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Direct Ranging in Multi-path Channels Using OFDM Pilot Signals

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Abstract—OFDM ranging is becoming important for positioning using terrestrial wireless networks. Conventional ranging methods rely on a two-step approach: range related parameters, such as the time of arrival (TOA), the bias induced by non-line-of-sight (NLOS) propagations etc., are first estimated, based on which the range is then inferred. In multi-path conditions, two-step range estimators which employ the correlator-based estimator or the energy detector lead to poor ranging accuracy when applied in non-ultra-wideband scenarios due to a bias. More advanced ranging schemes that estimate all multi-path components using a multidimensional search procedure provide higher ranging accuracy but have a prohibitive complexity. In this work, we propose a novel direct ranging technique that uses a point process formulated channel model. Based on this model, we derive an approximate maximum likelihood estimator of the range. In contrast to the estimator which requires a multidimensional search procedure, the proposed estimator does not demand the knowledge of the exact number of multi-path components and these components are separable. If the power delay spectrum of the multi-path channel and the signal-to-noise-ratio (SNR) are known, the complexity of the proposed estimator is tractable. We show by means of Monte Carlo simulations that this estimator outperforms the correlator-based estimator.

Index Terms—OFDM, point processes, Gaussian approximations, direct ranging technique.

I. Introduction

Accurate localization is becoming important for terrestrial wireless systems, in particular for OFDM systems such as WLAN, LTE and its extension LTE-A [1] [2]. One approach to improve the localization accuracy is to design high precision ranging techniques [3] [4]. State-of-the-art ranging techniques follow a two-step approach. First, parameters, such as the received signal strength, the TOA, the bias induced by NLOS propagations etc., are estimated from the received signal. Then, these estimates are used for ranging [3]. Since some of these information bearing parameters are readily available in communication systems, two-step ranging methods are very popular.

Two-step approaches employing OFDM signals have been considered in [4] [5] [6]. Wang *et al.* [6] proposed a maximum-likelihood ranging method based on OFDM signals for a scenario with separable multi-path components. Due to the assumed separability (in the delay domain) of these components, the obtained estimator converges to the correlator-based estimator [3]. However, for the separability condition to hold, a large system bandwidth is needed and even in this case it is not guaranteed that all paths are separable. In addition, because this method relies on the detection and estimation of the line-of-sight (LOS) path, it is sensitive to fading of early non-separable components [3]. MUltiple SIgnal Classification (MUSIC) algorithm was applied by Zhao *et al.* in [5] to estimate the delays of all multi-path components assuming their exact number is known. In [4], Wang *et al.* derived the Cramér-Rao bound (CRB) on the mean square error (MSE) of the range estimator and investigated how the OFDM signal parameters and the spacing between the multi-path components affect the bound. The CRB is a lower bound for the MSE of the estimator proposed in [7], which requires estimation of the delays of all separable multi-path components. The bound in [4] and the methods in [5] [6] [7] require the separability of the multi-path components and the knowledge of their exact number, which is generally difficult to estimate reliably. Furthermore, the required multidimensional search is impractical for a realistic number of multi-path components. As an alternative to the two-step approach, ranging can be performed in one step—referred to as "direct" ranging—avoids the need for the detection of the first-path and the estimation of its parameters. Despite the ability to bypass both the first-path detection problem and the path separability requirement, direct ranging has attracted little attention in the literature.

In this contribution, we address the problem of direct ranging using OFDM pilot signals in multi-path channels. The objective is to obtain a ranging estimator with low complexity, which does not rely on first-path detection, any separability condition, and the knowledge of the number of multi-path components. To that end, we formulate a multi-path channel model using a point process approach [8] [9] [10]. The channel transfer function is reformulated such that the range parameter is factored out to make it accessible for direct estimation. We then propose a direct ranging method using a Gaussian approximation of the channel transfer function. The method avoids the requirement of knowing the exact number of multi-path components and relaxes the constraint on their separability. Given the SNR and the RMS delay spread of the channel, the proposed estimator is computationally tractable. Simulation results demonstrate that the proposed estimator outperforms the correlator-based estimator.

