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Robust Multivariate Regression

Hasthika S. Rupasinghe Arachchige Don
hasthika@siu.edu

ROBUST MULTIVARIATE LINEAR REGRESSION

by

Rupasinghe Arachchige Don Hasthika Sriyantha

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A Research Paper
Submitted in Partial Fulfillment of the Requirements for the
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Dr. David J. Olive, Chair

Dr. Salah-Eldin Mohammed

Dr. Bhaskar Bhattacharya

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A robust multivariate linear regression estimator can be obtained by replacing the least squares estimator with the robust `hbreg` estimator. Then the robust multivariate linear regression estimator is asymptotically equivalent to the classical multivariate linear regression estimator since the probability that the robust estimator is equal to the classical estimator goes to one in probability as the sample size $n \to \infty$ for a large class of iid zero mean error distributions. This paper discusses the robust estimator and some tests using the robust estimator that are asymptotically equivalent to those using the classical estimator.

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CHAPTER 1

INTRODUCTION

Olive (2013b), using results from Su and Cook (2012) and Kakizawa (2009), derived a useful prediction region for the classical multivariate linear regression model, and gave F approximations to the widely used Wilk, Pillai, and Hotelling Lawley test statistics. This paper will show that these large sample tests also work for the robust multivariate linear regression estimator that replaces least squares with the `hbreg` estimator. This section reviews the multivariate linear regression model and the results from Olive (2013b). Section 1.3 reviews the `hbreg` estimator and derives the robust estimator and section 1.4 gives some examples and simulations.

1.1 THE MULTIVARIATE LINEAR REGRESSION MODEL

The *multivariate linear regression model* $\boldsymbol{y}_i = \boldsymbol{B}^T \boldsymbol{x}_i + \boldsymbol{\epsilon}_i$ for $i = 1, ..., n$ has $m \geq 2$ response variables $Y_1, ..., Y_m$ and p predictor variables $x_1, x_2, ..., x_p$. The ith case is $(\boldsymbol{x}_i^T, \boldsymbol{y}_i^T) = (x_{i1}, x_{i2}, ..., x_{ip}, Y_{i1}, ..., Y_{im})$ where the constant $x_{i1} = 1$ could be omitted from the case. The model is written in matrix form as $\boldsymbol{Z} = \boldsymbol{X}\boldsymbol{B} + \boldsymbol{E}$ where the matrices are defined below. The model has $E(\boldsymbol{\epsilon}_k) = \boldsymbol{0}$ and $\text{Cov}(\boldsymbol{\epsilon}_k) = \boldsymbol{\Sigma}_{\boldsymbol{\epsilon}} = (\sigma_{ij})$ for $k = 1, ..., n$. Also $E(\boldsymbol{e}_i) = \boldsymbol{0}$ while $\text{Cov}(\boldsymbol{e}_i, \boldsymbol{e}_j) = \sigma_{ij} \boldsymbol{I}_n$ for $i, j = 1, ..., m$ where $\boldsymbol{I}_n$ is the $n \times n$ identity matrix and $\boldsymbol{e}_i$ is defined below. Then the $p \times m$ coefficient matrix $\boldsymbol{B} = \begin{bmatrix} \boldsymbol{\beta}_1 & \boldsymbol{\beta}_2 & \dots & \boldsymbol{\beta}_m \end{bmatrix}$ and the $m \times m$ covariance matrix $\boldsymbol{\Sigma}_{\boldsymbol{\epsilon}}$ are to be estimated, and $E(\boldsymbol{Z}) = \boldsymbol{X}\boldsymbol{B}$ while $E(Y_{ij}) = \boldsymbol{x}_i^T \boldsymbol{\beta}_j$.

The $n \times m$ matrix of response variables

$$\boldsymbol{Z} = \begin{bmatrix} \boldsymbol{Y}_1 & \boldsymbol{Y}_2 & \dots & \boldsymbol{Y}_m \end{bmatrix} = \begin{bmatrix} \boldsymbol{y}_1^T \\ \vdots \\ \boldsymbol{y}_n^T \end{bmatrix}.$$

The $n \times p$ design matrix of predictor variables

$$\boldsymbol{X} = \begin{bmatrix} \boldsymbol{v}_1 & \boldsymbol{v}_2 & \dots & \boldsymbol{v}_p \end{bmatrix} = \begin{bmatrix} \boldsymbol{x}_1^T \\ \vdots \\ \boldsymbol{x}_n^T \end{bmatrix}$$

where $\boldsymbol{v}_1 = \mathbf{1}$.

The $n \times m$ matrix of errors

$$\boldsymbol{E} = \begin{bmatrix} \boldsymbol{e}_1 & \boldsymbol{e}_2 & \dots & \boldsymbol{e}_m \end{bmatrix} = \begin{bmatrix} \boldsymbol{\epsilon}_1^T \\ \vdots \\ \boldsymbol{\epsilon}_n^T \end{bmatrix}.$$

Least squares is the classical method for fitting the multivariate linear model. The *least squares estimators* are $\hat{\boldsymbol{B}} = (\boldsymbol{X}^T\boldsymbol{X})^{-1}\boldsymbol{X}^T\boldsymbol{Z} = \begin{bmatrix} \hat{\boldsymbol{\beta}}_1 & \hat{\boldsymbol{\beta}}_2 & \dots & \hat{\boldsymbol{\beta}}_m \end{bmatrix}$. The *predicted values* or *fitted values* $\hat{\boldsymbol{Z}} = \boldsymbol{X}\hat{\boldsymbol{B}} = \begin{bmatrix} \hat{\boldsymbol{Y}}_1 & \hat{\boldsymbol{Y}}_2 & \dots & \hat{\boldsymbol{Y}}_m \end{bmatrix}$. The *residuals*

$$\hat{\boldsymbol{E}} = \boldsymbol{Z} - \hat{\boldsymbol{Z}} = \boldsymbol{Z} - \boldsymbol{X}\hat{\boldsymbol{B}} = \begin{bmatrix} \hat{\boldsymbol{\epsilon}}_1^T \\ \hat{\boldsymbol{\epsilon}}_2^T \\ \vdots \\ \hat{\boldsymbol{\epsilon}}_n^T \end{bmatrix} = \begin{bmatrix} \hat{\boldsymbol{r}}_1 & \hat{\boldsymbol{r}}_2 & \dots & \hat{\boldsymbol{r}}_m \end{bmatrix}.$$

