A PHENOMENOLOGICAL MODEL OF MACROCRACK INITIATION AND GROWTH IN CYCLICALLY DEFORMED MATERIAL

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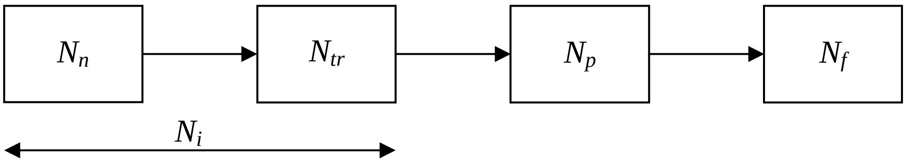
Abstract. Role of the process zone in fatigue macrocrack initiation and propagation is considered. An examples of the assessment of the fatigue macrocrack initiation period, N_i, and the failure period, N_f, of a notched specimens on the base of unified model using fatigue macrocrack growth rates only are shown. Some backgrounds and well-known fatigue phenomena (Kitagawa's and Smith's diagrams, etc) are discussed.

1.Introduction

The duration of the fatigue fracture process in structural components is considered in the common case as the total of initiation, N_i, and propagation, N_p, periods, where the period N_i includes both microcrack nucleation, N_n, and microcrack to macrocrack transition, N_{tr}, periods [1, 2]. For the time being we have some problems in quantitative estimation of the period N_i for notched components because the relevant parameters of this stage (Table 1) are still not well established. First of all it is connected with the determination of the fatigue (effective) stress concentration factor and the local stress range and also with the knowledge of the criterion of initial fatigue macrocrack length (moment of micro- to macrocrack transition).

Table 1. Different stages of fatigue failure and their relevant parameters



Macrocrack initiation period	Macrocrack growth period	Final failure
$a_i = a_n + a_{tr}$	$a_i < a < a_c$	a_c
$\Delta\sigma_y^* = K_f \cdot \Delta\sigma_{\text{nom}}$	ΔK	ΔK_{fc} (K_{1c})
Local stress range as fatigue (effective) stress concentration factor and nominal stress	Stress intensity factor range	Cyclic fracture toughness

It is suggested, in traditional approach, that fatigue macrocrack initiation and propagation are different process, therefore they have been considered separately. However, from our point of view, only one essential difference exists: at the macrocrack growth stage the crack closure effect appears, but at the initiation stage it is absent [2]. Nevertheless the stages of the fatigue failure of materials, i. e. macrocrack initiation and propagation stages, might be considered as a similar process.

The present paper introduces our process zone approach to fatigue fracture of materials for unified description of the entire fatigue life of notched structural component and for explanation of some well-known fatigue phenomena.

2. Fatigue macrocrack initiation and growth

Summarizing our own and the literature data it was considered [2] that due to the lower yield strength of the surface layer, as well as by the peculiar properties of a free surface (an easy exit for dislocations, an easy initiation and coalescence of point defects, etc), the process of plastic deformation and microstructural damage accumulation is localized in these surface layers. This facilitates the formation of a specific below-surface zone of a size (depth) determined by the characteristic parameter d^* (Fig. 1). This below-surface region is some process zone for next fatigue crack initiation [2].

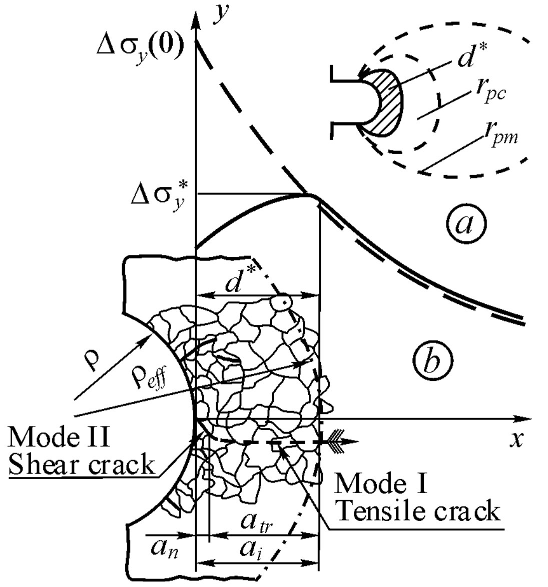


Fig. 1. Schemes for (*a*) plastic zone formation and stress distribution in the vicinity of the notch and (b) microstructurally short and physically small fatigue cracks growing within the process zone d^*.

