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# Tracking Controller for Intrinsic Output Saturated Systems in Presence of Amplitude and Rate Input Saturations

E. Chater, F. Giri, Josep M. Guerrero, F.Z. Chaoui, M. Haloua

**Abstract:** We consider the problem of controlling plants that are subject to multiple saturation constraints. Especially, we are interested in linear systems whose input is subject to amplitude and rate constraints of saturation type. Furthermore, the considered systems output is also subject to an intrinsic saturation. Then, aiming to design global (and asymptotic) output tracking of compatible output-reference trajectories, we propose a suitable saturated linear controller following the direct approach (where the constraints are taken into account during the regulator design). The (non-linear) closed-loop control system is analyzed using input-output stability tools. Thus, conditions guaranteeing  $l_2$ -tracking performances are formally defined. Interestingly, the proposed controller is shown to ensure perfect output-reference tracking in presence of varying with  $l_2$ -vanishing rate inputs. On the other hand, in the case of arbitrary inputs, the proposed controller guarantees that the less changing the inputs are the better the output-reference tracking.

## I. INTRODUCTION

Studies that address stabilization in presence of amplitude and rate saturation started to appear only in the 90's. A first semi-global stabilization result is given in [1]. Then, using a low and high gain approach, solutions to global and semi-global stabilization problems are provided in [2]. The global stabilization problem was also stated using scheduled low gain state feedback [3]. Nonetheless, it should be noticed that the above works have only considered continuous-time systems and the rate limitation was considered in the modelling of the actuator using the so-called position-feedback-type model [4], which seems to be unsuitable when dealing with the rate saturation phenomenon in discrete-time context. Thus, in order to deal with the rate saturation problem, an alternative approach has been proposed for discrete-time systems [5]. The idea consists of introducing a rate limiter inside the controller, which was performed by adding a nonlinear integrator in the controller structure. On the other hand, the asymptotic stabilization of linear systems with input and output saturation was also dealt in many works (e.g. [6], [7] and [8]).

Presently, considering the discrete time context, this paper develops a control strategy for output saturated linear

systems that are driven by an actuator whose output is also subject to amplitude and rate saturations. To deal with all these constraints, in addition to possible external disturbances, similar ideas to the ones proposed in [5], [8] and [11] are used. However, the above papers have not tackled the tracking issue in presence of the above three constraints with disturbance rejection, as either the focus was only put on closed-loop asymptotic stabilization [5] either the study did not consider all these constraints simultaneously ([8], [11]).

Therefore, in this paper, we consider the problem of controlling linear systems that are subject to multiple saturation constraints. Then, aiming to ensure tracking of any compatible output-reference sequences, a suitable nonlinear controller is designed. Thereafter, using input-output tools [9], the closed loop system is analyzed, which allows getting the necessary real positivity conditions that guarantee  $l_2$ -tracking performances. Roughly, it is shown that perfect output-reference tracking could be ensured in presence of varying with  $l_2$ -vanishing rate inputs (output references and external disturbances). Moreover, in case these inputs are arbitrary, the tracking error power is shown to be bounded (up to a multiplicative gain  $\alpha$ ) by the inputs rate power, which means that, the less changing (in the mean) the inputs are, the better the average (output-reference) tracking quality.

This paper is organized as follows: the control problem statement is formally given in Section 2; the controller design is given in Section 3 and the closed-loop is analyzed in Section 4. Then, the controller performances are illustrated by an example in Section 5 and a conclusion ends the paper.

For clarity, notations that are used throughout this paper are as follows:  $q^{-1}$  is the backward shift operator,  $\Delta$  stands for the polynomial  $(1 - q^{-1})$ , which means that  $\Delta X(q^{-1}) = (1 - q^{-1})X(q^{-1})$  for any polynomial  $X(q^{-1})$  and  $\{\Delta s(t)\}$  represents the increment sequence  $\{s(t) - s(t-1)\}$  of any real sequence  $\{s(t)\}$ .

