FATIGUE LIFE OF CONSTRUCTIONAL MATERIALS UNDER BENDING WITH TORSION FOR CRACK PROPAGATION

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Abstract The paper presents description of the equivalent range of the stress intensity factor in the case of bending with torsion of specimens where initiation and propagation of fatigue cracks are observed. Calculations with the Matlab-Simulink program were performed in order to estimate fatigue life. The calculations were realized for variable amplitudes of the resultant moment 13 and 16 N·m. The Forman's relation was also used for the calculations. It has been shown that the Forman's relation is suitable for description of fatigue life in the tested range of fatigue crack propagation.

1. Introduction

Stress intensity coefficients are often applied for description of the stress state in elements of structures where fatigue cracks occur and develop [1-2]. They are used in many relationships describing crack propagation rate or fatigue life in the range of fatigue crack propagation [3]. The known relationships for fatigue life estimation usually use stress intensity coefficients under simple loading states [4]. Under complex loading states, for example bending with torsion, it is necessary to determine the equivalent value of the stress intensity factor, including all the components of stress occurring in the loaded element [5]. Thus, the existing relationships describing fatigue life should be verified.

The authors of this paper try to describe the equivalent range of the stress intensity factor in the case when a specimen is simultaneously loaded by the torsional and bending moments. Next, fatigue life is going to be estimated in the range of fatigue crack propagation. The Forman's relation is used for estimation.

2. The tests

The tests were performed at the fatigue test stand MZGS-100. At this stand, it is possible to realize courses, variable in cycles, under bending, torsion and combined bending with torsion. The test results have been presented in [6]. The tests were done for two amplitudes of the resultant moment, M_{aw0} = 13 and 16 N· m. A scheme of the specimen loading has been shown in Fig. 1 [6]. For calculations, it was assumed that the amplitude of the resultant moment M_{aw0} changed together with the propagating crack. That change was caused by a change of the specimen rigidity while fatigue crack propagation [7].

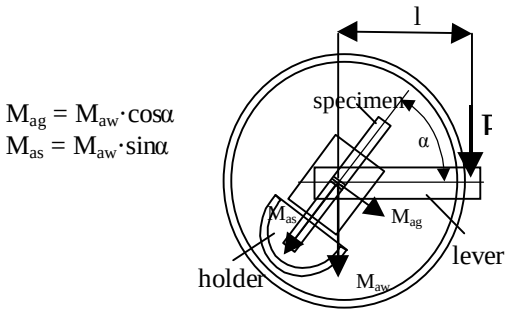

Fig. 1 A scheme of specimen loading

3. Description of the stress state

Fig. 2 shows stress distribution on the plane of fatigue crack propagation in the specimen, caused by bending and torsional loading.

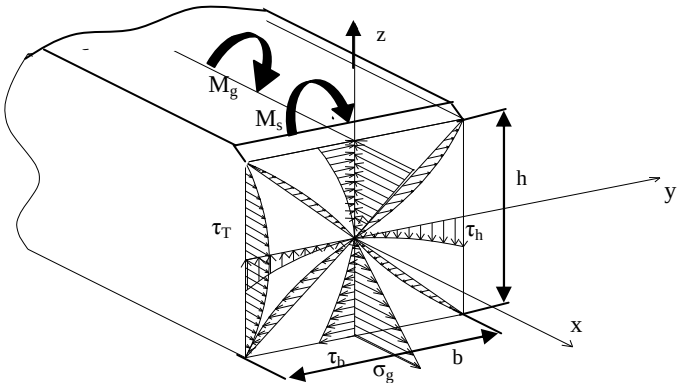

Fig. 2 Stress distribution in the specimen section, i.e. on the plane of fatigue crack propagation; $\tau_s = \tau_b > \tau_h$ – shear stresses caused by the torsional moment, τ_T – shear stress caused by shearing force, σ_g – normal stress caused by the bending moment

Normal stresses, σ_g and shear stresses, τ_s and τ_T, used for determination of the stress intensity factor are expressed as

$$\sigma_g = \frac{6M_{aw}\cos\alpha}{b(h-a)^2}, \tag{1}$$

$$\tau_s = \tau_b = \frac{M_{aw}\sin\alpha}{k_2 b(h-a)^2}, \tag{2}$$

$$\tau_T = \frac{P}{A}, \tag{3}$$

where $M_{aw0} = P \cdot l$, P – loading force for $M_{aw0} = 13$ N · m equal to 65 N and $M_{aw0} = 13$ N · m equal to 80 N, l – lever length, a – actual length of crack, b – specimen width, h – specimen height, k_2 – coefficient depending on the ratio b / (h-a) [7], A – section area of the specimen in the place of notch.

