

Rock Slopes from Mechanics to Decision Making

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ABSTRACT: Rock slope instabilities are discussed in the context of decision making for risk assessment and management. Hence, the state of the slope and possible failure mechanism need to be defined first. This is done with geometrical and mechanical models for which recent developments are presented. This leads with appropriate consideration of uncertainties to risk determination and to the description of tools for risk management through active and passive countermeasures, including warning systems. The need for sensitivity analysis is then demonstrated, and final comments address updating through information collection.

1 INTRODUCTION

This paper addresses rock slopes, specifically rock slope instabilities. In this context, the mechanics underlying instabilities and corresponding models are very briefly reviewed to provide a basis for the major topic of this paper: Decision making as applied to risk management of rock slopes.

Before further outlining the structure and context of this paper, it is necessary to define the rock slope instabilities that will be considered. There are several differing definitions, e.g. Goodman and Kieffer (2000); Cruden and Varnes (1996); Varnes (1958), and what will be used here are the two categories, “rock slides” and “rock falls”. Possible subcategories for rock slides are planar wedge-, rotational slides; toppling will also be considered in this category. For rockfalls, movement of single blocks in form of falling, jumping and rolling will be included as well as rock avalanches consisting of multiple rock blocks interacting in a flow-like mechanism. Simply looking at these instability processes will quickly lead to the conclusion that it is often not possible to clearly separate them, e.g. a rotational slide may end up as an avalanche. In the mechanism discussion in Section 4 some simplifying assumptions will be made to identify the major underlying mechanisms.

The eventual goal of this paper and, most importantly, of most practical applications is to assess and manage risk associated with rock slope instabilities by making appropriate decisions. Hence, it is advantageous to organize this paper using the classic flow chart for decision making under uncertainty, based

on Pratt et al. (1965), and expanded and implemented by the authors (Einstein and Karam, 2001) (Figure 1).

As one can see, one collects information on the state-of-nature, then models the phenomenon deterministically and probabilistically to end up with risk, which is the basis for decisions in form of various risk management options. Consequently, the following sections will first discuss information collection, then geometric and mechanical modelling, followed by risk determination and risk management. Given that this is a relatively short keynote paper, only snapshots of what is done with emphasis on new technologies/procedures can be provided. The authors hope that the readers understand that many relevant references can, therefore not be mentioned.

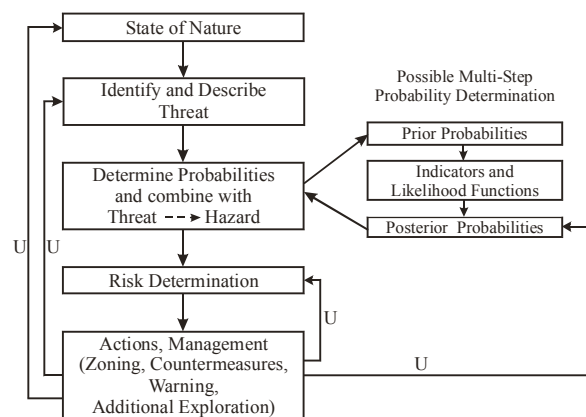


Figure 1. Decision-Making under Uncertainty. Based on Pratt et al. (1965) and Expanded by Einstein and Karam (2001) (U = Updating).

2 INFORMATION COLLECTION ON THE STATE-OF-NATURE

One can differentiate two major aspects on which information needs to be collected:

1. The geometric and geologic/geotechnical characteristics.

2. Movements. In most cases, movements or, more generally expressed, changes are obtained from repeated observations of geometric and geologic conditions.

The geometric and geologic features, in addition to the surface geometry, are the location and extent of discontinuities (faults, fractures, bedding planes). Modern technology using e.g. LIDAR InSAR, Ground Based Radar but also photography and total stations provide the information for sophisticated image analysis and eventually two - or three-dimensional models. Good examples of extensive use of these technologies are discussed e.g. by Blikra (2008), Ferrero et al. (2007). What is right now still somewhat problematic, is obtaining detailed information at depth where one has to still rely on boreholes possibly supplemented by geophysical ground penetrating techniques.

3 GEOMETRIC MODELLING

As indicated above, three dimensional models of the rock surface with intersecting discontinuities can be relatively easily created. These are actually good examples of deterministic models. As hinted at above, the situation is quite different when going into depth, where the information both on geometric and geologic/geotechnical aspects is uncertain. Regarding geometry, this concerns mostly the location, shape and size of fractures and to a lesser extent, the boundaries of different lithologies. Fracture (discontinuity) geometry, in particular, cannot be deterministically described at this point. Geologic/geotechnical information is somewhat easier to obtain deterministically, e.g. from bore cores or water level observations. Nevertheless, there is some spatial variation also with regard to these characteristics. Models have to reflect the spatial variation of geometric and geologic characteristics and the information collection procedures have to consider/correct for random errors and biases. For a review of uncertainties in rock mechanics and - engineering and how to handle them, see Einstein (2003), also Mauldon (1995) and Zhang and Einstein (1998).

Spatial uncertainty e.g. of fractures can be captured with stochastic models such as Fracman, (see e.g. Dershowitz & La Pointe (2007)), which is in wide commercial use, and Geofrac (Ivanova and Einstein (2004)), which can consider the underlying geologic genesis (Fig. 2). These probabilistic geometric models can be used together with mechanistic

models (see Section 4) to represent rock mass behavior, namely deformation including failure but also flow.