## II. CONTROL PROBLEM STATEMENT

In discrete-time context (*i.e.* for  $t \in \mathbb{N}$ ), the controlled plant is described as follows:

$$x(t) = (1 - A(q^{-1})) \text{sat}(x(t), x_M) + B(q^{-1})u(t) \quad (1)$$

$$y(t) = \text{sat}(x(t), x_M) + \eta(t) \quad (2)$$

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$$u(t) = \text{sat}(v(t), u_M) \quad (3)$$

$$|\Delta u(t)| \leq u_D \quad (4)$$

The rate constraint (4) is assumed to be imposed either by the actuator either by the system dynamics.

In (1),  $A(q^{-1})$  and  $B(q^{-1})$  are polynomial of the form:

$$A(q^{-1}) = 1 + a_1 q^{-1} + \dots + a_{na} q^{-na} \quad (5)$$

$$\begin{aligned} B(q^{-1}) &= q^{-d} b_0 + b_1 q^{-d-1} + \dots + b_{nb} q^{-d-nb} \\ &:= q^{-d} B^*(q^{-1}) \end{aligned} \quad (6)$$

In (1-4),  $y(t)$  and  $x(t)$  are the saturated and the unsaturated plant output, respectively.  $\eta(t)$  is a bounded signal accounting for modeling errors and external disturbances.  $v(t)$  and  $u(t)$  denote the actuator input and output, respectively.  $\Delta u(t)$  is the increment of  $u(t)$ .  $u_M$ ,  $u_D$  and  $x_M$  are the maximal magnitudes of  $u(t)$ ,  $\Delta u(t)$  and  $x(t)$  respectively.  $na$ ,  $nb$  and  $d$  are nonnegative integers. The  $a_i$ 's and  $b_i$ 's are arbitrary real numbers with  $b_0 \neq 0$ .

**Remark 1.** The considered plant model (1-2) does account for the intrinsic feedback that characterizes most physical output saturated linear systems. Accordingly, the internal state  $x(t)$  does depend not only on the input  $u(t)$  but also on its saturated value  $\text{sat}(x(t), x_M)$ . This fact is emphasized by the feedback loop in the block diagram of Fig. 1. However, if  $x(t)$  stops saturating for at least  $na$  successive sampling periods, the plant may simply be described by the following linear model,

$$A(q^{-1})y(t) = B(q^{-1})u(t) + A(q^{-1})\eta(t) \quad (7)$$

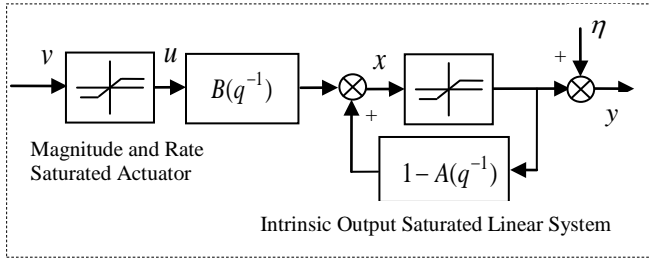


Fig. 1: Actuator and output saturated linear system

Now, taking into account the above saturation constraints, our aim is to design a controller able to achieve the following objectives:

- i). (Global) bounded input bounded output stability, provided the disturbance  $\eta(t)$  is bounded.
- ii). The output  $y(t)$  must match (as closely as possible) any compatible output-reference trajectory  $y_r(t)$ .

Let us recall that global stabilization of input saturated systems is only possible for asymptotically or marginally stable systems [10]. Then, in view of our objective, the considered class of systems should necessarily meet this

requirement. That is, the achievement of closed-loop stability, together with the output-reference matching property, requires the following assumption:

**A1.**  $A(q^{-1})$  is Schur.

On the other hand, even if the disturbance signal  $\eta(t)$  is absent, the output-reference trajectories should be compatible with the input and output (amplitude) limitations. In this sense, this leads to the necessary condition:

$$\text{Sup}_t (|y_r(t)|) \leq \min_t (\gamma_\infty (B(q^{-1})/A(q^{-1})) u_M, x_M) \quad (8)$$

where  $\gamma_\infty (B(q^{-1})/A(q^{-1}))$  denotes the  $l_\infty$ -gain of the linear operator  $B(q^{-1})/A(q^{-1})$ .

