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Fatigue Load Modeling and Control for Wind Turbines based on Hysteresis Operators

J.J. Barradas Berglind, Rafael Wisniewski and Mohsen Soltani

Abstract— The focus of this work is on fatigue load modeling and controller design for the wind turbine level. The main purpose is to include a model of the damage effects caused by the fatigue of the wind turbine components in the controller design process. This paper addresses an online fatigue estimation method based on hysteresis operators, which can be used in control loops. Furthermore, we propose a model predictive control (MPC) strategy that incorporates the online fatigue estimation through the objective function, where the ultimate goal in mind is to reduce the fatigue load of the wind turbine, while tracking a desired power reference. The outcome is an adaptive or self-tuning MPC scheme for wind turbine fatigue load reduction. The results of the proposed strategy are then compared against a baseline MPC.

I. Introduction

Fatigue damage in general terms can be understood as the weakening or breakdown of a material subject to stress, especially a repeated series of stresses. From a materials perspective, it can also be thought of as the (elastoplastic) deformations that cause damage on a certain material or structure, compromising its integrity as a result. In other words, fatigue is a phenomenon that occurs on a microscopic scale, manifesting itself as deterioration or damage in a component or structure. Therefore, it has been of great interest in different fields, and has been studied extensively, see [1] for a very detailed history of fatigue. It could be argued that two major turning points on the history of fatigue came firstly with the contributions of Wöhler [2], who as early as 1860 suggested the design for the finite fatigue life and the so-called Wöhler curve (or S-N curve of stress versus number of cycles to failure); and secondly, with the linear damage accumulation rule by Palmgren [3] and Miner [4], which still forms the basis for theoretical damage calculation.

Motivation. Perhaps the most recognized and used measure of fatigue is the so-called rainflow counting (RFC) method. It was first introduced by Endo [5], and its complex sequential structure decomposes arbitrary sequences of loads into cycles; its name comes from an analogy with roofs collecting rainwater, used to explain the algorithm. Typically, to compute a lifetime estimate from a given structural stress input, the RFC method is applied by counting cycles and extrema, followed by the Palmgren-Miner rule [3], [4] to calculate the expected damage. However, the RFC method is a very nonlinear numerical algorithm and not a mathematical

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function; thus it can only be used as a post-processing tool or, in other words, it is always performed offline. Hence, it is not possible to use RFC for real-time control since it requires a time series of stress and not only instantaneous measurements, as it is the case in control loops. Alternatives to the RFC method are the usage of stochastic processes [6] and the so-called frequency domain approximations or spectral methods [7]. As mentioned in [8], the purpose of the RFC method is to identify the closed hysteresis loops in the stress signals. In [9] an equivalence between symmetric RFC and a particular hysteresis operator is provided, allowing then to incorporate a fatigue estimator online within the control loop, in contrast to the RFC case; this is sketched in Fig. 1.
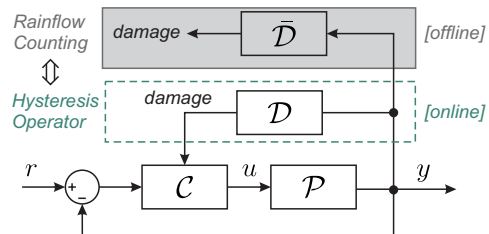

Fig. 1. Damage calculation comparison between RFC ($\bar{\mathcal{D}}$) and Hysteresis operator ($\mathcal{D}$) in a control loop.

Contribution. The fatigue estimation method presented here is based on a hysteresis operator, leaning on the equivalence between symmetric RFC and Preisach operator given in [9], taking a wind turbine control perspective. The intention of this paper is to consider the incurred damage in the turbine components introduced by varying wind load; control might modify the damaging load in this case by suitably adjusting the pitch angle or the generator torque. The approach taken in this paper, gives an estimated fatigue model which: (1) can be used in real-time control and is not limited by computational power, and (2) gives a closed mathematical formulation of fatigue that can be used in control design and optimization. The contributions of the present work are twofold: firstly, to show a fatigue estimation method based on hysteresis operators that can be implemented online for control purposes; and secondly, to design and synthesize an automatic tuning mechanism for the weighting matrices of a wind turbine fatigue load reduction predictive controller.

Related work. In the wind turbine control context, current control methods are based on minimization of certain norms of the stress on different components of the wind turbine, which are hoped to reduce fatigue, but are not a trustful characterization of the damage [10], [11]. Other

approaches, such as taking the variance of the stress are not a direct representation of fatigue, as mentioned in [12]. In [12], controllers for a single wind turbine were designed by approximating fatigue load with an analytical function based on spectral moments, where it is assumed that the stress is an output from a given linear system with Gaussian white noise input; measurements are not used, and thus the method is not directly applicable for online fatigue estimation. In [13] and [14] loading reductions are achieved by controlling the pitch of each blade independently. In [15], an algorithm for wind farm control is presented, which aims at optimizing power production and reducing structural loads. Since the fatigue estimation method proposed here involves a hysteretic element, control problems with hysteresis come into play. Optimal control problems with hysteresis were studied in [16] using necessary conditions for Pontryagin's extremum principle. In [17], the dynamic programming equations for systems with hysteresis on the control input were introduced, and in [18] optimization problems for scalar discrete time systems with Preisach hysteresis are considered.