II. System Model

We consider a single-input single-output OFDM setup with N active sub-carriers. An OFDM symbol with time duration T is generated by multiplexing a sequence of data symbols and known pilot symbols onto N orthogonal sub-carriers. The

adjacent sub-carrier spacing is defined as $\Delta f = \frac{1}{T}$. A cyclic prefix with duration T_{cp} is appended to prevent inter-symbol and inter-carrier interference. We index the N active sub-carriers with the set $\mathbf{I} = \{1, 2, \ldots, N\}$. Of these sub-carriers, $N_{\text{p}} = |\mathbf{I}_{\text{p}}|$ are pilots indexed by $\mathbf{I}_{\text{p}} \subseteq \mathbf{I}$.

We address estimation of the range parameter d based on the pilot signals. The multi-path channel is assumed to be time-invariant during the transmission of each OFDM symbol. Removing the cyclic prefix and concatenating the received pilot signals in the observation vector $\mathbf{y}$, we obtain the signal model in the frequency domain:

$$\mathbf{y} = \mathbf{A}\mathbf{h}(d) + \mathbf{w}, \tag{1}$$

where $\mathbf{A} = \text{diag}\{a_n : n \in \mathbf{I}_{\text{p}}\}$ is a diagonal matrix with a_n denoting the nth pilot symbol, the vector $\mathbf{h}(d) = [h(d; n\Delta f) : n \in \mathbf{I}_{\text{p}}]$, contains the samples of the channel frequency response, and $\mathbf{w}$ is a white circular-symmetric complex Gaussian noise vector with component variance σ^2. We define the SNR as $\frac{E_s}{\sigma^2}$ with $E_s = E[|a_n|^2]$.

The channel frequency response is modeled as a sum of delayed and attenuated multi-path components [10]:

$$h(d; f) = \underbrace{q\alpha_0 e^{-j2\pi f(\tau_0 + \frac{d}{c})}}_{\text{LOS term}} + \underbrace{\sum_{l=1}^{L} \alpha_l e^{-j2\pi f(\tau_l + \frac{d}{c})}}_{\text{Tail}}, \tag{2}$$

where path l has complex gain α_l and excess delay τ_l and c is the speed of light. The indicator q specifies the settings of the LOS path component. For LOS channels i.e. $q \neq 0$, q adjusts the power of the LOS component. When $q = 0$, the system operates in NLOS conditions. The delay of the LOS path is $\frac{d}{c}$ and thus we set the LOS excess delay equal to zero: $\tau_0 = 0$. The excess delays of the NLOS paths form a point process $\mathcal{T} = \{\tau_1, \tau_2, \ldots\}$ with intensity function $\rho(\tau)$. Note that the number of multi-path components L, i.e. the cardinality of $\mathcal{T}$, is not necessarily deterministic or finite under such channel formulation. We also assume that

$$E[\alpha_l | \tau_l] = 0, \quad E[\alpha_l \alpha_{l'}^* | \tau_l, \tau_{l'}] = \begin{cases} \sigma_\alpha^2(\tau_l), & l = l' \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

For convenience, we reformulate (2) as the product of a range-dependent factor $r(d; f)$ and a factor $\varepsilon(f)$ independent of d:

$$h(d; f) = \underbrace{e^{-j2\pi f \frac{d}{c}}}_{r(d;f)} \underbrace{\left(q\alpha_0 + \sum_{\tau_l \in \mathcal{T}} \alpha_l e^{-j2\pi f \tau_l}\right)}_{\varepsilon(f)}. \tag{4}$$

The assumption that $\varepsilon(f)$ is independent of d is a simplification which may or may not be realistic. Here, we employ it to simplify the forthcoming derivations. We leave investigation of more sophisticated distance dependent channel models such as presented in [11] to future works. Defining the diagonal matrix $\mathbf{R}(d) = \text{diag}\{r(d; n\Delta f) : n \in \mathbf{I}_{\text{p}}\}$ and the vector $\boldsymbol{\varepsilon} = [\varepsilon(n\Delta f) : n \in \mathbf{I}_{\text{p}}]^T$, the channel vector reads