Therefore, process zone size ought to be a constant of materials under the given test conditions (stress amplitude, temperature, environment, etc). It was suggested and analytically confirmed [2, 3] that maximum of the local stress range, $\Delta\sigma_y^*$, is located at the characteristic (critical) distance d^* from the notch tip (Fig. 1 *b*). The process zone, especially transition boundary between the below-surface zone d^* and the bulk of the material, gives rise to the main physical barrier for microstructurally short (of length a_n) and physically small (of length a_{tr}) microcracks (Fig. 1 *b*), because the deeper they propagate from the surface the more intence is the magnitude of the prior strain of the material (Fig. 1 *a*). If the stress intensity at the microcrack tip does not help to overcome the mesobarrier at d^*, the crack becomes non-propagating. This phenomenon is usually observed at a stress just below the fatigue limit of the material. As the stress increases, a system of initial microcracks develops either by the growth of the most favorably oriented cracks or by the merger with microcracks ahead until one of them becomes dominant and overcomes the boundary of the process zone at the critical distance d^* (Fig. 1 *b*). At the moment, when one of the cracks grows beyond d^*, the fracture process is entirely controlled by the fatigue process inherent to the tip of the dominant crack. In the vicinity of its tip, the own region of elastic-plastic deformation is formed, the closure effect for a crack of length $a_i = d^*$ starts [2] and its growth rate is determined by relationship (da/dN vs. ΔK) for long crack, i.e. the small crack transforms to a macrocrack of initial length a_i [2, 4]. Thus, the criterion of the micro- to macrocrack transition is

$$a_i = d^*, \tag{1}$$

which takes place after N_i loading cycles called the period of the fatigue macrocrack initiation. Cracks of length $a < d^*$ might be assumed safe, because their initial propagation always experiences a pronounced retardation before they become a macrocrack, or they become non-propagating altogether [2].

Process zone size determines also the local stress range, $\Delta\sigma_y^*$ (Fig 1 *b*) and the fatigue (effective) stress concentration factor, K_f. On the bases of well-known equations of Peterson and Neuber we have been proposed the formula for K_f-value [3]

$$K_f = 1 + \frac{K_t - 1}{1 + \sqrt{d^* / \rho_{eff}}}, \tag{2}$$

where K_t is the theoretical (elastic) stress concentration factor, $\rho_{eff} = \rho + d^*$ is the effective notch radius. Thus, the reality of coefficient K_f seems to be conditioned by the accuracy of the process zone size d^* definition as a linear (structural) parameter of materials [3]. From this point of view our d^*-parameter, ρ^*-parameter of Neuber [5], a_p-parameter of Peterson [6] and x_{eff}–parameter of Pluvinage et al [7] are similar and represent the critical distance theories of fracture. The known results [8] confirmed that critical point and critical area methods, which are similar to our approach, ensure the best prediction of the fatigue limit of notched component.

The above-mentioned considerations of the fatigue macrocrack initiation process make it possible to deduce, that it is a two-parameter process and it is determined by the local stress range, $\Delta\sigma_y^*$, and the process zone size, d^*. The relationships $\Delta\sigma_y^*$ versus N_i and d^* versus N_i (Fig. 2) are the fundamental (basic) characteristics of a structural material [2]. Using these basic characteristics we can estimate the period to macrocrack of length $a_i = d^*$ initiation in cyclically loaded structural element with arbitrary component and notch shape. As well one can establish the threshold magnitude of the local stress range, $(\Delta\sigma_y^*)_{th}$, when the initiation of the fatigue macrocrack of length $a_i \geq d^*$ at a stress concentrator is not realized. Below $(\Delta\sigma_y^*)_{th}$ value microcracks of length less than the process zone size d^* take place, but these cracks are non-propagating [2, 3].