Outline. The remainder of the paper is organized as follows: In Section II damage calculation with hysteresis operators is addressed, together with some properties of a particular hysteresis operator. Subsequently, Section III elaborates on the wind turbine model to be used in a control scheme, which is designed and implemented in Section IV. Simulation results are presented in Section V to illustrate the proposed control strategy. Lastly, conclusions are given in Section VI.

II. DAMAGE CALCULATION VIA HYSTERESIS OPERATORS

The purpose of the RFC algorithm is to identify the closed hysteresis loops in the stress signals [8]; as explained in [9], if one associates values to individual cycles or hysteresis loops, one assumes that the underlying process is rate independent. The previous implies that only the hysteresis loops themselves are important, but not the speed with which they are traversed, so in the case of damage, it does not matter how fast the stress occurs but its magnitude. Rate independent processes are mathematically formalized as hysteresis operators, for details see [19], [20], [21]. The result we will lean on, is the equivalence provided in [9], [21] between symmetric RFC and a Preisach hysteresis operator.

A. Damage Calculation Equivalence

Firstly, the notion of string is introduced as a way to model the stresses for damage calculation.

Definition 1 (Strings): Let $s = (v_0, \cdots, v_N) \in S$ be a given string, which represents an arbitrary load sequence. Let S be the space of finite sequences in $\mathbb{R}$, i.e., $S = \{(v_0, v_1, \cdots, v_N) : N \in \mathbb{N}_0, v_i \in \mathbb{R}, 0 \le i \le N\}$, $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$.

Following the interpretation of RFC given in [21], we intorduce $\mathcal{N}(\mu, \tau)$ with values μ and τ, chosen such that the input string s with $v_{2k} = \mu$ and $v_{2k+1} = \tau$ for $k \in \mathbb{N}_0$, destroys the specimen after $\mathcal{N}(\mu, \tau) = \tilde{\mathcal{N}}(|\tau - \mu|)$ cycles; the resulting curve is the so called Wöhler curve since the ansatz

$$\tilde{\mathcal{N}}(\mu, \tau) = \kappa_1 |\mu - \tau|^{\kappa_2} \quad (1)$$

exhibits a straight line in a log-log scale, where κ_1 and κ_2 are scaling constants and $|\mu - \tau|$ is a given stress range. Subsequently, the Palmgren-Miner rule is used to identify and count cycles for an arbitrary load sequence $s \in S$, such that the damage accumulation is obtained as

$$D_{ac}(s) := \sum_{\mu < \tau} \frac{c_{per}(s)(\mu, \tau)}{\mathcal{N}(\mu, \tau)}, \quad (2)$$

where $c_{per}(s)(\mu, \tau)$ is the rainflow count associated with a fixed string s, counting between the values of μ and τ.

Before presenting the equivalence between RFC and the Hysteresis method, the basic Relay operator, Variation, and Preisach Hysteresis operators will be introduced.

Definition 2 (Relay Hysteresis Operator): Let $\mu, \tau \in \mathbb{R}$ with $\mu < \tau$ and $w_{-1} \in \{0, 1\}$ be given. We define the Relay operator $\mathcal{R}_{\mu,\tau} : S \to S$ by

$$\mathcal{R}_{\mu,\tau}(v_0, \cdots, v_N) = (w_0, \cdots, w_N), \quad (3)$$

with
$$w_i = \begin{cases} 1, & v_i \ge \tau, \\ 0, & v_i \le \mu, \\ w_{i-1}, & \mu < v_i < \tau. \end{cases}$$

Definition 3 (Variation): For any $s = (v_0, \cdots, v_N) \in S$, we define its variation $\mathrm{Var} : s \to \mathbb{R}$ by

$$\mathrm{Var}(s) = \sum_{i=0}^{N-1} |v_{i+1} - v_i|. \quad (4)$$

Within the context of fatigue analysis, there is no reason to consider arbitrarily large input values. Consequently, the relevant threshold values for the relays $\mathcal{R}_{\mu,\tau}$ then lie within the triangle

$$P = \left\{ (\mu, \tau) \in \mathbb{R}^2, -M \le \mu \le \tau \le M \right\}, \quad (5)$$

known as the Preisach plane, where M is an a priori bound for admissible input values.

Definition 4 (Preisach Hysteresis Operator): Let the density function ρ with compact support in P, i.e., set to zero outside the triangle P, be given. We define the Preisach operator $\mathcal{W} : S \to S$ as

$$\mathcal{W}(s) = \int_{\mu < \tau} \rho(\mu, \tau) \mathcal{R}_{\mu,\tau}(s) d\mu d\tau. \quad (6)$$

Here, the integral is understood to be componentwise with respect to the elements of the string $\mathcal{R}_{\mu,\tau}(s)$.