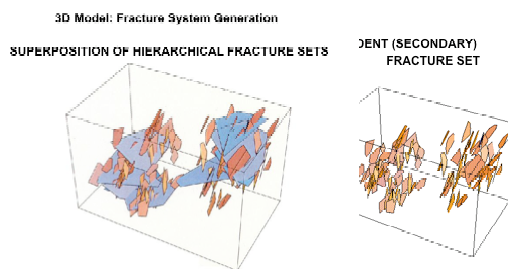


Figure 2. Stochastic Hierarchical Modeling of Fractures Using Geofrac. (Ivanova and Einstein, 2004)

4 MECHANISTIC MODELS

Although the title of this paper mentions “mechanics” but not the just discussed geometry, it will become quickly apparent that the two cannot be separated. Before addressing this issue, a few purely mechanics oriented comments need to be made: Rock slope instabilities usually involve two basic mechanism:

1. Detachment, including failure followed by: 2. Movement of the entire slope or of individual blocks (Fig. 3).

A few possible detachment mechanisms are shown in Figure 4. Movement can then occur in form of translational or rotational sliding, through toppling, through falling, jumping or rolling of single blocks or interacting blocks within an avalanche. Clearly this is a simplification in that e.g. several of the detachment mechanisms in Figure 3 can occur simultaneously and a large initially coherent rock mass may break up during movement, i.e. detachment and movement mechanisms may interact. In the following paragraphs, one example each of the detachment mechanism and the movement mechanism will be illustrated with physical experiments and associated numerical/analytical models. A final example will be the application of numerical models to an actual rockslide. These examples have been chosen since they represent the most recent information on the particular mechanisms or, in the last case, an example of interaction between model and in situ observations.

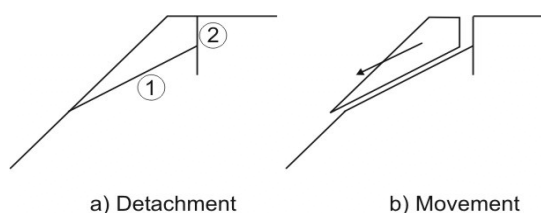


Figure 3. Examples of detachment and movement mechanisms: a) detach, start to slide at 1, separate in tension at 2, b) move.

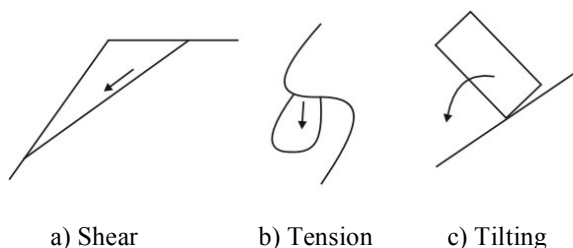


Figure 4. Detachment Mechanisms.

One of the major problematic issues regarding rock slope stability is the fact that fractures (joints) are not persistent and for detachment both for sliding and rockfalls to happen, the intact rock bridges need to fail. In other words, fracture coalescence needs to occur. An extensive investigation of crack coalescence is being conducted at MIT revealing a number of different coalescence modes (Figure 5). These observations come from laboratory scale tests on gypsum, marble and granite representing a reasonably wide range of grain sizes (textures) and mineral composition. The scale is clearly smaller than fracture coalescence in rock masses. So these observations might serve as a scaled model, and Wong and Einstein (2007) have shown that such a scaling is to some extent acceptable. Also and very importantly, the small scale mechanisms are an actual part of the failure of intact rock bridges consisting of the creation, propagation and eventual coalescence of cracks and fractures. Read's research group, see e.g. (Yan et al. (2007)) has been able to model the mechanisms numerically using the hybrid FE/DE (ELFEN) model. Similar reasonably satisfactory modeling results were obtained by Bobet (1998) using a boundary element model. Pierce et al., (2007) show how the Particle Flow Code can be used to model these cracking mechanisms. Although reasonably successful, some investigators, (e.g. Wong (2010); Silva (2009)), have shown that one cannot correctly represent all material/geometric combinations with these models. Clearly, the aim is to combine the stochastic geometry models with the mechanical models to completely represent slope failures. Initial steps have been taken in the past, (see e.g. Lee et al. (1992)) where slopes with stochastic joint patterns as shown in Figure 2 can be combined with mechanistic models to predict the probability of failure of a rock slope.

A similar combination of laboratory experiments and of numerical methods to investigate the mechanics is used by Manzella (Manzella and Labiouse (2009, 2007), Manzella (2009, 2008)) for rock avalanches. The mechanisms were experimentally simulated by assembling masses of differently shaped granular material or blocks and letting this mass slide down a ramp ending in a horizontal surface. The mass disintegrates and spreads during this process and depending on grain size, material, friction

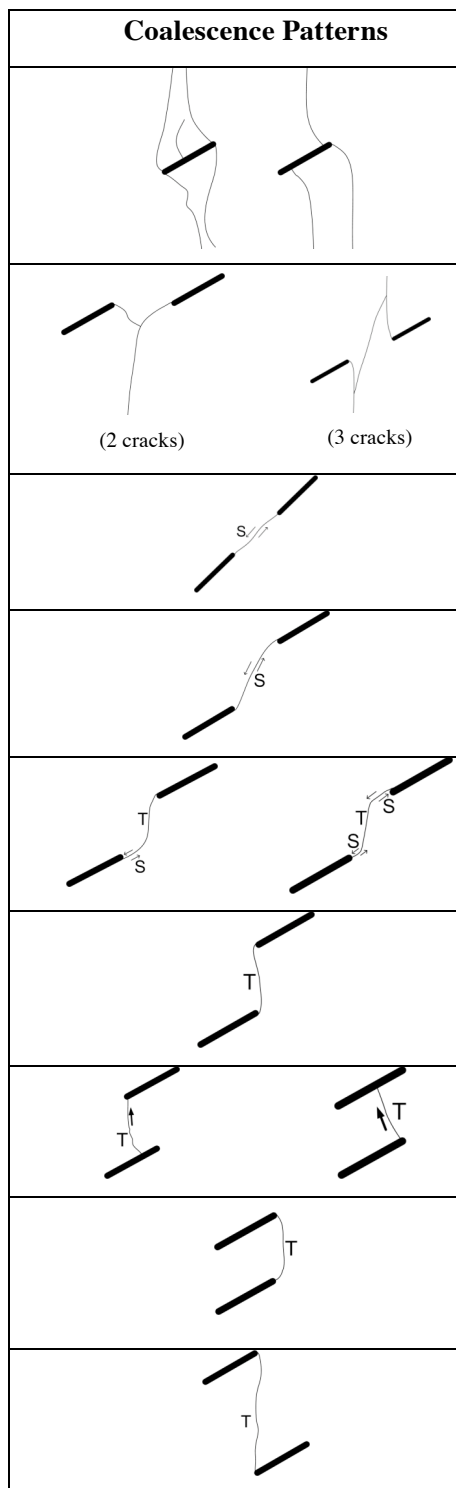


Figure 5. Crack Coalescence Patterns for Gypsum, Marble and Granite (Wong and Einstein, 2009). T= Tension Cracks; S=Shear Cracks

angle, volume, fall height and ramp inclination different runout distances, spreads, and pile heights result (Figure 6). After detailed characterization of the material including nano-indentation technologies, numerical modeling using the discrete element code, MIMES, (Williams and O'Connor, 1999) led to satisfactorily comparable results albeit, only in two-dimensions for the time being.



