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Making Meaning of Creativity and Mathematics Teaching

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Published in:

Nordic research in mathematics education

Publication date:

2015

Document Version

Early version, also known as pre-print

[Link to publication from Aalborg University](#)

Citation for published version (APA):

Misfeldt, M. (2015). Making Meaning of Creativity and Mathematics Teaching. In Nordic research in mathematics education: Proceedings of NORMA14, Turku, June 3–6, 2014 (Chapter 2, pp. 25-35). University of Turku, Department of Teacher Education. (Studies in Subject Didactics).

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MAKING MEANING OF CREATIVITY AND MATHEMATICS TEACHING

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Creativity and innovation are important 21st-century skills, and mathematics education contributes to the development of these skills. However, it is far from clear how we as mathematics educators should respond to the need to contribute to our students' development of creativity and innovation. One reason is that it is not clear what relation such creative and innovative skills have to mathematics, and how we should teach them. In this paper, I review different conceptions of creativity in mathematics education and investigate what mathematical innovation and creativity "are" in the mathematical classroom. I show how different conceptions of mathematical innovation and creativity dominate different parts of the mathematics education literature, and explain how these differences can be viewed as framing mathematical creativity toward different domains.

CREATIVITY AND INNOVATION IN THE MATHEMATICS CLASSROOM

Enhancing students' creativity and innovation is an important educational goal within and across specific school topics. Enhanced creativity and innovation supposedly empower students to cope with their lives, increase their potential value in the labor market, and allow them to participate in aesthetic and joyful experiences (Loveless, 2002; Organization for Economic Cooperation and Development [OECD], 2010). Mathematics education thus faces the challenge of how to develop curriculum and teaching that enhance creativity and innovation.

The broad discussion of the goal of education simultaneously acknowledges mathematics as a core subject and suggests the need for a close connection between mathematics and valued general skills and competencies such as creativity and innovation (Partnership for 21st Century Skills, 2009). However, creativity and innovation have different meanings, and therefore, addressing creativity and innovation when teaching mathematics is not easy. In the Partnership for 21st-Century Skills (P21), creativity and innovation are defined as the ability to create new and worthwhile ideas (alone and in collaboration with others), and acting on these ideas to make useful contributions (Partnership for 21st-Century Skills, 2009). However, it is far from clear what makes an idea worthwhile and what constitutes a useful contribution. Creativity has previously been described as an innate property of an individual who is virtuous in a specific field, such as a psychological tendency to divergent thinking, as a situated tendency to act in interesting and non-foreseen ways (Joas, 1996), an acknowledgement of aesthetic and constructive considerations (Loveless, 2002), and as a matter of connecting disciplinary knowledge to practical and professional situations (Shaffer, 2004). These views of creativity are related to

each other and to the P21 framework definition of creativity and innovation. When such fluffy concepts meet a national mathematics curriculum, many interpretations and practices are possible. “Creativity enhancing mathematics teaching” can thus signify everything from virtuous teaching of gifted students (Sriraman & Lee, 2011) to an inclusive pedagogical trend that acknowledge aesthetic concerns and students as producers in the mathematics classroom (Resnick, 2012).

This situation calls for two questions to be answered: What is creativity in mathematics education? And how should we teach for and assess creativity in mathematics education? In this paper, I mainly address the first question by surveying the existing mathematics education literature on creativity in the classroom.

Creativity as domain specific?

An obvious starting point is to ask whether creativity and innovation are domain-specific or domain-independent abilities. In a review of the domain specificity of creativity in mathematics education research, Plucker and Zabelina (2008) revealed strong evidence that creativity to some extent is domain specific; no person is likely to be creative in all domains of life. However, in the same review, the researchers presented many arguments for the opposite position—namely, that creativity is an inert ability. The domain specificity that we *observe* can be explained by the fact that significant results of creativity require strong knowledge and skills in an area. Thus, creativity might very well be a general ability, but using this ability to obtain results acknowledged as creative requires the possession of skills and knowledge. As an example, using divergent thinking (Guilford, 1967; Torrance, 1963) as a measure of creativity takes domain independence for granted when the nature of creativity is addressed. Whether creativity is domain specific or domain independent, it makes sense to review mathematical creativity as a phenomenon in its own right, but this phenomenon might or might not be explainable as an independent ability or as the sum of skills in an area and a general creative ability.