Then for each sequence of stresses $s = (v_0, \cdots, v_N) \in S$ with $\|s\|_\infty \le M$ and $v_0 = v_N$ the total damage $D_{ac}(s)$ associated to s satisfies

$$D_{ac}(s) = \sum_{\mu < \tau} \frac{c_{per}(s)(\mu, \tau)}{\mathcal{N}(\mu, \tau)} = \mathrm{Var}(\mathcal{W}^{per}(s)). \quad (7)$$

The left-hand side of (7) amounts to symmetric RFC, such that $\mathcal{N}(\mu, \tau)$ denotes the number of times a repetition of the input cycle (μ, τ) leads to failure. The right-hand side of (7) corresponds to the variation of the periodic Preisach operator (the periodic version of (6)) and its density function ρ is a function of $\mathcal{N}(\mu, \tau)$. The role of the periodic operator is to include the rainflow residual, for details refer to Theorem
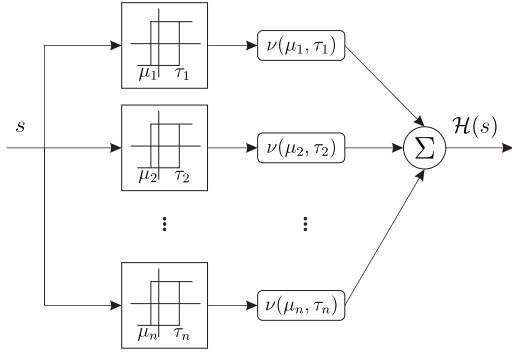
Fig. 2. The discretized Preisach operator.

Fig. 3.   Discretization of the Preisach plane with a) $L = 2$ and b) $L = 3$.

2.12.6 in [21]. The interpretation of this result is that the RFC method counts the number of oscillations at each range of amplitude, and this is precisely what $\mathrm{Var}(\mathcal{W}^{per}(s))$ represents, i.e., the oscillations of s between μ and τ.

B. Approximation by Discretization

The Preisach operator in (6) can be discretized or approximated by a weighted sum of relay hysterons, i.e., $\mathcal{H} = \sum_i \nu(\mu_i, \tau_i) \mathcal{R}(\mu_i, \tau_i)$, for $i \in \mathbb{N}$, as described in [20], resulting in a weighted sum of $L(L+1)/2$ relays, where L is called the discretization level. This follows the reasoning that (6) could be thought of as a weighted superposition of relays, and since the integral is restricted to $\mu < \tau$ then the relays should lie inside the triangle P. The approximation is depicted in Fig. 2, where every relay has an individual weighting factor $\nu(\mu_i, \tau_i)$. Fig. 3 shows the Preisach plane P for two cases of discretization level: a) $L = 2$, which amounts to 3 relays; and b) $L = 3$, giving rise to 6 relays. This corresponds to a uniform discretization and it is assumed that the density distribution inside each cell is concentrated at the center, shown as small blue circles.

C. Preisach Density Function

In order to use the Preisach operator, its density or weighting function $\rho(\mu, \tau)$ needs to be known in general. An identification procedure and a summary of other identification methods can be found in [22]. As addressed in [23], it is also possible to obtain this density function through estimation techniques, for the case of linear systems preceded by Preisach hysteresis. Moreover, while discretizing the Preisach operator, $\rho(\mu, \tau)$ is captured by the weightings on each relay $\nu(\mu_i, \tau_i)$. In other words, the density ρ might be thought of as a variable gain depending on μ and τ.

III. Wind Turbine Model

A. Wind Turbine Model

The plant model $\mathcal{P}$ to be controlled is based on the standard NREL 5MW wind turbine [24]. The wind turbine model to be used assumes that the gearbox is perfectly stiff, while transferring deformations on a low-speed shaft. The low-speed shaft is modeled by a rotational moment of inertia, and a viscously damped rotational spring. The inertia J_r represe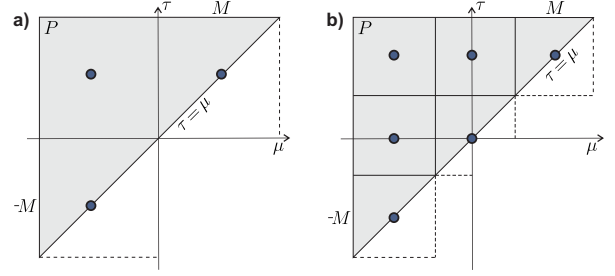nts the inertia of the rotor and shaft. Stiffness and damping of the drive train are combined into one spring and one damper on the rotor side with coefficients K_θ and B_θ, respectively. The rotational moment of inertia in the generator side, J_g, represents the collective inertias of the high-speed shaft, the gearbox, and the rotor of the generator. In this model θ stands for the shaft torsion, ω_g corresponds to the generator angular velocity, and ω_r to the rotor angular velocity. From this model, the following set of differential equations are derived:

$$J_r \dot{\omega}_r = T_r - K_\theta \theta - B_\theta \dot{\theta} \tag{8}$$

$$J_g \dot{\omega}_g = -T_g + \frac{K_\theta}{N_g} \theta + \frac{B_\theta}{N_g} \dot{\theta} \tag{9}$$

$$\dot{\theta} = \omega_r - \frac{\omega_g}{N_g}, \tag{10}$$

where T_g is the generator torque, and N_g is the gear ratio. Additionally, the aerodynamic rotor torque T_r is given by

$$T_r(\beta, v_r, \omega_r) = \frac{\pi}{2} \rho_a R_r^2 \frac{v_r^3}{\omega_r} C_P(\lambda(\omega_r, v_r), \beta), \tag{11}$$