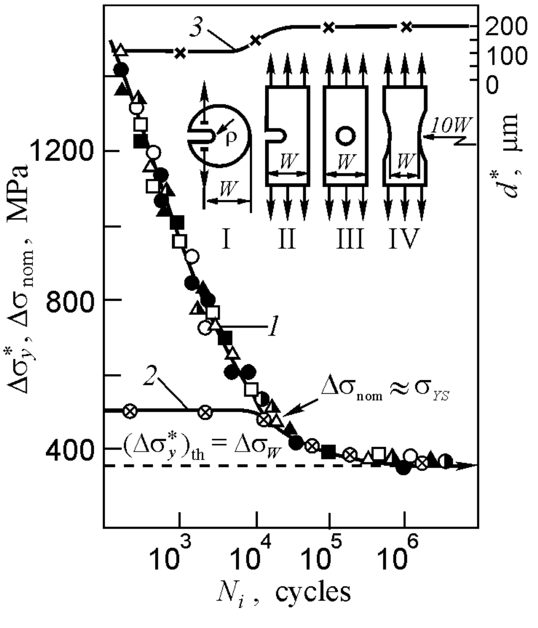


Fig. 2. Relationships $\Delta\sigma_y^*$ vs. N_i, (curve *1*) and d^* vs. N_i (curve *3*) for notched specimens and $\Delta\sigma_{nom}$ vs. N_i (curve *2*) for smooth specimens of D16chAT1 aluminium alloy. Note: specimen type I - W = 64 mm, ρ = 0.75 mm (△), 6.5 mm (○); type II - W = 64 mm, ρ = 0.75 mm (▲), 6.5 mm (●); W = 30 mm; ρ = 0.75 mm (◭), 2.0 mm (◑); type III - W = 30 mm, ρ = 0.75 mm (□); W = 20 mm, ρ = 0.75 mm (■); type IV - W = 10 mm (⊗).

The macrocrack propagation period was modelled as successively repeated similar events near the crack tip [4] based on the macrocrack initiation data, established for notched specimens, which are shown above. According to our model [4], during cyclic loading in the vicinity of the existing macrocrack tip, which might be considerent as a sharp notches of effective radius $\rho_{eff}^{cr} = d^*$ (Fig. 3), the process zone of size d^* is formed. Within this zone the macrocrack increment of length $\Delta a = d^*$ takes place (Fig. 3). The increment formation corresponds to that occurring for initial macrocrack formation near the notch ($\rho > d^*$), which is described by above-mentioned relationships ($\Delta\sigma_y^*$ vs. N_i) and (d^* vs. N_i). The principal condition that has to be used as a basis for the proposed approach is the equality of the local stress range, $\Delta\sigma_y^*$, in the vicinity of both notch and macrocrack tip:

$$(\Delta\sigma_y^*)_{notch} = (\Delta\sigma_y^*)_{crack}. \tag{3}$$

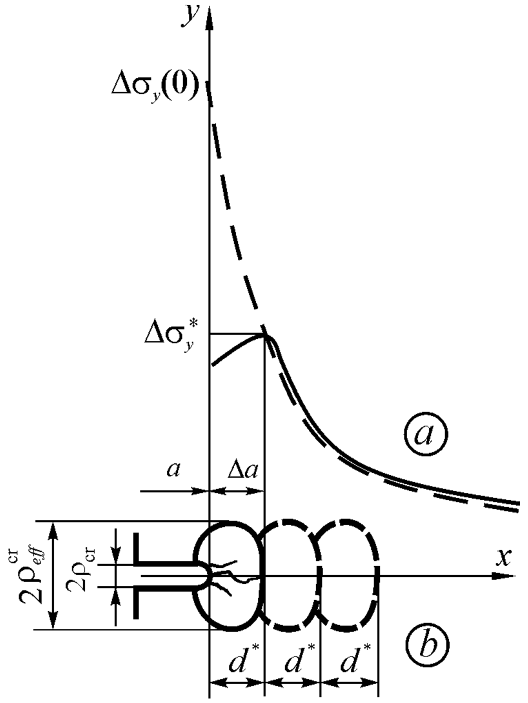


Fig. 3. Stress distribution near the macrocrack tip (*a*) and (*b*) a scheme of the macrocrack growth.

