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**DEPARTMENT OF CIVIL ENGINEERING**  
AALBORG UNIVERSITY

# **Manual for wave generation and analysis software in Matlab**

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Aalborg University  
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# **Manual for wave generation and analysis software in Matlab**

by

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# Preface

This Manual is for the included wave generation and analysis software and graphical user interface. The package is made for Matlab and is meant for educational purposes. The code is free to use under the GNU Public License (GPL). It is still in development and should be considered as such.

If you have questions, suggestions, or additions to the code you can contact the author.

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# Chapter 1

## Manual for wave generation and analysis software in Matlab

The wave generation and analysis software presented here is an open source software package for Matlab. While the code is inspired by the Wavelab by Andersen (2010) and DIWASP by MetOcean Solutions LTD (2002) software it is written independently. The software by this author is for educational purposes only and should not be used commercially. The software has a Graphical User Interface (GUI) facade. The interface calls separate generation and analysis scripts that can run without the GUI.

To start the program browse to the *GUI* folder and open the `GenerationGUI.m` or `AnalysisGUI.m` in Matlab, then click *Run* or press *F5*.

### 1.1 Wave generation

To generate unimodal waves<sup>1</sup> a Random Phase Method (RPM) is used. The method uses inverse Fourier transformation to calculate the coefficients in the discrete spectrum based on Frigaard and Andersen (2010). For the bimodal spectrum the much slower superposition of regular waves is used.

Running the wave generation software the *GenerationGUI* opens as seen in Fig. 1.1. The output formats implemented is a generic type based on Hawkes et al. (1997) “*Comparative Analyses of Multidirectional Wave Basin Data*” and another format readable by Wavelab.

Clicking the *Load Array* button opens the *WGArray* window. After selecting the nodes to be used for the array and the wave parameters the target spectra can be examined using the *Plot* option as seen in Fig. 1.2. The text based outputs from the software is put into structures for a clean workspace. The two structures *geninput*

---

<sup>1</sup>single peak spectrum, i.e. only swell waves or wind waves

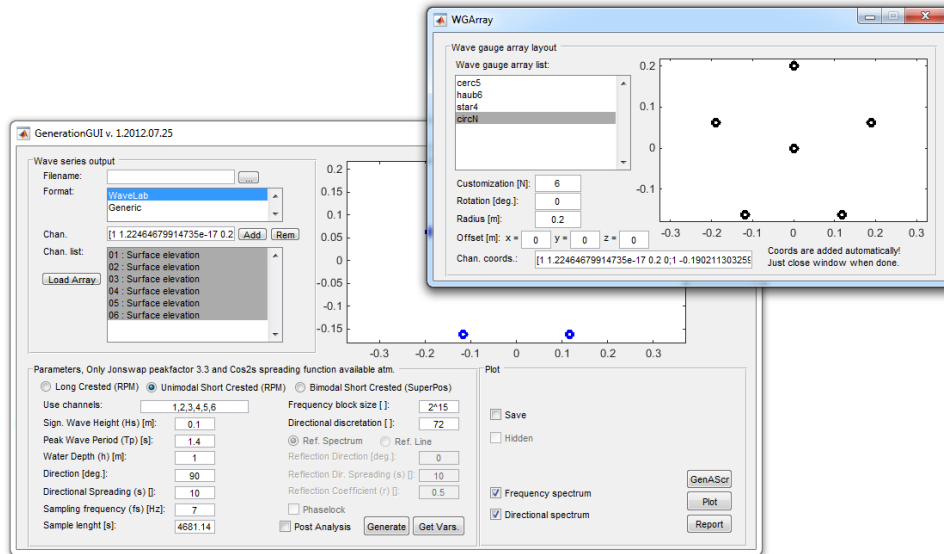


Figure 1.1: Wave Generation GUI.

and *generation* contain the inputs to the generation scripts and the output respectively. To run the wave analysis on the generated wave series the *Post Analysis* checkbox can be ticked. Alternately the saved file can be analyzed manually later.

## 1.2 Wave analysis

The 3D wave analysis methods referred to as Bayesian Directional Method (BDM) and Maximum Likelihood Method (MLM) has been implemented as seen in Fig. 1.3. The methods and implementations are explained in Section 2.

To do the analysis the water depth and the sampling frequency used in the samples are needed. The selection of incident and reflected wave directions are only used to process reflection coefficients and the calculation of separate incident and reflected significant wave heights.

The output from the analysis software can be plotted similar to the wave generation in Fig. 1.2. The two structures *anainput* and *analysis* contain the inputs to the model and the output respectively.

