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Published in:
Proceedings of the 2015 IEEE International Telecommunications Energy Conference (INTELEC)

DOI (link to publication from Publisher):
10.1109/INTLEC.2015.7572307

Publication date:
2015

Document Version
Early version, also known as pre-print

Link to publication from Aalborg University

Citation for published version (APA):
Mostaan, A., \& Soltani, M. (2015). A Family of Four Quadrant DC/DC Converters with Reduced Number of Components. In Proceedings of the 2015 IEEE International Telecommunications Energy Conference (INTELEC) Osaka, Japan: IEEE Press. DOI: 10.1109/INTLEC.2015.7572307

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# A Family of Four Quadrant DC/DC Converters with Reduced Number of Components 

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#### Abstract

A family of four quadrant DC/DC converters is presented in this paper. Compare with existing four quadrant DC/DC converters that have been introduced in literature, the proposed converters have lower number of components. There are two bidirectional switches, two coupled inductors and one capacitor in proposed converters that can lead to lower cost and also smaller occupied space. The proposed converters are analyzed in detailed and their voltage gain is obtained. It is shown that to obtain high voltage gain, it is not necessary to increase the transformer turn ratio, therefore the leakage inductance effect is minimized in these converters. The effectiveness of the proposed converters is validated with simulation using MATLAB/SIMULINK.


Keywords- four quadrant DC/DC converter, coupled inductors, $Z$ source network, voltage gain

## I. Introduction

DC/DC converters are widely used in telecommunication equipment power supplies, photovoltaic system, portable electronic devices and many other industrial applications [12]. Conventional DC/DC converters such as buck boost and buck-boost converter with one active switch and one diode can work only in one quadrant mode. In some applications particularly when battery is used as an energy storage resource, a bidirectional $\mathrm{DC} / \mathrm{DC}$ converter is required in order to provide an effective path to charge or discharge the battery. The bidirectional DC/DC converter can be realized easily if the active switch (transistor) and passive switch (diode) in conventional DC/DC converter are replaced with bidirectional switches [3-4]. In addition, in recent years many bidirectional DC/DC converters with high performance and high voltage gain have been introduced [5-8]. However, these converters do not provide a bipolar output voltage. In some special applications such as DC motor derive systems, a four quadrant $\mathrm{DC} / \mathrm{DC}$ converter is required that provides the fourquadrant operation modes that are include: forward and backward motoring mode and also forward and backward breaking mode. Four quadrant DC/DC converters have been found many applications, particularly in electrical transportation systems that the motor energy should back to source in breaking mode. In addition four quadrant $\mathrm{DC} / \mathrm{DC}$ converters can be utilized as one DC/AC inverter if their duty cycle is a sine wave. Traditionally, full bridge DC/DC converter has been used as a four quadrant $\mathrm{DC} / \mathrm{DC}$ converter for DC motor derive systems. But, there are four bidirectional switches in its structure that lead to its circuit complexity and also high cost. In addition, the input voltage and load do not
share the same ground that could lead to high leakage current when this converter is used as one single phase DC/AC inverter and injects AC current to grid [9]. An improved type of four quadrant $\mathrm{DC} / \mathrm{DC}$ converter with two switches, three inductors and three capacitors is introduced in [10]. The input voltage and load share the common ground in this topology that definitely is advantage in comparison to the H-bridge converter. But the number of inductors and capacitors are high in the converters that have been introduced in [10] that increase the cost and circuit complexity. In addition in [11], a family of four quadrant $\mathrm{DC} / \mathrm{DC}$ converters based on Z source and Quasi source converter is introduced. There are two switches, two inductors and two capacitors in their structure.

In this paper, a family of four quadrant $\mathrm{DC} / \mathrm{DC}$ converters based on Trans Z source inverter [12] and $\Gamma$ - Z source inverter are presented. The number of components is lower than other existing four quadrants $\mathrm{DC} / \mathrm{DC}$ converters. It is shown that it is not necessary to increase the transformer turn ratio in order to obtain high voltage gain. Therefore the effect of leakage inductor is minimized in proposed converters.

This paper is organized as follows: in section II, the proposed converters are introduced and their voltage gain in steady state is obtained. Small signal model for one of these converters (first topology) is obtained in section III. Simulation results are presented in section IV and finally, the conclusion is followed in section V .