**Remark 2.** Assumption (A1) only ensures that all zeros of  $z^{na} A(z^{-1})$  should be inside the open disk. Of course, this allows that the plant dynamics could be inversely unstable.

Therefore, this tracking problem is not an obvious issue as one is presently facing two difficulties: the presence of saturations and the possible non-minimum phase nature of the plant dynamics. Then, even if the actuator saturations were absent, perfect matching of arbitrary-shape output-references would not be possible since non-minimum phase dynamics are involved.

Now, as global asymptotic tracking are sought, let us make use of the fact that the controlled subsystem should be asymptotically stable. First, by including an integrator in the controller, it turns out that perfect tracking may be achievable in the absence of saturations for constant inputs. Then, the tracking objective may make the tracking quality dependent on the input rates  $\Delta y_r(t)$  and  $\Delta \eta(t)$ . This fact could be formalized by requiring that the mapping [11]:

$$\delta(t) \rightarrow \tilde{y}(t) \text{ is } l_2\text{-stable with finite gain,} \quad (9)$$

where

$$\delta(t) := [\Delta y_r(t) \ \Delta \eta(t)]^T, \text{ input rate vector} \quad (10)$$

$$\tilde{y}(t) := \{y(t) - y_r(t)\}, \text{ output tracking error.} \quad (11)$$

Indeed, the  $l_2$ -stability of (9) means that there exists a pair of positive real constants  $(\alpha, \beta)$  so that, for any bounded input  $\delta(t)$  and any integer  $T > 0$ , one gets:

$$\left( \sum_{t=0}^T |\tilde{y}(t)|^2 \right)^{1/2} \leq \alpha \left( \sum_{t=0}^T \|\delta(t)\|^2 \right)^{1/2} + \beta \quad (12)$$

Then, using the inequality  $(a+b)^{1/2} \leq a^{1/2} + b^{1/2}$  with  $(a, b > 0)$ , these statements, referred to ' $l_2$ -tracking performance', entail the following features:

- i) If  $\delta(t) \in l_2$ , one gets from (10) and (12):

$$\begin{aligned} \|\tilde{y}\|_2 &:= \limsup_{T \rightarrow \infty} \left( \sum_{t=0}^T |\tilde{y}(t)|^2 \right)^{1/2} \leq \alpha \limsup_{T \rightarrow \infty} \left( \sum_{t=0}^T \|\delta(t)\|^2 \right)^{1/2} + \beta \\ &\leq \alpha \left( \|\Delta y_r(t)\|_2 + \|\Delta \eta(t)\|_2 \right) + \beta \quad (13) \end{aligned}$$

ii) In the general case (i.e.  $\delta(t) \in l_\infty$ ), dividing both sides of (12) by  $T^{1/2}$  and letting  $T \rightarrow \infty$ , one gets:

$$\begin{aligned} \limsup_{T \rightarrow \infty} \left( \frac{1}{T} \sum_{t=0}^T |\tilde{y}(t)|^2 \right)^{1/2} &\leq \alpha \limsup_{T \rightarrow \infty} \left( \frac{1}{T} \sum_{t=0}^T (\Delta y_r(t))^2 \right)^{1/2} \\ &\quad + \alpha \limsup_{T \rightarrow \infty} \left( \frac{1}{T} \sum_{t=0}^T (\Delta \eta(t))^2 \right)^{1/2} \end{aligned}$$

Then, it follows that:

$$\|\tilde{y}(t)\|_{ap} \leq \alpha \left( \|\Delta y_r(t)\|_{ap} + \|\Delta \eta(t)\|_{ap} \right) \quad (14)$$

where the following notation is used:

$$\begin{aligned} \|s\|_{ap} &:= \limsup_{T \rightarrow \infty} \left( \frac{1}{T} \|s_T\|^2 \right)^{1/2} \\ &= \limsup_{T \rightarrow \infty} \left( \frac{1}{T} \sum_{t=0}^{T-1} |s(t)|^2 \right)^{1/2} \quad (15) \end{aligned}$$

for any real sequence  $s(t)$ .