Mathematical creativity

Poincaré’s 1908 essay (1946) marks a starting point for deliberate investigation of the creative mathematical process. Introspectively, he describes how conscious and subconscious processes interact in the creation of mathematical insights. Hadamard (1954) builds on this essay and develops an explicit stage model for mathematical creative processes. The stages are preparation, incubation, illumination, and verification. Hadamard describes how when a creative individual, in the incubation stage, leaves his or her desk, is sleeping, or thinking about something other than the addressed problem, all of a sudden a solution or insight appears. Using such stage models gives a picture of mathematical practice as a lonely thinking process, where long periods of work and experiences of getting stuck are continued by not thinking about a certain problem and finally (perhaps) suddenly seeing a solution.

If we use this stage model, we tend to view creativity in the mathematical field as *problem solving* where the problems are very hard and subconscious inspirational processes play an important role in their solution. The bestselling book *How to Solve It* by Polya (1945) is perhaps the most well-known account of mathematical thinking that follows this stage model. The book is a handbook for how to work with problems in the mathematics classroom, but the book's view on what creativity is is shared in the mathematical field by Poincaré and Hadamard.

Haylock (1987) reviews mathematical creativity in order to develop a framework for assessing creativity in the mathematical classroom. He notes several authors suggest that *divergent thinking* (Guilford, 1967; Torrance, 1963) is a good measure for creativity, whereas others build on the *incubation model* proposed by Hadamard, and again others are more focused on the nature of mathematical theory. In his description of mathematical creativity in schoolchildren, he suggests that there is a need to transcend what he calls *algorithmic fixation* (that creativity follows a certain recipe) and *content universe fixation* (you are creative in a specific area of knowledge), and suggests that this can be done by focusing on *problem solving*, *problem posing*, and *redefinition situations*.

Ervynck (1991) describes creative work in the context of undergraduate-level mathematics. He develops a stage model that relies to some extent on Hadamard, and distinguishes the first “technical stage” consisting of gaining familiarity with the involved concepts, followed by an “algorithmic” stage where more complex techniques related to the problem are performed, and finally a “creative” or constructive stage where a full-fledged mathematical construction is developed. Ervynck respects aspects of Hadamard's stage model but adds a conceptual focus on mathematical creativity, respecting concerns about the process object duality, and aligning reification processes (Sfard, 1991) and creativity. Ervynck thus defines mathematical creativity as:

the ability to solve problems and/or to develop thinking in structures, taking account of the peculiar logico-deductive nature of the discipline, and of the fitness of the generated concepts to integrate into the core of what is important in mathematics. (Ervynck, 1991, p. 47)

The focus on problems is explicitly supplemented by respect for the logical and theoretical nature of mathematics, which addresses theory building and problem solving. Furthermore, Ervynck suggests that the nature of mathematical creativity might differ according to the mathematical subjects involved.

Sriraman (Sriraman, 2008; Sriraman & Lee, 2011) has investigated mathematical creativity among research mathematicians, and his empirical results confirm stage modes similar to the one described by Hadamard. Although Sriraman surveys different individual psychological and social approaches to creativity, his main empirical project is to challenge/confirm Hadamard's stage model. Sriraman finds:

In trying to better understand the process of creativity, the author finds that the Gestalt model proposed by Hadamard (1945) is still applicable today. This study has attempted to add some detail to the preparation–incubation–illumination–verification model of Hadamard (1945), by taking into account the role of imagery, the role of intuition, the role of social interaction, the use of heuristics, and the necessity of proof in the creative process. (Sriraman, 2008, p. 25)

In that sense, Sriraman considers mathematical creativity a complex phenomenon related to various possible constructs and theories, but maintains that the stage model is at the center of the consideration of mathematical creativity.

Creativity and giftedness

A research environment surrounds “creativity and gifted education.” This environment can be followed in the conference series creativity and giftedness in mathematics education as well as in the ICME and CERME groups. A relatively large body of knowledge describes the relation between mathematical performance, divergent thinking, problem solving and problem posing (Leikin & Pitta-Pantazi, 2012; Silver, 2013; Sriraman, Haavold, & Lee, 2013), and on the relation between mathematical creativity and culture (Massarwe, Verner, & Bshouty, 2011). In the Nordic community, a recent Norwegian dissertation has continued the discussion of mathematical creativity in this tradition (Haavold, 2013).