Figure 6. Physical Modeling of Runout/Pileup of Rock Avalanche. (from Manzella and Labiouse. 2007, 2008)

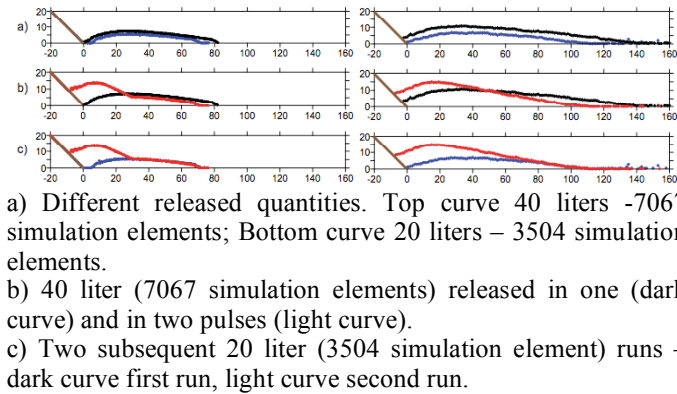


Figure 7. Rock Avalanche Experiments (left side) and Numerical Simulations (right side) (From Manzella, 2008, 2009).

This is shown in Figure 7 where the experiments (left column) and numerical results (right column) show similar effects on deposit morphology and runout for different released quantities and progressive failure.

The last example involves the application of numerical models and a comparison with field observations in a stepwise process leading to a final acceptable result, (Kveldsvik et al. (2009)). This is based on Kveldsvik's doctoral research on the Åknes rock slope in Norway, which is moving. He used DDA (Discontinuous Deformation Analysis, see Shi, 1988) and UDEC (Universal Distinct Element Code, see Cundall, 1980) in this analysis. In the DDA work, "DDA Backward" was used. Specifically, blocks based on geologic interpretation of the slope were used as input together with measured displacements. The difference between measured and modeled displacements was used to modify the block geometry. All in all, 10 models were examined to lead from the initial block geometry to the finally chosen one (Figures 8a, b). The finally chosen model reflects the observed fact that the upper part of the slope moves more than the lower one. The UDEC analysis was conducted on a vertical cross section (Figure 9). In this case measured friction angles, ground water levels and fracture patterns were used as input to determine at which depth it is most likely that failure along the slope parallel foliation fractures and less steeply inclined outcropping fractures occurs. So in combination the two models

provide reasonable information on possible mechanisms. Nevertheless, as Kveldsvik et al., (2009) point out, there are quite a few uncertainties as to which of the failure mechanisms actually acts. In the particular case of Åknes continued displacement- and water level measurements can help in making safety relevant decisions, even if the models cannot (yet?) be used in a complete prediction.

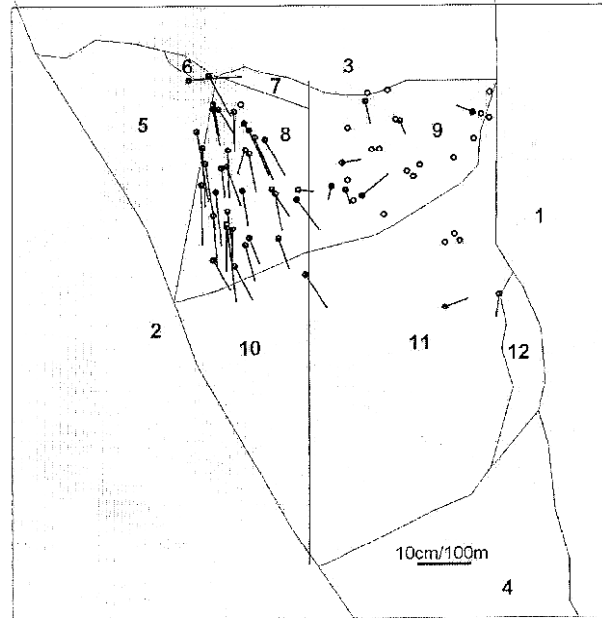


Figure 8a. Åknes Rock Slope - DDA Analysis. Initial Block Model and Annual Average Slope Displacements Derived from Photogrammetry 1983-2004 (From Kveldsvik et al., 2009).

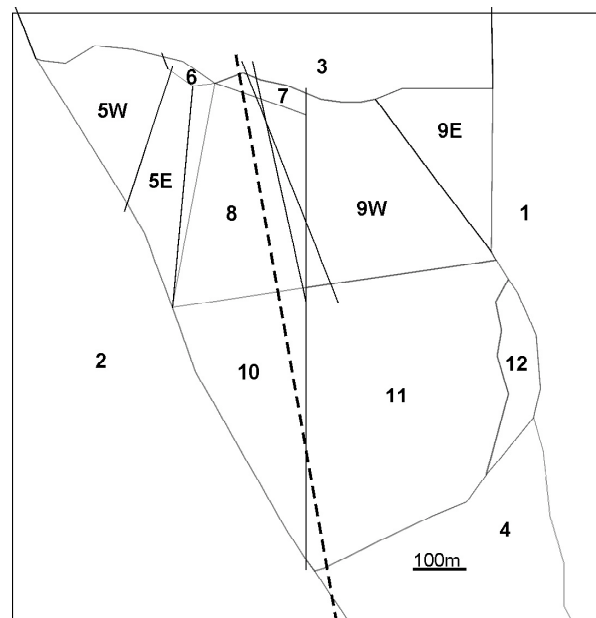


Figure 8b. Åknes Rock Slope - DDA Analysis Proposed Final Block Model. Note Differences in Upper Part Compared to Figure 8a. From Kveldsvik et al. (2009).