$$\mathbf{h}(d) = \mathbf{R}(d)\boldsymbol{\varepsilon}. \tag{5}$$

Following the assumptions (3), $E[\boldsymbol{\varepsilon}] = \mathbf{0}$ and thus $E(\mathbf{y}) = \mathbf{0}$. With these results, the covariance matrix of the observation vector $\mathbf{y}$ is given by

$$\mathbf{C}_{\mathbf{y}}(d) = \mathbf{A}\mathbf{R}(d)\mathbf{C}_{\boldsymbol{\varepsilon}}\mathbf{R}^H(d)\mathbf{A}^H + \sigma^2 \mathbb{I}, \tag{6}$$

where $\mathbf{C}_{\boldsymbol{\varepsilon}} = E[\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}^H]$ with $(\cdot)^H$ denoting conjugate transpose and $\mathbb{I}$ being the identity matrix. Inspired by [9], $\mathbf{C}_{\boldsymbol{\varepsilon}}$ can be computed from an underlying channel model. Entry (m, n) of $\mathbf{C}_{\boldsymbol{\varepsilon}}$ reads

$$[\mathbf{C}_{\boldsymbol{\varepsilon}}]_{mn} = q^2 \sigma_\alpha^2(0) + E\left[\sum_{\tau_l, \tau_{l'} \in \mathcal{T}} \alpha_l \alpha_{l'}^* e^{-j2\pi \Delta f(m\tau_l - n\tau_{l'})}\right].$$

By the law of total expectation, conditioning on the point process $\mathcal{T}$, and utilizing (3), we obtain

$$[\mathbf{C}_{\boldsymbol{\varepsilon}}]_{mn} = q^2 \sigma_\alpha^2(0) + E\left[\sum_{\tau_l \in \mathcal{T}} \sigma_\alpha^2(\tau_l) e^{-j2\pi \Delta f(m-n)\tau_l}\right].$$

Applying Campbell's theorem [12] yields

$$[\mathbf{C}_{\boldsymbol{\varepsilon}}]_{mn} = \int_0^\infty \underbrace{\sigma_\alpha^2(\tau)(\rho(\tau) + q^2\delta(\tau))}_{P(\tau)} e^{-j2\pi(m-n)\Delta f \tau} d\tau$$
$$= \mathcal{F}\{P(\tau)\}((m-n)\Delta f), \tag{7}$$

where $\delta(\cdot)$ denotes the Dirac delta function, $P(\tau)$ is the delay power spectrum (the average delay power profile) and $\mathcal{F}$ denoting the Fourier transform [9]. In practice, the delay power spectrum can be evaluated empirically or approximated using an appropriated channel model [13] [14]. We further assume that $\rho(\tau) = \rho$ with ρ being a constant. Thus, we assume a constant arrival rate for the delays induced in the "Tail". With this assumption, (7) reads

$$[\mathbf{C}_{\boldsymbol{\varepsilon}}]_{mn} = q^2 \sigma_\alpha^2(0) + \rho \underbrace{\int_0^\infty \sigma_\alpha^2(\tau) e^{-j2\pi(m-n)\Delta f \tau} d\tau}_{g_{mn} = \mathcal{F}\{\sigma_\alpha^2(\tau)\}((m-n)\Delta f)}. \tag{8}$$

To gain some insight into the impact of the properties of the channel model on $\mathbf{C}_{\boldsymbol{\varepsilon}}$, we consider the following three example models.

Example 1. *The number L is fixed and $\sigma_\alpha^2(\tau) = \frac{1}{L}$. The delays in the "Tail" are drawn independently and uniformly on $[0, T_{\text{cp}}]$. In this case, $\mathcal{T}$ is a Binomial point process. Hence, $\rho = \frac{L}{T_{\text{cp}}}$. Consequently, (8) reads*

$$[\mathbf{C}_{\boldsymbol{\varepsilon}}]_{mn} = q^2 \frac{1}{L} + \text{sinc}((m-n)\Delta f T_{\text{cp}}) e^{-j\pi(m-n)\Delta f T_{\text{cp}}} \tag{9}$$

with $\text{sinc}(x) = \sin(\pi x)/(\pi x)$. In the LOS scenario, the covariance matrix depends on the exact number of paths L, which is generally unknown in practice. In the NLOS scenario, i.e. $q = 0$, the covariance matrix looses the dependency on L since the first term in (9) vanishes due to the somewhat artificial assumption $\sigma_\alpha^2(\tau) = \frac{1}{L}$. Note that the involved assumptions are similar to those used to derive the robust Wiener filter [15].

