of global factors as instruments may alleviate the weak instrument problem. Section 4 explains the characterisation of steady states as long horizon expectations. Section 5 discusses how a cointegrating global vector autoregression (GVAR) model based on that of Dees, di Mauro, Pesaran and Smith (DdPS, 2007) can be used to provide instruments and theory-consistent estimates of the steady states. It is shown that if the variables in the system are I(1), the long horizon expectations in a linear system happen to be the same as the permanent or trend component obtained from a multivariate version of the Beveridge-Nelson (1981) decomposition. Section 6 presents estimates of the NKPC for 8 countries under a variety of assumptions about available instruments and measures of steady state. Section 7 provides some concluding comments.

## 2  Identification and Estimation of the Phillips Curve

Consider a standard closed economy NKPC model. For countries $i = 1, 2, ..., N$ and time periods $t = 1, 2, ..., T$, the NKPC relates the deviations from steady state of inflation, $\widetilde{\pi}_{it}$ and a driving output or marginal cost variable, $\tilde{y}_{it}$, by an equation of the form:

$$\widetilde{\pi}_{it} = \beta_{bi}\widetilde{\pi}_{i,t-1} + \beta_{fi}E(\widetilde{\pi}_{i,t+1} \mid \mathfrak{I}_{i,t-1}) + \gamma_i\widetilde{y}_{it} + \varepsilon_{it}, \tag{1}$$

where $E(\widetilde{\pi}_{i,t+1} \mid \mathfrak{I}_{i,t-1})$ denotes expectations formed conditional on information at time $t-1$.[1] All variables are measured as deviations from their respective steady states. Steady state values of inflation and output are denoted by $\pi_{it}^P$ and $y_{it}^P$, with deviations from the steady states given by $\widetilde{\pi}_{it} = \pi_{it} - \pi_{it}^P$, and $\widetilde{y}_{it} = y_{it} - y_{it}^P$.

The parameters are non-linear functions of underlying structural parameters. For instance, following Gali, Gertler and Lopez-Salido (2005) suppose that there is staggered price setting, with a proportion of firms, $(1 - \theta_i)$, resetting prices in any period, and a proportion $\theta_i$ keeping prices unchanged. Of those firms able to adjust prices only a fraction $(1 - \omega_i)$ set prices optimally on the basis of expected marginal costs. A fraction $\omega_i$ use a rule of thumb based on lagged inflation. Then for a subjective discount factor, $\lambda_i$, we have

$$\begin{aligned} \beta_{fi} &= \lambda_i\theta_i\phi_i^{-1}, \ \beta_{bi} = \omega_i\phi_i^{-1}, \\ \gamma_i &= (1 - \omega_i)(1 - \theta_i)(1 - \lambda_i\theta_i)\phi_i^{-1}, \end{aligned} \tag{2}$$

where $\phi_i = \theta_i + \omega_i[1 - \theta_i(1 - \lambda_i)]$. If $\omega_i = 0$, all those who adjust prices do so optimally, then

$\beta_{fi} = \lambda_i$, and $\beta_{bi} = 0$. If the discount factor, $\lambda_i = 1$, then $\beta_{fi} + \beta_{bi} = 1$ in either case. Since the discount factor is likely to be very close to unity, this case is worth attention. Notice that there is no reason for these parameters to be the same across countries with very different market institutions and property rights (which will influence $\lambda_i$), so we allow them to be heterogeneous from the start.

Traditionally, the driving variable has been a measure of unemployment or the output gap. More recently measures of marginal cost and the share of labour have been used. We will use the output gap because it is the variable that is relevant to policy, which relates inflation to aggregate demand not marginal cost; it is the variable that appears in the standard three equation macro model; and it is the variable that is available for all the countries in our sample. We will compare the performance of two measures of the output gap obtained using either the HP filter or the GVAR measure of the steady states, but there are various issues of identification and estimation to be considered first.

It is common to assume that inflation is stationary, and that its steady state is a constant, say $\overline{\pi}_i$, then equation (1) becomes the special case

$$\pi_{it} = (1 - \beta_{bi} - \beta_{fi})\overline{\pi}_i + \beta_{bi}\pi_{i,t-1} + \beta_{fi}E(\pi_{i,t+1} \mid \mathfrak{I}_{i,t-1}) + \gamma_i\widetilde{y}_{it} + \varepsilon_{it}. \tag{3}$$

The solution of the model depends on the process generating $\widetilde{y}_{it}$ and $\varepsilon_{it}$. It is typically assumed that $\varepsilon_{it}$ is a martingale difference process, and $\widetilde{y}_{it}$ follows a stationary time series process. Consistent estimation of the NKPC critically depends on the nature of the $\widetilde{y}_{it}$ process. The empirical literature typically assumes that suitable instruments (or moment conditions) exist and uses GMM to estimate the following version of the NKPC

$$\widetilde{\pi}_{it} = \beta_{bi}\widetilde{\pi}_{i,t-1} + \beta_{fi}\widetilde{\pi}_{i,t+1} + \gamma_i\widetilde{y}_{it} + \xi_{i,t+1}, \tag{4}$$

or

$$\widetilde{\pi}_{it} = \boldsymbol{\theta}'_i\mathbf{x}_{i,t+1} + \xi_{i,t+1},$$

where

$$\xi_{i,t+1} = \varepsilon_{it} - \beta_{fi}\upsilon_{i,t+1},$$

and $\upsilon_{i,t+1}$ is the expectations error of inflation, $\mathbf{x}_{i,t+1} = (\widetilde{\pi}_{i,t-1}, \widetilde{\pi}_{i,t+1}, \widetilde{y}_{it})'$, and $\boldsymbol{\theta}_i = (\beta_{bi}, \beta_{fi}, \gamma_i)'$.

The estimation of (4) requires at least three instruments that are

(a) not correlated with $\xi_{i,t+1}$, namely

$$E(\mathbf{z}_{i,t-1}\xi_{i,t+1} \mid \mathfrak{I}_{i,t-1}) = 0,$$

where $\mathbf{z}_{i,t-1}$ denotes the $s \times 1$ vector of instruments, and at the *same time* are

(b) sufficiently correlated with $\mathbf{x}_{i,t+1}$, such that

$$p\lim_{T \to \infty} \left(T^{-1}\sum_{t=1}^{T}\mathbf{x}_{i,t+1}\mathbf{z}'_{i,t-1}\right) = \text{Full Rank Matrix}.$$

Given the nature of the RE hypothesis there are no difficulties finding instruments that satisfy condition (a). Condition (b) is more problematic and whether it holds critically depends on the nature of the $\widetilde{y}_{it}$ process. To determine if the NKPC is identified requires solving the RE model.

## 2.1 Alternative Solutions

In the case where there are no feedbacks from inflation to output gap, $\beta_{bi}, \beta_{fi} \geq 0$, $\beta_{fi}\beta_{bi} \leq 1/4$ and $\beta_{bi} + \beta_{fi} \leq 1$, the NKPC (1) has the unique solution,

$$\widetilde{\pi}_{it} = \lambda_{bi}\widetilde{\pi}_{i,t-1} + \frac{\gamma_i}{1 - \lambda_{bi}\beta_{fi}} \sum_{j=0}^{\infty} \lambda_{fi}^{-j} E\left(\widetilde{y}_{i,t+j} \,|\, \mathfrak{I}_{i,t-1}\right) \tag{5}$$

$$+ \frac{\gamma_i}{1 - \lambda_{bi}\beta_{fi}} \left[\widetilde{y}_{it} - E\left(\widetilde{y}_{it} \,|\, \mathfrak{I}_{i,t-1}\right)\right] + \frac{\varepsilon_{it}}{1 - \lambda_{bi}\beta_{fi}},$$

where $\lambda_{bi}$ and $\lambda_{fi}$ are roots of

$$\beta_{fi}\lambda_i^2 - \lambda_i + \beta_{bi} = 0,$$

with $|\lambda_{bi}| \leq 1$ and $|\lambda_{fi}| > 1$. The condition $\beta_{bi} + \beta_{fi} < 1$ ensures that $|\lambda_{bi}| < 1$, and $|\lambda_{fi}| > 1$. If $\beta_{bi} + \beta_{fi} = 1$, then $\lambda_{bi} = 1$ and $\lambda_{fi} = \beta_{fi}^{-1}(1 - \beta_{fi}) > 1$ if $\beta_{fi} < 1/2$. Since by construction $\widetilde{y}_{it}$ is a stationary process, then in this case inflation will be $I(1)$. Finally, if $\beta_{bi} + \beta_{fi} > 1$, the RE solution will be indeterminate and there exists a multplicity of solutions, characterized in terms of an arbitrary martingale difference process. To see this consider the inflation expectations errors

$$\mathfrak{m}_{i,t+1} = \widetilde{\pi}_{i,t+1} - E(\widetilde{\pi}_{i,t+1} \,|\, \mathfrak{I}_{i,t-1})$$

and note that under the RE hypothesis $\mathfrak{m}_{i,t+1}$ is a martingale difference process such that $E(\mathfrak{m}_{i,t+1} \,|\, \mathfrak{I}_{i,t-1}) = 0$. Using $\mathfrak{m}_{i,t+1}$, a general solution for the inflation process can be written as

$$\widetilde{\pi}_{it} = \beta_{fi}^{-1}\widetilde{\pi}_{i,t-1} - \beta_{fi}^{-1}\beta_{bi}\widetilde{\pi}_{i,t-2} - \beta_{fi}^{-1}\gamma_i\widetilde{y}_{i,t-1} + \mathfrak{m}_{it} - \beta_{fi}^{-1}\varepsilon_{i,t-1}. \tag{6}$$

When $\beta_{bi} + \beta_{fi} > 1$, (6) is a stable solution but it is not unique; different stable solutions can be obtained for different choices of the martingale difference process, $\mathfrak{m}_{it}$. As a result estimation of the parameters of the NKPC will depend on which of the many possible proxies for the expectational errors, $\mathfrak{m}_{it}$, are used.