Clearly, determining models through back-analysis of observed behavior and recalibrating and checking them with continuous observations is the ideal approach. It is hampered by the fact that rock slope instabilities are ill-defined problems leading to differ-

ent models satisfying the same observations. This problem limits the use of the models in making predictions and is known as model uncertainty, which has been discussed extensively by the authors, (Einstein and Karam (2001), Karam (2005), and Sousa (2010)).

So both the comments on geometry and geometric models, and on the mechanisms and mechanical models indicate that it is necessary to assess and consider uncertainties through probabilistic modeling, which as indicated in Figure 1 is the next phase of the decision making process.

5 PROBABILISTIC MODELS AND RISK DETERMINATION

Model uncertainty, which was just discussed is only one of the sources of uncertainty affecting slope instability and geotechnical engineering in general:

1. Inherent spatial and temporal variability
2. Measurement errors (random or systematic)
3. Statistical fluctuation
4. Model uncertainty
5. Omissions

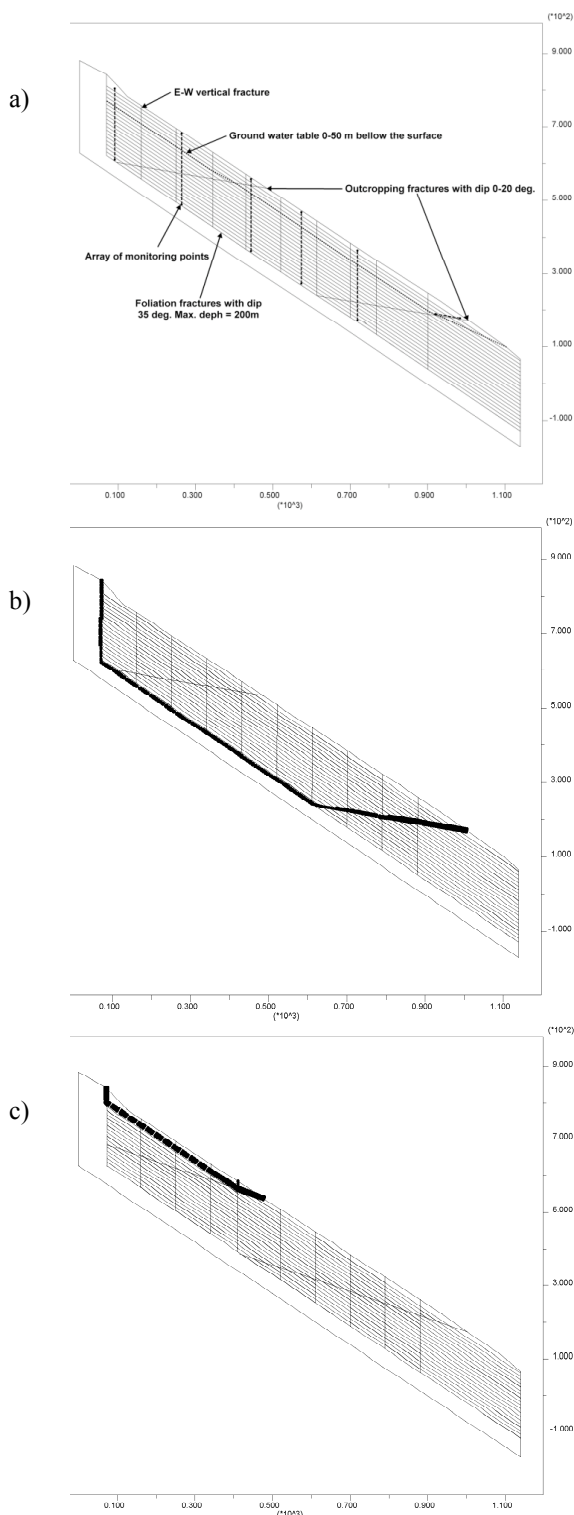
Another way of describing uncertainties is

- Epistemic uncertainty (lack of knowledge)
- Aleatory uncertainty (randomness)

The different types of uncertainties have been discussed in the past, (Baecher (1978); Einstein and Baecher (1987); Lacasse and Nadim (1998)) so only a few additional points are made here:

- The uncertainties, except omissions, have both an epistemic and a random component
- A good example of inherent spatial uncertainty are the joint patterns discussed earlier

Given all this, probabilistic models of rock slope stability, also called rock slope reliability models, have been in use for many years, e.g. CANMET (1976); Piteau and Martin (1977); Call and Nicholas (1978). Low (1996) developed a spreadsheet based approach for wedge instabilities, one of the classic rock slope stability problems. This approach allows one to consider uncertainties in the geometry of the wedge, in the material properties (unit weight, cohesion and friction angle) and in water level to derive the Hasofer and Lind (1974) reliability index. This approach has been extended to single plane dynamic problems by Christian and Urzua (1998). Other probabilistic approaches use Monte Carlo simulation, e.g. Piteau and Martin (1977), dynamic programming, e.g. Lee and Einstein, (1992) or FORM, e.g. Duzgun et al., (2003); for a review of these and other approaches, see Nadim et al. (2005).



- a) UDEC Model
- b) Largest shear displacement of fractures at 200 m depth and outcropping fractures
- c) Largest shear displacement of fractures at 20 m depth and outcropping fracture

Figure 9. Åknes Rock Slope – UDEC Modelling of Potential Displacement along Foliation Fractures and Outcropping Fractures. From Kveldsvik et al., (2009).

In this paper a simple approach for a rock-slide as shown in Figure 10 will be used (simplifications include: infinite slope, linear Coulomb failure criterion, groundwater level as shown in Figure 10). This leads, with the uncertainties listed in Figure 10, to the probability of failure shown in Figure 11, namely $P(FS < 1) = 0.207$. Combining the hazard with consequences leads to risk:

$$\text{Risk} = \text{Probability of Threat} \times \text{Worth of Loss} \\ = \text{Hazard} \times \text{Consequences.}$$

Otherwise expressed:

$$R = P[T] \times u(\underline{X})$$

where

R = Risk

P[T] = Hazard

$U(\underline{X})$ = utility of consequences where (\underline{X}) is a vector of attributes (see Keeney and Raiffa, (1976), Baecher, (1981))

This expression can be expanded to express the fact that consequences are uncertain by including the so-called vulnerability, which can be expressed by the conditional probability $P[X|T]$ leading, in the case of a single attribute X, to:

$$R = P[T] \times P[X|T] \times U(X)$$

All this has been discussed in detail earlier, e.g. Einstein (1997); Fell (2005). The reason for repeating this here is because it serves as a basis for the application of the decision making process in Section 6. Before doing this it has to be pointed out that, instead of working with risk, which implies associating values with the consequences (monetary values or utilities) one can work with hazard. This is often desirable if one cannot or does not want associate values with consequences, e.g. in case of life loss. In the rock slope instability domain, applications using hazard only are e.g. the rockfall hazard systems (New York State, 1990) or FN diagrams (Fig. 12).