Example 2. *The number L is fixed and motivated by experimental observations [11], we assume that*

$$\sigma_\alpha^2(\tau) = C \exp(-\frac{\tau}{\lambda}), \tag{10}$$

where C is a positive constant and λ denotes the RMS delay spread of the "Tail" of the multi-path channel. We reuse the assumptions invoked in Example 1 except the assumption on $\sigma_\alpha^2(\tau)$. Assuming that $\int_{T_{cp}}^\infty \sigma_\alpha^2(\tau)d\tau$ is negligible, (8) reads

$$[\mathbf{C}_\varepsilon]_{mn} = q^2 C + \frac{L}{T_{cp}} g_{mn}, \tag{11}$$

where

$$g_{mn} = C \frac{1 - e^{-(j2\pi(m-n)\Delta f + \frac{1}{\lambda})T_{cp}}}{j2\pi(m-n)\Delta f + \frac{1}{\lambda}}. \tag{12}$$

Notice that the covariance matrix depends on L.

Example 3. *$\mathcal{T}$ is modeled as a homogeneous Poisson point process on $[0, T_{cp}]$ with rate ρ and exponential power decay for $\sigma_\alpha^2(\tau)$. This is a special case of Turin's model [8]. Then L is a Poisson random variable with mean $\mu_L = E[L] = \rho T_{cp}$. Assuming that $\int_{T_{cp}}^\infty \sigma_\alpha^2(\tau)d\tau$ is negligible and utilizing (10), (8) reads*

$$[\mathbf{C}_\varepsilon]_{mn} = q^2 C + \rho g_{mn} \tag{13}$$

with g_{mn} defined as in (12). We observe that the covariance matrix $\mathbf{C}_\varepsilon$ does not depend on the exact number of paths of a specific channel realization but depends on the intensity ρ and λ. The intensity ρ and λ may be provided by an appropriate channel model.

III. DIRECT MAXIMUM LIKELIHOOD RANGING VIA GAUSSIAN APPROXIMATIONS

The direct maximum likelihood estimator of d based on the observation $\mathbf{y}$ reads

$$\hat{d}_{ML} = \arg\max_d p(\mathbf{y}|d), \tag{14}$$

where $p(\mathbf{y}|d)$ denotes the likelihood function of d given $\mathbf{y}$. Estimator (14) is a "direct" range estimator since no intermediate parameters such as delays, complex gains, etc. are estimated. To implement (14), the likelihood function $p(\mathbf{y}|d)$ needs to be computed. In the considered case, however, $p(\mathbf{y}|d)$ is unknown. Instead, estimator (14) may be approximated as in [4] and [7] via a two-step approach. These methods, however, require the knowledge of the number of path components, which is generally unknown and hard to estimate.

Here, we follow the alternative approach of approximating $p(\mathbf{y}|d)$ with a Gaussian pdf $\tilde{p}(\mathbf{y}|d)$ with the same first- and second-order moments. This approximation is exact if ε is a Gaussian random vector. It is a reasonable approximation in a multi-path channel where L is large and $\sigma_\alpha^2(\tau)$ is a constant. In more realistic channels with an exponential power decay, the Gaussian approximation can be inaccurate. Since the first- and second-order moments of $\mathbf{y}$ are known by (6), this approximation leads to an estimator that can be derived analytically. Using $\tilde{p}(\mathbf{y}|d)$ instead of $p(\mathbf{y}|d)$ in (14) yields

$$\hat{d}_{AML} = \arg\max_d \ln \tilde{p}(\mathbf{y}|d), \tag{15}$$

where the log-likelihood $\ln \tilde{p}(\mathbf{y}|d)$ is of the form [16]

$$\ln \tilde{p}(\mathbf{y}|d) \propto -\ln\det(\mathbf{C}_\mathbf{y}(d)) - \mathbf{y}^H \mathbf{C}_\mathbf{y}^{-1}(d)\mathbf{y} \tag{16}$$