## 2.2 Weak Instruments

Returning to the determinate case, in the absence of feedbacks where $\widetilde{y}_{it}$ does not depend (directly or indirectly through a third variable) on past values of $\widetilde{\pi}_{it}$, future inflation $\widetilde{\pi}_{i,t+1}$ and $\widetilde{y}_{it}$ do not depend on $\widetilde{\pi}_{i,t-1}$, $\widetilde{\pi}_{i,t-2}$ or earlier. As a result apart from $\widetilde{\pi}_{i,t-1}$ that enters (1), the use of inflation lagged two or more periods, namely, $\widetilde{\pi}_{i,t-2}$, $\widetilde{\pi}_{i,t-3}$, ..., cannot help identification and as a result do not contribute to meeting the full rank condition. Nevertheless, many papers in the literature routinely use second and higher order inflation lags as instruments. For example, Gali and Gertler (1999) use four lags of inflation, Batini, Jackson, and Nickell (2005, p. 1067) use five lags of

inflation, and Gali, Gertler and Lopez-Salido (2005) use four lags of inflation. Beyer *et al.* (2007) state that "as is usual" they use three lags of inflation, the output gap and the interest rates as instruments, but comment that it is questionable whether lags higher than one should be included.

As noted originally in Pesaran (1981, 1987, Ch. 7) and emphasised recently by Mavroeidis (2005), Beyer et al. (2007) and Nason and Smith (2008) among others, identification of the structural parameters critically depends on the process generating $\widetilde{y}_{it}$. For example, suppose that $\widetilde{y}_{it}$ follows the $AR(1)$ process

$$\widetilde{y}_{it} = \rho_i \widetilde{y}_{i,t-1} + v_{it}.$$

Then the RE solution is given by

$$\widetilde{\pi}_{it} = a_{i1} \widetilde{\pi}_{i,t-1} + a_{i2} \widetilde{y}_{i,t-1} + u_{it}, \tag{7}$$

where

$$a_{i1} = \lambda_{bi} = \frac{1 - \sqrt{1 - 4\beta_{fi}\beta_{bi}}}{2\beta_{fi}},$$

$$a_{i2} = \frac{1}{1 - \lambda_{bi}\beta_{fi}} \left( \frac{\gamma_i \rho_i}{1 - \rho_i \lambda_{fi}^{-1}} \right), \quad u_{it} = \frac{\varepsilon_{it} + \gamma_i v_{it}}{1 - \lambda_{bi}\beta_{fi}}.$$

The reduced form for $(\widetilde{\pi}_{it}, \widetilde{y}_{it})$ is a $VAR(1)$ that allows consistent estimation of the three parameters, $a_{i1}, a_{i2}$, and $\rho_i$, whilst we have four unknown coefficients, $\beta_{fi}, \beta_{bi}, \gamma_i$, and $\rho_i$. In this case the structural parameters $\beta_{fi}, \beta_{bi}$ and $\gamma_i$ are not identified. In other words although $\widetilde{\pi}_{i,t-s}, \widetilde{y}_{i,t-s}$ for $s = 2, 3, ...$, are uncorrelated with $\xi_{i,t+1}$, their use as instruments will not help in identification. This is because once $\widetilde{\pi}_{i,t-1}$ and $\widetilde{y}_{i,t-1}$ are included as instruments the additional lags do not contribute any further to the identification. Notice that the regression of the right hand side endogenous variables on the instruments may not be informative, (7) may fit very well even though the model is not identified.

More specifically, since $\widetilde{y}_{it}$ follows an $AR(1)$ process using $\mathbf{z}_{i,t-1} = (\widetilde{\pi}_{i,t-1}, \widetilde{y}_{i,t-1})'$ as instruments does not ensure that the rank condition will be met. Similarly, adding $\widetilde{\pi}_{i,t-s}, \widetilde{y}_{i,t-s}$ for $s = 2, 3, ...$, to the set of the instruments does not help either. This is because the RE solution for the inflation variable only depends on the first order lags of $\widetilde{\pi}_{it}$ and $\widetilde{y}_{it}$, and the additional lags of these variables will be uncorrelated with current and future inflation variable. For identification we need the order of the $AR(p)$ process for the output gap to be at least equal to two. In general if the output gap, $\widetilde{y}_{it}$, is $AR(p)$, the form for the RE solution is $ARDL(1, p-1)$ in $\widetilde{\pi}_{it}$ and $\widetilde{y}_{it}$. In the case where $\widetilde{y}_{it}$ follows the $AR(2)$ process

$$\widetilde{y}_{it} = \rho_{i1} \widetilde{y}_{i,t-1} + \rho_{i2} \widetilde{y}_{i,t-2} + v_{it},$$

then the extra instrument $\widetilde{y}_{i,t-2}$ exactly identifies the model. But the identification can be "weak" if $\rho_{i2}$ is not statistically significant. Weak instruments make GMM and the usual tests for over-identification unreliable, e.g. Stock et al. (2002).

So far we have assumed that the driving variable is exogenous in the sense that there are no feedbacks from lagged inflation to $\widetilde{y}_{it}$. However, it can be readily shown that allowing for feedbacks

from $\tilde{\pi}_{i,t-1}$ into $\tilde{y}_{it}$ will not resolve the weak instrument problem, unless it is assumed that the order of the lagged inflation term in the $\tilde{y}_{it}$ equation is greater than the order of the lagged inflation term in the NKPC equation. For example, augmenting the $AR(1)$ equation of the output gap with lagged inflation, namely

$$\tilde{y}_{it} = \rho_{iy}\tilde{y}_{i,t-1} + \rho_{i\pi}\tilde{\pi}_{i,t-1} + v_{it}, \tag{8}$$

does not alter the dynamic form of the inflation process and as before the lagged inflation terms, $\tilde{\pi}_{i,t-s}$ , $s = 2, 3, ...$ will not be valid instruments for future inflation in the NKPC equation. Notice that with (8) the present value condition (5) will no longer be valid since the future values of $\tilde{y}_{i,t+j}$ will be functions of future inflation.

The solution procedure discussed here for a closed economy is a special case of the global RE framework to be discussed in the next section. Nason and Smith (2008) set up a standard three equation New Keynesian model in which the output gap is influenced by inflation through the effect of inflation on the interest rate in the IS curve through the monetary policy rule. They show that the solution is a first order VAR so the identification problem remains. Of course, expanding the model to include more domestic variables or more lags could in principal solve the identification problem, but in practice the instruments are likely to be weak.

## 3  Identification by Global Factors

While the usual NKPC  may only be weakly identified, adopting a global context provides other instruments.[2]  As before to check identification, we need to find a solution for the rational expectations model, in this case for each country. Consider a multicountry version of the familiar three equation macro model comprising a NKPC, an optimising IS curve and a Taylor rule, for example discussed in Pesaran and Smith (2006). For each country $i$ we specify that

$$\begin{aligned} \mathbf{A}_{i0}\widetilde{\mathbf{x}}_{it} &= \mathbf{A}_{i1}\widetilde{\mathbf{x}}_{i,t-1} + \mathbf{A}_{i2}E_{t-1}(\widetilde{\mathbf{x}}_{i,t+1}) \\ &\quad + \mathbf{A}_{i3}\widetilde{\mathbf{x}}_{it}^* + \mathbf{A}_{i4}\widetilde{\mathbf{x}}_{i,t-1}^* + \mathbf{A}_{i5}E_{t-1}(\widetilde{\mathbf{x}}_{i,t+1}^*) + \boldsymbol{\varepsilon}_{it}, \end{aligned} \tag{9}$$

where $\widetilde{\mathbf{x}}_{it} = (\tilde{\pi}_{it}, \tilde{y}_{it}, \tilde{r}_{it})'$, and $\mathbf{x}_{it}^* = (\tilde{\pi}_{it}^*, \tilde{y}_{it}^*, \tilde{r}_{it}^*)'$ is the associated vector of foreign variables constructed as weighted cross section averages, defined as before by $\widetilde{\mathbf{x}}_{it}^* = \sum_{j=1}^{N} w_{ij}\widetilde{\mathbf{x}}_{jt}$ with $w_{ii} = 0$. Here expectations are taken with respect to a common global information set formed as the union intersection of the individual country information sets, $\mathcal{I}_{i,t-1}$. This formulation is sufficiently general for our purposes and represents an open economy version of the familiar three equation DSGE model composed of a NKPC, an output gap equation and an interest rule equation. Foreign variables need to appear in some equations of the domestic system, but not every equation and we will assume below that they do not appear in the Phillips Curve. Also, $\widetilde{\mathbf{x}}_{it}$ could contain other variables such as credit, money, or real equity prices, and need not include the same variables across $i$. In general, we assume that $\widetilde{\mathbf{x}}_{it}$ is a $k_i \times 1$ vector.