where R_r is the radius of the rotor, ρ_a is the air density, v_r is the effective wind speed, and C_P represents the aerodynamic efficiency in terms of the collective blade pitch angle β and the tip speed ratio λ, that is a rational function defined as $\lambda(\omega_r, v_r) := R_r \omega_r / v_r$. Thrust force will be transferred to the tower top through the nacelle, resulting in tower fore-aft motion. It is possible to simplify the tower fore-aft dynamics by the second order differential equation

$$M_t \ddot{y} + B_t \dot{y} + K_t y = F_t, \tag{12}$$

with y being the tower top displacement and M_t, B_t, and K_t being the identified mass, damping, and stiffness of the model; lastly F_t is the thrust force in the rotor given by

$$F_t(\beta, v_r, \omega_r) = \frac{\pi}{2} \rho_a R_r^2 v_r^2 C_T(\lambda(\omega_r, v_r), \beta), \tag{13}$$

where C_T represents the thrust coefficient in terms of the collective blade pitch angle β and the tip speed ratio λ. In the simulation model the functions $C_P(\lambda, \beta)$ and $C_T(\lambda, \beta)$ are implemented as look-up tables. The electrical power output is given as $P_{out} = \eta \omega_g T_g$, with η being the generator efficiency.

B. Linearization and Discretization

For controller design, we take $x = (\omega_g, \omega_r, \theta)$ as the vector of states, $u = (\beta, T_g)$ as the vector of the control inputs, and the average ambient wind speed on the rotor area as

a disturbance, i.e., $d = v_r$. Linearizing (8-11) around an operating point (x^*, u^*, d^*), we get the state space equation

$$\dot{\bar{x}} = A\bar{x} + B\bar{u} + E\bar{d}, \qquad (14)$$

where the deviation variables with respect to the chosen operating point are $\bar{x} = x - x^*$, $\bar{u} = u - u^*$, and $\bar{d} = d - d^*$. We will make use of the discretized version of (14),

$$\bar{x}_{k+1} = A_d\bar{x}_k + B_d\bar{u}_k + E_d\bar{d}_k. \qquad (15)$$

where $\bar{x}_k$ and $\bar{u}_k$ correspond to the vectors of states and control inputs, respectively, and $\bar{d}_k$ represents the wind disturbance. We will use $\bar{x}_k = (\bar{\omega}_{g|k}, \bar{\omega}_{r|k}, \bar{\theta}_k)$ in the sequel.

IV. Fatigue Load Model Predictive Control Strategy for Wind Turbines

In this Section we will propose a modified model predictive control (MPC) strategy that takes into account the fatigue load calculation method introduced in Section II. MPC is an optimization-based control technique widely used for controller design of complex systems involving constraints [25], [26]. The main idea behind MPC is to use the system dynamics to predict the state evolution, and together with a cost functional obtain a constrained optimal control problem. Solving this control problem at a given time step gives an optimal control sequence from which only the first is implemented; then, the same process is repeated for the subsequent steps. In the present we will use MPC since constraints can be handled directly; also because it has been successfully used for wind turbine control, and due to the afnity it has with the fatigue estimation technique proposed in the sequel.

A. Control Problem Formulation

Consider the baseline MPC strategy $\mathcal{C}_b$, where the following optimization problem is solved

Problem 1 (Baseline MPC strategy, $\mathcal{C}_b$):

$$\min_U J(\bar{u}_k, \bar{x}_k) = \sum_{k=0}^{N_p - 1} \bar{x}_k^\top Q \bar{x}_k + \sum_{k=0}^{N_u} \bar{u}_k^\top R \bar{u}_k + \bar{x}_{N_p}^\top Q_t \bar{x}_{N_p}$$

$$s.t. \begin{cases} \bar{x}_0 = x(t) - x^*, \\ \bar{x}_{k+1} = A_d\bar{x}_k + B_d\bar{u}_k, \text{ for } k = 0, 1, \ldots, N_p - 1, \\ \bar{u}_k = \bar{u}_{N_u}, \text{ for } N_u + 1 \leq k \leq N_p - 1, \\ u_{\min} \leq \bar{u}_k \leq u_{\max}, \text{ for } k = 0, 1, \ldots, N_u, \\ |\bar{u}_{k+1} - \bar{u}_k| \leq \Delta u_{\max}, \text{ for } k = 0, 1, \ldots, N_u - 1, \\ x_{\min} \leq \bar{x}_k \leq x_{\max}, \text{ for } k = 1, \ldots, N_p, \end{cases}$$

$$(16)$$

over $U := \{\bar{u}_0, \ldots, \bar{u}_{N_u}\}$, for prediction horizon $N_p \in \mathbb{N}$, and control horizon $N_u \in \mathbb{N}$, such that $N_p > N_u$. The problem optimization is parametrized with the current state measurement and subject to the discretized system dynamics, with state, input and slew rate constraints. The running cost on states and inputs is given by the weighting matrices $Q = Q^\top \succ 0$, $R = R^\top \succ 0$, and $Q_t = Q_t^\top \succ 0$, is used to weight the terminal state cost.

