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Abstract

As the increasing of power electronics based systems, not only the component-based analysis but also a systematic interaction analysis with each other systems are being important. Especially studies about the stability and the harmonics interaction are critical in order to establish the required performance in a power network. However, the traditional small-signal modeling approaches are not enough to represent the complexity of the interaction due to the time varying components of such systems. This paper compares the small-signal modeling methods for harmonic analysis of AC - DC converters and discusses their advantages and limitations. The Harmonic State Space (HSS) model based on the Linear Time-Periodically varying (LTP) system and the traditional small signal model based on the Linear Time-Invariant (LTI) system are evaluated. The results show that the HSS model presents superior performance for the AC-DC converters with a low ratio of switching to the fundamental frequency, and it provides an effective way to reveal the harmonic interaction and stability analysis for the future power electronics based power systems.

1. Introduction

The harmonic coupling and the overall system stability, which are challenged by the increasing use of power electronic devices in power systems, are becoming important issues. A large number of trains in Switzerland were shut down by protection relay due to high harmonic current [2]. The results were announced that it was due to an unexpected harmonic instability, which is not taken into account in the original analysis and design. Besides that, the harmonic instability had also happened in the SVC, HVDC system [4] for several decades. Lastly, a large offshore wind farm is having a problem because of unexpected harmonic resonance caused by the topology of the cable collection network and different operating points of each wind turbine converter [5]. Hence, the importance of harmonic instability as well as the unexpected operation of the power converter caused harmonics is being more significant because of the complexity of future power system having a large share of the power converters. Originally, the interaction between converter systems, the cable, and the analysis of network in terms of stability and harmonics are mainly studied by the classical power system studies. However, such studies are becoming almost mandatory in the power electronics field, i.e micro-grid, smart grid, renewable energy, wind farms etc, in order to make a cost-effective product as well as to keep a renewable system stable.

Hence, in order to know how harmonics affect to the power electronics based systems, it is also required looking into which harmonic sources are mainly influencing to the output harmonics of power converters. If it is assumed that the harmonics generated by the high switching frequency are eliminated by AC-DC converter filters, the main source of the low-order harmonics is non-linear components like transformer, inductor, and other magnetic devices. Even though these components are designed and selected under some design guide line in order to operate at a linear region, the saturation or the operation at non-linear operating ranges may happen, frequently. Besides, the non-linear loads composed by diodes or thyristors are also one of the largest harmonic sources to the AC-DC converter because of their grid dependent switching. Additionally, a voltage source converter having a low pulse ratio (e.g. 450 Hz Carrier / 900 Hz sampling frequency with 50 Hz fundamental) can bring an amplified voltage and current harmonics in control loop [6]. Lastly, the unbalance of the three phase system is also one of the harmonic sources which should be taken into account. These situations can be found in the wind power system because of unbalanced condition of transmission system and different operation trajectories of the converter [7]. However, they have difficulties in terms of harmonics analysis since parts of these harmonic sources are non-linear and others are linear. Even though various kinds of modeling and simulation methods are developed for several decades, they have their own advantages and disadvantages by themselves according to the purpose of usage. Conclusively, the optimized modeling methods for harmonics analysis are needed to represent all harmonics in the model as well as to explain their phenomenon, simultaneously.

This paper presents a comparative analysis of modeling methods of AC-DC converter for steady-state harmonic coupling analysis and harmonic stability studies. Three modeling methods, namely, state-space averaging, generalized averaging and harmonic domain based model, for power electronics systems are compared in terms of advantage and discuss the limitation for harmonic studies. Besides that the possibility of the analysis is also discussed not only in the steady state harmonic coupling situation but also in the dynamic stability analysis based on impedance based analysis method.