To solve the above rational expectations model a statistical model for $(\widetilde{\mathbf{x}}_{it}^*, \boldsymbol{\varepsilon}_{it})$ is clearly required. In the DSGE literature the foreign variables, $\widetilde{\mathbf{x}}_{it}^*$, are typically assumed to be strictly exogenous,

excluding any feedbacks from the lagged $\widetilde{\mathbf{x}}_{it}$. However, due to the presence of common factors and dominant country effects $\widetilde{\mathbf{x}}_{it}^*$ is unlikely to be strictly exogenous, and one needs to derive a globally consistent RE solution. This can be achieved by linking up the $N$ country-specific DSGE models using the equations for $\widetilde{\mathbf{x}}_{it}^*$. To see this let $\widetilde{\mathbf{z}}_{it} = (\widetilde{\mathbf{x}}_{it}', \widetilde{\mathbf{x}}_{it}^{*\prime})'$ and write the $N$ country-specific DSGE models as

$$\mathbf{A}_{iz0}\widetilde{\mathbf{z}}_{it} = \mathbf{A}_{iz1}\widetilde{\mathbf{z}}_{i,t-1} + \mathbf{A}_{iz2}E_{t-1}\left(\widetilde{\mathbf{z}}_{i,t+1}\right) + \boldsymbol{\varepsilon}_{it}, \text{ for } i = 1, 2, ..., N. \tag{10}$$

where $\mathbf{A}_{iz0} = (\mathbf{A}_{i0}, -\mathbf{A}_{i3})$, $\mathbf{A}_{iz1} = (\mathbf{A}_{i1}, \mathbf{A}_{i4})$ and $\mathbf{A}_{iz2} = (\mathbf{A}_{i2}, \mathbf{A}_{i5})$. But given that $\widetilde{\mathbf{x}}_{it}^* = \sum_{j=1}^{N} w_{ij}\widetilde{\mathbf{x}}_{jt}$, there must be a 'link' matrix $\mathbf{W}_i$ such that

$$\widetilde{\mathbf{z}}_{it} = \mathbf{W}_i\widetilde{\mathbf{x}}_t,$$

where $\widetilde{\mathbf{x}}_t = (\widetilde{\mathbf{x}}_{1t}', \widetilde{\mathbf{x}}_{2t}', ..., \widetilde{\mathbf{x}}_{Nt}')'$ is the $k \times 1$ vector of the endogenous variables in the global economy $(k = \Sigma_{i=1}^{N} k_i)$, and hence (10) can be written as

$$\mathbf{A}_{iz0}\mathbf{W}_i\widetilde{\mathbf{x}}_t = \mathbf{A}_{iz1}\mathbf{W}_i\widetilde{\mathbf{x}}_{t-1} + \mathbf{A}_{iz2}\mathbf{W}_iE_{t-1}(\widetilde{\mathbf{x}}_{t+1}) + \boldsymbol{\varepsilon}_{it}.$$

Stacking these models now yields

$$\mathbf{A}_0\widetilde{\mathbf{x}}_t = \mathbf{A}_1\widetilde{\mathbf{x}}_{t-1} + \mathbf{A}_2E_{t-1}(\widetilde{\mathbf{x}}_{t+1}) + \boldsymbol{\varepsilon}_t, \tag{11}$$

where

$$\mathbf{A}_j = \begin{pmatrix} \mathbf{A}_{1zj}\mathbf{W}_1 \\ \mathbf{A}_{2zj}\mathbf{W}_2 \\ \vdots \\ \mathbf{A}_{Nzj}\mathbf{W}_N \end{pmatrix}, \text{ and } \boldsymbol{\varepsilon}_t = \begin{pmatrix} \boldsymbol{\varepsilon}_{1t} \\ \boldsymbol{\varepsilon}_{2t} \\ \vdots \\ \boldsymbol{\varepsilon}_{Nt} \end{pmatrix}, \text{ for } j = 0, 1, 2,$$

and $\mathbf{A}_0$ is a non-singular matrix.

The solution properties of the RE model, (11), depends on the roots of the quadratic matrix equation[3]

$$\mathbf{A}_2\boldsymbol{\Phi}^2 - \mathbf{A}_0\boldsymbol{\Phi} + \mathbf{A}_1 = \mathbf{0}.$$

There will be a globally consistent RE solution if there exists a real matrix solution to the above equation such that all the eigenvalues of $\boldsymbol{\Phi}$ and $(\mathbf{I}_k - \mathbf{A}_2\boldsymbol{\Phi})^{-1}\mathbf{A}_2$ lie inside or on the unit circle. In such a case the unique solution is given by

$$\widetilde{\mathbf{x}}_t = \boldsymbol{\Phi}\widetilde{\mathbf{x}}_{t-1} + \mathbf{v}_t, \tag{12}$$

where $\mathbf{v}_t = \mathbf{A}_0^{-1}\boldsymbol{\varepsilon}_t$. This solution shows that all first order lags of inflation rates, output gaps and interest rates can be used as instruments. However, as in the closed economy example above, higher order lags are not valid instruments, because they do not appear in the solution.

The dimension of $\widetilde{\mathbf{x}}_t$ in (12) rises with $N$ and, given the length of available time-series, it is unlikely to be feasible to estimate the reduced form parameters of the solution directly even for moderate values of $N$. For the same reason maximum likelihood estimation of the full rational

expectation model, (11), will not be feasible either. However, as Chudik and Pesaran (2009, CP) show, the RE solution can be decomposed into conditional country-specific models under certain restrictions on the degree of dependence across countries and over time as the number of countries increases. The degree of cross dependence is governed by the size of the off diagonal elements of $\boldsymbol{\Phi}$ and the largest eigenvalue of the covariance matrix of the reduced form errors, $\mathbf{v}_t$. CP show that when the column and row matrix norms of $\boldsymbol{\Phi}$ are bounded in $N$ and the degree of error cross dependence is weak then as $N \rightarrow \infty$, (12) reduces to country-specific VAR models and global interdependencies can be ignored. However, in cases where one or more column norms of $\boldsymbol{\Phi}$ are unbounded in $N$ and/or there are strong error dependencies, for a valid decomposition of the infinite dimensional VAR the country specific models must also be conditioned on the dominant observed or unobserved factors.

In practice, due to common technology and commodity price shocks in the world economy, one would expect $\mathbf{v}_t$ to contain common factors which can be expressed as, $\mathbf{v}_t = \boldsymbol{\Gamma}\mathbf{f}_t + \boldsymbol{\eta}_t$, where $\mathbf{f}_t$ is the $m \times 1$ vector of common factors ($m$ being fixed as $N \rightarrow \infty$), and $\boldsymbol{\eta}_t$ are the remaining errors that are weakly cross correlated. In this case CP show that the country specific reduced form models for $i = 1, 2, ..., N$ can be written as

$$\widetilde{\mathbf{x}}_{it} = \boldsymbol{\Phi}_{ii}\widetilde{\mathbf{x}}_{i,t-1} + \boldsymbol{\Psi}_{i0}\widetilde{\mathbf{x}}_{it}^* + \boldsymbol{\Psi}_{i1}\widetilde{\mathbf{x}}_{i,t-1}^* + \mathbf{e}_{it}. \tag{13}$$

where $\boldsymbol{\Phi}_{ii}$ is the $i$ th block of $\boldsymbol{\Phi}$. The $\widetilde{\mathbf{x}}_{it}^*$ are weighted averages of the $\widetilde{\mathbf{x}}_t$, such as $\widetilde{\mathbf{x}}_{it}^* = \sum_{j=1}^{N} w_{ij}\widetilde{\mathbf{x}}_{jt}$, which act as proxies for the $\mathbf{f}_t$, with the weights assumed to be granular, such that for each $i$, $\Sigma_{j=1}^{N} w_{ij}^2 \rightarrow 0$, as $N \rightarrow \infty$. Although all the elements of $\widetilde{\mathbf{x}}_t$ are endogenous, when $N$ is sufficiently large and there are only a few dominant economies and/or common factors, the weighted cross section averages of the foreign variables, the $\widetilde{\mathbf{x}}_{it}^*$, can be treated as weakly exogenous for the purpose of estimation. Although $\mathbf{x}_{it}^*$ and $\mathbf{v}_{it}$ are correlated for a fixed $N$, they become uncorrelated as $N \rightarrow \infty$. This also implies that current values of the $\widetilde{\mathbf{x}}_{it}^*$ can be used as instruments in estimation of the structural equations. So for small open economies, $\widetilde{\pi}_{it}^*$, $\tilde{y}_{it}^*$, and $\widetilde{r}_{it}^*$ are valid instruments when estimating their NKPC equations in which these variables do not appear directly. The weak exogeneity assumption for these variables is testable and, as reported below, is accepted in our application.