These quantities can be found from the m multiple linear regressions of Y_j on the predictors: $\widehat{\boldsymbol{\beta}}_j = (\boldsymbol{X}^T\boldsymbol{X})^{-1}\boldsymbol{X}^T\boldsymbol{Y}_j$, $\widehat{\boldsymbol{Y}}_j = \boldsymbol{X}\widehat{\boldsymbol{\beta}}_j$ and $\hat{\boldsymbol{r}}_j = \boldsymbol{Y}_j - \hat{\boldsymbol{Y}}_j$ for $j = 1, ..., m$. Hence $\hat{\epsilon}_{i,j} = Y_{i,j} - \hat{Y}_{i,j}$ where $\hat{\boldsymbol{Y}}_j = (\hat{Y}_{1,j}, ..., \hat{Y}_{n,j})^T$. Finally,

$$\hat{\boldsymbol{\Sigma}}_{\boldsymbol{\epsilon},d} = \frac{(\boldsymbol{Z} - \hat{\boldsymbol{Z}})^T(\boldsymbol{Z} - \hat{\boldsymbol{Z}})}{n-d} = \frac{(\boldsymbol{Z} - \boldsymbol{X}\hat{\boldsymbol{B}})^T(\boldsymbol{Z} - \boldsymbol{X}\hat{\boldsymbol{B}})}{n-d} = \frac{\hat{\boldsymbol{E}}^T\hat{\boldsymbol{E}}}{n-d} = \frac{1}{n-d}\sum_{i=1}^n \hat{\boldsymbol{\epsilon}}_i\hat{\boldsymbol{\epsilon}}_i^T.$$

The choices $d = 0$ and $d = p$ are common. If $d = 1$, then $\hat{\boldsymbol{\Sigma}}_{\boldsymbol{\epsilon},d=1} = \boldsymbol{S}_r$, the sample covariance matrix of the residual vectors $\hat{\boldsymbol{\epsilon}}_i$ since the sample mean of the $\hat{\boldsymbol{\epsilon}}_i$ is $\mathbf{0}$. Let $\hat{\boldsymbol{\Sigma}}_\epsilon = \hat{\boldsymbol{\Sigma}}_{\epsilon,p}$ be the unbiased estimator of $\boldsymbol{\Sigma}_{\boldsymbol{\epsilon}}$.

The $\boldsymbol{\epsilon}_i$ are assumed to be iid. Some important joint distributions for $\boldsymbol{\epsilon}$ are completely specified by an $m \times 1$ population *location* vector $\boldsymbol{\mu}$ and an $m \times m$ symmetric positive definite population *dispersion* matrix $\boldsymbol{\Sigma}$. An important model is the elliptically contoured $EC_m(\boldsymbol{\mu}, \boldsymbol{\Sigma}, g)$ distribution with probability density function

$$f(\boldsymbol{z}) = k_m |\boldsymbol{\Sigma}|^{-1/2} g[(\boldsymbol{z} - \boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1} (\boldsymbol{z} - \boldsymbol{\mu})]$$

where $k_m > 0$ is some constant and g is some known function. The multivariate normal (MVN) $N_m(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ distribution is a special case.

Some additional notation will be useful. Assume that $\boldsymbol{x}_1$, ..., $\boldsymbol{x}_n$ are iid from a multivariate distribution. The classical estimator $(\overline{\boldsymbol{x}}, \boldsymbol{S})$ of multivariate location and dispersion is the sample mean and sample covariance matrix where

$$\overline{\boldsymbol{x}} = \frac{1}{n} \sum_{i=1}^{n} \boldsymbol{x}_i \quad \text{and} \quad \boldsymbol{S} = \frac{1}{\mathrm{n}-1} \sum_{\mathrm{i}=1}^{\mathrm{n}} (\boldsymbol{x}_{\mathrm{i}} - \overline{\boldsymbol{x}})(\boldsymbol{x}_{\mathrm{i}} - \overline{\boldsymbol{x}})^{\mathrm{T}}. \tag{1.1}$$

Let the $p \times 1$ column vector T be a multivariate location estimator, and let the $p \times p$ symmetric positive definite matrix $\boldsymbol{C}$ be a dispersion estimator. Then the ith *squared sample Mahalanobis distance* is the scalar

$$D_i^2 = D_i^2(T, \boldsymbol{C}) = (\boldsymbol{x}_i - T)^T \boldsymbol{C}^{-1} (\boldsymbol{x}_i - T) \tag{1.2}$$

for each observation $\boldsymbol{x}_i$. Notice that the Euclidean distance of $\boldsymbol{x}_i$ from the estimate of center T is $D_i(T, \boldsymbol{I}_p)$. The classical Mahalanobis distance uses $(T, \boldsymbol{C}) = (\overline{\boldsymbol{x}}, \boldsymbol{S})$. Following Johnson (1987, pp. 107-108), the population squared Mahalanobis distance

$$U \equiv D^2(\boldsymbol{\mu}, \boldsymbol{\Sigma}) = (\boldsymbol{x} - \boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1} (\boldsymbol{x} - \boldsymbol{\mu}), \tag{1.3}$$

and for elliptically contoured distributions, U has probability density function (pdf)

$$h(u) = \frac{\pi^{p/2}}{\Gamma(p/2)} k_p u^{p/2-1} g(u). \tag{1.4}$$

1.2 TESTING

Following Olive (2013b), next consider testing a linear hypothesis $H_0 : \boldsymbol{LB} = \boldsymbol{0}$ versus $H_1 : \boldsymbol{LB} \neq \boldsymbol{0}$ where $\boldsymbol{L}$ is a full rank $r \times p$ matrix. Let $\boldsymbol{H} = \hat{\boldsymbol{B}}^T \boldsymbol{L}^T [\boldsymbol{L}(\boldsymbol{X}^T \boldsymbol{X})^{-1} \boldsymbol{L}^T]^{-1} \boldsymbol{L}\hat{\boldsymbol{B}}$. Let the error or residual sum of squares and cross products matrix be

$$\boldsymbol{W}_e = \hat{\boldsymbol{E}}^T \hat{\boldsymbol{E}} = (\boldsymbol{Z} - \hat{\boldsymbol{Z}})^T (\boldsymbol{Z} - \hat{\boldsymbol{Z}}) = \boldsymbol{Z}^T \boldsymbol{Z} - \boldsymbol{Z}^T \boldsymbol{X} \hat{\boldsymbol{B}} = \boldsymbol{Z}^T [\boldsymbol{I}_n - \boldsymbol{X}(\boldsymbol{X}^T \boldsymbol{X})^{-1} \boldsymbol{X}^T] \boldsymbol{Z}.$$

Then $\boldsymbol{W}_e/(n-p) = \hat{\boldsymbol{\Sigma}}_{\boldsymbol{\epsilon}}$. Let $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_m$ be the ordered eigenvalues of $\boldsymbol{W}_e^{-1}\boldsymbol{H}$. Then there are four commonly used test statistics.