2. Modeling Method for Power Converters

This section will provide a comprehensive comparison of the principal modeling methods in terms of the harmonic analysis in power converters. This section will also introduce the advantages and limitations of those methods in terms of harmonic analysis capabilities. For the time being, the widely used state-space variable averaging modeling method [8, 9] will be analyzed. After this, the generalized average model and the dynamic phasor model will be compared as these methods are proposed to enhance the validity of state-space variable averaging modeling method. Finally, the HD, EHD, DHD and HSS modeling methods will be compared to demonstrate the effectiveness of the modeling in terms of the harmonic modeling, stability analysis and also the accuracy of the time-domain waveforms.

2.1. State-space averaging modeling

The state-space variable averaging model is originally proposed in order to simplify the dc-dc converter characteristics in terms of the analysis of the input and output characteristic. In this process, a moving average filter in (1) is used to eliminate the switching ripple component over one switching period. Even though the filter eliminates the switching ripple terms under the assumption the switching frequency is high, the results are non-linear large signal components [8].

$$\bar{x}(t) = \frac{1}{T}\int_{t-T}^{t} x(s)ds \tag{1}$$

Hence, in order to linearize this non-linear characteristic at the dc operating point, a small signal perturbation is injected to excite the small variation of output signals under the assumption that the Taylor series is valid. Based on this approximation, it is assumed in the modeling procedure that the small signal component of the PWM is a continuous fixed duty ratio modulation. As a result, the critical signal performance can be represented into a transfer function in order to derive the frequency characteristics of the input and the output. This can make it possible to use a more general analysis method, such as Bode diagram and Nyquist plot.

For instance, the basic grid connected converter with an *LCL* filter topology using the state space averaging and small signal method were used to analyze the harmonic interaction as well as the overall stability [10, 11]. A block diagram of a 3-phase grid connected converter is shown as Fig. 1-(a). Even though the switching sequence function of the converter has a non-linear characteristic, the operation characteristics of the frequencies lower than the switching frequency can be modeled by an average model [12]. By supposing a constant dc voltage and a fixed operating point, the switching sequence of the converter can be represented as a unity gain (G_{inv}), which is composed by the magnitude of dc-link voltage (V_{dc}), the carrier (V_{pwm}) and a PWM delay function (G_d) as shown in Fig. 1-(b). Using these assumptions, the transfer function block diagram can be derived as shown in Fig. 1-(c). This transfer function can be represented into a Norton equivalent circuit model to analyze the interaction and harmonic stability between the input and output admittance [13] as shown in Fig. 1-(d). However, the state-space variable averaging modeling can not be used properly in the case of a low switching frequency pulse ratio and the state variable dependent switching, e.g in the case of using a diode, thyristor, as the large signal terms in the switching frequency are linearized like a moving average filter. The main features of the state space averaging modeling can be summarized as follows.

- This modeling method is strong to represent the performance of DC-DC converters having a high switching frequency. This is enough to represent the low order harmonic under the assumption, where there is no coupling between switching and the low order harmonics since most switching harmonics are eliminated by filter.
- However, the possibility of interaction caused by the switching frequency in parallel connected inverters should be taken into account. Besides, it is found that the generalized averaging model that includes the harmonic components in the averaging model can make the model more accurate.
- Even though there are several proposals to represent the non-linear characteristic in the state-space averaging model [14, 15], there are still model mismatches because of the time varying operation trajectories. The model can not be used properly for the analysis of low switching frequency. Hence, there is limitation to be combined with other power converters having a low switching frequency in a large system.

2.2. Generalized averaging and dynamic phasor modeling

The state space averaging method can make the model simple since this method averages the switching ripple as well as simplify the information of the dc-circuit. However, it restricts the application to specific applications operating at high switching frequency and having a small ripple in a fixed operating point. In order to deal with the performance limitation of the state space averaging based model, the generalized averaging model and the dynamic phasor model are proposed and given by the following Fourier series based equations.

$$\langle X\rangle_k(\mathrm{t}) = \frac{1}{\mathrm{T}}\int_0^T x(t-T-\tau)e^{-jk\omega_s(t-T+\tau)}d\tau \quad \text{(2-(a))}$$

$$\mathrm{x(t-T+\tau)} = \sum_k \langle X\rangle_k(t)e^{jk\omega_s(t-T+\tau)} \quad \text{(2-(b))}$$