In description of fatigue crack propagation a range of the stress intensity factor was applied. It was defined from [8]:

$$\Delta K_{eq} = \sqrt{\Delta K_I^{\,2} + \Delta K_{II}^2 + \frac{1}{(1-\upsilon)}\Delta K_{III}^{\,2}}, \tag{4}$$

where: ΔK_I, ΔK_{II}, ΔK_{III} – ranges of stress intensity coefficients for loading modes I, II and III.

From analysis of the stress state in the specimen it appears that the stress τ_T causing longitudinal shearing is very low in comparison with stresses σ_g and τ_s ($\tau_T \approx 1\%$ of σ_g) and it does not strongly influence the process of fatigue crack development, so $\Delta K_{II} = 0$ is assumed for further considerations.

The equivalent range of stress intensity factor takes the form:

$$\Delta K_{eq} = \sqrt{\Delta K_I^{\,2} + \frac{1}{(1-\upsilon)}\Delta K_{III}^{\,2}}, \tag{5}$$

where:

$$\Delta K_I = M_{K_I}\Delta\sigma_g\sqrt{\pi a};\,[1],$$

$$\Delta K_{III} = M_{K_{III}}\Delta\tau_s\sqrt{\pi a};\,[1],$$

M_{kI} – correction coefficient $$M_{KI} = \frac{5}{\sqrt{20 - 13\frac{a}{h} - 7\left(\frac{a}{h}\right)^2}}$$ [11],

M_{kIII} – correction coefficient $$M_{KIII} = \sqrt{2\frac{h}{a} tg\left(\frac{\pi a}{2h}\right)}$$ [10].

4. Estimation of fatigue life

The Forman's relation [9] was used for fatigue life estimation. The equivalent range of stress intensity factor in the form [5]:

$$\frac{da}{dN} = \frac{C(\Delta K_{eq})^m}{(1-R)K_C - \Delta K_{eq}}. \tag{6}$$

was introduced to the Forman's relation.
After integration of Eq. (6), we obtain the following relation for fatigue life estimation:

$$N = \int_{a_0}^{a_k} \frac{(1-R)K_C - \Delta K_{eq}}{C_1(\Delta K_{eq})^{m1}} da, \tag{7}$$

where: R = -1, $K_C = K_{IC}$ = 259MPa·m^{0,5}

Parameters C_1 and m_1 from Eq. (7) were calculated on the basis of test results, assuming the criterion of the maximum accuracy of theoretical curve fitting to the experimental points with the method of least squares of the distance from the theoretical curve. The same scale of axes of the normalized coordinate system was assumed in relation of the test data range. A distance between the experimental points and the theoretical curve was defined as minimum of the length of interval d_i [8] joining the experimental point M_i and the theoretical curve Γ (Fig.3).

$$d_i = \sqrt{\left(\frac{a_{\exp_i} - a_{obl_i}}{a^{\max}_{\exp_i}}\right)^2 + \left(\frac{N_{\exp_i} - N_{obl_i}}{N^{\max}_{\exp_i}}\right)^2}. \tag{8}$$

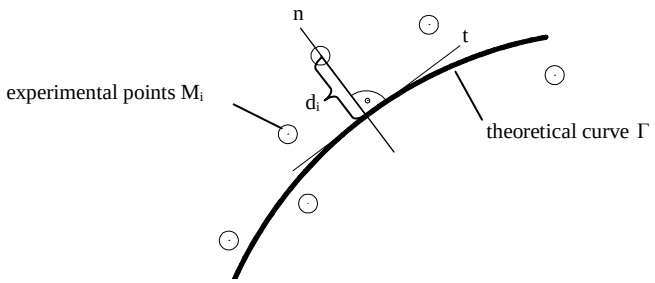

Fig. 3 Distance r_i between experimental points M_i and the curve Γ

Eqs. (6) and (7) were solved with simulation methods and the Matlab-Simulink program. Values of the parameters C_1 and m_1, and life N_{obl} are presented in Table 1.