Table I
SIMULATION SETTINGS

OFDM system: $N = 512$, $N_p = 103$, $\Delta f = 15$ kHz, $T_p = T + T_{cp} = 66.7 + 5.4 = 72.1\,\mu$s
Estimation range $d_{obs} \in [0, 7\,\text{km}]$; True range: $d = 1\,\text{km}$.   Results obtained from 10000 Monte Carlo trials are displayed.

with $x \propto z$ denoting $x = z + constant$ and $\det(\cdot)$ denoting the determinant. Using the eigenvalue decomposition $\mathbf{C}_\varepsilon = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^H$, we can recast (6) as

$$\mathbf{C}_\mathbf{y}(d) = \mathbf{R}(d)\mathbf{G}\mathbf{R}^H(d),$$

with $\mathbf{G} = \mathbf{A}\mathbf{U}(\mathbf{\Lambda} + \mathbf{I}\sigma^2/E_s)\mathbf{U}^H\mathbf{A}^H$. Since $\mathbf{R}(d)$ is unitary, the determinant $\det(\mathbf{C}_\mathbf{y}(d)) = \det(\mathbf{G})$ does not depend on d and can be dropped. Thus,

$$\ln \tilde{p}(\mathbf{y}|d) \propto -\mathbf{y}^H\mathbf{R}(d)\mathbf{G}^{-1}\mathbf{R}^H(d)\mathbf{y}, \tag{17}$$

where

$$\mathbf{G}^{-1} = \mathbf{A}\mathbf{U}(\mathbf{\Lambda} + \frac{\sigma^2}{E_s}\mathbf{I})^{-1}\mathbf{U}^H\mathbf{A}^H. \tag{18}$$

Since the matrices $\mathbf{A}\mathbf{U}$ and $\mathbf{\Lambda}$ can be pre-computed and stored, the inversion of $\mathbf{G}$ amounts to compute the diagonal matrix $(\mathbf{\Lambda} + \frac{\sigma^2}{E_s}\mathbf{I})^{-1}$. This circumvents the brute force inversion of $\mathbf{C}_\mathbf{y}(d)$ in (16) and thereby reduces the complexity of the estimator. We remark that $\mathbf{C}_\varepsilon$ and thus $\mathbf{U}$ and $\mathbf{\Lambda}$ depend on the parameters of $P(\tau)$. It can be shown that the non-coherent correlator-based estimator [2] [3] [6] is a limiting case of the proposed estimator (15) when $q \to \infty$, which implies a single path channel.

IV. NUMERICAL PERFORMANCE EVALUATION

We first evaluate the performance of estimator (15) in a multi-path channel with different parameter settings and contrast it with the performance of the non-coherent correlator-based estimator [2] [6]. We omit the comparison with the energy detector, which is sensitive to the selected threshold value and provides inaccurate TOA estimates [3] [13]. We also omit the comparison with multidimensional search approach, because these estimators require access to L, which is assumed to be unknown in this work [4] [7]. We then report the performance of estimator (15) when there is a mismatch between the channel assumptions made for its derivation and the real channel conditions in which it is used. Specifically, we say that there is a mismatch if a LOS (NLOS) condition prevails in the channel, while the used estimator is the one derived under the assumption of NLOS (LOS). Otherwise there is a match. Remember that the factor q controls which of the LOS ($q = 1$) or NLOS ($q = 0$) condition holds. Table I summarizes the settings used for the simulations of the considered OFDM system. Pilots with equal power are placed either with equal spacing (Uniform pilot pattern) or randomly (Random pilot pattern) in an OFDM symbol. A random pilot pattern is generated by sampling N_p pilots uniformly at random without replacement from $\mathbf{I}$. In the Monte Carlo simulation, we use the channel model in Section II Example 3.
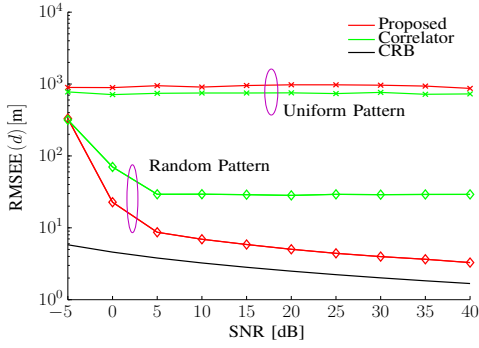
Figure 1. LOS scenario: RMSEE obtained by using different estimators and pilot patterns for a multi-path channel with RMS delay spread $\lambda = 50$ ns and average number of paths $\mu_L = 480$.
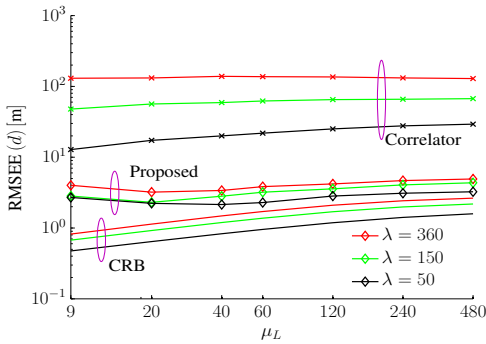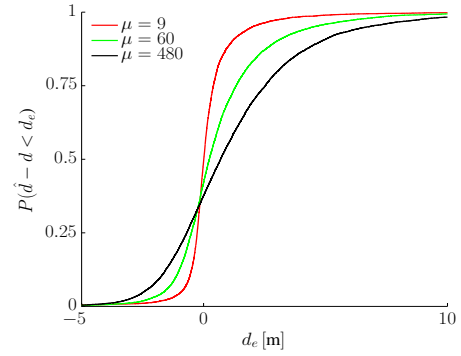