The Fourier coefficient has more possibilities in terms of the representation of whole signals instead of

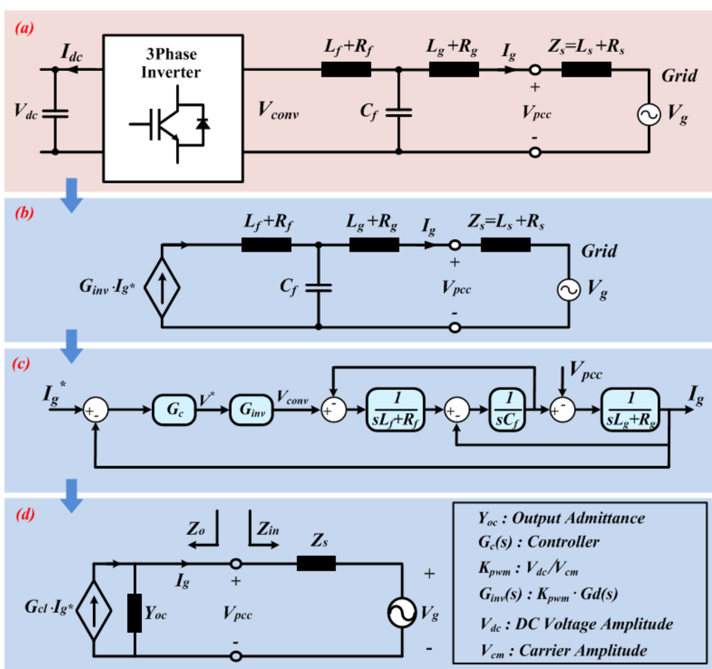


Fig. 1. Procedure diagram for state-space averaging method (L_f=converter side filter inductor, R_f=converter side filter inductor-parasitic resistance, L_g=grid side filter inductor, R_g= grid side filter inductor-parasitic resistance, C_f=filter capacitor, Z_s=grid impedance) (a) Circuit diagram of 3 phase grid connected inverter with grid impedance, (b) Circuit diagram of the state space averaging model, (c) Block diagram of the small signal model, (d) Block diagram of the impedance based model.

using a DC signal, if that signal has a periodic property (ω_s). Hence, in order to handle the entire signal property compared to the state-space averaging model, the Fourier series are adopted. If an arbitrary signal (*x(t)*) is decomposed by a Fourier series in the periodic range (*T*), each Fourier coefficient can be obtained. Also, the original signal can be restored.

First (2) is obtained from the same assumption, where the high switching components are averaged using a moving average filter (1).

$$\frac{\mathrm{d}}{\mathrm{dt}}x(t) = f\big(x(t), u(t)\big) \tag{3}$$

where, x(t), u(t) are the state variable and input signal. At this point in order to deal with the harmonic components, the Fourier coefficient vector (2) can be substituted directly into (3). Then, the time domain equation having a Fourier coefficient can be taken as given in (4), where $\langle d/dt\, X\rangle_k, \langle f(x,u)\rangle_k$ are the derivative terms of the state vector Fourier coefficient and the input-state vector Fourier coefficient.

$$\left\langle \frac{d}{dt}x\right\rangle_k = \langle f(x,u)\rangle_k \tag{4}$$

It is required to unfold (4) in order to solve it. Hence, it is needed to get the information about the differentiation terms. If the state variable vector $\langle X\rangle_k$is differentiated in the time domain, the differentiated state variable vector ($\langle d/dt\, x\rangle_k$) and complex vector information ($jk\omega_s\langle x\rangle_k$), which is obtained from exponential term of the Fourier coefficients, can be obtained as shown in (5)