The Roy's maximum root statistic is $\lambda_{max}(\boldsymbol{L}) = \lambda_1$.

The Wilk's Λ statistic is $\Lambda(\boldsymbol{L}) = |(\boldsymbol{H} + \boldsymbol{W}_e)^{-1} \boldsymbol{W}_e| = |\boldsymbol{W}_e^{-1}\boldsymbol{H} + \boldsymbol{I}|^{-1} = \prod_{i=1}^m (1+\lambda_i)^{-1}$.

The Pillai's trace statistic is $V(\boldsymbol{L}) = tr[(\boldsymbol{H} + \boldsymbol{W}_e)^{-1}\boldsymbol{H}] = \sum_{i=1}^m \frac{\lambda_i}{1+\lambda_i}$.

The Hotelling-Lawley trace statistic is $U(\boldsymbol{L}) = tr[\boldsymbol{W}_e^{-1}\boldsymbol{H}] = \sum_{i=1}^m \lambda_i$.

Theorem 1, Olive (2013b). The Hotelling-Lawley trace statistic

$$U(\boldsymbol{L}) = \frac{1}{n-p} [vec(\boldsymbol{L}\hat{\boldsymbol{B}})]^T [\hat{\boldsymbol{\Sigma}}_{\boldsymbol{\epsilon}}^{-1} \otimes (\boldsymbol{L}(\boldsymbol{X}^T\boldsymbol{X})^{-1}\boldsymbol{L}^T)^{-1}][vec(\boldsymbol{L}\hat{\boldsymbol{B}})]. \tag{1.5}$$

Some notation is useful to show (1.5) and to show that $(n-p)U(\boldsymbol{L}) \xrightarrow{D} \chi^2_{rm}$ under mild conditions if H_0 is true. Following Henderson and Searle (1979), let matrix $\boldsymbol{A} = [\boldsymbol{a}_1 \ \ \boldsymbol{a}_2 \ \ \ldots \ \ \boldsymbol{a}_p]$. Then the vec operator stacks the columns of $\boldsymbol{A}$ on top of one another so

$$vec(\boldsymbol{A}) = \begin{pmatrix} \boldsymbol{a}_1 \\ \boldsymbol{a}_2 \\ \vdots \\ \boldsymbol{a}_p \end{pmatrix}.$$

Let $\boldsymbol{A} = (a_{ij})$ be an $m \times n$ matrix and $\boldsymbol{B}$ a $p \times q$ matrix. Then the Kronecker product of

$\boldsymbol{A}$ and $\boldsymbol{B}$ is the $mp \times nq$ matrix

$$\boldsymbol{A} \otimes \boldsymbol{B} = \begin{bmatrix} a_{11}\boldsymbol{B} & a_{12}\boldsymbol{B} & \cdots & a_{1n}\boldsymbol{B} \\ a_{21}\boldsymbol{B} & a_{22}\boldsymbol{B} & \cdots & a_{2n}\boldsymbol{B} \\ \vdots & \vdots & \cdots & \vdots \\ a_{m1}\boldsymbol{B} & a_{m2}\boldsymbol{B} & \cdots & a_{mn}\boldsymbol{B} \end{bmatrix}.$$

An important fact is that if $\boldsymbol{A}$ and $\boldsymbol{B}$ are nonsingular square matrices, then $[\boldsymbol{A} \otimes \boldsymbol{B}]^{-1} = \boldsymbol{A}^{-1} \otimes \boldsymbol{B}^{-1}$. The following assumption is important.

Assumption D1: Let h_i be the ith diagonal element of $\boldsymbol{X}(\boldsymbol{X}^T\boldsymbol{X})^{-1}\boldsymbol{X}^T$. Assume $\max_{1 \le i \le n} h_i \to 0$ as $n \to \infty$, assume that the zero mean iid errors have finite fourth moments, and assume that $\frac{1}{n}\boldsymbol{X}^T\boldsymbol{X} \xrightarrow{P} \boldsymbol{W}^{-1}$.

Then for the least squares estimator, Su and Cook (2012) show that if assumption D1 holds, then $\hat{\boldsymbol{\Sigma}}_{\boldsymbol{\epsilon}}$ is $\sqrt{n}$ consistent and $\sqrt{n}\ \ vec(\hat{\boldsymbol{B}} - \boldsymbol{B}) \xrightarrow{D} N_{pm}(\boldsymbol{0}, \boldsymbol{\Sigma}_{\boldsymbol{\epsilon}} \otimes \boldsymbol{W})$.

Theorem 2, Olive (2013b). If assumption D1 holds and if H_0 is true, then $(n-p)U(\boldsymbol{L}) \xrightarrow{D} \chi^2_{rm}$.

Kakizawa (2009) shows, under stronger assumptions than Theorem 2, that for a large class of iid error distributions, the following test statistics have the same χ^2_{rm} limiting distribution when H_0 is true, and the same noncentral $\chi^2_{rm}(\omega^2)$ limiting distribution with noncentrality parameter ω^2 when H_0 is false under a local alternative. Hence the three tests are robust to the assumption of normality. The limiting null distribution is well known when the zero mean errors are iid from a multivariate normal distribution. See Khattree and Naik (1999, p. 68): $(n-p)U(\boldsymbol{L}) \xrightarrow{D} \chi^2_{rm}$, $(n-p)V(\boldsymbol{L}) \xrightarrow{D} \chi^2_{rm}$, and $-[n - p - 0.5(m - r + 3)] \log(\Lambda(\boldsymbol{L})) \xrightarrow{D} \chi^2_{rm}$. Results from Kshirsagar (1972, p. 301) suggest that the chi-square approximation is very good if $n \ge 3(m^2 + p^2)$ for multivariate normal errors.

Theorems 1 and 2 are useful for relating multivariate tests with the partial F test for multiple linear regression that tests whether a reduced model that omits some of the predictors can be used instead of the full model that uses all p predictors. The partial F

test statistic is

$$F_R = \left[\frac{SSE(R) - SSE(F)}{df_R - df_F}\right] / MSE(F)$$

where the residual sums of squares $SSE(F)$ and $SSE(R)$ and degrees of freedom df_F and df_r are for the full and reduced model while the mean square error $MSE(F)$ is for the full model. Let the null hypothesis for the partial F test be $H_0 : \boldsymbol{L\beta} = \boldsymbol{0}$ where $\boldsymbol{L}$ sets the coefficients of the predictors in the full model but not in the reduced model to 0. Seber and Lee (2003, p. 100) shows that

$$F_R = \frac{[\boldsymbol{L}\hat{\boldsymbol{\beta}}]^T(\boldsymbol{L}(\boldsymbol{X}^T\boldsymbol{X})^{-1}\boldsymbol{L}^T)^{-1}[\boldsymbol{L}\hat{\boldsymbol{\beta}}]}{r\hat{\sigma}^2}$$

is distributed as $F_{r,n-p}$ if H_0 is true and the errors are iid $N(0, \sigma^2)$. Note that for multiple linear regression with $m = 1$, $F_R = (n-p)U(\boldsymbol{L})/r$ since $\hat{\boldsymbol{\Sigma}}_\epsilon^{-1} = 1/\hat{\sigma}^2$. Hence the scaled Hotelling Lawley test statistic is the partial F test statistic extended to $m > 1$ predictor variables by Theorem 1.