Constructionism

A review of the notion of mathematical creativity shows that research mathematicians’ self-reported practices and experiences play an important role in the conceptualization of the field, and that the conception of mathematical creativity as “sudden insights solving hard problems” is important. However, at least one large-scale project worked with a different conception of creativity in relation to mathematics. In the late ’70s, Papert and his colleagues at the Massachusetts Institute of Technology introduced a different take on what primary and lower secondary mathematics education could be in a technological era. This approach, built on pupils’ creative and aesthetic work in a computer-based mathematical environment, was developed by Seymour Papert in the context of primary and middle school pupils working with the Logo program (Papert, 1980). Papert developed the interpretive educational framework known as *constructionism*. According to constructionism, learning mathematics is especially effective when pupils develop and construct artifacts for which they care. Moreover, computer-based environments (“microworlds”) can be designed in such a way that they are especially well suited to supporting such epistemic constructions. From the point of view of mathematical creativity, the Logo project is interesting for two reasons; rather than considering mathematical creativity as an end, creative and constructive work is considered a vehicle for learning mathematics, and technology is seen as a possible catalyst for this constructive and creative pedagogy.

Papert relates his educational ambition explicitly to creativity and to the workings of the research mathematician, by directly comparing an art class with the typical mathematics classroom:

In the math class students are generally given little problems which they solve or don't solve pretty well on the fly. In this particular art class they were all carving soap, but what each student carved came from wherever fancy is bred and the project was not done and dropped but continued for many weeks. It allowed time to think, to dream, to gaze, to get a new idea and try it and drop it or persist, time to talk, to see other people's work and their reaction to yours--not unlike mathematics as it is for the mathematician, but quite unlike math as it is in junior high school. I remember craving some of the students' work and learning that their art teacher and their families had first choice. I was struck by an incongruous image of the teacher in a regular math class pining to own the products of his students' work! An ambition was born: I want junior high school math class to be like that. (Harel & Papert, 1991)

The vision can be realized by mathematics instruction in which children develop digital artifacts and during that work are guided toward powerful and important mathematical ideas.

Epistemic games

The vision of building things with mathematics and technology can be aligned with a competence and labor market-oriented approach to mathematics education that has had an increasing international influence during the last decade (Niss et al., 2002). In the labor market, the value of being able to create an artifact with technology and mathematics is increasingly important. Papert did not address this aspect, but the relation to work life has been systematically addressed by Papert's graduate student Shaffer (2006), who has systematically investigated and designed a didactical transposition from mathematical and scientific activities outside academia, into what he calls epistemic games. Shaffer's main idea is to copy professional working situations in computer-supported practice-simulating games, to create a new kind of learning that enhances important innovative skills. One example is the epistemic game *Escher's World* (2006) in which high school students, in an after-school activity, act as professionals in a design studio creating visual layouts by using the tool the Geometer's Sketchpad. Creating visual layouts in the Geometer's Sketchpad activates creative competencies and mathematical concepts (Shaffer, 2002, 2006), and the activity resembles an important work life activity involving mathematics. However, this activity has little to do with conceptualizing mathematical creativity as a sudden inspiration that leads to valuable solutions to existing problems.

OVERVIEW OF CONCEPTIONS OF MATHEMATICAL CREATIVITY

I propose a conceptualization of creativity in mathematics education that contests two metaphors for creativity, *as problem solving* and *as construction*, as well as creativity enacted *inside* mathematics and *with* mathematics. These pairs of concepts should be

understood as continuums spanning the field of creativity in mathematics education, rather than as dichotomies that classify mathematics education activities in a binary fashion. The overview builds on a pragmatist approach to categorization, valuing a continuous rather than a dichotomist approach to differences between phenomena.

By creativity and innovation *as problem solving*, I mean the idea that a creative/innovative mathematical activity involves finding new and valuable solutions to hard problems. This metaphor for creativity is predominant in the descriptions by Hadamard (1954) and Pólya (1945) and in the area of gifted education (e.g., Leikin & Pitta-Pantazi, 2012). And it is present to a larger or smaller extent in much of the mathematics education literature (Silver, 2013; Sriraman, 2008).