The global perspective provides a very large number of potential instruments and there is the danger that the IV estimator will just closely approximate the biased OLS estimator. Thus one needs to map the large number of potential instruments into a smaller number that satisfy the two conditions for valid instruments. Kapetanios and Marcellino (2007) and Ng and Bai (2009) investigate estimating factor models and using the estimated factors as instruments. The GVAR provides an alternative mapping by measuring the factors, $\mathbf{f}_t$, by $\widetilde{\mathbf{x}}_{it}^*$, the trade weighted averages of the foreign variables corresponding to a particular domestic variable. Other mappings such as principal components are possible.

# 4 Characterisation of Steady States as Long-horizon Expectations

The steady state values used in the log-linearised version of DSGE models are usually assumed either to be constants or measured by a statistical procedure such as the HP filter. The standard procedure thus does not use any economic information about the steady state and could be mis-specified in the presence of stochastic trends. The usual assumption that steady state inflation is a constant may be misleading because there are shifts in the steady state because of changing inflation targets by the monetary authorities and because of the possibility of a stochastic trend in inflation since many estimates suggest that $\beta_{fi} + \beta_{bi}$ is close to or equal to unity. In either case the evolution of mean inflation needs to be modelled, possibly in terms of other factors. With respect to output, most statistical filters, like the HP filter, are two sided, using information about future values of the variables in calculation of the steady state values, rather than using the information available to the agents at their decision time.[4] This not only raises problems for forecasting with models using HP filtered data, it also does not represent agents' judgement about equilibrium output at the time. There is also a danger, as Harvey and Jaeger (1993) point out, that the HP filter can induce spurious serial correlation. However, our argument is not specific to the HP filter and equally applies to band pass and other filters that are not consistent with the underlying economic model.

There are a number of ways that one could estimate economic measures of the steady states, including by use of the Kalman filter, but here we consider using the long horizon expectations from an economic model. Denote the steady state of a vector of variables as $\mathbf{x}_t^P$ and the deviations from steady state as $\widetilde{\mathbf{x}}_t = \mathbf{x}_t - \mathbf{x}_t^P$. If a unique steady state exists and is stable, then one would expect $\widetilde{\mathbf{x}}_t$ to be a stationary process such that the long-horizon expectations of the deviation of $\mathbf{x}_t$ from its steady state must be zero, namely

$$\lim_{h \to \infty} E_t \left( \widetilde{\mathbf{x}}_{t+h} \right) = \lim_{h \to \infty} E_t \left( \mathbf{x}_{t+h} - \mathbf{x}_{t+h}^P \right) = \mathbf{0}. \tag{14}$$

This implies that

$$\lim_{h \to \infty} E_t \left( \mathbf{x}_{t+h} \right) = \lim_{h \to \infty} E_t \left( \mathbf{x}_{t+h}^P \right). \tag{15}$$

Steady state values must be consistent with the underlying DSGE model and should satisfy certain time consistency properties. In the absence of deterministic trends, a definition of steady state that meets both of these two criteria is given by the long-horizon expectations

$$\mathbf{x}_t^P = \lim_{h \to \infty} E_t \left( \mathbf{x}_{t+h} \right), \tag{16}$$

where expectations are taken with respect to the underlying DSGE model and it is assumed that the information set is non-decreasing with time. Under this specification and recalling from (15) that $\lim_{h \to \infty} E_t \left( \mathbf{x}_{t+h} \right) = \lim_{h \to \infty} E_t \left( \mathbf{x}_{t+h}^P \right)$ it now readily follows that

$$\mathbf{x}_t^P = \lim_{h \to \infty} E_t \left( \mathbf{x}_{t+h}^P \right).$$

This ensures that the steady states are time consistent, in the sense that [5]

$$E_t(\mathbf{x}_{t+s}^P) = E_t \left[ \lim_{h \to \infty} E_{t+s} \left( \mathbf{x}_{t+s+h}^P \right) \right] = \lim_{h \to \infty} E_t \left[ E_{t+s} \left( \mathbf{x}_{t+s+h}^P \right) \right] = \lim_{h \to \infty} E_t \left( \mathbf{x}_{t+s+h}^P \right),$$

and hence

$$\lim_{h\to\infty} E_t\left(\mathbf{x}^P_{t+s+h}\right) = \lim_{h\to\infty} E_t\left[\lim_{h'\to\infty} E_{t+s+h}\left(\mathbf{x}_{t+s+h+h'}\right)\right] = \lim_{h\to\infty}\left[\lim_{h'\to\infty} E_t\left(\mathbf{x}_{t+s+h+h'}\right)\right] = \mathbf{x}^P_t.$$

This result also establishes that, in the absence of deterministic components, steady state values satisfy the martingale property, $E_t(\mathbf{x}^P_{t+1}) = \mathbf{x}^P_t$. This is not a property which is satisfied by most statistical measures of steady states such as those produced by the HP filter, but is a natural requirement of any coherent definition of the steady state.

If the elements of $\mathbf{x}_t$ are stationary, $\mathbf{x}^P_t$ will be a vector of constants, while if they show deterministic trends, $\mathbf{x}^P_t$ will have a deterministic trend and the long horizon expectation will have to be adjusted accordingly. For example, in the case of linear trends $\mathbf{x}^P_t$ should be defined as

$$\mathbf{x}^P_t = \lim_{h\to\infty} E_t(\mathbf{x}_{t+h} - \boldsymbol{\gamma} h),$$

where $\boldsymbol{\gamma}$ is the vector of trend coefficients. If the elements of $\mathbf{x}_t$ are $I(1)$ and possibly cointegrated, then $\mathbf{x}^P_t$ turns out to be the same as the permanent component in a multivariate version of the Beveridge-Nelson decomposition of $\mathbf{x}_t$ as pointed out by Garratt, Robertson and Wright (2006). See also Garratt, Lee, Pesaran and Shin (2006, Ch. 10). The details of the calculation of the steady states for the GVAR are discussed below.

## 5 Measuring Steady states with the GVAR

### 5.1 The GVAR

Above we derived the solution to a global RE model in terms of deviations from steady state and showed that for small open economies linear combinations of the foreign variables may be treated as weakly exogenous, a testable assumption. We also showed that the steady states could be measured as the long-horizon forecasts from a dynamic model of the variables. Thus in the relation $\widetilde{\mathbf{x}}_t = \mathbf{x}_t - \mathbf{x}^P_t$ the $\mathbf{x}^P_t$ can be represented by functions of current and lagged $\mathbf{x}_t$. Therefore our procedure is to approximate the global unrestricted reduced form in terms of the original variables, use this to measure the steady states and then estimate structural models using deviations from these steady states and the foreign variables as instruments.

The reduced form for each country in terms of deviations from steady state is given by (13). Replacing the steady states by distributed lags of the original variables and adding the deterministic terms gives the solution of the DSGE as a set of reduced forms for each country which can be expressed as a VARX* in the original variables:

$$\mathbf{x}_{it} = \mathbf{b}_{0i} + \mathbf{b}_{1i}t + \boldsymbol{\Phi}_{ii}\mathbf{x}_{i,t-1} + \boldsymbol{\Psi}_{i0}\mathbf{x}^*_{it} + \boldsymbol{\Psi}_{i1}\mathbf{x}^*_{i,t-1} + \mathbf{e}_{it}. \tag{17}$$

The parameters of this VARX* will reflect both the parameters in (13) and the parameters determining the steady state values in terms of the original variables. This VARX* can be estimated separately for each country conditional on $\mathbf{x}^*_{it}$, taking into account the possibility of cointegration

both within $\mathbf{x}_{it}$ and across $\mathbf{x}_{it}$ and $\mathbf{x}_{it}^*$. Although estimation is done on a country by country basis, the individual VARX* models can be combined into a cointegrating global vector autoregression, GVAR, using the weighting matrices $\mathbf{W}_i$ and following the same process as going from (10) to (12). This provides a feasible way to estimate the parameters of the reduced form in terms of the original variables, rather than deviations from steady state as in (12). The steady states can be calculated from the GVAR as long-horizon forecasts, the details are given below, and the deviations from steady states then used in structural modelling.