Table 1. Values of the parameters C_1, m_1 and life N_{cal}

Material	M_{aw0} [N·m]	Ratio M_{ag}/M_{as}	C_1	m_1	N_{cal} [cycles *10^5]
18G2A	13	M_{as} = 0	$0{,}186\cdot10^{-3}$	1,20	2,41
		7,59	$0{,}164\cdot10^{-3}$	1,21	2,93
		3,73	$0{,}117\cdot10^{-3}$	1,24	3,50
		2,41	$0{,}165\cdot10^{-3}$	1,04	4,47
		1,73	$0{,}154\cdot10^{-3}$	0,95	6,11
		1,30	$0{,}235\cdot10^{-4}$	1,56	6,86
		1,00	$0{,}456\cdot10^{-4}$	1,31	8,05
	16	M_{as} = 0	$0{,}205\cdot10^{-3}$	1,18	1,51
		7,59	$0{,}140\cdot10^{-3}$	1,36	1,58
		3,73	$0{,}160\cdot10^{-3}$	1,26	1,79
		2,41	$0{,}331\cdot10^{-3}$	1,00	1,97
		1,73	$0{,}381\cdot10^{-4}$	1,66	2,58
		1,30	$0{,}210\cdot10^{-3}$	1,00	3,16
		1,00	$0{,}226\cdot10^{-3}$	0,90	4,01

5. Analysis of the calculation results

Figs. 4 and 5 show the graphs of the experimental and calculation lives, N_{exp} and N_{cal}, versus the crack length a. The solid line means lives N_{cal} calculated from Eq. (7), and the points express experimental lives N_{exp} for three specimens. It can be seen that as the fatigue crack length a increases, difference between experimental and calculation results becomes greater. The maximum difference between N_{exp} and N_{cal} for loading by amplitude of the resultant moment M_{aw0} = 13 N·m does not exceed 11% (Fig. 4d). Under loading by the resultant moment M_{aw0} = 16 N·m, this difference does not exceed 15% (Fig. 5a).

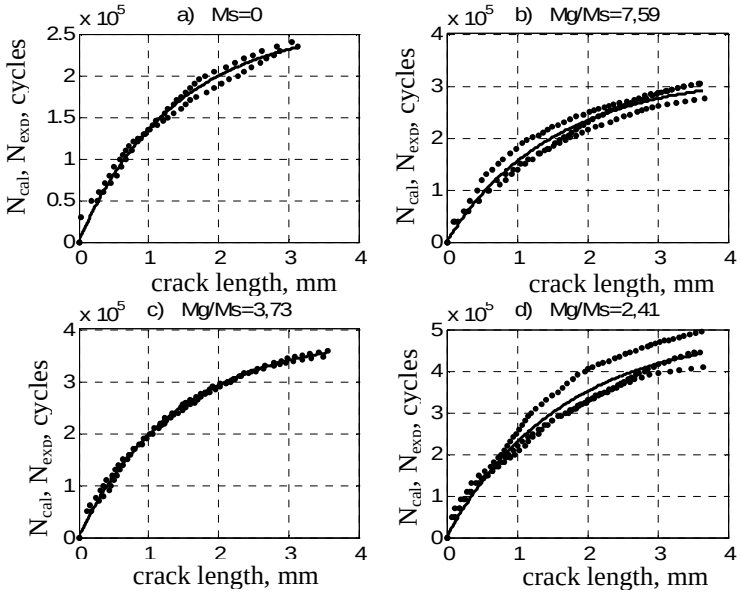

Fig. 4 Graphs of the experimental and calculation lives, N_{exp} and N_{obl}, versus the crack length, a, for 18G2A steel and amplitude of the resultant moment M_{aw0} = 13 N·m for ratios M_{ag}/M_{as}: a) M_{as} = 0, b) 7.59, c) 3.73, d) 2.41