## II. Proposed Converters Structure

## A. Analysis in steady state

In order to increase the voltage gain in Z source inverter, Trans Z source [12] and $\Gamma$-Z source inverter [13] have been introduced. These inverters are shown in Fig. 1. The proposed four quadrants $\mathrm{DC} / \mathrm{DC}$ converters are realized with some modifications on Trans Z source and $\Gamma-\mathrm{Z}$ source inverters. The proposed converters are shown in Fig. 2. It is obvious that there are two bidirectional switches (MOSFET and its anti parallel diode), two coupled inductors and one capacitor in the proposed converters. The two switches operate complementary and the duty cycles for $S_{1}$ and $S_{2}$ are $D$ and 1$D$, respectively. Therefore, there are two equivalents circuits in one switching cycle for proposed converters. As one example, the topology that is shown in Fig. 2a is analyzed to show the converter working stages.

(a) First topology

(b) Second topology

(c) Third topology

(d) Fourth topology

Fig.2. proposed four quadrants $\mathrm{DC} / \mathrm{DC}$ converters
In fact, the topologies that are shown in Fig. 2a and Fig. 2b are realized with some modifications on Trans Z source inverter (Fig. 1a). While the topologies that are shown in Fig.

2c and Fig. 2d are realized with some modifications on the $\Gamma$ $Z$ source inverters (Fig. 1b).

## A): steady state analysis of the first topology

In our analysis the coupled inductors are modeled with ideal transformer that its turn ratio is $1: \mathrm{n}$. Also the magnetic inductance $\left(\mathrm{L}_{\mathrm{m}}\right)$ is in parallel with primary winding. The equivalent circuits during the first state and second state are shown in Fig. 3a Fig. 3b, respectively. From Fig. 3a it is clear that the voltage across the $L_{m}$ is equal to input voltage when S1 is on

$$
\begin{equation*}
V_{L m}=V_{W 1}=V_{i n} \tag{1}
\end{equation*}
$$

Similarly, using Fig. 3b when S 2 is on the voltage across the $\mathrm{L}_{\mathrm{m}}$ is

$$
\begin{equation*}
V_{L m}=V_{W 1}=\frac{V_{O}-V_{i n}}{n} \tag{2}
\end{equation*}
$$

Using the volt-second law across the magnetic inductance during a switching cycle we have

$$
\begin{equation*}
D V_{i n}+\frac{(1-D)\left(V_{0}-V_{i n}\right)}{n}=0 \tag{3}
\end{equation*}
$$

That result

$$
\begin{equation*}
\frac{V_{0}}{V_{\text {in }}}=\frac{1-D(1+n)}{1-D} \tag{4}
\end{equation*}
$$

From (4) it is obvious that the output voltage is positive if $D(1+n)<1$ and negative if $D(1+n)>1$. The voltage gain of this converter versus duty cycle (D) for different transformer turn ratio is shown in Fig.4. From Fig. 4 it is clear that the proposed converter works only in buck mode when the positive voltage is required and the voltage gain is always lower than one. In contrast, this converter works in buck-boost mode when negative voltage is required. In this case the voltage gain can change from zero to infinite. In addition, Fig. 4 shows that the voltage gain of this converter can increase when the transformer turn ratio is raised. It should be noticed that the voltage gain is increased only when the output voltage is negative and converter always works in buck mode when positive output voltage is required in every transformer turn ratio. Therefore, in this paper the transformer turn ratio is selected to $\mathrm{n}=1$ in order to minimize the leakage inductance effect. If $\mathrm{n}=1$, using (4) the voltage gain can be written as

$$
\begin{equation*}
\frac{V_{0}}{V_{\text {in }}}=\frac{1-2 D}{1-D} \tag{5}
\end{equation*}
$$

The voltage gain of this converter under $\mathrm{n}=1$ is equal to other four-quadrant DC/DC converters that have been introduced in [10-11], while the number of components are lower in proposed converter. There is only one capacitor in this converter, while there are three capacitors in converters that are introduced in [10] and there are two capacitors in converters that have been introduced in [11].
As mentioned earlier, the voltage gain of this converter can be positive or negative that depends on duty cycle and transformer turn ratio. When $\mathrm{n}=1$, the output voltage is positive when $\mathrm{D}<0.5$ and negative when $\mathrm{D}>0.5$. Therefore, the voltage polarity and direction of magnetic inductor is different in two cases as shown in Fig. 5. Fig. 5(a, b) show the voltages and currents direction when $\mathrm{D}<0.5$ and Fig. 5(c, d) show the voltages and current direction when $\mathrm{D}>0.5$.