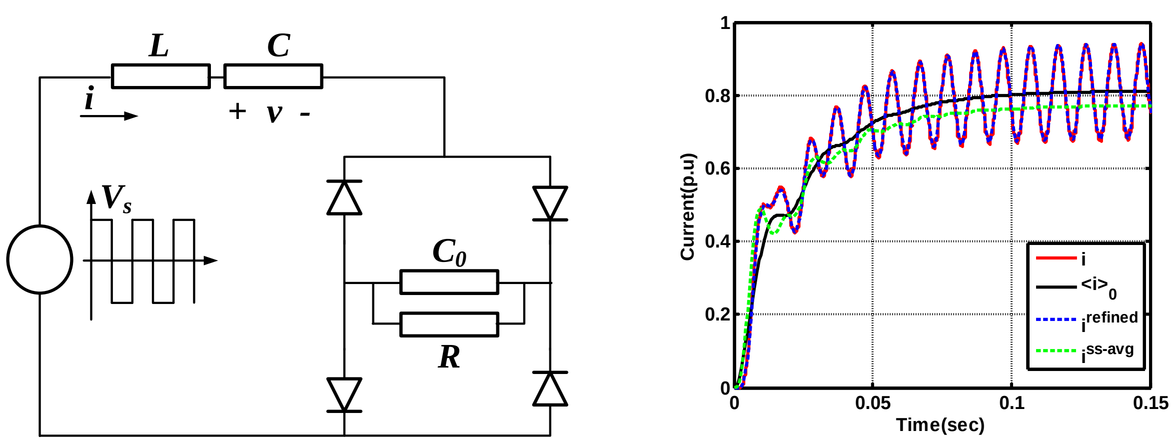
$$\frac{\mathrm{d}}{\mathrm{dt}}\langle X\rangle_k(t) = \left\langle \frac{d}{dt}x\right\rangle_k(t) - jk\omega_s\langle x\rangle_k(t) \tag{5}$$

From the obtained differentiation information, (6) can be calculated by substituting (2) into (5). This procedure is also directly used in the output state space equation (*y(t)=g(x,u)*).

$$\frac{\mathrm{d}}{\mathrm{dt}}\langle X\rangle_k = -jk\,\langle X\rangle_k + \langle f(x,u)\rangle_k \tag{6}$$

In (6), note that it can bring a similar result with the conventional state-space variable averaging modeling method, when the index of harmonic order k is equal to "0". It means that the only dc component is considered in the modeling. However, if the dc terms are not enough to derive the modeling results, it is possible to consider other harmonic components. This selection is only dependent on the most significant harmonic components. Simulation results are shown in Fig 2, where the switching frequency is a significant harmonic order.

If multi harmonic components should be considered, the equation becomes non-linear. The number of state variable vectors and the number of input can be different. Hence, the procedure for linearization is essentially required to make it as a linearized model [3]. The describing function can be used to linearize the non-linear equation. However, that means the advanced modeling method is also using the same



(a) (b)

Fig. 2. (a) Circuit diagram of DC-DC resonant converter (b) Comparison of generalized averaging model ($i^{refined}$) with traditional state-space averaged model ($i^{ss\text{-}avg}$) and non-linear time domain model (i) [3]

assumption as in the case with the describing function [16]. It should be enough to be used in the modeling of several power converter applications for the purpose of control. However, this modeling method has also some limitations in order to analyze the harmonic components and to figure out the relationship between the harmonic components. The main features of the generalized averaging method can be summarized as follows:

- In terms of modeling accuracy, the generalized averaging model can show better results compared to the state space averaging model as shown in Fig. 2. Hence, by including a harmonic term can derive more accurate modeling results can be derived.
- This method can be applied to the modeling of diode / thyristor applications [17] as well as the single phase applications in order to achieve similar results with non-linear time domain simulations.
- This method can also be applied for the stability analysis, which can not be modelled by the traditional state-averaging model. In [18, 19], it is verified that the stable region of the converter calculated by the state space-averaging model is not accurate because of the existence of other harmonics in the systems. However, except for the critically significant harmonic components, the other harmonic components are linearized by means of a describing function. Therefore the possibility of harmonic coupling modeling is not possible.
- The generalized averaging model (≈Dynamic Phasor) can not be applied to the modeling of non-linear passive components. Hence, there are limitations to explain the harmonics generated by the non-linear passive components.