B. Parameter Identification for Damage Calculation

The goal is to control a wind turbine $\mathcal{P}$ reducing both power fluctuations and the incurred fatigue, see [10], [11], [27]. However, the inclusion of the fatigue damage given by (7) into (16) is not straightforward, due to the Preisach hysteresis operator. As explained in [28, ch.6], Preisach hysteresis operators involve discontinuities, non-smooth nonlinearities, and memory effects. Therefore, optimal control problems involving hysteresis are typically hard.

In order to facilitate this task, we propose a least-squares based parameter estimator $\mathcal{K}$ that will provide tuning parameters to a modified MPC strategy, with the discretized shaft torsion $\bar{\theta}_k : [0, T] \to \mathbb{R}$ as an input; the estimator $\mathcal{K}$ is depicted in Fig. 4. We introduce the operator $\tilde{\mathcal{F}}$ written as

$$\tilde{\mathcal{F}}(\bar{\theta}_k, \dot{\bar{\theta}}_k) = a\bar{\theta}_k^2 + g\dot{\bar{\theta}}_k^2, \qquad (17)$$

and using the substitution $\alpha_k \equiv \bar{\theta}_k^2$ and $\gamma_k \equiv \dot{\bar{\theta}}_k^2$, we define $\mathcal{F} : \mathbb{R}^2 \to \mathbb{R}$, $(\alpha_k, \gamma_k) \mapsto \mathcal{F}(\alpha_k, \gamma_k)$, such that

$$\mathcal{F}(\alpha_k, \gamma_k) = \begin{bmatrix} a & g \end{bmatrix} \begin{bmatrix} \alpha_k & \gamma_k \end{bmatrix}^\top, \qquad (18)$$

where a and g are the parameters to be estimated. The aim is to approximate the accumulated damage given by

$$h(\bar{\theta}_k) := \text{Var}(\mathcal{H}(\bar{\theta}_k)), \qquad (19)$$

which is an implementation of the equivalence described in (7) with $\mathcal{H}$ being a discretized Preisach operator, by

$$\hat{h}(\alpha_k, \gamma_k) := \sum_{k=0}^{T} \mathcal{F}(\alpha_k, \gamma_k). \qquad (20)$$

Notice that (20) is a discrete integration of $\mathcal{F}(\alpha_k, \gamma_k)$, which is needed to capture the behavior of the variation operator, such that the error $e(k) = h(k) - \hat{h}(k)$ is minimized in the least squares sense by minimizing the criterion

$$V_N(a, g) = \frac{1}{N} \sum_{k=1}^{N} \left\| h(k) - \hat{h}(k) \right\|^2. \qquad (21)$$

An interpretation of this approach is to consider $\bar{\theta}_k^2$ as the variance of the shaft torsion and $\dot{\bar{\theta}}_k^2$ as the variance of the shaft torsion first derivative. Hence, while estimating the damage in the shaft, not only the variance of the shaft torsion is considered, but also the variance of its velocity. This ansatz will be evaluated in the sequel, which is equivalent to approximate (19) by the variance of the torsion given by the 0th spectral moment, and the variance of the torsion's first derivative given by the 2nd spectral moment [12].

The estimation of the parameters a and g in the least-squares sense will be carried out using an ARX model [29] due to its simplicity, the small number of parameters to be estimated, and its consistency with the MPC formalism. Hence, we will use the one-input two-output ARX model

$$\tilde{A}(q^{-1})h(k) = \tilde{B}(q^{-1}) \begin{bmatrix} \alpha(k) & \gamma(k) \end{bmatrix}^\top + e(k)$$
$$= \mathcal{F}(\alpha_k, \gamma_k) + e(k), \qquad (22)$$

where q^{-1} is the backward shift operator: $q^{-1}h(k) = h(k-1)$, $\tilde{B}(q^{-1}) = \begin{bmatrix} a & g \end{bmatrix} q^0$ gives the desired coefficients, and
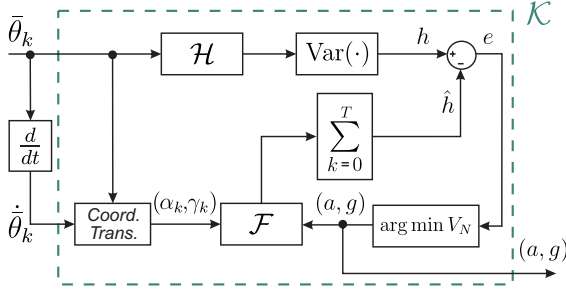
Fig. 4.   Fatigue parameter estimator $\mathcal{K}$.

the polynomial $\tilde{A}(q^{-1}) = 1 + \xi q^{-1}$ with $\xi \equiv -1$, accounts for the embedded discrete integrator introduced in (20). In Section V we will address two cases: varying coefficients $a(k)$ and $g(k)$ obtained online via recursive identification with a forgetting factor, and constant a and g coefficients identified a priori. Note that by adopting discrete time here we approximate the string setting introduced in Section II.