**Remark 3.**

i) Recall that, in discrete-time context, if signals  $\Delta y_r(t)$  and  $\Delta \eta(t)$  do belong to  $l_2$  then, they converge to zero. Thus, (13) implies that  $\tilde{y}(t) \rightarrow 0$ , which means that

perfect asymptotic tracking will be achieved for varying inputs with  $l_2$ -vanishing rate.

ii) In case of not  $l_2$ -vanishing rate inputs, the statement (14) means that the tracking error power is bounded (up to a multiplicative gain  $\alpha$ ) by the power of the input rate. That is, the less changing the inputs are, the better the average tracking quality. Indeed, this is an appealing control feature since, except for boundedness, no assumption is made on the inputs  $\{y_r(t)\}$  and  $\{\eta(t)\}$ .

Now, as the inputs  $y_r(t)$  and  $\eta(t)$  are bounded, there exists a positive real  $\mu_{ap}$  such that:

$$\left( \|\Delta y_r(t)\|_{ap} + \|\Delta \eta(t)\|_{ap} \right) = \mu_{ap} \quad (16)$$

Thus, the control objective can be reformulated as follows:

*Find a controller such that there exist real constants,  $\mu > 0$  and  $K > 0$ , so that if  $0 \leq \mu_{ap} \leq \mu$  then  $\|\tilde{y}(t)\|_{ap} \leq K \mu_{ap}$ .*

### III. CONTROLLER SYNTHESIS

First, let us notice that, by considering the tracking error  $\tilde{y}(t)$ , the initial tracking problem could be transformed into a

regulation one. Then, the tracking error  $\tilde{y}(t)$  and the control input increment  $\Delta u(t)$  will represent the main performance indices of the proposed control strategy.

Now, as  $A(q^{-1})$  and  $B^*(q^{-1})$  are not necessarily coprime, let us factorize these polynomials:

$$\begin{cases} A(q^{-1}) = \Lambda(q^{-1})A_0(q^{-1}) \\ B^*(q^{-1}) = \Lambda(q^{-1})B_0(q^{-1}) \end{cases} \quad (17)$$

Then, using assumption (A1),  $\Lambda(q^{-1})$  is Schur and may be of the form:

$$\Lambda(q^{-1}) = 1 + \lambda_1 q^{-1} + \dots + \lambda_m q^{-m} \quad (18)$$

with  $0 \leq m \leq \min(na, nb)$ .

In addition, let  $C(q^{-1})$  be any Schur polynomial of the form:

$$C(q^{-1}) = 1 + c_1 q^{-1} + \dots + c_{nc} q^{-nc} \quad (19)$$

with  $0 \leq nc \leq 2n - 1 - m$  and  $n = \max(na + 1, nb + d)$ .

To alleviate the text, let us define:

$$P(q^{-1}) = C(q^{-1})\Lambda(q^{-1}) \quad (20)$$

Therefore, as  $\Delta A_0(q^{-1})$  and  $B_0(q^{-1})$  are coprime, solving the following Diophantine-Bezout equation:

$$\Delta A(q^{-1})R(q^{-1}) + B(q^{-1})S(q^{-1}) = P(q^{-1}), \quad (21)$$

guarantees the existence and uniqueness of polynomials pair:

$$\begin{cases} R(q^{-1}) = 1 + r_1 q^{-1} + \dots + r_{n-1} q^{-n+1} \\ S(q^{-1}) = s_0 + s_1 q^{-1} + \dots + s_{n-1} q^{-n+1} \end{cases} \quad (22)$$

Thus, using these notations, the proposed regulator is given by:

$$w(t) = (1 - R(q^{-1}))\Delta u(t) - S(q^{-1})\tilde{y}(t) \quad (23)$$

$$w'(t) = \text{sat}(w(t), u_D) \quad (24)$$

$$v(t) = w'(t) + u(t-1) \quad (25)$$

$$u(t) = \text{sat}(v(t), u_M) \quad (26)$$

In (23-26),  $w(t)$  and  $w'(t)$  are respectively the computed and the saturated values of the control input rate. Similarly,  $v(t)$  and  $u(t)$  are respectively the computed and the saturated values of the control input.