The GVAR model we use is developed from that described in DdPS.[6] VARX* models of the form (17) are estimated for 33 countries[7], covering over 90% of world output, linked within a unified GVAR framework. Relative to the DdPS model (a) the estimation period has been extended to cover 1979Q4-2006Q4 (rather than to 2003Q4); (b) 8 European countries have been treated separately rather than aggregated into a euro block, to allow examination of the Phillips Curve for each of the major industrial economies and (c) real equity prices and long interest rates have been excluded from the model for consistency with the standard DSGE based macro model. The variables included are given in Table 1 below, with the US treated differently given its importance in the world economy and the fact that US dollar is used as a reference currency.

This version of the GVAR model has 131 endogenous variables, 82 stochastic trends and 49 cointegrating relations. All the roots of the global VAR model in the 33 countries either lie on or inside the unit circle. The moduli of the largest non unit eigenvalue is 0.926. It has fewer cointegrating relations than DdPS because excluding the long interest rate removes the term structure relationship, which is likely to be I(0). The lag orders for the domestic variables, $p_i$, and foreign variables, $q_i$, is selected based on the Akaike criterion with $p_{\max i} = 2$ and $q_{\max i} = 1$. The individual country models are estimated subject to reduced rank restrictions as described in DdPS and the cointegrating relations obtained are based on the trace statistic at the 95% critical value.

For estimation, $\mathbf{x}_{it}^*$ are treated as "long-run forcing" or $I(1)$ weakly exogenous with respect to the parameters of the conditional model. This assumption can be tested by regressing the $\mathbf{x}_{it}^*$ on the error correction terms for country $i$ and testing whether these terms are statistically significant. Only for 8.4 per cent of the cases was this restriction rejected (which is only slightly larger than the nominal 5% level used when carrying out the tests), in particular the hypothesis of weak exogeneity cannot be rejected for foreign output and inflation in the US model. The model uses the exactly identified cointegrating vectors. Discussion of the effect of imposing over-identifying restrictions on the long run relations can be found in Dees, Holly, Pesaran and Smith (2007), but are likely to be of second order importance for the estimation of the parameters of the NKPC equation, which is the focus of the present analysis.

## 5.2   Estimation of the Steady States

In this section we discuss how we obtain the estimate of, say, the output gap as the deviation of output from its steady state, $\widetilde{y}_{it} = y_{it} - y_{it}^P$, from the decomposition of the variables in the GVAR

into their permanent, $\mathbf{x}_t^P$, and transitory components, $\mathbf{x}_t^C$ (or equivalently $\tilde{\mathbf{x}}_t$). Denote the $k \times 1$ vector of endogenous variables in the global economy by $\mathbf{x}_t$, and consider the decomposition, $\mathbf{x}_t = \mathbf{x}_t^P + \mathbf{x}_t^C$, dividing the permanent component, $\mathbf{x}_t^P$, into deterministic and stochastic components, $\mathbf{x}_t^P = \mathbf{x}_{dt}^P + \mathbf{x}_{st}^P$. The permanent-deterministic component, $\mathbf{x}_{dt}^P$, is

$$\mathbf{x}_{dt}^P = \boldsymbol{\mu} + \mathbf{g}t,$$

where $\boldsymbol{\mu}$ and $\mathbf{g}$ are $k \times 1$ vectors of fixed constants, and $t$ is a deterministic time trend. The permanent-stochastic component, $\mathbf{x}_{st}^P$, is then *uniquely* defined as the 'long-horizon forecast' (net of the permanent-deterministic component)

$$\mathbf{x}_{st}^P = \lim_{h \to \infty} E_t \left( \mathbf{x}_{t+h} - \mathbf{x}_{d,t+h}^P \right) = \lim_{h \to \infty} E_t \left[ \mathbf{x}_{t+h} - \boldsymbol{\mu} - \mathbf{g}(t+h) \right], \tag{18}$$

and $E_t(.)$ denotes the expectations operator conditional on the information available at time $t$, taken to include at least $\{\mathbf{x}_t, \mathbf{x}_{t-1}, ..., \mathbf{x}_0\}$.[8]

The above decomposition has a number of attractive properties. The permanent stochastic component is identically equal to zero if the process generating $\mathbf{x}_t$ is trend stationary. On the other extreme $\mathbf{x}_{st}^P = \mathbf{x}_t$ if $\mathbf{x}_t$ is a pure unit root process and non-cointegrated. The GVAR provides a model of interest that lies somewhere in between these two extremes and allows derivation of permanent components that take account of unit roots and cointegration in the global economy. To illustrate some of these points and highlight the uniqueness of $\mathbf{x}_{st}^P$, as a simple example abstract from the deterministics and suppose that $\mathbf{x}_t$ follows a VAR of order 1 with the coefficient matrix $\boldsymbol{\Phi}$. It is then easily seen that $\mathbf{x}_{st}^P = \lim_{h \to \infty} E_t(\mathbf{x}_{t+h}) = \left( \lim_{h \to \infty} \boldsymbol{\Phi}^h \right) \mathbf{x}_t = \boldsymbol{\Phi}^{\infty} \mathbf{x}_t$. Hence, as indicated $\mathbf{x}_{st}^P = \mathbf{0}$, if the VAR(1) process is stationary and all eigenvalues of $\boldsymbol{\Phi}$ lie within the unit circle, $\mathbf{x}_{st}^P = \mathbf{x}_t$ if $\mathbf{x}_t$ is a unit root process with $\boldsymbol{\Phi} = \mathbf{I}_k$. But when $\mathbf{I}_k - \boldsymbol{\Phi}$ is rank deficient and some of the roots of $\boldsymbol{\Phi}$ lie exactly on the unit circle, $\mathbf{x}_{st}^P$ will be determined by the linear combinations of $\mathbf{x}_t$ that are not cointegrated.

The GVAR is constructed from the underlying country-specific models and its global error correction form is given by

$$\mathbf{G}\Delta\mathbf{x}_t = \mathbf{a} - \tilde{\boldsymbol{\alpha}}\tilde{\boldsymbol{\beta}}' \left[ \mathbf{x}_{t-1} - \boldsymbol{\gamma}(t-1) \right] + \sum_{i=1}^{p-1} \boldsymbol{\Gamma}_i \Delta\mathbf{x}_{t-i} + \mathbf{u}_t, \tag{19}$$

where $\mathbf{G}$ is a $k \times k$ matrix that reflects the contemporaneous interdependencies across countries, $\boldsymbol{\gamma}$ is a $k \times 1$ vector of fixed constants, $\tilde{\boldsymbol{\alpha}}$ is the $k \times r$ block-diagonal matrix of the global loading coefficients, with diagonal elements $\boldsymbol{\alpha}_j$, with $r = \sum_{i=1}^{N} r_i$ and $r_i$ is the cointegrating rank for country $i$, and $\tilde{\boldsymbol{\beta}}$ is the global $k \times r$ cointegrating matrix[9]: $\tilde{\boldsymbol{\beta}} = \left( \mathbf{W}_1' \boldsymbol{\beta}_1, \quad \mathbf{W}_2' \boldsymbol{\beta}_2, \quad ..., \quad \mathbf{W}_N' \boldsymbol{\beta}_N \right)$.

To derive the permanent components, we first write the global error correction model, (19), as the VAR(p) specification

$$\mathbf{x}_t = \mathbf{b}_0 + \mathbf{b}_1 t + \sum_{i=1}^{p} \boldsymbol{\Phi}_i \mathbf{x}_{t-i} + \boldsymbol{\varepsilon}_t, \tag{20}$$

where

$$\mathbf{b}_0 = \mathbf{G}^{-1}(\mathbf{a} - \tilde{\boldsymbol{\alpha}}\tilde{\boldsymbol{\beta}}'\boldsymbol{\gamma}), \ \mathbf{b}_1 = \mathbf{G}^{-1}\tilde{\boldsymbol{\alpha}}\tilde{\boldsymbol{\beta}}'\boldsymbol{\gamma}, \ \boldsymbol{\varepsilon}_t = \mathbf{G}^{-1}\mathbf{u}_t,$$

$$\boldsymbol{\Phi}_1 = \mathbf{G}^{-1}(\mathbf{G} + \boldsymbol{\Gamma}_1 - \tilde{\boldsymbol{\alpha}}\tilde{\boldsymbol{\beta}}'), \ \boldsymbol{\Phi}_i = \mathbf{G}^{-1}(\boldsymbol{\Gamma}_i - \boldsymbol{\Gamma}_{i-1}), \ i = 2, ...., p-1, \ \boldsymbol{\Phi}_p = -\mathbf{G}^{-1}\boldsymbol{\Gamma}_{p-1}.$$