By Theorem 2, for example, $rF_R \xrightarrow{D} \chi^2_r$ for a large class of nonnormal error distribution. If $Z_n \sim F_{k,d_n}$, then $Z_n \xrightarrow{D} \chi^2_k/k$ as $d_n \to \infty$. Hence using the $F_{r,n-p}$ approximation gives a large sample test with correct asymptotic level, and the partial F test is robust to nonnormality.

Similarly, using an $F_{rm,n-pm}$ approximation for the following test statistics gives large sample tests with correct asymptotic level by Kakizawa (2009) and similar power for large n. The large sample test will have correct asymptotic level as long as the denominator degrees of freedom $d_n \to \infty$ as $n \to \infty$, and $d_n = n - pm$ reduces to the partial F test if $m = 1$ and $U(\boldsymbol{L})$ is used. Then the three test statistics are

$$\frac{-[n-p-0.5(m-r+3)]}{rm} \ \log(\Lambda(\boldsymbol{L})), \quad \frac{n-p}{rm} \ V(\boldsymbol{L}), \text{ and } \ \frac{\mathrm{n-p}}{\mathrm{rm}} \ \mathrm{U}(\boldsymbol{L}).$$

Following Khattree and Naik (1999, p. 67) for the Roy's largest root test, if $h = \max(r, m)$, use

$$\frac{n-p-h+r}{h}\lambda_{max}(\boldsymbol{L}) \approx F(h, n-p-h+r).$$

Simulations in Olive (2013b) suggest that this approximation is good for $r = 1$ but poor for $r > 1$. Anderson (1984, p. 333) states that Roy's largest root test has the greatest power if $r = 1$ but is an inferior test for $r > 1$.

Multivariate analogs of tests for multiple linear regression can be derived with appropriate choice of $\boldsymbol{L}$. Assume a constant $x_1 = 1$ is in the model. The analog of the ANOVA F test for multiple linear regression is the MANOVA F test that uses $\boldsymbol{L} = [\boldsymbol{0} \;\; \boldsymbol{I}_{p-1}]$ to test whether the nontrivial predictors are needed in the model.

The F_j test of hypotheses uses $\boldsymbol{L}_j = [0, ..., 0, 1, 0, ..., 0]$, where the 1 is in the jth position, to test whether the jth predictor is needed in the model given that the other $p - 1$ predictors are in the model. This test is an analog of the t tests for multiple linear regression.

The MANOVA partial F test is used to test whether a reduced model is good where the reduced model deletes r of the variables from the full model. For this test, the ith row of $\boldsymbol{L}$ has a 1 in the position corresponding to the ith variable to be deleted. Omitting the jth variable corresponds to the F_j test while omitting variables $x_2, ..., x_p$ corresponds to the MANOVA F test. Using $\boldsymbol{L} = [\boldsymbol{0} \;\; \boldsymbol{I}_k]$ tests whether the last k predictors are needed in the multivariate linear regression model given that the remaining predictors are in the model.

1.3 ROBUST ESTIMATORS

Resistant Regression Estimators for Multiple Linear Regression

Consider the multiple linear regression model, written in matrix form as $\boldsymbol{Y} = \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{e}$. This model is a special case of the multivariate linear regression model with $m = 1$.

Resistant estimators are useful for detecting certain types of outliers. Resistant estimators are often created by computing several trial fits $\boldsymbol{b}_i$ that are estimators of $\boldsymbol{\beta}$. Then a criterion is used to select the trail fit to be used in the resistant estimator. Suppose $c \approx n/2$. The LMS(c) criterion is $Q_{LMS}(\boldsymbol{b}) = r^2_{(c)}(\boldsymbol{b})$ where $r^2_{(1)} \leq \cdots \leq r^2_{(n)}$ are the ordered squared residuals, and the LTS(c) criterion is $Q_{LTS}(\boldsymbol{b}) = \sum_{i=1}^{c} r^2_{(i)}(\boldsymbol{b})$. The LTA($c$) criterion is $Q_{LTA}(\boldsymbol{b}) = \sum_{i=1}^{c} |r(\boldsymbol{b})|_{(i)}$ where $|r(\boldsymbol{b})|_{(i)}$ is the ith ordered absolute residual. Three impractical high breakdown robust estimators are the Hampel (1975) least median of squares (LMS) estimator, the Rousseeuw (1984) least trimmed sum of squares (LTS) estimator, and the Hössjer (1991) least trimmed sum of absolute deviations (LTA) estimator. These estimators correspond to the $\hat{\boldsymbol{\beta}}_L \in \mathcal{R}^p$ that minimizes the corresponding criterion.

A good resistant estimator is the Olive (2005) *median ball algorithm* (MBA or `mbareg`). The Euclidean distance of the ith vector of predictors $\boldsymbol{x}_i$ from the jth vector of predictors $\boldsymbol{x}_j$ is

$$D_i(\boldsymbol{x}_j) = \sqrt{(\boldsymbol{x}_i - \boldsymbol{x}_j)^T(\boldsymbol{x}_i - \boldsymbol{x}_j)}.$$

For a fixed $\boldsymbol{x}_j$ consider the ordered distances $D_{(1)}(\boldsymbol{x}_j), ..., D_{(n)}(\boldsymbol{x}_j)$. Next, let $\hat{\boldsymbol{\beta}}_j(\alpha)$ denote the OLS fit to the $\min(p + 3 + \lfloor \alpha n/100 \rfloor, n)$ cases with the smallest distances where the approximate percentage of cases used is $\alpha \in \{1, 2.5, 5, 10, 20, 33, 50\}$. (Here $\lfloor x \rfloor$ is the greatest integer function so $\lfloor 7.7 \rfloor = 7$. The extra $p + 3$ cases are added so that OLS can be computed for small n and α.) This yields seven OLS fits corresponding to the cases with predictors closest to $\boldsymbol{x}_j$. A fixed number K of cases are selected at random without replacement to use as the $\boldsymbol{x}_j$. Hence $7K$ OLS fits are generated. We use $K = 7$ as the default. A robust criterion Q is used to evaluate the $7K$ fits and the OLS fit to all of the data. Hence $7K+1$ OLS fits are generated and the MBA estimator is the fit that minimizes the criterion. The median squared residual is a good choice for Q.