(a) First stage

(b) Second stage

Fig. 3.Equivalent circuits of first topology in one switching cycle


Fig.4. Voltage gain of the first topology versus the transformer turn ratio

(b) Equivalent circuit under $\mathrm{n}=1$ and $\mathrm{D}<0.5$ (second stage)

(c) Equivalent circuit under $\mathrm{n}=1$ and $\mathrm{D}>0.5$ (first stage)

(d) Equivalent circuit under $\mathrm{n}=1$ and $\mathrm{D}>0.5$ (second stage) Fig.5. Equivalent circuits of first topology under $n=1$
B): steady state analysis of the other topologies

Similar method can be used to determine the voltage gain in other topologies. Applying volt-second balance law on magnetic inductance, the voltage gain for other topologies (second, third and fourth topology) can be obtained. The voltage gain for second, third and fourth topology can be calculated from (6), (7) and (8), respectively

$$
\begin{align*}
\frac{V_{O}}{V_{i n}} & =\frac{(1-D)}{1-D(1+n)}  \tag{6}\\
\frac{V_{O}}{V_{i n}} & =\frac{1-n(1-D)}{(1-n)(1-D)}  \tag{7}\\
\frac{V_{O}}{V_{i n}} & =\frac{(1-n)(1-D)}{1-n(1-D)} \tag{8}
\end{align*}
$$

The voltage gain versus the transformer turn ratio for third topology is shown in Fig. 6. It is clear that the converter works in buck mode when positive voltage is required, while works as a buck-boost converter when negative voltage is required. The interesting feature of this converter is that the voltage gain of this converter is increased with lowering the transformer turn ratio and theoretically will become infinite when n approaches too one. Therefore the effect of the leakage inductance is minimized in this converter. Since low transformer turn ratio can be selected. With $n=2$, the voltage gain of this converter is similar to proposed converters that have been introduced in [10-11].

From fig. 4 and Fig. 6 it is clear that voltage gain in the first and third topologies is changed continuously when the duty cycle is changed from zero to one. In contrast, the voltage gain in the second and fourth topology is changed discontinuously when the duty cycle is changed from zero to one. Theoretically the voltage gain is infinite when $D=$ $1 /(n+1)$ in second topology and $D=(n-1) / n$ in the fourth topology. Fig. 7 shows the voltage gain of the second topology (under $\mathrm{n}=1$ ) and the fourth topology (under $\mathrm{n}=2$ ).

From fig. 7, it is clear that the output voltage in the second (under $\mathrm{n}=1$ ) and the fourth topologies (under $\mathrm{n}=2$ ) can be positive if $\mathrm{D}<0.5$, negative $\mathrm{D}>0.5$ and theoretically infinite if D approaches to 0.5


Fig.6. Voltage gain of third topology versus duty cycle under different transformer turn ratio


Fig.7. Voltage gain of the second topology (under $n=1$ ) and fourth topology (under $\mathrm{n}=2$ ) versus the duty cycle

## $C$ : Improved voltage gain in proposed topologies

Among the proposed topologies, the first and the third topologies can be suitable candidate for DC motor drive system, because their voltage gain changes continuously. But their output voltage is lower than input voltage when positive output voltage is required. In order to solve this problem, two phase version of these converters can be realized easily. As one example, Fig. 8 shows the two phase version of the third topology. There are four switches in this converter such as (S1, S3) and (S2, S4) that work complementary. From Fig. 8, the output voltage is equal to

$$
\begin{equation*}
V_{O}=V_{O 1}-V_{O 2} \tag{9}
\end{equation*}
$$

Using (7), we have

$$
\begin{equation*}
V_{O}=\frac{1-n(1-D)}{(1-n)(1-D)}-\frac{1-n D}{(1-n) D} \tag{10}
\end{equation*}
$$

That results in

$$
\begin{equation*}
V_{O}=\frac{2 D-1}{(1-n)(1-D) D} \tag{11}
\end{equation*}
$$

From (11), it is obvious that the voltage gain of this converter approaches to infinite when the transformer turn ratio approaches to one. Fig. 9 shows the voltage gain versus the duty cycle ratio with different transformer turn ratio.