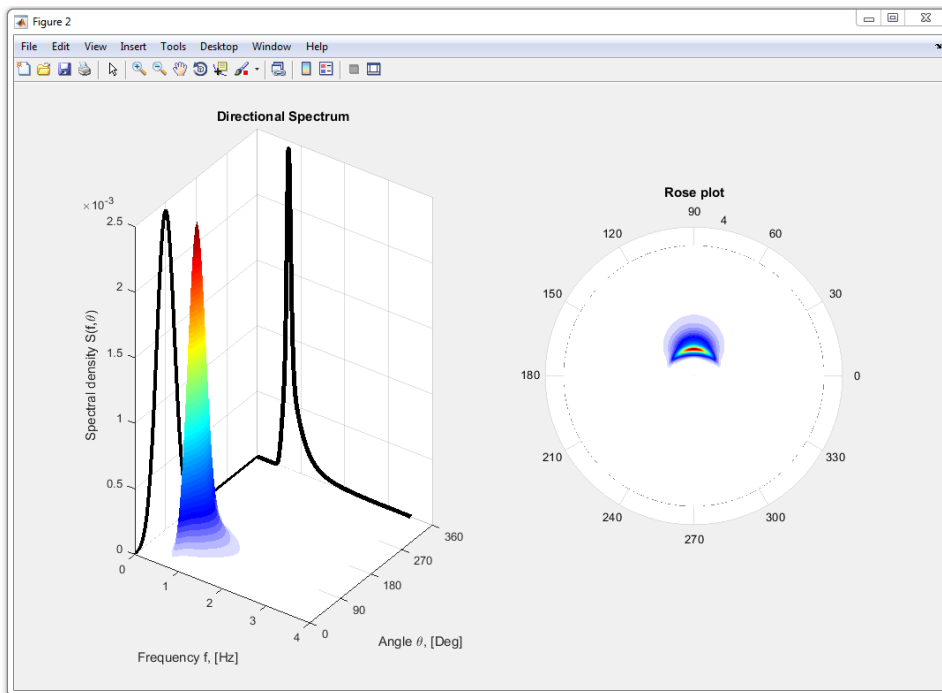


Figure 1.2: Plots of two common representations of the directional spectrum.

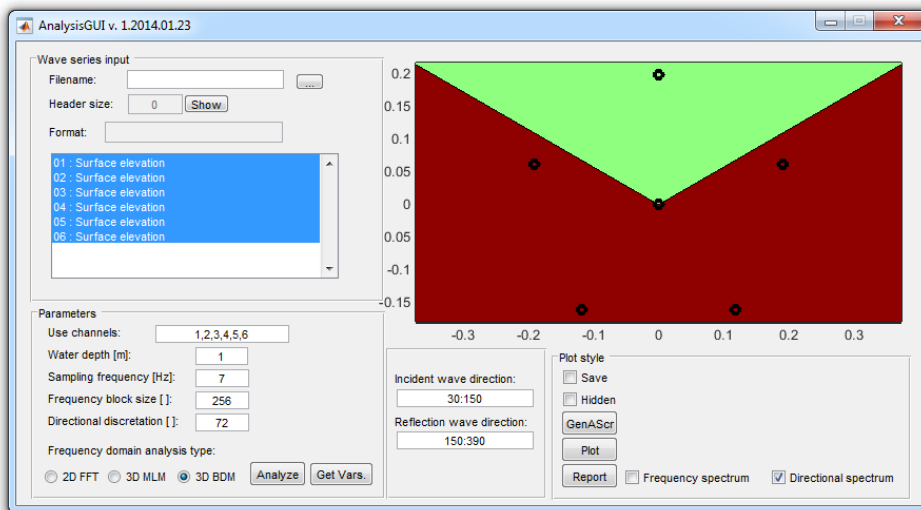


Figure 1.3: Wave analysis user interface.



## Chapter 2

# Implementation of 3D wave analysis

It is recommended to read the literature which outlines and explains the most commonly used methods to estimate the directional spectrum i.e. Hashimoto et al. (1987); Isobe et al. (1984); Davis and Regier (1977); Sand (1979); Benoit et al. (1997); Hawkes et al. (1997). This section will be an abridged version of the underlying theory and an explanation of how these methods are derived from the probabilistic methods.