Using (20) we can now write down the solution of $\mathbf{x}_t$ as

$$\mathbf{x}_t = \boldsymbol{\mu} + \mathbf{g}t + \mathbf{C}\left(1\right)\mathbf{s}_{\varepsilon t} + \mathbf{C}^*\left(L\right)\boldsymbol{\varepsilon}_t, \tag{21}$$

where

$$\boldsymbol{\mu} = \mathbf{x}_0 - \mathbf{C}^*(L)\boldsymbol{\varepsilon}_0, \ \mathbf{s}_{\varepsilon t} = \sum_{j=1}^{t} \boldsymbol{\varepsilon}_j, \ \mathbf{C}^*\left(L\right) = \sum_{j=0}^{\infty} \mathbf{C}_j^* L^j,$$

$$\mathbf{C}_j = \mathbf{C}_{j-1}\boldsymbol{\Phi}_1 + \mathbf{C}_{j-2}\boldsymbol{\Phi}_2 + \cdots + \mathbf{C}_{j-p}\boldsymbol{\Phi}_p, \text{ for } j = 1, 2, ...,$$

with $\mathbf{C}_0 = \mathbf{I}_k$, $\mathbf{C}_1 = -(\mathbf{I}_k - \boldsymbol{\Phi}_1)$, and $\mathbf{C}_j = \mathbf{0}$ for $j < 0$; $\mathbf{C}_j^* = \mathbf{C}_{j-1}^* + \mathbf{C}_j$, for $j = 1, 2, ...,$ with $\mathbf{C}_0^* = \mathbf{C}_0 - \mathbf{C}(1)$, and $\mathbf{C}(1) = \sum_{j=0}^{\infty} \mathbf{C}_j$. Hence, it is easily seen that

$$\mathbf{x}_{st}^P = \lim_{h \to \infty} E_t\left[\mathbf{x}_{t+h} - \boldsymbol{\mu} - \mathbf{g}(t+h)\right] = \mathbf{C}\left(1\right) \sum_{j=1}^{t} \boldsymbol{\varepsilon}_j, \tag{22}$$

which is the multivariate version of the Beveridge-Nelson (BN) stochastic trend component. Note that $\mathbf{x}_{st}^P$ is uniquely determined from the time series observations on $\mathbf{x}_t$ and its lagged values. The identification problem with the BN decomposition discussed in the literature relates to separating the $k$ shocks, $\boldsymbol{\varepsilon}_t$, into permanent (supply) or transitory (demand) shocks. A general discussion of this problem is provided by Pagan and Pesaran (2008).

The permanent-stochastic component can now be estimated directly from the parameters of the GVAR as $\hat{\mathbf{x}}_{st}^P = \hat{\mathbf{C}}\left(1\right) \sum_{i=1}^{t} \hat{\boldsymbol{\varepsilon}}_i$. The transitory component, $\hat{\mathbf{x}}_t^C$, the deviation from steady state, can then be estimated as

$$\hat{\mathbf{v}}_t = \mathbf{x}_t - \hat{\mathbf{x}}_{st}^P = \hat{\boldsymbol{\mu}} + \hat{\mathbf{g}}t + \hat{\mathbf{x}}_t^C \tag{23}$$

with $\hat{\boldsymbol{\mu}}$ and $\hat{\mathbf{g}}$ in turn estimated from the OLS regressions

$$\hat{\mathrm{v}}_{i,\ell t} = \mu_{i\ell} + g_{i\ell}t + \xi_{i,\ell t}, \ i = 1, 2, ..., N; \ \ell = 1, ..., k_i$$

for variable $\ell$ in country $i$. In this way we are also able to impose a number of trend restrictions of interest. For example, we set $g_{i\pi} = g_{i,r}^S = g_{i,r}^L = 0$ in all countries, as it does not seem reasonable to allow for long-run trends in inflation and interest rates. The estimated transitory component, $\hat{\mathbf{x}}_t^C$, is then the residual from the above regressions, that is $\hat{\mathbf{x}}_t^C = (\hat{\boldsymbol{\xi}}_1', \hat{\boldsymbol{\xi}}_2'..., \hat{\boldsymbol{\xi}}_N')'$.

In the empirical applications we consider two measures of output gaps: one based on the GVAR and computed as above which we denote by $\tilde{y}_{it}$, and the familiar HP measure denoted by $\tilde{y}_{it}^{HP}$. Measures of country-specific foreign output gaps are computed as $\tilde{y}_{it}^* = \sum_{j=1}^{N} w_{ij}\tilde{y}_{jt}$.

Note that in contrast to $\tilde{y}_{it}^{HP}$, the output gap measures, $\tilde{y}_{it}$ will reflect the structure of the full GVAR model of the economy, including the variables chosen, the lag orders selected, the cointegrating relations imposed and the treatment of deterministic elements. Changing any of these will change the estimated decomposition. This seems a desirable feature as compared to

statistical procedures like the HP filter where the estimate is invariant to the form of the economic model. However, where there is uncertainty about the form of the model and the appropriate sample to be used for estimation, one could use some form of model averaging to obtain a more robust decomposition.

# 6 Estimates of the NKPC

We have argued that the use of a global perspective has the advantage that it provides valid instruments that can avoid the problem of weak identification and allows the calculation of economic rather than statistical measures of the steady state. It is also important to evaluate the estimates of the NKPC on a number of countries rather than just the US, which is unusual in being large and relatively closed. We shall therefore present estimates for eight developed industrial countries, though one could also consider all 33 countries in the GVAR. The GVAR is estimated over 1979Q1-2006Q4, but since some observations are lost through initialising the steady state and the use of future inflation the estimation period for the NKPC is 1980Q1-2006Q3. Initially we follow the literature (a) in treating steady state inflation as a constant picked up by the intercept, and (b) in measuring steady state output by the HP filter using 1600 as its smoothing parameter, and denoting the deviation of output from its HP steady state by $\widetilde{y}_{it}^{HP}$. We begin by estimating the conventional formulation of the NKPC equation:

$$\pi_{it} = \alpha_i + \beta_{fi} E(\pi_{i,t+1} \mid \mathfrak{I}_{i,t-1}) + \beta_{bi}\pi_{i,t-1} + \gamma_i \widetilde{y}_{it}^{HP} + \varepsilon_{it},$$

for countries $i = 1, 2, ..., N$. As instruments we use one lag of the domestic variables suggested by the closed economy three equation model: i.e. an intercept, $\pi_{i,t-1}$, $\widetilde{y}_{i,t-1}^{HP}$, and $r_{i,t-1}$. Table 2 gives the coefficient estimates and t ratios based on Newey-West standard errors (using a Bartlett window of size 4). The table also gives the standard error of regression, $\hat{\sigma}_i$, and Generalised $R^2$ of Pesaran and Smith (1994), which measures the degree of the fit of the IV regressions. With a pure forward looking model $\beta_{fi}$ should be the discount factor, $\lambda_i$ in (2) and $\beta_{bi} = 0$. It is common in the literature to assume a discount rate of about 1% a quarter, so we also give Wald statistics for testing the joint hypothesis $\beta_{fi} = 0.99$, and $\beta_{bi} = 0$. The prior for $\gamma_i$ in the literature appears to be about 0.12, corresponding to about 30% of firms resetting prices in a quarter.

The estimates reported in Table 2 suggest that expectations are forward looking: $\beta_{fi}$ is large and $\beta_{bi}$ is small and a purely forward model is not rejected in four of the countries. The major problem with the estimates is that the coefficient of the output gap is negative in the case of four countries and statistically insignificant in all cases. This is a common finding in the literature and could be symptomatic of the weak instrument problem. Imposing the restriction that $\beta_{fi} = 0.99$ and $\beta_{bi} = 0$, where accepted, reduces the standard errors in some cases, but otherwise does not change the estimates much.

We then consider the effect of using the GVAR estimate of the output deviation, $\widetilde{y}_{it}$, instead of the HP estimate, $\widetilde{y}_{it}^{HP}$. The two estimates of the output deviation are rather different, though their

correlation is always positive, varying from 0.19 for Japan to 0.58 for Italy. The differences arise partly from the two sided nature of the HP filter, which tends to adjust before a shock, using future information which is not available to agents or the GVAR and partly because the GVAR estimate is multivariate using information in other series, while the HP only uses univariate information. The estimates using $\widetilde{y}_{it}$, with instruments: intercept, $\pi_{i,t-1}$, $\widetilde{y}_{i,t-1}$, $r_{i,t-1}$, are given in Table 3.

The results show a similar forward looking pattern for inflation, the fit as measured by $GR^2$ is very similar between Tables 2 and 3, though in Table 3 the coefficient of output is now negative only in Italy, but is never significant. Continuing to assume steady state inflation is constant and using the GVAR estimate of the deviation of output from its steady state, we extend the instrument set by adding current and lagged foreign variables. The instruments are then: intercept, $\pi_{i,t-1}$, $\widetilde{y}_{i,t-1}, r_{i,t-1}, \pi_{it}^*, \widetilde{y}_{it}^*, r_{it}^*, \pi_{i,t-1}^*, \widetilde{y}_{i,t-1}^*, r_{i,t-1}^*, \Delta poil_t$. The results are given in Table 4.