2.3. Harmonic domain (HD) and Harmonic state space (HSS) modeling

In order to obtain an accurate model and to analyze harmonic coupling, the Harmonic Domain (HD) based models are introduced. If the overall signals are linear and periodic, the components in the model can be converted into the periodic signals. In terms of steady state analysis, the harmonic domain is enough to investigate the steady state harmonic components and the coupling in the system, where every periodic signal can be represented into a time-varying Fourier series expansion. Hence the combination of each Fourier series matrix brings the same results with time domain response. Furthermore, it is possible to analyze the steady state harmonic interaction in HD. However, when this approach is compared to the traditional modeling methods, this method has a limitation in the transient analysis. In order to solve this problem, the Harmonic State Space (HSS) modeling is a more useful solution, where the Exponentially Modulated Periodic (EMP) signal can be used as a kernel function in HD as like “s” in the Laplace domain. As a result, it can make it possible to derive the time domain transient response of each harmonic component, where the summation of all transient harmonic components is equal to the response of the time domain simulation method. Consequently, the HSS model based on LTP theory can be compared with LTI model, where the conventional approaches are based on the LTI theory.
However, sometimes the numerical iteration method e.g Newton Raphson could be required for the three methods in the modelling process [20] in order to achieve a harmonic impedance matrix of non-linear passive components. Besides, the analysis results depend on the number of harmonic components and also the selection of the harmonic components. To adapt the theoretical proof into the practical power electronics based systems, the overall procedure to derive the HSS modeling is explained in the following. First, the state variable, state transition matrix and the other related input output matrixes are needed to be changed into periodically time varying signals as given in (7).

$$\begin{aligned} \dot{x}(t) &= A(t)x(t) + B(t)u(t) \\ y(t) &= C(t)x(t) + D(t)u(t) \end{aligned} \tag{7}$$

The main theorem of the HSS modeling starts from a periodic signal, which can be represented in the time interval $[t_0, t_0 + T]$ by its Fourier series.

$$x(t) = \sum_{k \in \mathbb{Z}} X_k e^{jk\omega_0 t} \tag{8}$$

where each Fourier coefficient can be calculated by

$$\mathrm{X}_k(t) = \frac{1}{T}\int_{t_0}^{t_0+T} x(t) e^{-jk\omega_0 t} dt$$

where $\omega_0 = \frac{2\pi}{T}$ and $t \in [t_0, t_0 + T]$

After then, it can be written as a matrix

$$x(t) = \Gamma(t)X$$

where,

$$\Gamma(t) = [e^{-jh\omega_0 t} \cdots e^{-j2\omega_0 t}, e^{-j\omega_0 t}, 1, e^{j\omega_0 t}, e^{j2\omega_0 t} \cdots e^{jh\omega_0 t}]$$
$$X = [X_{-h}(t) \cdots X_{-1}(t) X_0(t) X_1(t) \cdots X_h(t)]^{\mathrm{T}}$$

Based on the basic representation of the EMP signal characteristics [21], it is also possible to derive various mathematical expressions like a derivative, integral and the product of two signals in order to make a time varying differential equation of the power converter. The derivative of the time varying signal ($x(\mathrm{t}) = \Gamma(t)X$) is

$$\dot{x}(t) = \dot{\Gamma}(t)X + \Gamma(t)\dot{X} \tag{9}$$

From the principle of these kinds of transformation, the state space equation, which has a time varying state transition matrix and a time varying state variable, can be represented as the product of the matrix as

$$\begin{aligned}(\mathrm{s} + \mathrm{jm}\omega_0)\mathrm{X_n} &= \textstyle\sum_{-\infty}^{\infty} A_{n-m}X_m + \sum_{-\infty}^{\infty} B_{n-m}U_m \\ Y_n &= \textstyle\sum_{-\infty}^{\infty} C_{n-m}X_m + \sum_{-\infty}^{\infty} D_{n-m}U_m\end{aligned} \tag{10}$$