Relation between the directional spectrum  $S(f, \theta)$ , the frequency spectrum  $S(f)$  and the spreading function  $D(\theta|f)$  is shown in Eq. 2.1.

$$S(f, \theta) = S(f) \cdot D(\theta|f) \quad (2.1)$$

The spreading function is subject to the constraint in Eq. 2.2.

$$\int_{-\pi}^{\pi} D(\theta|f) d\theta = 1 \quad (2.2)$$

To do the 3D analysis it is necessary to estimate the directional wave spectrum  $S(f, \theta)$ . A relation between the directional wave number-spectrum  $S(k, \sigma)$  and the spectral matrix  $\Phi(f)$  is outline through Fourier Transformation in Eq. 2.3 Isobe et al. (1984).

$$\Phi_{mn}(\sigma) = \int_k H_m(k, \sigma) H_n^*(k, \sigma) e^{-ik(\mathbf{x}_{nm})} S(k, \sigma) dk \quad (2.3)$$

Where  $k$  is the wave number can be determined from the dispersion equation using either Chebyshev(used here) or Newton-Raphson convergence theorem.  $H$  is a transfer function which depends of the type of sensor data used,  $H = 1$  is used if the time series are from surface elevation measurement by Hashimoto et al. (1987); Benoit et al. (1997).  $\mathbf{x}_{nm}$  represents the distance between each gauge pair, which can be described by Cartesian or Polar coordinates in Eq. 2.4 and 2.5 respectively.

Eq. 2.1 is used to isolate the spreading function rather than the directional wave spectrum, cf. Sand (1979).

$$\frac{\Phi_{mn}(f)}{S(f)} = \int_{-\pi}^{\pi} H_m(f, \theta) H_n^*(f, \theta) e^{-ik((x_m - x_n) \cdot \cos(\theta) + (y_m - y_n) \cdot \sin(\theta))} D(\theta|f) d\theta \quad (2.4)$$

*or*

$$\frac{\Phi_{mn}(f)}{S(f)} = \int_{-\pi}^{\pi} H_m(f, \theta) H_n^*(f, \theta) e^{-ik r_{mn} \cos(\theta - \beta_{mn})} D(\theta|f) d\theta \quad (2.5)$$

The solution to Eqs. 2.3, 2.4 and 2.5 is the subject to a wide range of analyzing procedures. Those used in the software are explained in the following.

## 2.1 Maximum Likelihood Method

The first implementation of the Maximum Likelihood Method was introduced in the field of seismic research but is useful within marine research using wave gauge arrays as well, cf. Capon et al. (1967); Capon (1969).

$$\hat{P}(\gamma, k) = \left[ \sum_{m,n} \Phi_{mn}(\gamma)^{-1} e^{ik\mathbf{x}_{mn}} \right]^{-1} \quad (2.6)$$

where  $\hat{P}$  is the estimate of the power output subject to the Maximum Likelihood filter.  $\gamma$  is the normalized frequency  $\gamma = \omega T$ .  $\hat{P}$  is proportional to the directional wave spectrum, so under the constraint of Eq. 2.2 the expression can be rewritten to Eq. 2.7.

$$\hat{S}(k, \sigma) = \alpha \cdot \left[ \sum_{m,n} \Phi_{mn}(\sigma)^{-1} e^{ik\mathbf{x}_{mn}} \right]^{-1} \quad (2.7)$$

$\Phi_{mn}(\sigma)^{-1}$  being the inverse spectral density matrix and  $\alpha$  being a proportionality constant. To include other types of sensory data an extension was made to the MLM, which included the transfer function  $H$  as suggested by Isobe et al. (1984).

$$\hat{S}(k, \sigma) = \alpha \cdot \left[ \sum_{m,n} \Phi_{mn}(\sigma)^{-1} H_m^*(k, \sigma) H_n(k, \sigma) e^{ik\mathbf{x}_{mn}} \right]^{-1} \quad (2.8)$$

The implementation in the analysis code uses the spreading function as suggested earlier together with the constraint in Eq. 2.2 to determine  $\alpha$ . Then knowing the spreading of the waves the directional wave spectrum is determined by Eq. 2.1. One of the issues with the method is the inverted matrix that makes the method vulnerable to truncation errors in the spectrum where there is little energy.