Figure 3. LOS scenario: Empirical CDF of the range errors obtained using (15) when RMS delay spread $\lambda = 360$ ns and SNR $= 40$ dB.

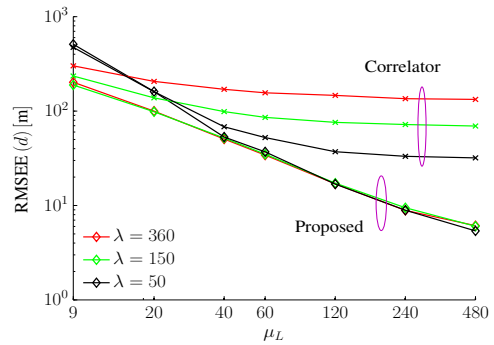Figure 2. LOS scenario: RMSEE versus average number of paths μ_L for different values of the RMS delay spread λ at SNR $= 40$ dB.

Figure 4. NLOS scenario: RMSEE versus the average number of paths μ_L for different values of RMS delay spread λ at SNR $= 40$ dB.

A. LOS scenario: Performance Evaluation Using Different Pilot Patterns and Estimators

Fig. 1 shows the simulated root mean square estimation error (RMSEE) of d using estimator (15) and the correlator-based estimator. We observe that for both estimators, the uniform pilot pattern leads to outliers due to high side-lobes in the respective objective functions. This effect does not occur when the random pilot pattern is used. We observe in this case that the proposed estimator outperforms the correlator-based estimator. We then compare the results with the CRB [16], which is computed under the assumption that ε and $\mathbf{y}$ are jointly Gaussian. Since such assumption is not fulfilled here, the simulated RMSEE does not meet the CRB.

B. Performance Evaluation Under Different Channel Settings

From this point on, we only report the results obtained by employing a random pilot pattern. Fig. 2 reports the simulated RMSEE in the LOS scenario. We observe that estimator (15) outperforms significantly the correlator-based estimator. As the average number of paths increases, the performance of the correlator-based estimator deteriorates. When μ_L is small, i.e. the Gaussian assumption is significantly violated, the RMSEE of estimator (15) noticeably deviates from the CRB. Such deviation becomes smaller as μ_L increases. Moreover, the accuracy of estimator (15) increases as the RMS delay

spread decreases. Fig. 3 depicts the cumulative distribution function (CDF) of the range errors in the LOS scenario. We observe that the medians are positive which indicates that positive errors are more frequent than negative errors. As μ_L decreases, the corresponding CDF shows a sharper slope and the median decreases accordingly. We remark that rare large outliers appear when μ_L is small, which lifts the overall RMSEE up as shown in Fig. 2.