Three ideas motivate this estimator. First, $\boldsymbol{x}$-outliers, which are outliers in the predictor space, tend to be much more destructive than Y-outliers which are outliers in the response variable. Suppose that the proportion of outliers is γ and that $\gamma < 0.5$. We would like the algorithm to have at least one "center" $\boldsymbol{x}_j$ that is not an outlier. The probability of drawing a center that is not an outlier is approximately $1 - \gamma^K > 0.99$ for $K \geq 7$ and this result is free of p. Secondly, by using the different percentages of coverages, for many data sets there will be a center and a coverage that contains no outliers. Thirdly, the MBA estimator is a $\sqrt{n}$ consistent estimator.

The Olive and Hawkins (2011) `hbreg` estimator is a robust estimator that is asymptotically equivalent to the least squares estimator for many error distributions. Assume that the multiple linear regression model contains an intercept and that the median absolute deviation (MAD) of the Y values from their median is finite. Make an OLS fit to the c_n cases whose Y values are closest to the median Y, and use this fit as the start for concentration: find the c_n cases with the smallest squared residuals and fit OLS to these cases. Use 10 concentration steps and let the attractor be the final estimator, denoted by $\hat{\boldsymbol{\beta}}_B$. It can be shown that $\hat{\boldsymbol{\beta}}_B$ is a high breakdown estimator.

With these preliminaries, we now define our high breakdown procedure. This is made up of three components.

1) The OLS estimator $\hat{\boldsymbol{\beta}}_C$ that is consistent for clean data.

2) The practical $\sqrt{n}$ consistent `mbareg` estimator $\hat{\boldsymbol{\beta}}_A$ that is effective for outlier identification.

3) The practical high-breakdown estimator $\hat{\boldsymbol{\beta}}_B$.

By selecting one of these three estimators according to the features each of them uncovers in the data, we may inherit the good properties of each of them.

The `hbreg` estimator $\hat{\boldsymbol{\beta}}_H$ is defined as follows. Pick a constant $a > 1$ and set $\hat{\boldsymbol{\beta}}_H = \hat{\boldsymbol{\beta}}_C$. If $aQ_L(\hat{\boldsymbol{\beta}}_A) < Q_L(\hat{\boldsymbol{\beta}}_C)$, set $\hat{\boldsymbol{\beta}}_H = \hat{\boldsymbol{\beta}}_A$. If $aQ_L(\hat{\boldsymbol{\beta}}_B) < \min[Q_L(\hat{\boldsymbol{\beta}}_C), aQ_L(\hat{\boldsymbol{\beta}}_A)]$, set $\hat{\boldsymbol{\beta}}_H = \hat{\boldsymbol{\beta}}_B$.

That is, find the smallest of the three scaled criterion values $Q_L(\hat{\boldsymbol{\beta}}_C)$, $aQ_L(\hat{\boldsymbol{\beta}}_A)$, $aQ_L(\hat{\boldsymbol{\beta}}_B)$. According to which of the three estimators attains this minimum, set $\hat{\boldsymbol{\beta}}_H$ to $\hat{\boldsymbol{\beta}}_C, \hat{\boldsymbol{\beta}}_A$ or $\hat{\boldsymbol{\beta}}_B$ respectively.

Large sample theory for `hbreg` is simple and given in the following theorem. Let $\hat{\boldsymbol{\beta}}_L$ be the LMS, LTS or LTA estimator that minimizes the criterion Q_L. Note that the impractical estimator $\hat{\boldsymbol{\beta}}_L$ is never computed. The following theorem shows that $\hat{\boldsymbol{\beta}}_H$ is asymptotically equivalent to $\hat{\boldsymbol{\beta}}_C = \hat{\boldsymbol{\beta}}_{OLS}$. The clean data are in general position if any p clean cases give a unique estimate of $\hat{\boldsymbol{\beta}}$. The LTA criterion will be used in the simulations.

Theorem 3, Olive and Hawkins (2011) Assume the clean data are in general position, and suppose that both $\hat{\boldsymbol{\beta}}_L$ and $\hat{\boldsymbol{\beta}}_C$ are consistent estimators of $\boldsymbol{\beta}$ where the regression model contains a constant. Then the `hbreg` estimator $\hat{\boldsymbol{\beta}}_H$ is high breakdown and is asymptotically equivalent to $\hat{\boldsymbol{\beta}}_C$ since the probability that $\hat{\boldsymbol{\beta}}_H = \hat{\boldsymbol{\beta}}_C$ goes to one as $n \to \infty$.

Robust Multivariate Linear Regression

The classical multivariate linear regression estimator is found from m least squares multiple linear regressions of Y_j on the predictors. The robust multivariate linear regression estimator is found from m `hbreg` multiple linear regressions of Y_j on the predictors. By Theorem 3, the probability that the robust estimator is equal to the classical estimator goes to one as $n \to \infty$ for a large class of error distributions.

Hence the large sample Wilk's test, Pillai's test and Hotelling Lawley test using the robust estimator are asymptotically equivalent to their analogs using the classical estimator for a large class of error distributions. The next section investigates whether reasonable sample sizes result in good results for the robust estimator.

1.4 PLOTS, EXAMPLES AND SIMULATIONS

Plots

A *response plot* for the jth response variable is a plot of the fitted values $\widehat{Y}_{ij}$ versus the response Y_{ij} where $i = 1, ..., n$. The identity line with slope one and zero intercept is added to the plot as a visual aid. A *residual plot* corresponding to the jth response variable is a plot of $\hat{Y}_{ij}$ versus r_{ij}.

Make the m response and residual plots for the multivariate linear regression model. In a response plot, the vertical deviations from the identity line are the residuals $r_{ij} = Y_{ij} - \hat{Y}_{ij}$.

The plotted points in the response plot should cluster about the identity line in each of the m response plots. If outliers are present or if the plot is not linear, then the current model or data need to be changed or corrected. The response and residual plots are used just as for multiple linear regression where $m = 1$. See Olive and Hawkins (2005) and Cook and Weisberg (1999, p. 432).