**Remark 4.**

*In case the saturation nonlinearities are no longer active, it is readily seen that (23-26) become:*

$$R(q^{-1})\Delta u(t) + S(q^{-1})\tilde{y}(t) = 0, \quad (27)$$

which corresponds to a unitary feedback linear regulator.

#### IV. CONTROL SYSTEM ANALYSIS

The performances of the proposed regulator, especially its ability to track compatible slowly varying output-reference trajectories will now be established. The following theorem shows that the control system is actually bounded input bounded output (BIBO) stable and gives necessary conditions that ensure  $l_2$ -tracking performances to the overall control system.

**Theorem 1.** Consider the system (1-4), subject to assumption (A1), in closed-loop with the saturated controller (23-26). Suppose that the disturbance  $\eta(t)$  is bounded, the output-reference trajectory  $y_r(t)$  verifies the compatibility condition (8) and the pair  $(y_r(t), \eta(t))$  is slowly varying in the mean with a mean rate  $\mu_{ap}$  as defined in (16). Let  $P(z^{-1})$  be any Schur polynomial of the form (20). Then,

1) All signals of the closed-loop remain bounded whatever their finite initial conditions.

2) If  $P(z^{-1})$  is chosen such that,

$$\inf_{0 \leq \omega < 2\pi} \operatorname{Re} \left( \frac{\Delta A(e^{-j\omega})}{P(e^{-j\omega})} \right) > 0, \quad (28)$$

$$\inf_{0 \leq \omega < 2\pi} \operatorname{Re} \left( \frac{\Delta R(e^{-j\omega})}{P(e^{-j\omega})} \right) > 0 \quad (29)$$

$$\frac{1 + \gamma_a}{1 - \gamma_a} \frac{1 + \gamma_b}{1 - \gamma_b} \gamma_2 \left( \frac{\Delta B(e^{-j\omega}) S(e^{-j\omega})}{(P(e^{-j\omega}))^2} \right) < 1 \quad (30)$$

where

$$\gamma_a = \gamma_2 \left( \frac{\Delta A(e^{-j\omega}) - P(e^{-j\omega})}{\Delta A(e^{-j\omega}) + P(e^{-j\omega})} \right) \quad (31)$$

$$\gamma_b = \gamma_2 \left( \frac{\Delta R(e^{-j\omega}) - P(e^{-j\omega})}{\Delta R(e^{-j\omega}) + P(e^{-j\omega})} \right) \quad (32)$$

Then, the  $l_2$ -tracking performance (9) is achieved and consequently, the controller (23-26) enjoys the tracking features (14). Accordingly,

a) if  $\Delta y_r \in l_2$  and  $\Delta \eta \in l_2$  then,  $\{\tilde{y}\}, \{\Delta y\}, \{\Delta x\}, \{w - \Delta u\}$  and  $\{v - u\}$  belong to  $l_2$  (33)

b) if  $y_r \in l_\infty$  or  $\eta \in l_\infty$ , there exists a constant  $K > 0$ , such that :

$$\left\{ \begin{array}{l} \|\tilde{y}\|_{ap} \leq K\mu_{ap}, \{\Delta y\} \leq K\mu_{ap}, \{\Delta x\} \leq K\mu_{ap} \\ \|w - \Delta u\|_{ap} \leq K\mu_{ap}, \|v - u\|_{ap} \leq K\mu_{ap} \end{array} \right\} \quad (34)$$

Because of space limitation, the proof of the above theorem is omitted. Nevertheless, the corresponding author invites any reader to contact him if he is interested in the

details of the proof.

#### Remarks 4.

- i) As finite-order linear systems are concerned,  $l_2$ -stability is equivalent to global exponential stability. Then, if the mapping (9) is  $l_2$ -stable, it is globally asymptotically stable. Thus, the  $l_2$ -tracking feature (33) entails the achievement of perfect asymptotic tracking for varying inputs with  $l_2$ -vanishing rate.
- ii) The tracking property (34) is an appealing control feature as it holds in presence of arbitrary inputs. Thus, the average tracking quality depends on the power of the input rates. Nonetheless, as the linear dynamics is non-minimum phase, the closed-loop system could not track well very fast-changing sequences.