Fig. 4 reports the simulated RMSEE in the NLOS scenario. Contrary to the LOS scenario, the RMSEE decreases as μ_L increases and the proposed estimator's performance becomes insensitive to the RMS delay spread when this parameter is large enough. When μ_L and the RMS delay spread of the channel are small, in which case the Gaussian assumption is significantly violated, both estimators yield large errors. However, compared to the correlator-based estimator, estimator (15) exhibits a promising performance gain when μ_L and the RMS delay spread of the channel are large. Fig. 5 depicts the CDF of the range errors in NLOS scenarios. We notice that μ_L affects the proposed estimator's performance. Contrary to what was observed in Fig. 3, the median increases as μ_L decreases. When $\mu_L = 9$, the median is at around 80 m, which explains the high RMSEE in Fig. 4.
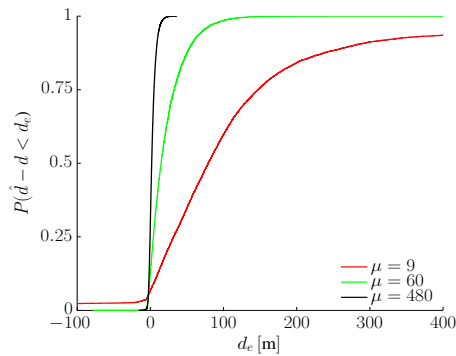
Figure 5. NLOS scenario: Empirical CDF of the range errors obtained using (15) when $\lambda = 360$ ns and SNR $= 40$ dB.
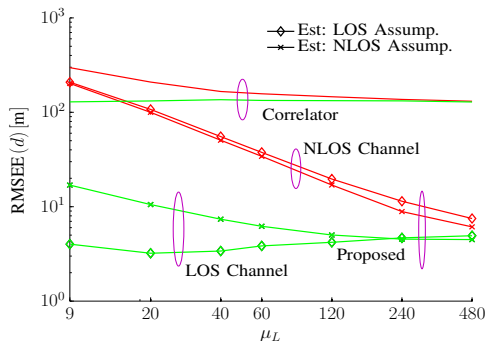

Figure 6. RMSEE versus average number of paths μ_L for RMS delay spread $\lambda = 360$ ns and SNR $= 40$ dB. Green and red curves indicate the real channel conditions. "Est: LOS Assump." shows the RMMSE of estimator (15) assuming that the real propagation channel is in LOS conditions while "Est: NLOS settings" denotes the RMMSE of estimator (15) assuming that the real channel is in NLOS conditions.

C. Performance Comparison in Conditions with Model Match and Mismatch

Fig. 6 reports the simulated RMSEE when the assumptions used to derive estimator (15) match or mismatch the real channel propagation conditions. Clearly, estimator (15) still outperforms the correlator-based estimator and both estimators benefit from the LOS propagation channel, which leads to lower RMSEEs. In case of a mismatch, estimator (15) performs worse than when there is a match. This is particularly noticeable in LOS conditions with small μ_L. Except for the matched case when the real channel is in LOS conditions, the RMSEE decreases as μ_L increases due to the Gaussian assumption becomes more realistic.

V. Conclusion

Using a channel model formulated as a point process, we demonstrate that the proposed approximate maximum likelihood estimator outperforms the correlator-based estimator. In the single-path scenario, the correlator-based estimator coincides with the proposed estimator. The proposed estimator does not require first-path detection, path separability, nor estimation of the number of path components. Though the invoked Gaussian assumptions is not fulfilled in typical channel conditions, the proposed estimator achieves promising range accuracy. An additional finding is that both proposed and correlator-based estimators achieve higher estimation accuracy when a random pilot pattern is employed rather than the uniform pilot pattern, as currently used in LTE.

Given the SNR and the covariance matrix of the channel, the complexity of the proposed estimator is tractable. The estimator accuracy depends on the RMS delay spread and the average number of path components. The proposed estimator achieves promising results in the LOS scenario even if there is a mismatch between the assumptions used to derive the proposed estimator and the real channel conditions. In the NLOS scenario, the average number of path components limits the estimator's performance in both matched and mismatched cases.

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