Fig.8. Two phase version of the third topology


Fig.9. Voltage gain versus duty cycle under different transformer turn ratio for two phase version of third topology

From Fig. 9 it is clear that this converter can work as a buckboost converter if positive or negative voltage is required. In addition, the output voltage is increased with lowering the transformer turn ratio that lead to lower leakage inductance. Although in order to eliminate the leaking inductance effect, an active-clamp with a small auxiliary switch and small series capacitor (about $1 \mu F$ ) can be added in parallel with transformers winding. The active- clamp eliminates turn off spike voltage efficiency [14].

## D: Four- quadrant operation of the proposed converters

 The proposed converters have both capability of providing bipolar output voltage and a bi-directional character. Each switch consists of an active device (MOSFET) and its antiparallel diode. The four quadrant operation of the first topology under $\mathrm{n}=1$ are shown in Fig. 10. In the first and second quadrant the output voltage is positive while $\mathrm{D}<0.5$. The output voltage is negative in third and fourth quadrant while $\mathrm{D}>0.5$. E can be considered as the motor or other source with internal resistance R. In the first and third quadrant the energy is transferred from the input voltage to load, while in the second and fourth quadrant the energy is transferred from load to input.
(a) Operation modes in first quadrant: $\mathrm{D}<0.5$ and $\mathrm{V}_{\mathrm{O}}>\mathrm{E}$

(b) Operation modes in second quadrant: $\mathrm{D}<0.5$ and $\mathrm{V}_{\mathrm{O}}<\mathrm{E}$

S1

(c) Operation modes in third quadrant: $\mathrm{D}<0.5$ and $\left|\mathrm{V}_{\mathrm{O}}\right|>|\mathrm{E}|$

(d) Operation modes in third quadrant: $\mathrm{D}<0.5$ and $\left|\boldsymbol{V}_{\boldsymbol{O}}\right|<|\boldsymbol{E}|$

Fig. 10. Four quadrant operation of the first topology
III. Small Signal Model Of The Proposed

## CONVERTERS

In this section, dynamic model of the first topology is obtained using the state-space average modeling method. The load is modeled with a voltage source (E) and an internal resistance ( R ). The current value is positive when the energy is transferred from source to load and is negative when the energy is transferred from load to source as shown in Fig. 11.


Fig.11. First topology as a bi-directional DC/DC converter
The state equation during the first state ( S 1 is ON and S 2 is OFF) and second state (S1 is OFF and S2 is ON) can be written as (12) and (13) respectively

$$
\begin{align*}
& \left\{\begin{array}{c}
L_{m} \frac{d i_{m}}{d t}=V_{i n} \\
C \frac{d v_{O}}{d t}=-\left(\frac{V_{O}-E}{R}\right)
\end{array}\right.  \tag{13}\\
& \left\{\begin{array}{c}
L_{m} \frac{d i_{m}}{d t}=\frac{\left(V_{i n}-V_{O}\right)}{n} \\
C \frac{d v_{O}}{d t}=\frac{i_{m}}{n}-\left(\frac{V_{0}-E}{R}\right)
\end{array}\right. \tag{14}
\end{align*}
$$

According above equations, the average state equations are

$$
\left\{\begin{array}{c}
L_{m} \frac{d<i_{m}>}{d t}=<V_{i n}>\left[D+\frac{(1-D)}{n}\right]-\frac{(1-D)<V_{O}>}{n}  \tag{15}\\
C \frac{d<v_{o}>}{d t}=\frac{(1-D)<i_{m}>}{n}-\left(\frac{<V_{O}>-E}{R}\right)
\end{array}\right.
$$

In above equation, symbol $<x>$ is used to represent the average value of a variable $x$, where $x$ indicates voltage or current. The resulting average model of this converter is shown in Fig. 12