The Rousseeuw and Van Driessen (1999) DD plot is a plot of classical Mahalanobis distances versus robust Mahalanobis distances. Results from Olive (2002) suggest the plotted points in the DD plot will cluster about the identity line if the $\boldsymbol{\epsilon}_i$ are iid from a multivariate normal $N_m(\boldsymbol{0}, \boldsymbol{\Sigma}_{\boldsymbol{\epsilon}})$ distribution and about some line through the origin with slope greater than one for a large class of elliptically contoured distributions. Make a DD plot of the residuals $\hat{\boldsymbol{\epsilon}}_i$ to check the error distribution. Make a DD plot of the continuous predictor variables to check for $\boldsymbol{x}$-outliers.

The Olive and Hawkins (2010) RMVN estimator $(T_{RMVN}, \boldsymbol{C}_{RMVN})$ is an easily computed $\sqrt{n}$ consistent estimator of $(\boldsymbol{\mu}, c\boldsymbol{\Sigma})$ under regularity conditions (E1) that include a large class of elliptically contoured distributions, and $c = 1$ for the $N_p(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ distribution. The RMVN estimator also gives useful estimates of $(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ for $N_p(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ data even when certain types of outliers are present, and will be the robust estimator used in the DD plots. Also see Zhang, Olive and Pi (2012).

Consider the DD plot applied to the $\hat{\boldsymbol{z}}_i$ based on the robust estimator. The nonparametric region based on the robust estimator uses the sample mean and sample covariance matrix applied to the $\hat{\boldsymbol{z}}_i$. The DD plot will have a vertical line at the cutoff $D_{(U_n)}$. Hence points to the left of the line correspond to cases that are in the nonparametric region. The RMVN estimator can be applied to the $\hat{\boldsymbol{z}}_i$. The region that uses $D_i(T_{RMVN}, \boldsymbol{C}_{RMVN}) \leq D_{(U_n)}(T_{RMVN}, \boldsymbol{C}_{RMVN})$ will be called the semiparametric region, while the parametric MVN region uses $D_i(T_{RMVN}, \boldsymbol{C}_{RMVN}) \leq \sqrt{\chi^2_{p,q_n}}$ where $P(W \leq \chi^2_{p,q_n}) = q_n$ if $W \sim \chi^2_p$. These two regions are only conjectured to be large sample prediction regions, but are added to the DD plot as visual aids. Cases below the horizontal line that crosses the identity line correspond to the semiparametric region while cases below the horizontal line that ends at the identity line correspond to the parametric MVN region. A vertical line dropped down from this point of intersection does correspond to a large sample prediction region for multivariate normal data. Note that $\hat{\boldsymbol{z}}_i = \hat{\boldsymbol{y}}_f + \hat{\boldsymbol{\epsilon}}_i$, and adding a constant $\hat{\boldsymbol{y}}_f$ to all of the residual vectors does not change the Mahalanobis distances, so the DD plot of the residual vectors can be used to display the prediction regions.

Examples and Simulations

Example 1. Cook and Weisberg (1999, p. 351, 433, 447) give a data set on 82 mussels sampled off the coast of New Zealand. Let $Y_1 = \log(S)$ and $Y_2 = \log(M)$ where S is the shell mass and M is the muscle mass. The predictors are $X_2 = L$, $X_3 = \log(W)$ and $X_4 = H$: the shell length, log(width) and height. Figures 1.1 and 1.2 give the response and residual plots for Y_1 and Y_2. For Y_1, case 79 sticks out while for Y_2, cases 8, 25 and 48 are not fit well. Figure 1.3 shows the DD plot of the residual vectors. Cases 8, 48 and 79 have especially large distances. For this data set, the classical and robust estimators were identical, and hence the Cook (1977) distances can be computed. Highlighted cases had Cook's distance $> \min(0.5, 2p/n)$. The response, residual and DD plots are effective for finding influential cases, for checking linearity and whether the error distribution is multivariate normal or some other elliptically contoured distribution, and for displaying the nonparametric prediction region.

Example 2. Buxton (1920) gives various measurements of 88 men. *Height* and *head length* were the response variables while an intercept, *nasal height, bigonal breadth,* and *cephalic index* were used as predictors in the multiple linear regression model. Observation 9 was deleted since it had missing values. Five individuals, numbers 62–66, were reported to be about 0.75 inches tall with head lengths well over five feet! Figure 1.4 shows the response and residual plots corresponding to Y_1 for the robust estimator. The response plot for the classical estimator, not shown, has the identity line tilted slightly above most

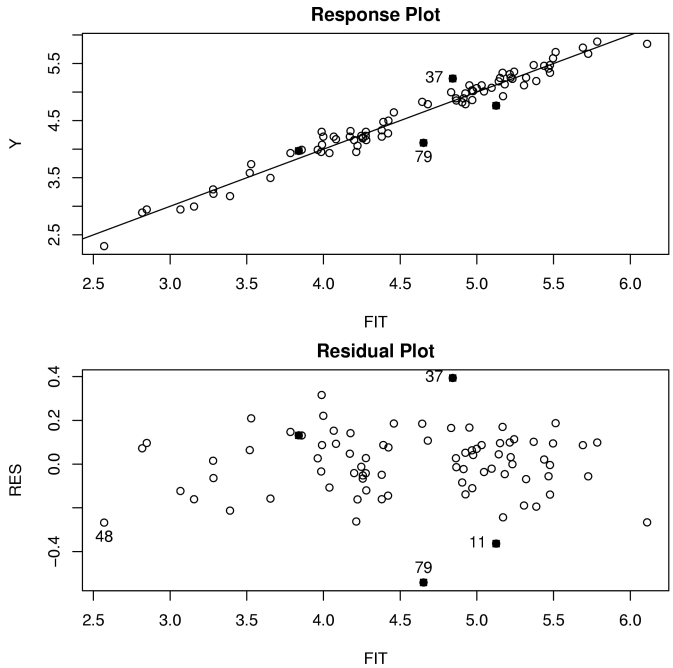

Figure 1.1. Response and Residual Plots for $Y_1 = \log(W)$.

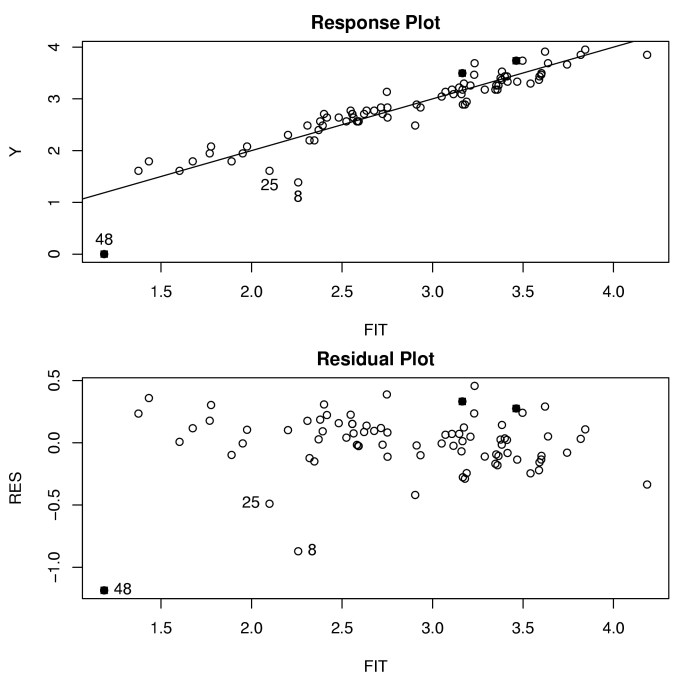

Figure 1.2. Response and Residual Plots for $Y_2 = \log(M)$.