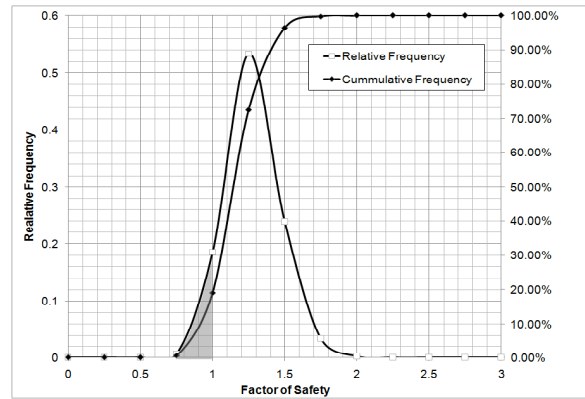


Figure 11. Probability of Failure for Rock Slope shown in Fig. 10. Hazard = $P[\text{Threat}] = P[\text{Factor of safety} < 1] = 0.207$

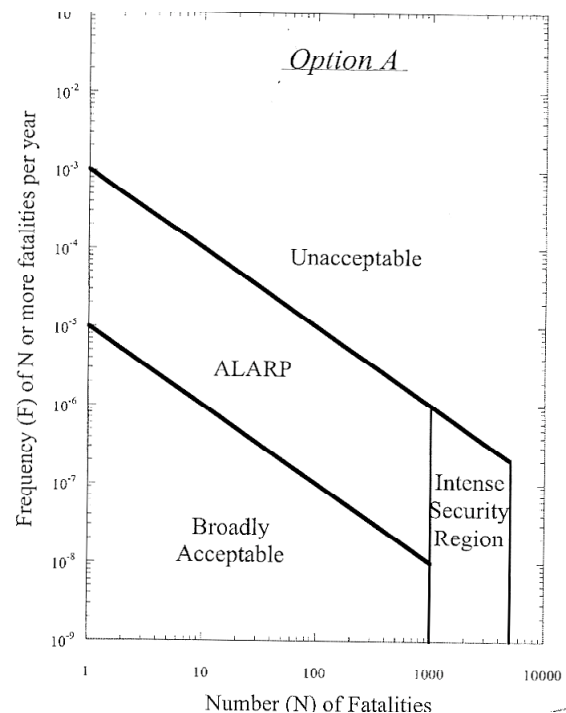


Figure 12. FN Diagram – Example for Hong Kong (Ho et al., 2000). ALARP = As Low as Reasonably Practical

6 RISK MANAGEMENT

As seen in Figure 1, this is the final block of the decision-making flow diagram. It represents the most important practical aspect since it is here where “something can be done about risk”; it is also central to this paper through the application of the decision-making approach and application of new concepts. Possible management actions are, as shown with the decision tree in Figure 13: No action, active and passive countermeasures as well as warning systems, where the latter can be considered a type of passive countermeasure. An additional action, which will be treated separately, is collecting new information. Decision trees have the advantage of systematically

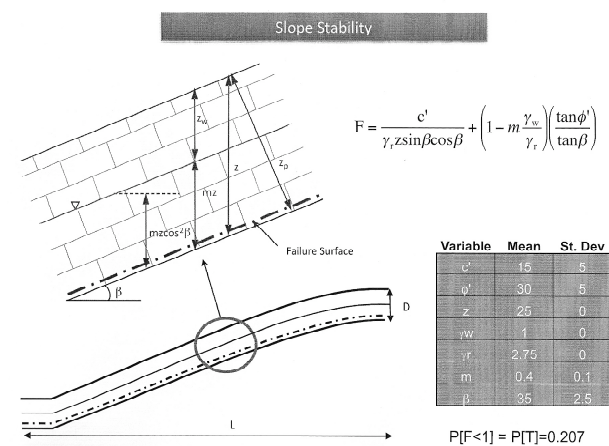


Figure 10. Example Rock Slope Stability Problem (c' = effective cohesion, ϕ' = effective friction angle, γ_w = water unit weight, γ_r = rock mass unit weight).

organizing the process. They can become quite involved for complex processes, however. In the following, the rock slope failure example of Figure 10 will be used to demonstrate the different risk management actions.

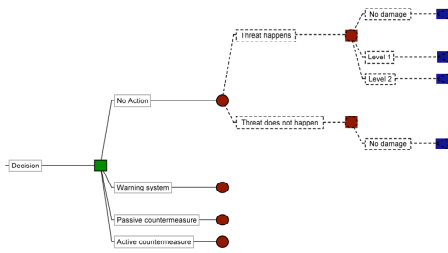


Figure 13. Decision Tree for Rock Slope Instability Problems.

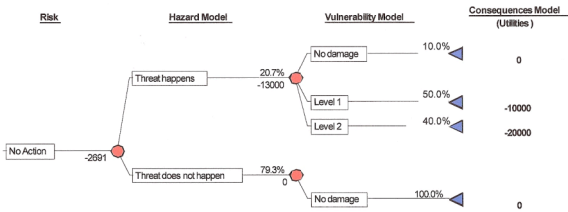


Figure 14. Decision Tree Rock Slope Instability- No Action.

The no-action decision tree illustrated in Figure 14 illustrates the base case, i.e. failure occurring with a particular probability (20.7%) and having different vulnerabilities or, as in Figure 14, different probabilities of the various damage levels where these damage levels are expressed by different utilities. By multiplying and summing the numbers in Figure 14, one obtains the risk of “-2691” without any countermeasures.