#### V. NUMERICAL EXAMPLE

Let us consider a system that is described by (1-4) with:

$$\left. \begin{array}{l} q^{-d} B^*(q^{-1}) = q^{-1}(1 - 3q^{-1}) \\ A(q^{-1}) = 1 - 0.8q^{-1} \end{array} \right\} \quad (35)$$

It is readily seen that the linear dynamics (35) does comply with assumption (A1). However, this system is obviously non-minimum phase.

The (actuator and system) saturation constraints are characterized by:

$$u_M = 1, u_D = 1 \text{ and } x_M = 10 \quad (36)$$

Then, using the proposed design control strategy, the polynomial  $P(q^{-1})$  is chosen bearing in mind conditions (28-30). Accordingly, the following (non-unique) choice is made:

$$P(q^{-1}) = (1 - 0.55q^{-1})^3 \quad (37)$$

Solving equation (21) gives:

$$\left. \begin{array}{l} R(q^{-1}) = 1 + 0.3423q^{-1} \\ S(q^{-1}) = -0.1923 + 0.1467q^{-1} \end{array} \right\} \quad (38)$$

From the Nyquist plots, it can be easily seen that conditions (28-29) are verified. Moreover, it can easily be checked that:

$$\frac{1 + \gamma_a}{1 - \gamma_a} \frac{1 + \gamma_b}{1 - \gamma_b} \gamma_2 \left( \frac{\Delta B(e^{-j\omega}) S(e^{-j\omega})}{(P(e^{-j\omega}))^2} \right) = 0.8103 < 1 \quad (39)$$

which means that the condition (30) is verified.

The reference trajectory  $\{y_r(t)\}$  is a periodic square signal switching between -10 and +10, which is the maximum allowed compatible value. Then, the resulting control performances are illustrated by in Figs. (3-5).

The system output tracking is then ensured as, in steady state,  $y(t)$  is almost confounded with its reference  $y_r(t)$  (Fig. 3).

In addition, the control action  $u(t)$  and its rate  $\{\Delta u(t)\}$  are shown to remain inside their physical limits (Fig. 4-5). Now, in order to appreciate the relevance of conditions (28-30), the choice

$$P(q^{-1}) = 1 - 2.4q^{-1} + 1.92q^{-2} - 0.512q^{-3} \quad (40)$$

not satisfying condition (30) is considered. The resulting performance deterioration is clearly illustrated in (Fig. 6-7).

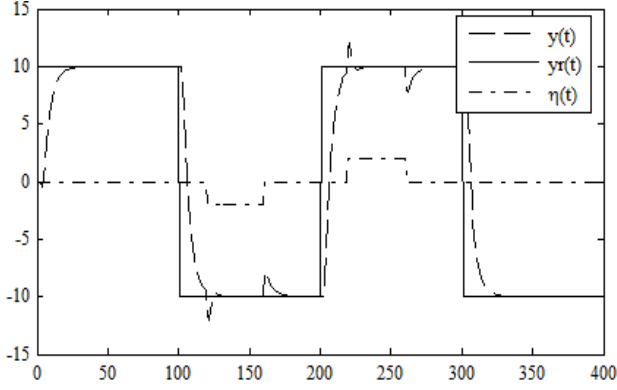


Fig. 3: Output  $y(t)$ , reference  $y_r(t)$  and disturbance  $\eta(t)$  when conditions (28-30) are not verified.

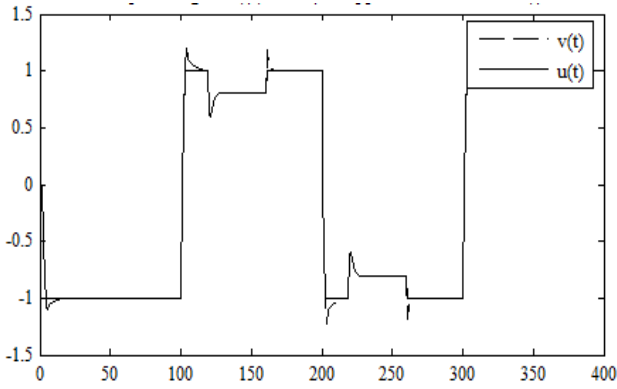


Fig. 4: Computed input  $v(t)$  vs Applied control input  $u(t)$  when conditions (28-30) are not verified.