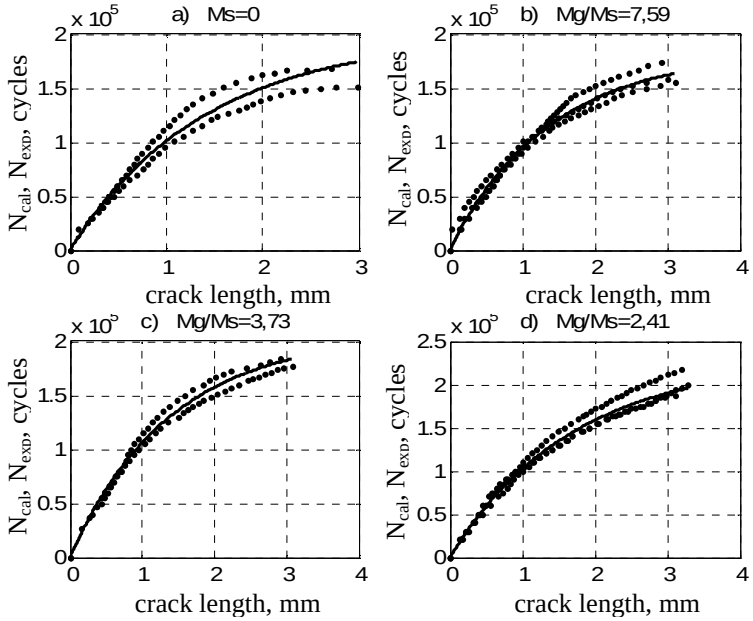

Fig. 5 Graphs of the experimental and calculation lives, N_{exp} and N_{obl}, versus the crack length, a, for 18G2A steel and amplitude of the resultant moment M_{aw0} = 16 N·m for ratios M_{ag}/M_{as}: a) M_{as} = 0, b) 7.59, c) 3.73, d) 2.41

Figs. 6 and 7 present comparison of the calculated and experimental lives, N_{obl} and N_{exp}. The maximum values of the experimental life are determined for critical lengths of fatigue cracks a_{kr}. Above the critical lengths, it is not possible to use stress description in order to determine the fatigue life because of high plasticization of the material. The given values are included into the interval a_{kr} = 3.45 ÷ 3.66 mm for loading amplitude M_{aw0} = 13 N·m, and a_{kr} = 2.92 ÷ 3.23 mm for loading amplitude M_{aw0} = 16 N·m and the depend on the ratio M_{ag}/M_{as}.

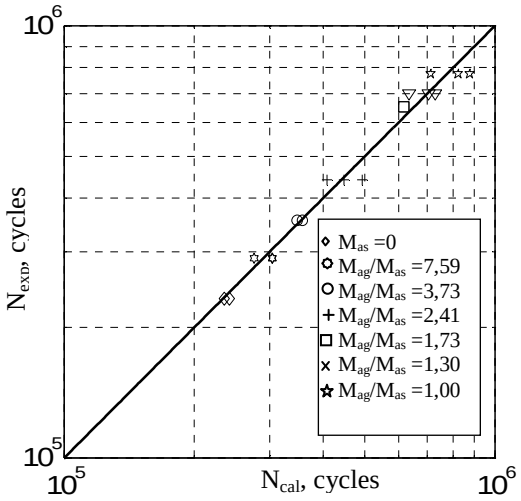

Fig. 6 Comparative graph for experimental and calculated lives, N_{exp} and N_{cal}, for amplitude of the resultant moment M_{aw0} = 13 N·m

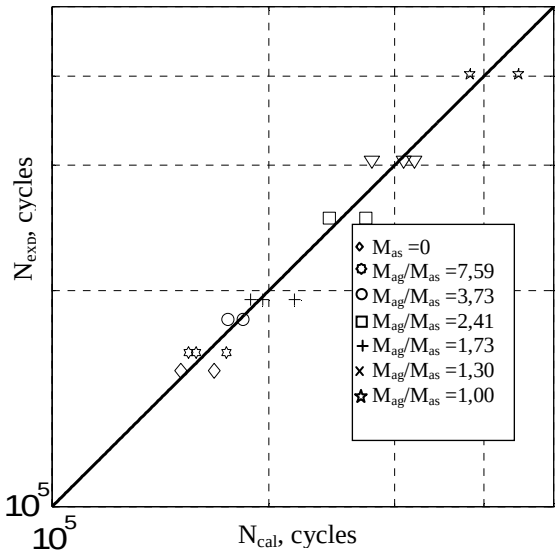

Fig. 7 Comparative graph for experimental and calculated lives, N_{exp} and N_{cal}, for amplitude of the resultant moment M_{aw0} = 16 N·m

It can be seen that under both loading amplitudes the results are included into a narrow scatter band. Thus, the applied Forman's relation and the equivalent range of stress intensity factor (5) allow to obtain good conformity of the results. In this order, the coefficients C_1 and m_1 should be determined from the experimental data. The least conformity is observed for ratios M_{ag} / M_{as} = 1.30 and 1.00 for both considered moment amplitudes.

6. Conclusions

From the performed calculations and their analysis the following conclusions can be drawn:

1. The relation describing fatigue life based on the Forman's relation and including the equivalent range of stress intensity factor ΔK_{eq} provides suitable estimation of fatigue life of specimens made of 18G2A steel.
2. In order to estimate fatigue life with Eq. (7) it is necessary to determine coefficients C_1 and m_1, depending on material properties and the ratios of amplitudes of bending and torsional moments.

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