## 2.2 Bayesian Directional Method

The Bayesian Directional Method is the method suggested by default as it has shown to be more accurate and reliable by Hashimoto et al. (1987); Hashimoto and Kobune (1988). This method only makes assumption about local smoothness of the spectrum. By introducing the sample data functions  $x$  and  $y$  and implicitly assuming that the expression is subject to a priori information Bayes' Theorem is expressed in Eq. 2.9.

$$p(y|x) = \frac{p(y)p(x|y)}{p(x)} \quad (2.9)$$

Considering  $x$  as the measured data function the model distributions  $p(x|y)$  is interpreted as the likelihood function  $L(x, \sigma^2)$ .  $p(y)$  is the prior information redefined as  $p(x|u^2, \sigma^2)$  and  $p(y|x)$  is the posterior distribution,  $p_{post}(x|u^2, \sigma^2)$ . This leads to the proportionality in 2.10.

$$p_{post}(x|u^2, \sigma^2) \propto L(x, \sigma^2)p(x|u^2, \sigma^2) \quad (2.10)$$

Where the prior distribution is given by 2.11 and the likelihood function by 2.12.

$$p(x|u^2, \sigma^2) = \left( \frac{u}{\sqrt{2\pi}\sigma} \right)^K \exp \left( -\frac{u^2}{2\sigma^2} \sum_{k=1}^K (x_k - 2x_{k-1} + x_{k-2})^2 \right) \quad (2.11)$$

$$L(x, \sigma^2) = \frac{1}{(2\pi\sigma^2)^N} \exp \left( -\frac{1}{2\sigma^2} \sum_{i=1}^{2N} \left( \phi_i - \sum_{k=1}^K \alpha_{i,k} \exp(x_k) \right)^2 \right) \quad (2.12)$$

where

$$M = N(N + 1)/2 \quad (2.13)$$

$$\phi_i(f) = \Re \left( \frac{\Phi_{mn}(f)}{S(f)W_{mn}(f)} \right), \quad i = 1, \dots, M \quad (2.14)$$

$$\phi_i(f) = \Im \left( \frac{\Phi_{mn}(f)}{S(f)W_{mn}(f)} \right), \quad i = M + 1, \dots, 2M \quad (2.15)$$

$$\alpha_{i,k}(f) = \Re \left( \frac{\Delta\theta H_m(f, \theta_k) H_n^*(f, \theta_k) e^{ik\mathbf{x}_{nm}}}{W_{mn}(f)} \right), \quad i = 1, \dots, M \quad (2.16)$$

$$\alpha_{i,k}(f) = \Im \left( \frac{\Delta\theta H_m(f, \theta_k) H_n^*(f, \theta_k) e^{ik\mathbf{x}_{nm}}}{W_{mn}(f)} \right), \quad i = M + 1, \dots, 2M \quad (2.17)$$

Where  $N$  is the number of wave guages,  $W$  is a weighting function,  $u$  is a hyperparameter and  $\sigma$  is the standard deviation. These are chosen through Akaike (1980)'s Bayesian Information Criterion (ABIC), by minimizing Eq. 2.18.

$$ABIC = -2\ln \int L(x, \sigma^2)p(x|u^2, \sigma^2)da \quad (2.18)$$



Which should be maximized to obtain the estimate of  $x$ . In the implementation  $x$  is discrete, which is distinguished by  $x_k$ , where  $k = 1, \dots, K$ . The directional spreading function is then obtained from Eq. 2.19.

$$\hat{D}(\theta|f) = \sum_{k=1}^K e^{x_k(f)} I_k(\theta) \quad (2.19)$$

$$I_k(\theta) = \begin{cases} 1 & : (k-1)\Delta\theta \leq \theta < k\Delta\theta \\ 0 & : \text{otherwise} \end{cases} \quad (2.20)$$

It is significant to note that the method to calculate  $W$  of the spectral density functions was greatly improved by Hashimoto (1997). The newer method is implemented here which is defined in Eq. 2.21.

$$W_{mn}(f) = \sqrt{(\Phi_{mm}(f) \cdot \Phi_{nn}(f) \pm C_{mn}(f)^2 \pm Q_{mn}(f)^2)/2N_a} \quad (2.21)$$

Where  $N_a$  is the number of ensemble averages. When calculating the weighting function Eq. 2.16 uses positive coincident spectral density and negative quadrature while 2.17 uses negative coincident spectral density and positive quadrature. It should be noticed that this new method can result in  $W_{mn}(f) = 0$ , and in this rare case it is revert back to the old method in the implementation.

In an attempt to get even better results a *relaxation* of the new estimates are performed when no new solution was found. For stability and to avoid long computational time, the number of iterations are limited which dramatically reduces the computational time at the cost of some accuracy. The expressions in Eq. 2.22 and 2.23 are used to reduce the computational time need for convergence further suggested by Hashimoto (1997).

$$u = ab^m \quad , a = 1.0, b = 0.5, m = 1, 2, \dots \quad (2.22)$$

$$x_0(i) = \ln(1/2\pi) \quad , i = 1, \dots, K \quad (2.23)$$

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