Countermeasures reduce risk but do so at a cost. Specifically, active countermeasures reduce the hazard, i.e. produce a lower probability of failure and a reduced risk R'

$$R'_i = P' [T] + P[X_i|T] + u (X_i) + u (C_{act})$$

where

$P' [T]$ = reduced probability of threat

C_{act} = cost of active countermeasures

Other terms as before

For active countermeasures to be worthwhile R' should be less than R , where R is the risk associated with “no action”. The decision tree for the case with active countermeasures is shown in Fig. 15. Examples of active countermeasures are bolts (anchors) and retaining structures.

With passive countermeasures, the vulnerability is reduced

$$R' = P [T] + P' [X|T] + u (X) + u (C_{pas})$$

where

$P' [X|T]$ = reduced vulnerability

C_{pas} = cost of passive countermeasures

Again R' should be less than R for passive countermeasures to be worthwhile. The corresponding decision tree is shown in Figure 16. Examples of passive countermeasures are, for instance, rockfall nets or protective sheds.

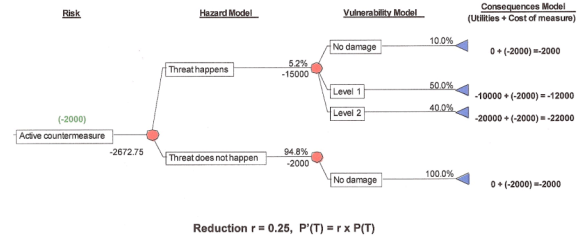


Figure 15. Decision Tree Rock Slope Instability – Active Countermeasure. The probability of Failure, i.e. the Hazard has been Reduced:

$P' [T] = r \times P[T]$; $r = 0.25$ in this example.

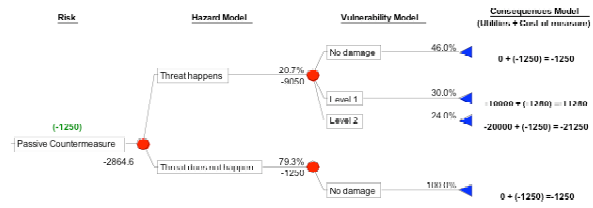


Figure 16. Decision Tree Rock Slope Instability – Passive Countermeasure. Vulnerabilities have been changed from No Action (Compare to Fig. 14).

For both active and passive countermeasures, one could, in principle, include the probability that the countermeasure is successful but one usually does not do this. This is different in warning systems and one of the reasons why they are treated separately, although they are also a passive countermeasure. There is a reasonable probability that on the one hand warning systems do not work or on the other hand, that false alarms are issued. These possibilities need to be included and, consequently, lead to rather involved decision trees as shown in Figure 17, which includes the reliability of the warning system (warning issued when it should – when it should not, warning not issued when it should not – when it should). It is quite evident when looking at Figure 17 that the tree is complete but also cumbersome. This is even more so as the entire decision tree actu-

ally consists of all trees in Figures 14 to 17! This is where Bayesian networks come in.

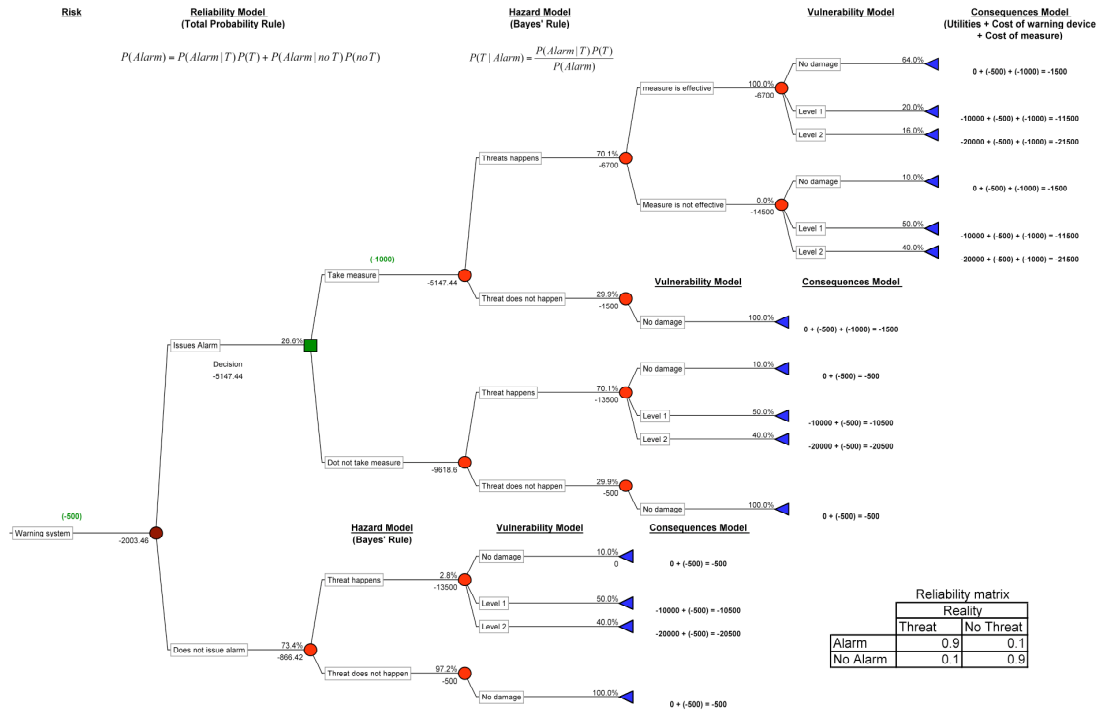


Figure 17. Decision Tree for Rock Slope Instability – Warning System.