C. Redefinition of the Cost Functional

In order to incorporate the fatigue estimator $\mathcal{K}$ in the control loop, the baseline MPC strategy $\mathcal{C}_b$ in (16) needs to be modified. Consequently, we propose an augmented MPC controller with a redefined cost functional

$$\bar{J}(\bar{u}_k, \bar{x}_k) := J(\bar{u}_k, \bar{x}_k) + \sum_{k=0}^{N_p-1} \left(a\bar{\theta}_k^2 + g\dot{\bar{\theta}}_k^2 \right), \quad (23)$$

where we reformulate the running cost on the states as $\tilde{Q} := Q + \bar{Q}(a, g)$, with Q from Problem 1, and $\bar{Q}$ obtained from the fatigue estimation parameters a and g. Thus, yielding

$$\bar{J}(\bar{u}_k, \bar{x}_k) = \sum_{k=0}^{N_p-1} \bar{x}_k^\top \tilde{Q}(a, g)\bar{x}_k + \sum_{k=0}^{N_u} \bar{u}_k^\top R\bar{u}_k + \bar{x}_{N_p}^\top Q_t \bar{x}_{N_p}. \quad (24)$$

Accordingly, we take (17) and make use of (10) such that

$$\tilde{\mathcal{F}}(\bar{\theta}_k, \dot{\bar{\theta}}_k) = a\bar{\theta}_k^2 + g\left(\bar{\omega}_{r|k} - \frac{\bar{\omega}_{g|k}}{N_g} \right)^2$$

$$= a\bar{\theta}_k^2 + g\bar{\omega}_{r|k}^2 - 2g\bar{\omega}_{r|k}\frac{\bar{\omega}_{g|k}}{N_g} + g\frac{\bar{\omega}_{g|k}^2}{N_g^2}, \quad (25)$$

from which we get the extra weight on the state running cost

$$\bar{Q} := \begin{bmatrix} g/N_g^2 & -g/N_g & 0 \\ -g/N_g & g & 0 \\ 0 & 0 & a \end{bmatrix}. \quad (26)$$

To lean upon convex optimization, $\bar{Q}$ should be positive semi-definite. By Schur complement, since

$$\Psi := \begin{bmatrix} g/N_g^2 & -g/N_g \\ -g/N_g & g \end{bmatrix} \succeq 0, \quad (27)$$

then it follows that

$$\bar{Q} = \begin{bmatrix} \Psi & 0 \\ 0 & a \end{bmatrix} \succeq 0, \quad (28)$$

for a, g positive. And on account of $Q \succ 0$ by design, we conclude that $\tilde{Q}(a, g) \succeq 0$.

D. Fatigue Reduction MPC strategy

The proposed fatigue reduction MPC strategy $\mathcal{C}_{fr}$ is given as follows

Problem 2 (Fatigue reduction MPC strategy, $\mathcal{C}_{fr}$):

$$\min_U \bar{J}(\bar{u}_k, \bar{x}_k) = \sum_{k=0}^{N_p-1} \bar{x}_k^\top \tilde{Q}(a, g)\bar{x}_k + \sum_{k=0}^{N_u} \bar{u}_k^\top R\bar{u}_k + \bar{x}_{N_p}^\top Q_t \bar{x}_{N_p}$$

$$s.t. \begin{cases} \bar{x}_0 = x(t) - x^*, \\ \bar{x}_{k+1} = A_d\bar{x}_k + B_d\bar{u}_k, \text{ for } k = 0, 1, \ldots, N_p - 1, \\ \bar{u}_k = \bar{u}_{N_u}, \text{ for } N_u + 1 \leq k \leq N_p - 1, \\ u_{\min} \leq \bar{u}_k \leq u_{\max}, \text{ for } k = 0, 1, \ldots, N_u, \\ |\bar{u}_{k+1} - \bar{u}_k| \leq \Delta u_{\max}, \text{ for } k = 0, 1, \ldots, N_u - 1, \\ x_{\min} \leq \bar{x}_k \leq x_{\max}, \text{ for } k = 1, \ldots, N_p, \end{cases}$$

$$(29)$$

over U, for prediction horizon N_p, and control horizon N_u, such that $N_p > N_u$. The running cost on states and inputs is given by weighting matrices $\tilde{Q}(a, g) = \tilde{Q}(a, g)^\top \succeq 0$, $R = R^\top \succ 0$, and the terminal state cost $Q_t = Q_t^\top \succ 0$.

V. Simulation Results

The proposed fatigue reduction MPC strategy for wind turbine control was implemented in `Matlab`. The wind turbine model $\mathcal{P}$ to be controlled is a non-linear model based on the standard NREL 5MW wind turbine [24] implemented in `Simulink`, driven to the operating point of a mean wind speed of $18m/s$. The plant model considers the tower dynamics and aerodynamics, as well as the rotational mode of the shaft used for controller design. The controller was synthesized with `CVX` [30], using the discretized and linearized dynamics in (15) with a sampling time $T_s = 0.15s$. The control horizon was set to $N_u = 20$ samples, the prediction horizon was set to $N_p = 50$ samples, and the simulation was performed for $200s$. The weightings on the running cost were chosen according to Bryson's rule [31, p.537] such that Q and R are diagonal matrices with elements $(1/30^2, 1/0.3^2, 1/0.001^2)$ and $(1/30^2, 1/0.1^2)$, respectively; and the terminal state cost as $Q_t = 100Q$. The limits on inputs, states and slew rate were considered as $u_{\max} = [90, 40700]$, $u_{\min} = [0, 40660]$, $x_{\max} = [142.9, 2.27, 8.5 \times 10^{-3}]$, $x_{\min} = [102.9, 0.27, 0.5 \times 10^{-3}]$ and $\Delta u_{\max} = [1.2, 2250]$. The wind disturbance $d = v_r$ was taken from wind series data, and the initial conditions were set to $x_0 = (\omega_g^*, \omega_r^*, \theta^*)$.