The conception of creativity and innovation as *construction* signifies the development of new and valuable artifacts as the primary sign of creativity/innovation. Such artifacts can be conceptual as well as material. The conception is prevalent in parts of the general literature on creativity in education (Loveless, 2002), as well as in the constructionist program (Papert, 1980; Resnick, 2012). But it is also present in the contribution by Ervynk (1991), emphasizing the construction of conceptual or theoretical artifacts as a reification process crucial for mathematical creativity.

The distinction between creativity and innovation *in* and *with* mathematics deals with the specific role that mathematics has in framing students' activities. Activities described as *with* mathematics contain explicit references to domains outside mathematics, and mathematics is typically used as a tool for obtaining something else. Constructionism (Papert, 1980; Resnick, 2012) and the epistemic game project (Shaffer, 2006) explicitly address creativity/innovation *with* mathematics. However, the area of mathematical modeling and application typically also tends to consider mathematics as a tool for making a difference outside mathematics (Lesh & Zawojewski, 2007). Opposed to this understanding, I will consider creativity/innovation *in* mathematics. Creativity and innovation *in* mathematics are used to describe situations where the problems and constructs that the students work with and the criteria for giving value to a certain construction, problem, or solution are internal to mathematics.

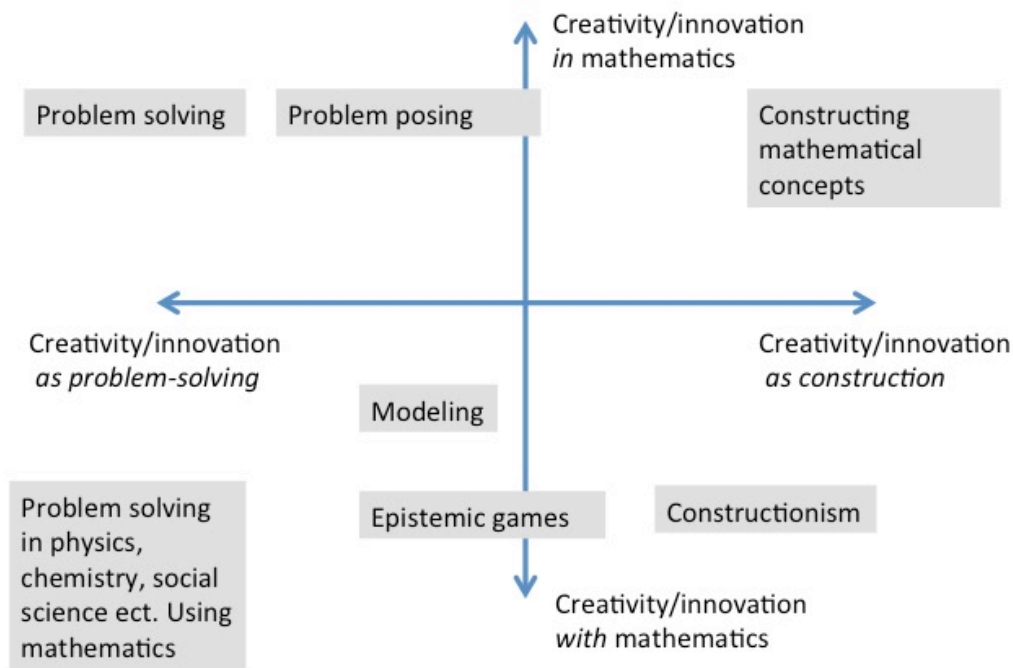


Figure 1: An overview of creativity in mathematics education.

If we look at the different and (partly conflicting) attempts to define creativity and innovation in relation to mathematics education, we can see that they are placed differently in the two-dimensional model. The discussions of the stage model for mathematical creativity are mainly framed as internal to mathematics and rely on a problem-solving metaphor for creativity. In contrast, Papert's constructionism is framed toward children's construction of artifacts meaningful in their own life worlds, and therefore, this conception can be viewed as creativity *with* mathematics building on a construction metaphor for creativity. The epistemic games project also mainly addresses creativity *with* mathematics. However, the domain for the students' activities is not their own interest and life world but a professional domain imported to the classroom from outside. Such a situation is typical, for example, in modeling activities where the problems dealt with are found outside mathematics. It is important to be sensitive toward how different domains frame mathematical activities in school, particularly if we want to improve our understanding of innovation and creativity in the mathematics classroom. Therefore, I suggest that we need more empirical knowledge about how various domains of knowledge frame students' work with mathematics in open scenarios related to problems and practices outside mathematics.