As a result, the procedure for the prediction of effective fatigue macrocrack growth curve *da/dN* vs. ΔK_{eff}, when the fatigue macrocrack closure effect is not taken into account, was proposed on the base of two formulae [4]:

$$da/dN = \Delta a/\Delta N = d^*/N_i; \tag{4}$$

$$\Delta K_{eff} = 0.886\Delta\sigma_y^*\sqrt{d^*}\ . \tag{5}$$

This procedure [Fig.4 (*a*) and (*b*)] was confirmed by experimental data [4].

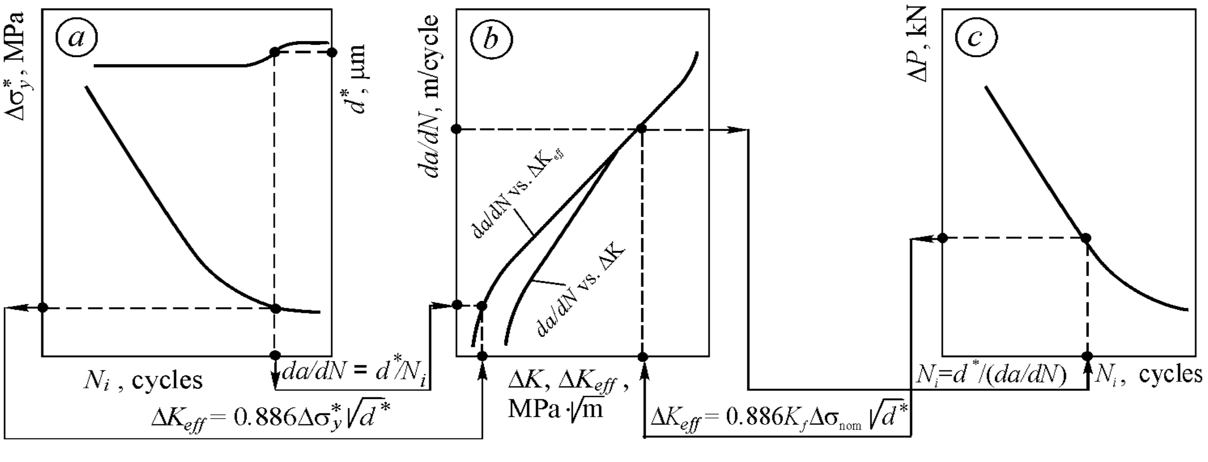


Fig. 4. A scheme for predictions [(*a*) and (*b*)] the effective fatigue macrocrack propagation curve and [(*b*) and (*c*)] the number of cycles to fatigue macrocrack initiation (length $a_i = d^*$) at the given load range Δ*P*.

3. Material endurance characteristics correlations

Correlation between fatigue fracture resistance characteristics can be estimated on the base of Eq. (5)

$$\Delta K_{th\ eff} = 0.886\left(\sigma_y^*\right)_{th}\sqrt{d^*}\ , \tag{6}$$

where $(\Delta\sigma_y^*)_{th}$ is the fatigue limit of notched specimens of arbitrary shape. Klesnil and Lukas proposed an equation of such kind for smooth specimens [9]:

$$\Delta K_{th} = \Delta\sigma_W^{sm}\sqrt{\pi a_0}\ , \tag{7}$$

where $\Delta\sigma_W^{sm}$ is fatigue limit of smooth specimen; a_0 is maximum length of non-propagating crack. The method for a_0-value estimation has been not developed [9]. As it was mentioned above, fatigue limit is conditioned by macrocrack initiation and before this event closure effect is absent. Besides, maximum length of non-propagating crack is $a_0 \le d^*$. Taking into account such consideration for smooth specimen we receive the following:

$$\Delta K_{th\ eff} = \Delta\sigma_W^{sm}\sqrt{\pi d^*}\ . \tag{8}$$

The best coincidence between experimental and calculational data for fatigue limit of titanium alloys using $\Delta K_{th\ eff}$ instead ΔK_{th} values was reported in [10].

The fatigue limit of cracked specimens, $\Delta\sigma_W^{cr}$, is also widely discussed in literature [11—13], for example, based on Kitagawa-Takahashi diagram [11]. El Haddad et al [12] proposed an empirical equation, introducing a material constant a_0':

$$\Delta\sigma_W^{cr} = \frac{\Delta K_{th}}{\sqrt{\pi\left(a_0' + a_{cr}\right)}}, \tag{9}$$

where a_{cr} is crack length. There has been considerable debate about the use of this equation and about the physical significance of a_0' [13]. In our opinion the Kitagawa-Takahashi diagram ought to be assembled in two separate parts, because for long fatigue cracks the closure effect exist, but for short (small) cracks it is not appeared [2]. Based on above-mentioned, the distinct point is conditioned by process zone size d^*, i.e. $a_{cr} = d^*$, and we must have following:

$$\begin{aligned} \Delta\sigma_W^{cr} &= \Delta\sigma_W^{sm} = \frac{\Delta K_{th\ eff}}{\sqrt{\pi d^*}}, \text{ when } a_{cr} < d^*; \\ \Delta\sigma_W^{cr} &= \frac{\Delta K_{th}}{\sqrt{\pi a_{cr}}}, \text{ when } a_{cr} > d^*. \end{aligned} \tag{10}$$

4. Fatigue life estimation

The above-described correlation between the macrocrack initiation and propagation stages [Fig. 4 (*a*) and (*b*)] allows one to perform a reverse calculation scheme [4]: assessment of the period to fatigue macrocrack initiation near the notch using *da/dN* versus ΔK_{eff} macrocrack propagation curve [Fig. 4 (*b*) and (*c*)]. Taking into account the specimen geometry, width *W*, thickness *t*, for the given load range ΔP the nominal stress range $\Delta\sigma_{nom}$ is calculated (Fig 4 *c*).Than, using the material constant d^*, Eq. (2) and Eq. (5) the value of ΔK_{eff} is estimated (Fig 4 *b*). It makes possible to determine the corresponding *da/dN* value from *da/dN* versus ΔK_{eff} curve, which can be represented analytically for the given material as:

$$da/dN = B(\Delta K_{eff} - \Delta K_{th\ eff})^n, \tag{11}$$

where $\Delta K_{th\ eff}$, *B* and *n* are material constants estimated experimentally. Then from Eq. (4) we have

$$N_i = \frac{d^*}{\left(da/dN\right)}. \tag{12}$$

Thus, the number of cycles N_i to initiation of a macrocrack of length $a_i = d^*$ can be assessed (Fig. 4 *c*). It is shown [4] that the result of calculation and experimental data are in good agreement.

Generally, fatigue life of a structural component can be established as:

$$N_f = N_i + N_p. \tag{13}$$

Process zone size d^* makes it possible to separate clearly main stages of the fatigue failure:

$$N_f = N_i\Big|_{a_i = d^*} + \int_{d^*}^{a_c} \frac{da}{F\left[\Delta K\left(\Delta P, a\right)\right]}, \tag{14}$$

where $a_c = f\,(\Delta K_{fc})$. Parameter ΔK_{fc} is the cyclic fracture toughness, which may be equal, less or larger than the static fracture toughness K_{1c} [14]. The function $F\,[\Delta K(\Delta P,\, a)]$ represents the fatigue macrocrack growth curve da/dN versus ΔK in the simple form of Paris equation or in other known forms, where subfunction $\Delta K = f\,(\Delta P,\, a)$ must be chosen in connection with the component shape (K-calibration). Period N_i to macrocrack initiation may be estimated by the direct experiment (as was shown above) or calculated by the proposed scheme [Fig. 4 (*b*) and (*c*)]. Thus, based on two experimentally obtained curves (da/dN vs. ΔK_{eff}) and (da/dN vs. ΔK) we can estimate the fatigue life N_f of a component at the given load range ΔP and component shape.