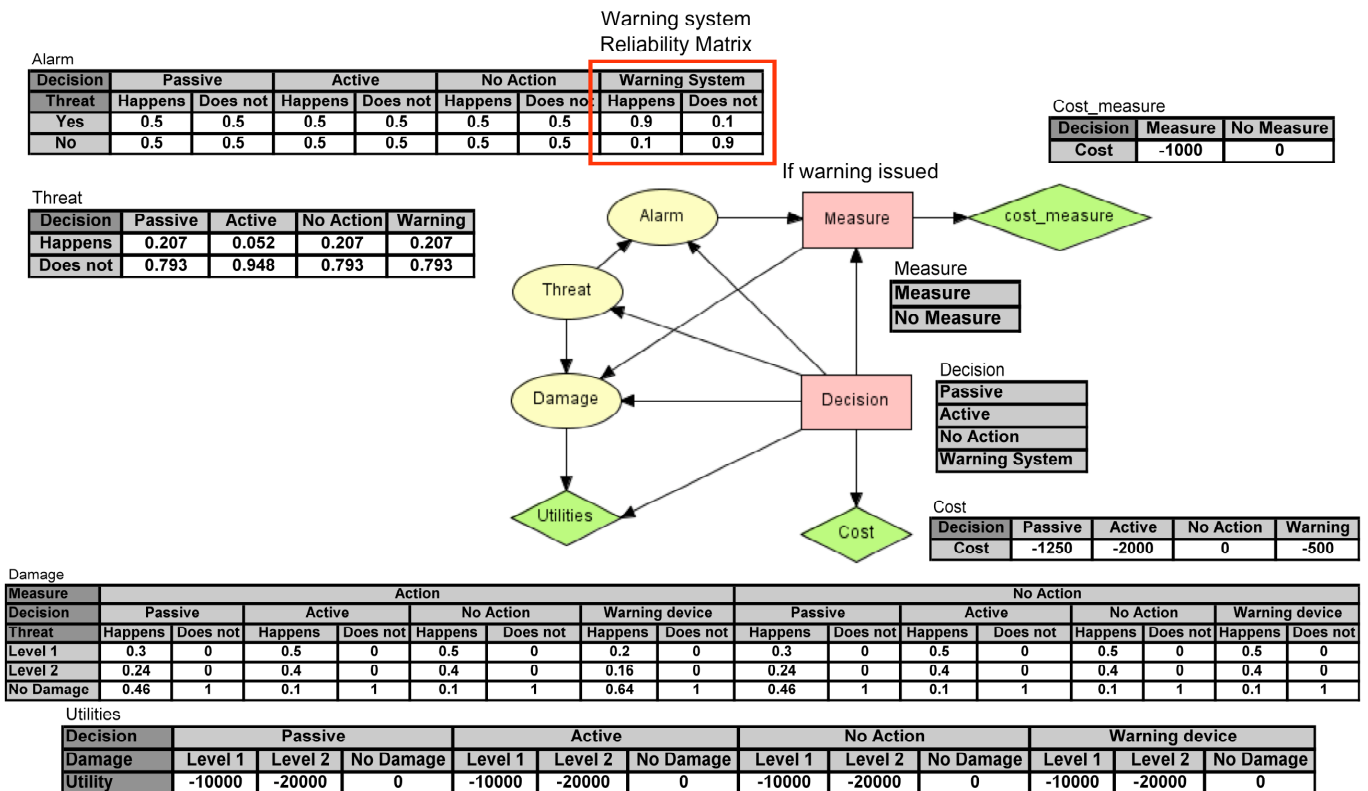
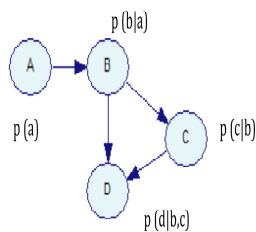


Figure 19. Bayesian Network Applied to Rock Slope Instability Problem with Possible Decisions No Action, Active Countermeasures, Passive countermeasures and Warning System.

A Bayesian network, also known as belief network, is a “graphical representation of knowledge for reasoning under uncertainty”, or as stated by Russell and Norvig (1995), “A concise graphical representation of the joint probability of the domain represented by random variables.” Bayesian networks encode conditional independencies between variables, which simplify and allow one to compute the joint probability of a domain more efficiently. This is illustrated in Figure 18. The application of the Bayesian network to the decision problem including warning systems is shown in Figure 19. In this figure the matrices for the warning system reliability, threat probability (hazard), cost of the countermeasures (active, passive, warning system) and cost of consequences are shown.

Typical warning systems for rock slope failures are based on observations of displacements as e.g. proposed by Blikra (2008) and shown in Figure 20.

The final result of the entire decision making process be this done with decision trees or Bayesian networks is shown in Figure 21. One ends up with final utilities for each of the actions and selects the one with the lowest (negative) utility, i.e., the lowest risk, which in this example is the warning system.



Joint Probability (chain rule):

$$p(a, b, c, d) = p(a) p(b|a) p(c|a, b) p(d|a, b, c)$$

Independency assumptions (chain rule for BN):

$$p(a, b, c, d) = p(a) p(b|a) p(c|b) p(d|b, c)$$

Figure 18. Principle of Bayesian Network.

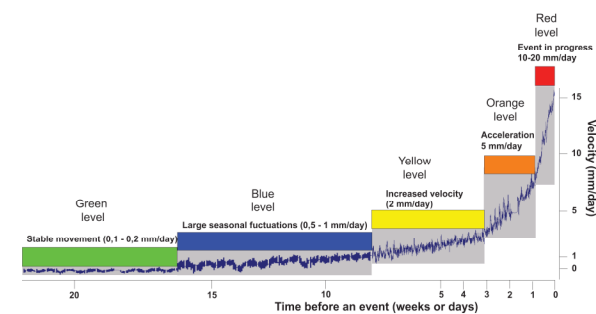


Figure 20. Warning System Based on Rock Slope Displacements for Åknes Rock Slope. Synthetic Figure Making Use of Historical Data at Åknes and Experience with Other Rock Slopes, from Blikra (2008)

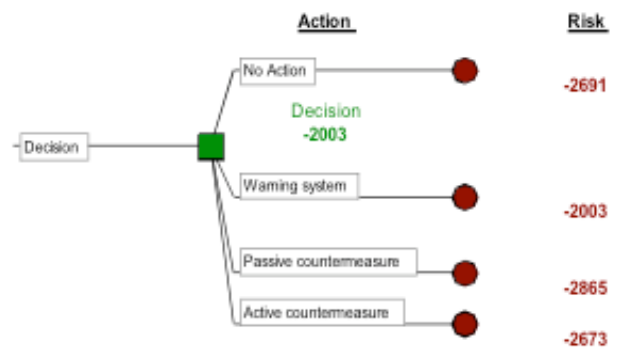
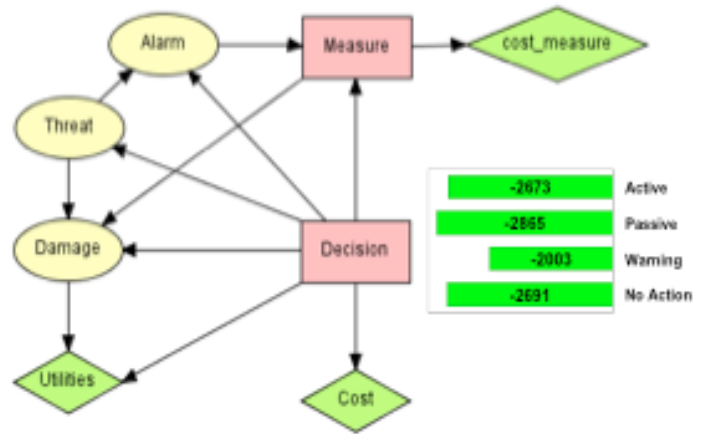