Adding the foreign instruments results in a substantial improvement in fit, as measured by $GR^2$, and in an increase in the precision of estimate of the coefficient of output, which is now significant in the US, UK and France. The purely forward looking specification is rejected strongly in Italy and marginally in Canada and the UK, but accepted elsewhere. If the restrictions $\beta_{fi} = 0.99$ and $\beta_{bi} = 0$ are imposed, the US estimate of $\gamma_i$ is 0.126 (very close to the usual prior) with a t ratio of 4.09.

Next we consider the effects of estimating the NKPC using inflation measured as a deviation from the GVAR estimate of steady state, $\widetilde{\pi}_{it}$ (rather than assuming the steady state inflation to be constant as above), which allows for possible unit root effects. Specifically

$$\widetilde{\pi}_{it} = \beta_{fi} E(\widetilde{\pi}_{i,t+1} \mid \mathfrak{I}_{i,t-1}) + \beta_{bi}\widetilde{\pi}_{i,t-1} + \gamma_i\widetilde{y}_{it} + \varepsilon_{it},$$

where we have dropped the intercept term since the deviations have zero means by construction. We use the full set of instruments with all variables in deviation form (i.e. $\widetilde{\pi}_{i,t-1}$, $\widetilde{y}_{i,t-1}$, $\widetilde{r}_{i,t-1}$, $\widetilde{\pi}_{it}^*$, $\widetilde{y}_{it}^*, \widetilde{r}_{it}^*, \widetilde{\pi}_{i,t-1}^*, \widetilde{y}_{i,t-1}^*, \widetilde{r}_{i,t-1}^*, \Delta poil_t$, and an intercept). The results are given in Table 5. $GR^2$ and $\hat{\sigma}_i$ are not strictly comparable with the earlier tables because the dependent variable is different.

The coefficient of output is now significant in four countries, strongly so in the US. The same three countries, Italy, Canada and the UK reject the purely forward looking model as in the previous table. If the forward looking restriction $\beta_{fi} = 0.99$, $\beta_{bi} = 0$ is imposed, both the size of estimate of $\gamma_i$ and its t ratio tend to increase: for the US the estimate of $\gamma_i$ is 0.127 (t= 4.39); Germany 0.043 (t=2.49), France 0.074 (t=2.49), Spain 0.065 (t=2.33). However, it appears to be the foreign instruments rather than the measurement of inflation as a deviation from its steady state or imposing the forward looking restriction which accounts for much of the increase in precision of the estimate of the output coefficient.

# 7    Conclusion

In this paper we have highlighted two main issues that surround the NKPC and its estimation in a global context; namely identification and measurement of steady states. We have argued that

both need to be approached from an economic theory perspective and cannot be resolved in a satisfactory manner by resort to purely statistical procedures. To determine instrument validity requires explicit solution of the rational expectations model and identification will depend on the form of the model for the driving processes. Similarly, to determine steady states requires an explicit long-run economic model. Unlike the HP filter, the GVAR estimates of the steady states as long-horizon expectations are model dependent. This seems to be a desirable feature as our estimate of steady state should reflect economic information.

The global perspective, using the GVAR as a framework, contributes to both issues and this was illustrated using estimates of the NKPC for 8 developed industrial countries. The GVAR provides global factors that are valid instruments and help alleviate the weak instrument problem. The GVAR steady states also reflect global influences and any long-run theoretical relationships that might prevail within and across countries in the global economy. The GVAR measure of the steady state performed better than the HP measure, and the use of foreign instruments substantially increased the precision of the estimates of the output coefficient as one might expect if there was a weak instrument problem. Measuring all variables, including inflation, as deviations from their steady states also produced some improvement. The US estimate of the output coefficient was very similar to those estimated elsewhere, at about 0.12, but estimated very precisely. As is common in the literature, the estimates suggested that future inflation had greater weight than past inflation and a pure forward looking model could not be rejected in case of 5 out of the 8 countries considered.

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# Notes

[1]Some authors condition agent's expectations on information sets dated at time $t$ rather $t-1$. For macroeconomic relations where considerable aggregation of information across heterogeneous agents is made, the use of information sets dated $t-1$ seems more appropriate. But our analysis can be readily modified to deal with dated $t$ information sets.

[2]Monacelli (2005) provides a theoretical discussion of the open economy NKPC, whilst Ihrig et al. (2007) give a review of recent empirical work on external influences on inflation, with an emphasis on whether these have changed with the process of globalisation.

[3]See, for example, Binder and Pesaran (1995,1997).

[4]Some like Beyer *et al.* (2007) recognise this problem and use one-sided HP filters.

[5]We are assuming that the limit and expectations operators can be interchanged.

[6]The original data (ending in 2003Q4) and codes for the DdPS model are available on the Journal of Applied Econometrics data archive (http://qed.econ.queensu.ca/jae/).

[7]The countries are Argentina, Australia, Austria, Belgium, Brazil, Canada, Chile, China, Finland, France, Germany, India, Indonesia, Italy, Japan, Korea, Malaysia, Mexico, Netherlands, New Zealand, Norway, Peru, Philippines, South Africa, Saudi Arabia, Singapore, Spain, Sweden, Switzerland, Thailand, Turkey, UK, USA.

[8]One could equally well have derived the long horizon forecast with respect to the information set at t-1. Here we have chosen to work with time $t$ long-horizon expectations so that, as we shall see, the permanent-stochastic component coincides with that obtained in the Beveridge-Nelson decomposition. This should help the comparability of our results with those in the literature.

[9]Note that for the deterministic trend properties of the variables to be the same in the global model as in the underlying country-specific models $\tilde{\boldsymbol{\alpha}}\tilde{\boldsymbol{\beta}}'\boldsymbol{\gamma} = \left( \begin{array}{cccc} (\boldsymbol{\alpha}_1\boldsymbol{\beta}_1'\mathbf{W}_1\boldsymbol{\gamma})', & (\boldsymbol{\alpha}_2\boldsymbol{\beta}_2'\mathbf{W}_2\boldsymbol{\gamma})', & \ldots, & (\boldsymbol{\alpha}_N\boldsymbol{\beta}_N'\mathbf{W}_N\boldsymbol{\gamma})' \end{array} \right)'$ where $\boldsymbol{\alpha}_i$ and $\boldsymbol{\beta}_i$ are the loading coefficients and the cointegrating matrix, respectively, of the individual country models.

# Tables

**Table 1: Domestic and Foreign Variables Included in the Individual Country Models**

|  | All Countries Excluding US | | US | |
|---|---|---|---|---|
| Variables | Endogenous | Foreign | Endogenous | Foreign |
| Real Output | $y_{it}$ | $y_{it}^*$ | $y_{us,t}$ | $y_{us,t}^*$ |
| Inflation | $\pi_{it}$ | $\pi_{it}^*$ | $\pi_{us,t}$ | $\pi_{us,t}^*$ |
| Real Exchange Rate | $ep_{it}$ | - | - | $ep_{us,t}^*$ |
| Short-Term Interest Rate | $r_{it}$ | $r_{it}^*$ | $r_{us,t}$ | - |
| Oil Price | - | $p_t^o$ | $p_t^o$ | - |

**Table 2: Estimates (and t ratios) for the NKPC, using the HP estimate of the output deviation, $\widetilde{y}_{it}^{HP}$; constant steady state inflation, and domestic instruments**

|  | $\hat{\beta}_{fi}$ | $\hat{\beta}_{bi}$ | $\hat{\gamma}_i$ | $GR^2$ | $\hat{\sigma}_i$ | $Wald$ |
|---|---|---|---|---|---|---|
| US | 0.838* | 0.317* | 0.017 | 0.516 | 0.005 | 19.989 |
|  | (4.55) | (2.97) | (0.54) |  |  |  |
| Japan | 1.234* | -0.093 | 0.022 | 0.486 | 0.006 | 1.739 |
|  | (5.49) | (-0.61) | (0.50) |  |  |  |
| Germany | 0.846* | 0.156 | 0.035 | 0.520 | 0.004 | 3.133 |
|  | (5.58) | (1.60) | (0.99) |  |  |  |
| UK | 0.930* | 0.222 | -0.003 | 0.698 | 0.004 | 27.802 |
|  | (3.22) | (1.26) | (-0.08) |  |  |  |
| France | 1.049* | 0.024 | -0.019 | 0.766 | 0.005 | 4.015 |
|  | (3.14) | (0.08) | (-0.50) |  |  |  |
| Italy | 0.516* | 0.477* | -0.013 | 0.831 | 0.004 | 26.776 |
|  | (3.98) | (4.20) | (-0.31) |  |  |  |
| Spain | 0.948* | 0.063 | -0.010 | 0.721 | 0.005 | 1.331 |
|  | (4.68) | (0.38) | (-0.41) |  |  |  |
| Canada | 0.764* | 0.254* | 0.009 | 0.609 | 0.005 | 13.339 |
|  | (7.14) | (3.26) | (0.43) |  |  |  |

Notes: Intercept included but not reported. The equation is estimated by instrumental variables using an intercept, $\pi_{i,t-1}$, $\widetilde{y}_{i,t-1}^{HP}, r_{i,t-1}$ as instruments over 1980Q1-2006Q3; t statistics calculated using Newey-West standard errors are given in parentheses, $*$ indicates that a regression coefficient is significant at the 5% level, on a one tailed test. $GR^2$ is the generalised $R^2$ of Pesaran and Smith (1994), $\hat{\sigma}_i$ is the standard error of the regression. $Wald$ is a Wald statistic which is distributed $\chi^2(2)$ under the null that $\beta_{fi} = 0.99$ and $\beta_{bi} = 0$.