Below such calculations for cyclically loaded strips of width W = 20 mm, thickness t = 4.4 mm with central hole of diameter 2ρ = 3.2 mm (K_t = 2.6) and 2ρ = 5.0 mm (K_t = 2.4) made from B95pchT2 aluminum alloy (σ_{YS} = 456 MPa, σ_U = 510 MPa, δ = 12 %, d^* = 100 μm) are presented. Fatigue macrocrack growth curves for this alloy are shown in Fig. 5. They are represented analytically as:

$$da/dN = 6.53 \cdot 10^{-10}(\Delta K - 2.7)^{2.95}, \quad 2.7 < \Delta K < 10; \tag{15}$$

$$da/dN = 7.3 \cdot 10^{-12}\,(\Delta K\,)^{\,4.77}\,, \;\; 10 < \Delta K < \Delta K_{fc}\,; \tag{16}$$

$$da/dN = 1.39 \cdot 10^{-9}(\Delta K_{eff} - 1.8)^{2.66}, \;\; 1.8 < \Delta K_{eff} < 10. \tag{17}$$

Note, that the relationship (da/dN vs. ΔK) has one peculiarity because it consist of two plots described of Paris Eqs (15) and (16). The value of cyclic fracture toughness ΔK_{fc} based on experimental data (Fig. 5) is 34 $\mathrm{MPa}\cdot\sqrt{\mathrm{m}}$.

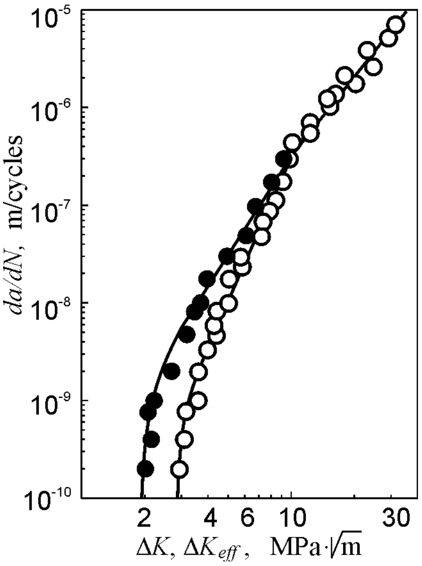


Fig. 5. Fatigue macrocrack growth rates for B95pchT2 aluminium alloy: ○ – da/dN vs. ΔK; ● – da/dN vs. ΔK_{eff} .

Experimental and calculational results for N_i, N_p and N_f , which are in agreement, are shown in Table 2. Summarizing this data we can note that proposed process zone approach is valid for the assessment of life-time for cyclically loaded notched components. Besides, the entire fatigue process can be considered from the unified position [4].

Table 2. Experimental and calculational fatigue macrocrack initiation (N_i) and propagation (N_p) periods and life-time (N_f) of the strip with a central hole at load ratio R = 0.1

Hole diameter, mm	Load range, ΔP, kN	Experimental data, cycles			Calculational data, cycles		
		N_i	N_p	N_f	N_i	N_p	N_f
3.2	9.0	135 000	15 000	150 000	132 300	10 300	142 600
	12.6	25 200	6 000	32 200	17 200	4 500	21 700
5.0	9.0	118 000	5 000	123 000	101 900	6 600	108 500
	13.5	15 300	3 300	18 600	10 100	2 200	12 300

5. Some backgrounds and known fatigue phenomena

The above-mentioned is the reason to conclude that process zone size d^* is the basic material parameter, which determinates:

- earlier stages of fatigue fracture through cyclic stress concentration factor K_f (see Eq. (2));
- condition for initial fatigue macrocrack of length a_i formation, when its accelerated growth starts (see Eq. (1));
- limiting condition for macrocrack propagation through effective threshold $\Delta K_{th\,eff}$ (see Eq. (6)).