The Preisach operator was approximated as a parallel connection of three relay operators, i.e., $\mathcal{H}$ with discretization level $L = 2$, and the thresholds were set to $(\mu_1, \tau_1) = (-0.05M, 0.05M)$, $(\mu_2, \tau_2) = (0.05M, 0.05M)$ and $(\mu_3, \tau_3) = (-0.05M, -0.05M)$, where M is the bound for the Preisach plane calculated as $M = \max\{|\bar{\theta}|\}$; the initial conditions of the relays were given according to:

$$w_{-1}(\mu, \tau) = \begin{cases} 1, & \mu + \tau < 0, \\ 0, & \mu + \tau \geq 0. \end{cases} \quad (30)$$

The relay weightings were chosen as $\nu_1 = \sigma$, $\nu_2 = \sigma^2$, $\nu_3 = \sigma^3$ for $\nu_1 + \nu_2 + \nu_3 = 1$. The estimation scheme for $\bar{Q}$ was
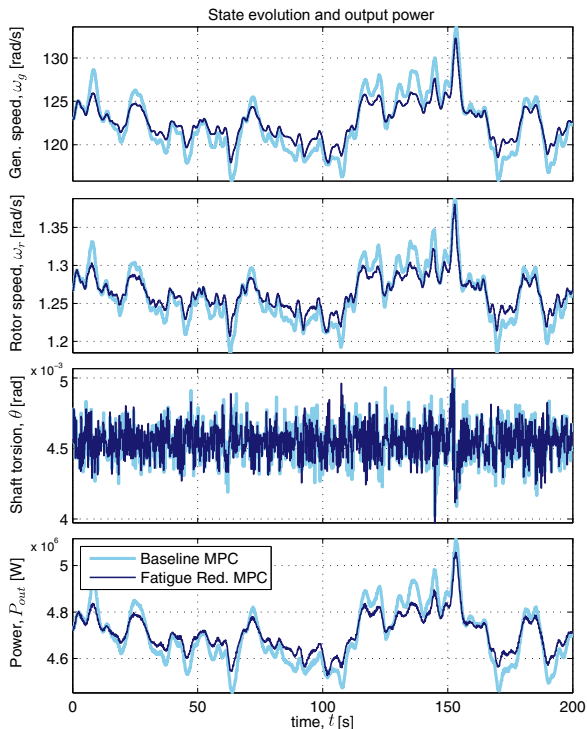
Fig. 5.   State evolution for $\mathcal{C}_b$ and $\mathcal{C}_{fr}$.

Fig. 6.   Control inputs and disturbances for $\mathcal{C}_b$ and $\mathcal{C}_{fr}$.

implemented recursively starting after $t = 3s$, such that the algorithm has enough points to calculate an initial damage estimate, until it reaches 50 samples, and then it keeps a window of such size to perform the estimation recursively.

The simulation results are presented in Fig. 5-7, for both $\mathcal{C}_b$ in Problem 1 and $\mathcal{C}_{fr}$ in Problem 2. In Fig. 5 it can be appreciated that both strategies effectively reduce the deviations in all the states, while showing some sensitivity to the wind, but $\mathcal{C}_{fr}$ reduces the deviations more effectively. In Fig. 6, some difference in the control signals can be seen, which suggests that $\mathcal{C}_{fr}$ tries to keep the oscillations in the shaft torsion θ within certain bounds to prevent damage from occurring. Furthermore, in Fig. 7 an instantaneous and accumulated damage comparison is shown, where it can be observed that the proposed control strategy $\mathcal{C}_{fr}$ effectively achieves damage reduction with respect to $\mathcal{C}_b$.

The proposed strategy $\mathcal{C}_{fr}$ is compared against the baseline strategy $\mathcal{C}_b$ in the context of Equivalent Damage Load to validate the previous results. The shaft torsion for both cases was ran through NREL's `MCrunch` post-processor [32]. Comparing the outcome, the proposed method achieves a damage reduction of 9.66% against the baseline scheme, thanks to the proposed damage load estimation. The reason for this additional validation is that in the process of incorporating the fatigue estimation into the MPC formalism the following assumptions are made: symmetric RFC applies, Preisach operator discretization, and fatigue approximation by least-squares.