In the s-domain, it can also be represented as

$$\begin{aligned}\mathrm{sX} &= (\mathrm{A} - N)X + BU \\ \mathrm{Y} &= \mathrm{C}X + DU\end{aligned} \tag{11}$$

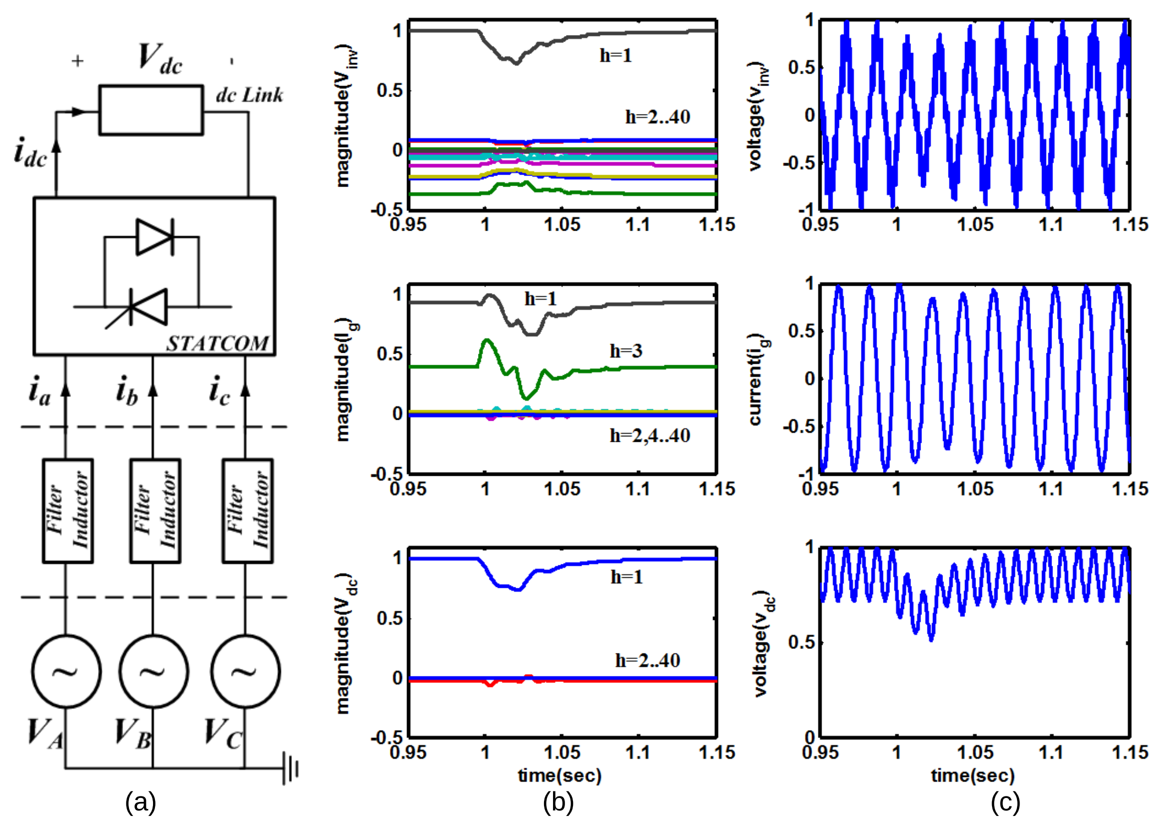


Fig. 3. STATCOM operation (a) Circuit diagram (b) Dynamic harmonics response of STATCOM Voltage and Current using EHD / DHD (c) time domain simulations using EHD / DHD method [1], where "h" is harmonic order (h=1 means 1st order harmonic, h=2..40 means that harmonic order starts from 2nd order to 40th order.).

where, $\mathrm{N}(= \mathrm{diag}(\mathrm{jm}\omega_0))$ is a matrix for dynamic transition of each Fourier coefficient using the state-space equation. From (7) ~(11), note that each state variable and state transition equation are a time varying matrix which means the harmonic frequency components decomposed by the Fourier series can be varied according to time but new periodically. The structure of the matrix will make it possible to analyze the harmonic coupling and interaction as each signal is already decomposed into a harmonic component in the time domain. It will also result in an output, which has decomposed harmonic components.