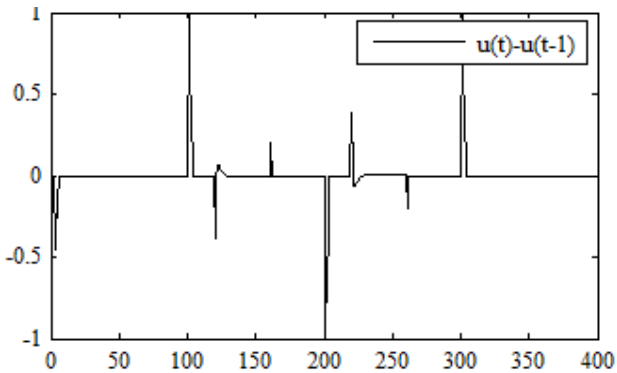


Fig. 5: Control input rate sequence  $\{u(t) - u(t-1)\}$  when conditions (28-30) are not verified.

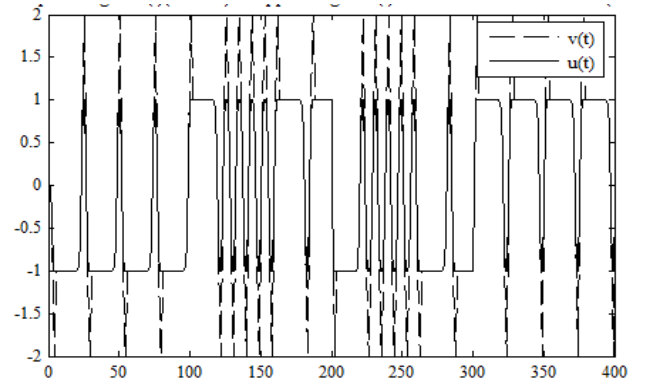


Fig. 6: Computed input  $v(t)$  vs Control input  $u(t)$  when conditions (28-30) are not verified.

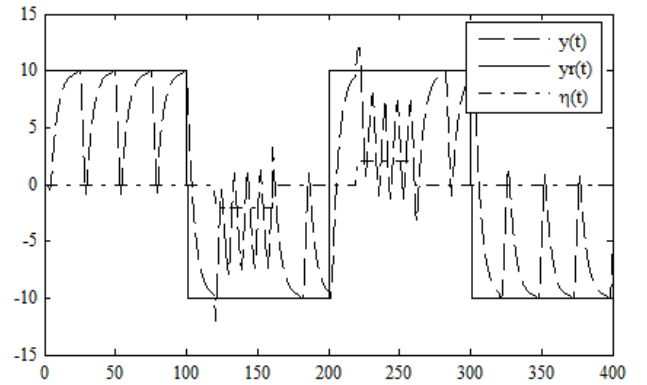


Fig. 7: Output  $y(t)$ , reference  $y_r(t)$  and disturbance  $\eta(t)$  when conditions (28-30) are not verified.

## VI. CONCLUSION

The problem of controlling systems subject to multiple saturations has been addressed. Thus, a magnitude and rate saturations on the control input and an intrinsic saturation on the plant output were all considered. First, a suitable saturated controller is designed, bearing in mind the aim to ensure global asymptotic tracking of compatible output-reference trajectories and the rejection of bounded disturbances. Then, using input-output stability tools (circle criterion and small gain theorem), the closed loop analysis shows that if the regulator parameters are chosen so that conditions (28-30) hold, then the entire control system is asymptotically stable, which guarantees perfect output-reference tracking if the inputs are varying with  $l_2$ -vanishing rate. In addition, in case of arbitrary bounded inputs, it turns out that the less changing the inputs are, the better the average tracking quality. Of course, this is an interesting result in presence of multiple saturations and non-minimum phase dynamics.

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