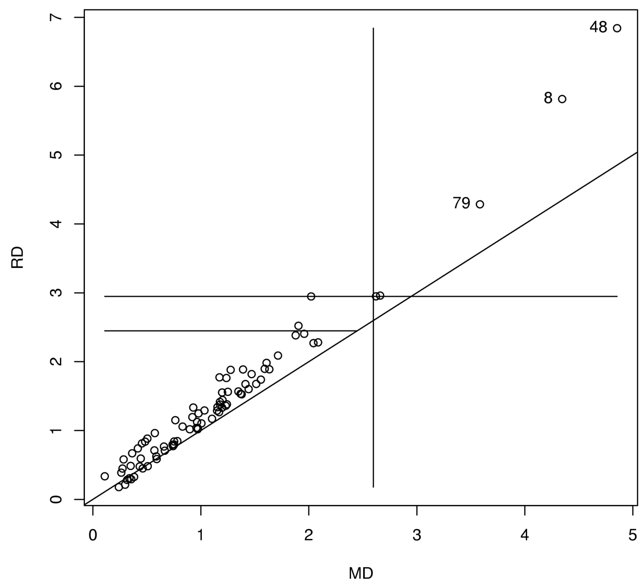

Figure 1.3. DD Plot of the Residual Vectors.

of the plotted points in the lower part of the plot, while the plotted points in the lower part of the residual plot follow a line with negative slope instead of the $r = 0$ line. Figure 1.5 shows the response and residual plots corresponding to Y_2 for the robust estimator. The response plot for the classical estimator, not shown, has the identity line tilted slightly below most of the plotted points in the upper part of the plot, while the plotted points in the upper part of the residual plot follow a line with negative slope instead of the $r = 0$ line. Figure 1.6 shows the DD plot. The tests of hypotheses for the robust estimator are not robust to outliers because all $n = 87$ residual vectors are used to make $\hat{\boldsymbol{\Sigma}}_{\boldsymbol{\epsilon}}$. As is typically the case, outliers can be detected with the plots using the classical or robust estimator.

A simulation was used to study the Wilk's Lambda test, the Pillai's trace test, the Hotelling Lawley trace test, and the Roy's largest root test for the F_j tests and the MANOVA F test for multivariate linear regression. These test statistics were computed with the robust estimator $\hat{\boldsymbol{B}}$ instead of the classical estimator. The first row of $\boldsymbol{B}$ was always $\mathbf{1}^T$ and the last row of $\boldsymbol{B}$ was always $\mathbf{0}^T$. When the null hypothesis for the MANOVA F test

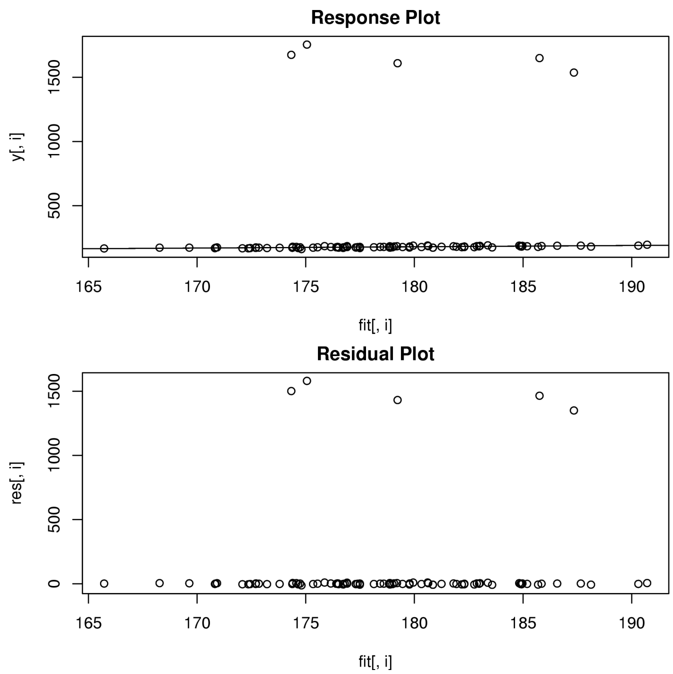

Figure 1.4. Response and Residual Plots for Y_1 = height.

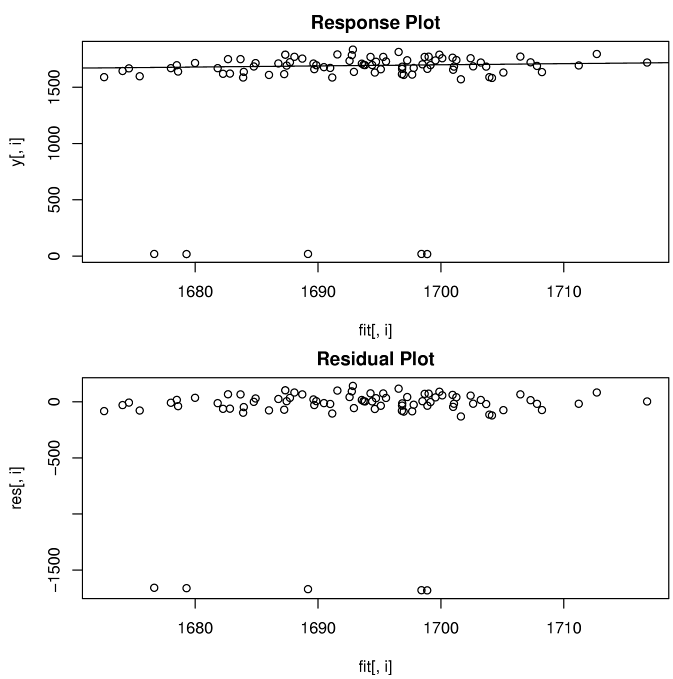

Figure 1.5. Response and Residual Plots for Y_2 = head height.