Figure 21. Decision Making for Rock Slope Instability Problem Using Bayesian Network and Decision Tree. Results of No Action, Active Countermeasure, Passive Countermeasure and Warning Systems.

The reader will correctly note that this decision is based on many numbers that can vary i.e. are uncertain. For instance the consequence costs can vary and so can the “active reduction factor r ” (Figure 15); other probabilistic values such as the probability of failure or the vulnerability or the warning system’s reliability can have other values (uncertainty of the uncertainty). It is, therefore, beneficial to conduct sensitivity analyses. Figure 22 investigates the effect of the probability of threat ($P[T] = \text{Hazard}$) against different actions. As to be expected, for very low failure probabilities, no action is preferred, otherwise it is the warning system, except for very high probabilities where active countermeasure are preferred. This is only one example and the sensitivity of the decision to other factors needs to be similarly investigated.

One last issue needs to be addressed in the context of decision making (under uncertainty) for rock slopes instability and natural threats in general. This relates to collecting additional information, which will lead to an updating of the entire decision process as indicated in Figure 1. There are two possibilities for updating with new information:

1. Information collected after decision
2. Information collected before decision

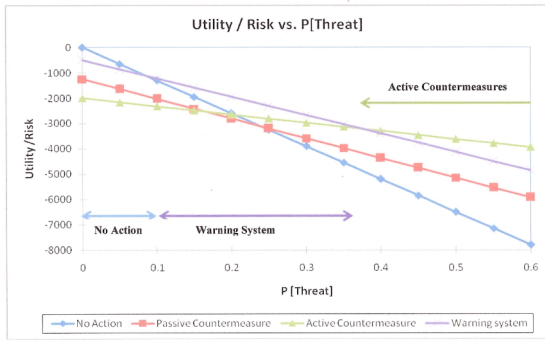


Figure 22. Rock Slope Instability Problem – Sensitivity Analysis for Varying Hazard $P[\text{Threat}]$.

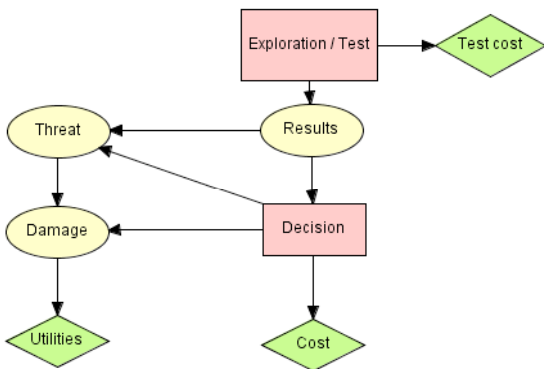


Figure 23. Bayesian Network for Decision Making Including “Virtual Exploration” for Rock Slope Instability Problem.

Updating with information collected after the decision is the standard procedure of e.g. mapping additional fractures, observing water levels in boreholes, and with this update the state of nature and/of the hazard. This may then lead to a different final decision.

Updating with information collected before the decision is a two step process involving so called virtual exploration as a first step and then deciding which countermeasures to take based on this virtual exploration. This requires an additional set of branches in the decision tree or an additional node in the Bayesian network, which represent the information from the virtual exploration influencing the hazard (Figure 23). This new information is uncertain, which has to be reflected in the virtual exploration. All this is considered with Bayesian updating in pre-posterior analysis and has been used for tunnel exploration, (Karam et al. (2007)). Efforts are under way at present to develop this also for rock slopes. The results will be a utility (cost) of the best (lowest negative utility) action given the virtual exploration. The difference between this number and the utility without virtual information is called Expected Value of Sample Information (EVSI). If EVSI is greater than the cost of the exploration, one will decide to conduct the exploration. This is schematically shown

in Figure 1 and completes the description of possible decision making for rock slope “instabilities”.

7 SUMMARY AND CONCLUSIONS

The decision making process as outlined in Figure 1 has been used to briefly review how one proceeds from information collection on the state of nature to risk management regarding rock slope instabilities be they slides or related instabilities, or rock falls. Some detailed comments were made about recent work on failure mechanisms and modeling them. It becomes apparent that while advances have been made regarding our understanding of failure mechanisms, much seems to be inconclusive or uncertain. This, together with other uncertainties, is the reason why the process of decision making under uncertainty is ideally suited to the rock slope instability problem. Consequently, the paper shows how this can be applied emphasizing risk management consisting of a choice of decisions between no action, active and passive countermeasures, and warning systems.

Practical tools to support the decision making process are the classic decision tree or Bayesian networks. The use of the latter represents a new development. Both tools allow one to conduct sensitivity analyses, which are absolutely essential when dealing with processes that include so many uncertainties. Both decision tools can also be used to assess the effect of additional information collection through exploration.

The main points are, therefore, that dealing with rock slope instabilities involves uncertainties and requires an appropriate decision making process, particularly when determining and managing the associated risks. Tools to do this, including newly developed ones, do exist. This should, however, not be interpreted that a better understanding of the underlying mechanism is not worthwhile. Research in this direction has - and will continue to advance the practice of dealing with rock slope instabilities.

ACKNOWLEDGEMENTS

Support for research and applications discussed in this paper came from: The National Science Foundation (USA), Swiss National Science Foundation, Portuguese Foundation for Science and Technology, International Centre for Geohazards at the Norwegian Geotechnical Institute, U.S. Department of Energy, MIT-Portugal Program.

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