The decrease of parameter d^*, for example, caused by the material degradation during long-term service [15], stipulates the increase of the fracture danger, since the K_f -value according to Eq. (2) will increase; values of a_i and $\Delta K_{th\,eff}$ according to Eqs (1) and (6) must decrease.

The logical conclusion, that some relation between process zone and well-known fatigue phenomena connected with the short cracks problem (Fig 6 *a, b*) and critical radius problem (Fig 6 *c, d*) exist, can be made on the base of experimental results.

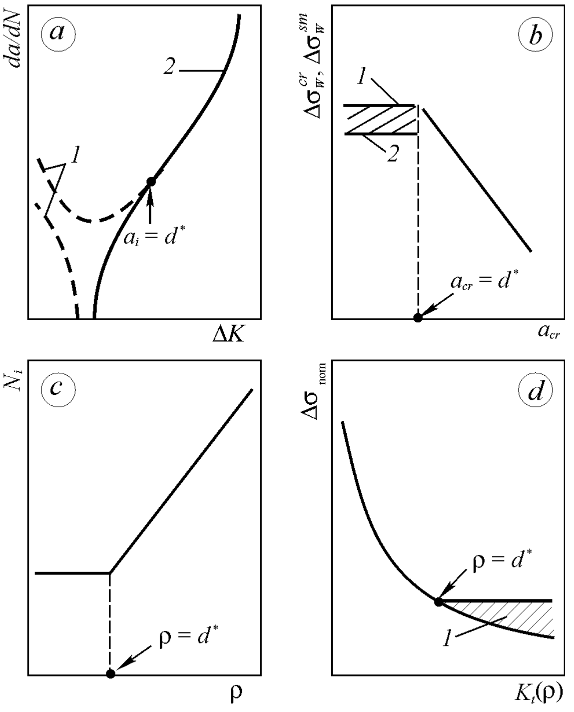


Fig. 6. Relation between process zone and well-known fatigue phenomena connected with (*a, b*) short cracks problem and (*c, d*) critical notch tip radius problem.

To our mind, the coincidence point of short and long fatigue crack growth rates (curves *1* and *2* on Fig. 6 *a*) [2, 4], the inflection (Fig 6 *b*) on Kitagawa-Takahashi diagram [11], the inflection (Fig. 6 *c*) corresponded to independence of fatigue macrocrack initiation period versus notch radius [2, 3, 16] and the inflection (Fig. 6 *d*) on Smith-Miller diagram [17] are determined by process zone size d^*. Note, that taking into account the above-mentioned expediency for the Kitagawa-Takahashi diagram dividing in two separate parts, line *1* on Fig 6 *b* is suitable for low-plasticity materials, where crack closure effect appears faintly and $\Delta K_{th} \approx \Delta K_{th\,eff}$; line *2* corresponds to plastic materials, where $\Delta K_{th\,eff} < \Delta K_{th}$. Region *1* on Smith –Miller diagram (Fig. 6 *d*) is connected with non-propagating cracks of length $a < d^*$.

7. Conclusions

- Process zone size d^* is a basic parameter of material depended on its microstructure, mechanical behavior and test conditions and it determinates various stages of the fatigue fracture of materials.
- The proposed unified model of fatigue fracture based on d^*-parameter makes it possible to assess of the life time of cyclically loaded notched structural component using the fatigue crack growth rates only.
- Process zone size d^* stipulates some peculiarities of well-known fatigue fracture relationships (Kitagawa-Takahashi and Smith-Miller diagrams, etc).

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