On the contrary to the other modeling method, the nonlinear components and switching components, which are regarded as time varying systems with a periodic excitation, can also be linearized around the nominal frequency 50 Hz or 60 Hz signals. This is the linear time periodic model, which is called as the Harmonic State Space (HSS) model. As an example, the Extended Harmonic Domain (EHD) / Dynamic Harmonic Domain (DHD) method are used in a STATCOM application in order to analyze the dynamics of the harmonic propagation as shown in Fig. 3. The modeling results show that each harmonic order has its own response according to the characteristic of the harmonic transfer function, where it is possible to investigate, which harmonics are coupled together in the harmonic transfer matrix. As a result, the summation of the harmonics responses in Fig. 3-(b) can derive same results with time domain simulation in Fig. 3-(c) by rotating harmonic vector with same frequency. The main feature of the HD based modeling is summarized as follows:

- The model can describe the relationship between AC and DC with transfer function. This shows exactly which harmonic components are coupled to each other in a single domain.
- It is compatible to be used in the analysis of multiple connected converters as well as large power electronics based network since this model is based on the state-space equation composed by matrixes.
- In, the non-linear passive components are modeled using a HD based modeling approach in order to analyze harmonics caused by the saturation of an inductor or a transformer in the network. This approach can directly be used in the dynamic simulation by considering the non-linear effects [1].

Table I. Performance comparison of harmonic analysis

	State Space Averaging	**Generalized Average (=Dynamic Phasor)**	**Harmonic State Space (= Harmonic Domain Base)**
Including Switching	Averaging	Averaging	O
Availability of a small signal analysis	O	O	O
AC-DC Harmonic coupling analysis	X	X	O
Harmonic unbalance analysis	X	X	O
Availability of including a nonlinear passive component	X	Δ	O
Modularity for a large network harmonic analysis	X	Δ	O
Purpose	Stability analysis, Control design	Stability analysis, Control design (with relatively large harmonics)	Stability analysis, Control design, Harmonic analysis
Main mathematics	Taylor series	Fourier series	Fourier / Taylor series
Model complexity	Low	High	High

O= possible, X=impossible, Δ=partially possible

- The different impedance profiles can be reflected into the HD based model. Hence, the model is also applicable for the analysis of unbalanced harmonics in 3-phase systems [22].
- By changing the base harmonic, the method can analyze the inter-harmonic coupling with same approaches [23].
- Even though this is a frequency domain model, the HSS, LTP model has various harmonic impedances inside the model. Hence, fast time domain simulations are also possible by means of convolution and rotation of a frequency vector at a specific frequency [24].

3. Summary and Conclusion

A summarizing table for each modeling method is shown in Table I. Compared to the traditional state-space averaging method using other approach, it is worth to note that time varying elements as well as the harmonics are important in order to derive an accurate model. The generalized averaging method is introduced to cover the possibilities of other harmonic components. This model is being the basis for other modeling approaches adopting a Fourier coefficient. However, this model has also a limitation, as it can only include relatively large harmonics and neglect the others by means of a describing function. Besides, the harmonic terms are derived from time-dependent window. This can demand a re-calculation of the model. In power system studies, the harmonic domain method is developed in order to be used in the analysis of harmonic coupling in a large network. Besides the HSS, EHD, DHD are developed based on the theory of LTP system. The model shows the possibilities of harmonic impedance as well as the coupling characteristic, which harmonics are coupled together. Besides, it can also be used in the impedance based analysis for small signal analysis. Furthermore, it can be combined with the harmonic impedance matrixes from other converters, loads, and non-linear components. Lastly, it can also be used for the time-domain simulation using a convolution procedure. Even though the HSS approach has also difficulties in the calculation procedure due to the size of matrix, it can show in a more advanced way for the harmonic analysis and identify harmonic instability problems.

4. Acknowledgement

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