MATHEMATICS EDUCATION AS FRAMED TOWARD DIFFERENT DOMAINS OF KNOWLEDGE

Any teaching situation involves a particular environment or situated task within which the students work. Since we focus on creativity and innovation, the environments are characterized by (1) a potential open-ended nature and (2) a relation to different situations outside school that the students somehow envision or relate to in their classroom work. The concept *frame* designates the socio-cognitive structures that make it possible for people to interpret their world and act within that world. In that sense, framing is the cognitive mechanism that allows the generation of a “situation” from the many and diverse sensory motor inputs that an individual receives (Goffman, 1974, p. 10). Thus, frames are crucial for making meaning in situations. Skovsmose relates frames to meaning in mathematics education (Kilpatrick, Hoyles, Skovsmose, & Valero, 2005), and suggests that meaning must be considered situated and oriented toward the task that the students address, while the mathematics education literature tends to consider meaning as oriented toward abstract mathematical concepts. Relating meaning to situations and tasks (and not only “concepts”) makes it imperative to relate meaning to the involved “spheres of practice,” and the concept *frame* let us do that systematically. Frames relate to *domains* in the sense that any educational situation is framed by our understanding of the practices. Figure 2 illustrates the relationship between four domains common in education: (1) the domain of schooling, (2) disciplinary domains (mathematics, physics), (3) professional domains (agriculture, engineering), and (4) everyday domains (games, social media, family life; Hanghøj et al., n.d.).

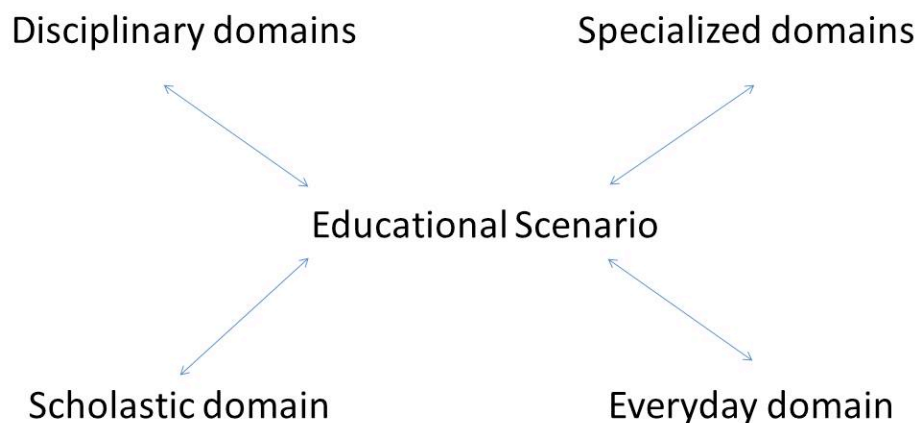


Figure 2: Relationship between the four domains (Hanghøj et al.).

It is a valuable route for research to investigate how the different conceptions of creativity and innovation are framed toward different domains. When students act “as” designers of educational games, “as” researchers, or “as” engineers, the students’ experience is related to their idea of this practice but is of course different from the lived experiences in these practices. Therefore, to answer the initial question of what creativity is in relation to the teaching of mathematics education, it makes

sense to investigate examples where out-of-school practices and domains frame activities in the mathematics classroom and ask how such examples allow students to enact creativity and innovation in relation to mathematics.

CONCLUSIONS

The purpose of this paper was to address questions about what creativity and innovation *is* in mathematics education, as well as how we should teach for and assess creativity and innovation in relation to the teaching and learning of mathematics.

In the paper, the literature regarding creativity in mathematics education was reviewed and considered on two continuums (as showed in Figure 1). One of the proposed continuums goes from creativity *in* mathematics to creativity *with* mathematics, and the other one goes from a *problem-solving metaphor* for creativity to a *construction metaphor*. Furthermore, it was suggested that creativity in mathematics education is always enacted in relation to one or several domains. In that sense, mathematical creativity understood in some of the intrinsic definitions (Ervynk, 1991; Hadamard, 1954) cannot in any meaningful way be considered *the same thing* as the more general of artistic or professional activities involving the development of new ideas (Partnership for 21st-Century Skills, 2009; Resnick, 2012).

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