Fig.12. Average model of the first topology
According to (15), the small signal equations can be obtained if all variations change a little around the quiescent point and multiplied AC terms are neglected [15]. The small signal equations of this converter are $\left\{\begin{array}{c}L_{m} \frac{d \hat{\iota}_{m}}{d t}=\left(V_{i n}-\frac{V_{i n}}{n}+\frac{V_{O}}{n}\right) \hat{d}+\left(D+\frac{(1-D)}{n}\right) \hat{v}_{i n}-\frac{(1-D)}{n} \hat{v}_{o} \\ C \frac{d \hat{v}_{O}}{d t}=\frac{1}{n}\left[(1-D) \hat{l}_{m}-I_{m} \hat{d}\right]-\frac{\hat{v}_{o}}{R}\end{array}\right.$
Using (16), the control to output transfer function can be obtained easily as shown in (17). This transfer function can be used to design an appropriate controller for this converter to regulate the output voltage.

$$
\begin{align*}
& \frac{\hat{v}_{O}}{\hat{d}}=\frac{-L_{m} I_{m} R S+R(1-D)\left(V_{i n}-\frac{V_{i n}+V_{O}}{n}\right)}{L_{m} C n R S^{2}+n L_{m} S+\frac{R(1-D)^{2}}{n}}  \tag{17}\\
& \text { IV. } \quad \text { SIMULATION RESULTS }
\end{align*}
$$

In order to verify the theoretical results, the first topology under $n=1$ is simulated using MATLAB/SIMULATION software. The circuit parameters that were used in simulation are $: V_{i n}=50 \mathrm{~V}, \quad L_{m}=1 \mathrm{mH}, \quad C=100 \mu F, \quad R=50 \Omega$, and $f_{S}=100 \mathrm{KHz}$ in these following cases:

1) $\mathrm{D}=0.25$ and $\mathrm{E}=20 \mathrm{~V}$ (operation in first quadrant)
2) $D=0.25$ and $E=40 \mathrm{~V}$ (operation in second quadrant)
3) $\mathrm{D}=0.75$ and $\mathrm{E}=-40 \mathrm{~V}$ (operation in third quadrant)
4) $\mathrm{D}=0.75$ and $\mathrm{E}=-120 \mathrm{~V}$ (operation in fourth quadrant)

The simulation results in these cases are shown in Fig. 13 to Fig. 16 respectively


Fig.13. Operation in first quadrant $\left(\mathrm{V}_{\mathrm{O}}>0, \mathrm{I}_{\mathrm{O}}>0\right.$ and $\left.\mathrm{I}_{\text {in }}>0\right)$


Fig.14. Operation in second quadrant $\left(\mathrm{V}_{\mathrm{O}}>0, \mathrm{I}_{\mathrm{O}}<0\right.$ and $\left.\mathrm{I}_{\text {in }}<0\right)$



Fig.15. Operation in third quadrant $\left(\mathrm{V}_{\mathrm{O}}<0, \mathrm{I}_{\mathrm{O}}<0\right.$ and $\left.\mathrm{I}_{\text {in }}>0\right)$



Fig.16. Operation in fourth quadrant $\left(\mathrm{V}_{\mathrm{O}}<0, \mathrm{I}_{\mathrm{O}}>0\right.$ and $\left.\mathrm{I}_{\text {in }}<0\right)$ The output voltage value is 33 V when $\mathrm{D}=0.25$ and is -100 V when $\mathrm{D}=0.75$. Both of these values are identical with theoretical results that are obtained from (5). The simulation results clearly shows that proposed converters can work in four quadrant modes, therefore can be good candidate for DC motor drive systems and also bi-directional DC/DC converter in renewable energy systems.

## V. CONCLUSION

A family of four quadrant $\mathrm{DC} / \mathrm{DC}$ converters has been introduced in this paper. The proposed converters have lower number of components in compared with other four quadrants DC/DC converters that have been introduced in literature. In addition, it is not required to increase the transformer turn ratio to obtain high voltage gain. Particularly in the second
and fourth topology the voltage gain is raised with lowering the transformer turn ratio that lead to lower leakage inductance. Also, two phase type of these converters have been introduced that can operate in buck-boost mode when positive or negative output voltage is required. The proposed converters can be good candidate in DC motor drive systems or as bidirectional DC/DC converters in renewable energy systems. Experimental test under closed-loop control system is a subject for future studies.

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