Finally, in Fig. 8 several accumulated damage curves are presented. Notice the correspondence to the damage curves
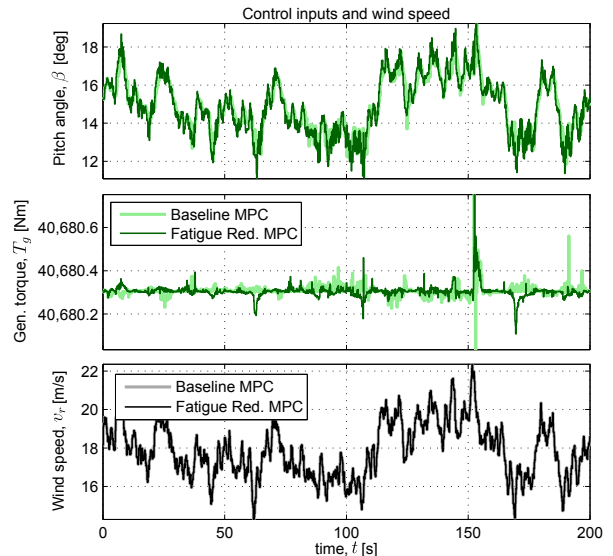
of $\mathcal{C}_b$ and $\mathcal{C}_{fr}$ from Fig. 7. The solid curves correspond to the accumulated damage h in (19) of the baseline MPC $\mathcal{C}_b$, the fatigue reduction with recursive identification (varying parameters) $\mathcal{C}_{fr}$ and the fatigue reduction with fixed parameters $\mathcal{C}_{fr2}$, i.e., with $a = 1.546 \times 10^6$ and $g = 48.92$ from a one shot offline a-priori identification. The dashed curves show the normalized approximated damage curves $\hat{h}$ in (20) provided by the estimator $\mathcal{K}$. From these results, we can conclude firstly, that the proposed estimation scheme approximates the variation of the discretized Preisach operator reasonably well; and secondly, that the recursive estimation outperforms the fixed parameter scheme, showing that the former successfully adapts according to the damage better. Even though $\mathcal{C}_{fr}$ outperforms $\mathcal{C}_{fr2}$, the latter could be used as an average damage reduction solution.

VI. Conclusions

As mentioned earlier, RFC is the most widely accepted method for fatigue calculation, but its nature makes it impossible to implement in real-time control applications, thus being used mainly as a post-processing tool instead. However, the equivalence between RFC and a Preisach operator introduced in [9] provides the opportunity to use the latter for online implementation in control loops. This equivalence applies to symmetric RFC and not all RFC methods are symmetric; for symmetric RFC the so-called Madelung rules apply, i.e., deletion pairs commute, meaning that it does not matter how the sequences are deleted. However, in our case the primal concern is to apply this technique online, so deletions are not possible.

A fatigue load reduction MPC strategy was designed and implemented, which incorporates a fatigue load model. This strategy was implemented on a non-linear model based on the standard NREL 5MW wind turbine, and it was compared against a baseline MPC strategy, achieving damage reduction in the shaft. Perhaps one shortcoming of our approach is the
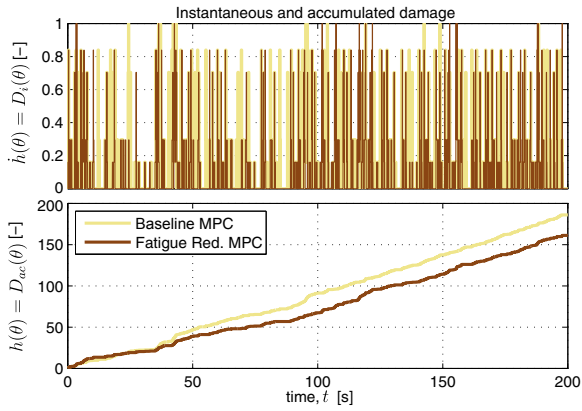
Fig. 7. Damage estimation on the shaft torsion for $\mathcal{C}_b$ and $\mathcal{C}_{fr}$.
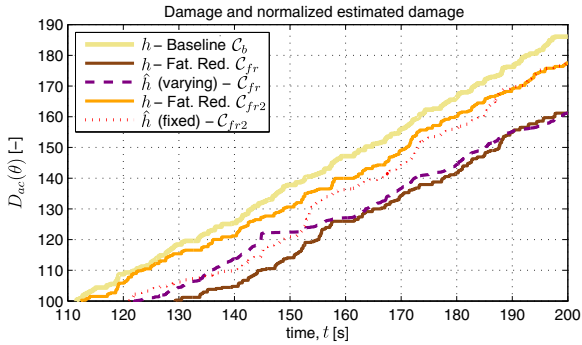

Fig. 8. Accumulated damage comparison: h (solid) and $\hat{h}$ (dashed).

fact that the residual cannot be considered directly (which would require the periodic version of the hysteresis operator), but the contribution of the residual would not be so large if a sufficiently large time window were considered. Other considerations to take into account are the hysteresis parameters, i.e., weightings, thresholds and discretization level. Regarding the thresholds, the tuning was done using RFC as a baseline, however, these parameters could be identified or fitted if the S-N parameters of the material are known.

Finally, it could be argued that linear identification methods are not enough to capture all the behavior of the hysteresis operator, due to its nonlinear nature, discontinuities, and memory effects. Nonetheless, the variation operator introduces some kind of regularization, thus allowing the identification to work by embedding a discrete integrator, and enabling the controller to achieve a significant damage reduction, which was also corroborated using wind turbine analysis tools. Lastly, the approach presented here, allows a self-tuning MPC strategy to be implemented in real-time control, effectively incorporating the damage of the components into the problem cost functional, via the weighting matrices.

VII. Acknowledgments

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