**Table 3: Estimates (and t ratios) for the NKPC, using the GVAR estimate of the output deviation from steady state, $\widetilde{y}_{it}$, constant steady state inflation and domestic instruments**

|  | $\hat{\beta}_{fi}$ | $\hat{\beta}_{bi}$ | $\hat{\gamma}_i$ | $GR^2$ | $\hat{\sigma}_i$ | $Wald$ |
|---|---|---|---|---|---|---|
| US | 0.861* | 0.293* | 0.016 | 0.513 | 0.005 | 10.14 |
|  | (4.46) | (2.16) | (0.44) |  |  |  |
| Japan | 1.295* | -0.111 | 0.13 | 0.475 | 0.006 | 3.74 |
|  | (5.75) | (-0.71) | (1.16) |  |  |  |
| Germany | 0.883* | 0.149 | 0.007 | 0.512 | 0.004 | 3.90 |
|  | (6.24) | (1.53) | (0.43) |  |  |  |
| UK | 0.929* | 0.166 | 0.039 | 0.685 | 0.004 | 10.50 |
|  | (2.62) | (0.66) | (1.12) |  |  |  |
| France | 1.054* | 0.015 | 0.012 | 0.765 | 0.005 | 4.03 |
|  | (2.81) | (0.05) | (0.32) |  |  |  |
| Italy | 0.493* | 0.498* | -0.025 | 0.83 | 0.004 | 19.43 |
|  | (2.62) | (3.04) | (-0.50) |  |  |  |
| Spain | 0.963* | 0.048 | 0.009 | 0.724 | 0.005 | 0.83 |
|  | (4.72) | (0.28) | (0.25) |  |  |  |
| Canada | 0.786* | 0.224* | 0.012 | 0.614 | 0.005 | 8.22 |
|  | (7.59) | (2.76) | (1.12) |  |  |  |

Notes: Intercept included but not reported. The equation is estimated by instrumental variables using an intercept, $\pi_{i,t-1}, \widetilde{y}_{i,t-1}$, $r_{i,t-1}$ as instruments over 1980Q1-2006Q3; t statistics calculated using Newey-West standard errors are given in parentheses, $*$ indicates that a regression coefficient is significant at the 5% level, on a one tailed test. $GR^2$ is the generalised $R^2$ of Pesaran and Smith (1994), $\hat{\sigma}_i$ is the standard error of the regression. $Wald$ is a Wald statistic which is distributed $\chi^2(2)$ under the null that $\beta_{fi} = 0.99, \beta_{bi} = 0$.

**Table 4: Estimates (and t ratios) for the NKPC using the GVAR estimate of the output deviation from steady state, $\widetilde{y}_{it}$, constant steady state inflation, and domestic and foreign instruments**

|  | $\hat{\beta}_{fi}$ | $\hat{\beta}_{bi}$ | $\hat{\gamma}_i$ | $GR^2$ | $\hat{\sigma}_i$ | $Wald$ |
|---|---|---|---|---|---|---|
| US | 0.991* | 0.175 | 0.080* | 0.717 | 0.005 | 5.150 |
|  | (7.94) | (1.62) | (2.65) |  |  |  |
| Japan | 0.776* | 0.158 | 0.044 | 0.453 | 0.005 | 2.943 |
|  | (6.02) | (1.41) | (0.66) |  |  |  |
| Germany | 0.983* | 0.096 | 0.008 | 0.614 | 0.004 | 3.224 |
|  | (6.66) | (0.93) | (0.47) |  |  |  |
| UK | 0.953* | 0.148* | 0.042* | 0.819 | 0.004 | 11.134 |
|  | (7.67) | (1.82) | (1.87) |  |  |  |
| France | 1.040* | 0.017 | 0.052* | 0.821 | 0.005 | 2.686 |
|  | (4.26) | (0.08) | (1.77) |  |  |  |
| Italy | 0.659* | 0.366* | 0.004 | 0.867 | 0.004 | 40.820 |
|  | (9.68) | (6.11) | (0.16) |  |  |  |
| Spain | 1.124* | -0.090 | 0.048 | 0.772 | 0.006 | 0.805 |
|  | (6.14) | (-0.59) | (1.24) |  |  |  |
| Canada | 0.811* | 0.207* | 0.013 | 0.681 | 0.005 | 8.488 |
|  | (8.00) | (2.80) | (1.24) |  |  |  |

Notes: Intercept included but not reported. The equation is estimated by instrumental variables using an intercept, $\pi_{i,t-1}, \widetilde{y}_{i,t-1}$, $r_{i,t-1}, \pi^*_{it}, \widetilde{y}^*_{it}, r^*_{i,t}, \pi^*_{i,t-1}, \widetilde{y}^*_{i,t-1}, r^*_{i,t-1}, \Delta poil_t$ as instruments over 1980Q1-2006Q3; t statistics calculated using Newey-West standard errors are given in parentheses, $*$ indicates that a regression coefficient is significant at the 5% level, on a one tailed test. $GR^2$ is the generalised $R^2$ of Pesaran and Smith (1994), $\hat{\sigma}_i$ is the standard error of the regression. $Wald$ is a Wald statistic which is distributed $\chi^2(2)$ under the null that $\beta_{fi} = 0.99$, $\beta_{bi} = 0$.

**Table 5: Estimates (and t ratios) for the NKPC using the GVAR estimate of the output deviation from steady state, $\widetilde{y}_{it}$, inflation deviation from steady state, $\widetilde{\pi}_{it}$, domestic and foreign instruments**

|  | $\hat{\beta}_{fi}$ | $\hat{\beta}_{bi}$ | $\hat{\gamma}_i$ | $GR^2$ | $\hat{\sigma}_i$ | $Wald$ |
|---|---|---|---|---|---|---|
| US | 0.797*† | 0.210* | 0.107* | 0.783 | 0.004 | 7.482 |
|  | (8.33) | (2.73) | (3.89) |  |  |  |
| Japan | 0.979* | 0.053 | 0.027 | 0.595 | 0.006 | 0.301 |
|  | (6.61) | (0.43) | (0.23) |  |  |  |
| Germany | 1.216* | 0.115 | 0.011 | 0.420 | 0.005 | 1.552 |
|  | (4.31) | (0.85) | (0.32) |  |  |  |
| UK | 0.915* | 0.181* | 0.035* | 0.796 | 0.004 | 12.165 |
|  | (9.49) | (2.82) | (1.66) |  |  |  |
| France | 0.922* | 0.105 | 0.057* | 0.802 | 0.004 | 1.835 |
|  | (6.27) | (0.74) | (1.76) |  |  |  |
| Italy | 0.644* | 0.367* | 0.036 | 0.902 | 0.004 | 62.206 |
|  | (13.03) | (7.89) | (1.61) |  |  |  |
| Spain | 1.182* | -0.164 | 0.071* | 0.954 | 0.004 | 3.104 |
|  | (9.91) | (-1.41) | (2.22) |  |  |  |
| Canada | 0.737* | 0.252* | 0.019* | 0.658 | 0.005 | 18.955 |
|  | (10.86) | (4.24) | (1.82) |  |  |  |

Notes: Intercept not included. The equation is estimated by instrumental variables using an intercept, $\widetilde{\pi}_{i,t-1}, \widetilde{y}_{i,t-1}, \widetilde{r}_{i,t-1}, \widetilde{\pi}_{it}^*, \widetilde{y}_{it}^*$, $\widetilde{r}_{it}^*, \widetilde{\pi}_{i,t-1}^*, \widetilde{y}_{i,t-1}^*, \widetilde{r}_{i,t-1}^*, \Delta poil_t$ as instruments over 1980Q1-2006Q3; t statistics calculated using Newey-West standard errors are given in parentheses, $*$ indicates that a regression coefficient is significant at the 5% level, on a one tailed test. $GR^2$ is the generalised $R^2$ of Pesaran and Smith (1994), $\hat{\sigma}_i$ is the standard error of the regression. $Wald$ is a Wald statistic which is distributed $\chi^2(2)$ under the null that $\beta_{fi} = 0.99$, $\beta_{bi} = 0$.