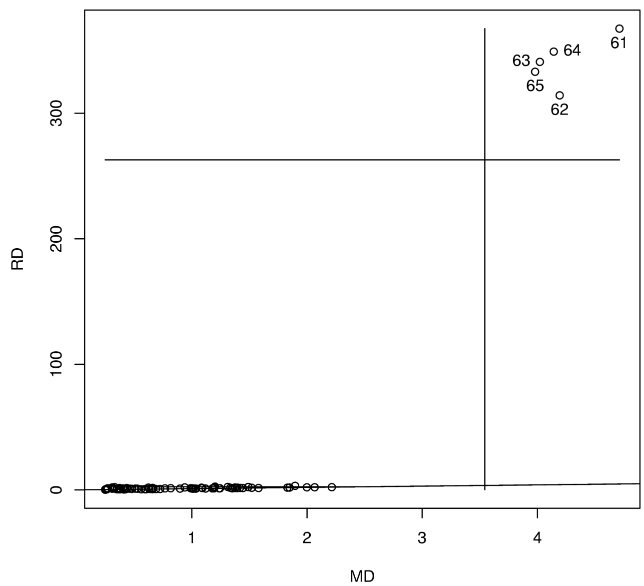

Figure 1.6. DD Plot of the Residual Vectors for the Buxton Data.

is true, all but the first row corresponding to the constant are equal to $\mathbf{0}^T$. When $p \geq 3$ and the null hypothesis for the MANOVA F test is false, then the second to last row of $\boldsymbol{B}$ is (1, 0, ..., 0), the third to last row is (1, 1, 0, ..., 0) etcetera as long as the first row is not changed from $\mathbf{1}^T$. First m iid errors $\boldsymbol{w}_i$ are generated such that the m errors are iid with variance σ^2. Let the $m \times m$ matrix $\boldsymbol{A} = (a_{ij})$ with $a_{ii} = 1$ and $a_{ij} = \psi$ where $0 \leq \psi < 1$ for $i \neq j$. Then $\boldsymbol{\epsilon}_i = \boldsymbol{A}\boldsymbol{w}_i$ so that $\hat{\boldsymbol{\Sigma}}_{\boldsymbol{\epsilon}} = \sigma^2 \boldsymbol{A}\boldsymbol{A}^T = (\sigma_{ij})$ where the diagonal entries $\sigma_{ii} = \sigma^2[1 + (m-1)\psi^2]$ and the off diagonal entries $\sigma_{ij} = \sigma^2[2\psi + (m-2)\psi^2]$ where $\psi = 0.10$. Hence the correlations $(2\psi + (m-2)\psi^2)/(1 + (m-1)\psi^2)$. As ψ gets close to 1, the data clusters about the line in the direction of $(1, ..., 1)^T$. Used $\boldsymbol{w}_i \sim N_m(\mathbf{0}, \boldsymbol{I}), \boldsymbol{w}_i \sim (1-\tau)N_m(\mathbf{0}, \boldsymbol{I}) + \tau N_m(\mathbf{0}, 25\boldsymbol{I})$ with $0 < \tau < 1$ and $\tau = 0.25$ in the simulation, $\boldsymbol{w}_i \sim$ multivariate t_d with $d = 7$ degrees of freedom, or $\boldsymbol{w}_i \sim$ lognormal - E(lognormal): where the m components of $\boldsymbol{w}_i$ were iid with distribution $e^z - E(e^z)$ where $z \sim N(0, 1)$. Only the lognormal distribution is not elliptically contoured.

The simulation used 5000 runs, and H_0 was rejected if the F statistic was greater than

$F_{d_1,d_2}(0.95)$ where $P(F_{d_1,d_2} < F_{d_1,d_2}(0.95)) = 0.95$ with $d_1 = rm$ and $d_2 = n - mp$ for the test statistics

$$\frac{-[n-p-0.5(m-r+3)]}{rm} \ \log(\Lambda(\boldsymbol{L}), \ \ \frac{n-p}{rm} \ V(\boldsymbol{L}), \text{ and } \frac{\mathrm{n-p}}{\mathrm{rm}} \ \mathrm{U}(\boldsymbol{L})$$

while $d_1 = h = \max(r, m)$ and $d_2 = n - p - h + r$ for the test statistic

$$\frac{n-p-h+r}{h}\lambda_{max}(\boldsymbol{L}).$$

Denote these statistics by *Wilk's Lambda*, *Pillai's Trace*, *Hotelling Lawley* and *Roy's*. Let the coverage be the proportion of times that H_0 is rejected. Want coverage near 0.05 when H_0 is true and coverage close to 1 for good power when H_0 is false. With 5000 runs, coverage outside of (0.04,0.06) suggests that the true coverage is not 0.05. Coverages are tabled for the F_1, F_2, F_{p-1}, and F_p test(denoted by b_1, b_2, b_{p-1}, and b_p) and for the MANOVA F test denoted by MANOVA F. Three types of error distributions are considered where *etypeI* for MVN $N_m(0, I)$, *etypeII* for $(1-eps)N_m(0, I) + epsN_m(0, 25I)$, eps = 0.1, 0.25, 0.4, and 0.6 are interesting, *etypeIII* for multivariate t_d with $d = dd$ the degrees of freedom = 1, 2, 3, 5, 7 are interesting. The null hypothesis H_0 was always true for the F_p test and always false for the F_1 test. When the MANOVA F test was true, H_0 was true for the F_j tests with $j \neq 1$. When the MANOVA F test was false, H_0 was false for the F_j tests with $j \neq p$, but the F_{p-1} test should be hardest to reject for $j \neq p$ by construction of $\boldsymbol{B}$ and the error vectors.

CHAPTER 2

TYPE I ERROR SIMULATIONS, ETYPE I

Table 2.1. Type I error rates: etypeI, p=2, m=2, n=20

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	0.9752			0.0600	0.0600
Pillai's Trace	0.9530			0.0360	0.0360
Hotelling-Lawley	0.9876			0.1040	0.1040
Roy's	0.9862			0.0938	0.0938

Table 2.2. Type I error rates: etypeI, p=2, m=2, n=70

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000			0.0666	0.0666
Pillai's Trace	1.0000			0.0622	0.0622
Hotelling-Lawley	1.0000			0.0758	0.0758
Roy's	1.0000			0.0730	0.0730

Table 2.3. Type I error rates: etypeI, p=2, m=2, n=120

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000			0.0498	0.0498
Pillai's Trace	1.0000			0.0468	0.0468
Hotelling-Lawley	1.0000			0.0570	0.0570
Roy's	1.0000			0.0548	0.0548

Table 2.4. Type I error rates: etypeI, p=2, m=2, n=170

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000			0.0458	0.0458
Pillai's Trace	1.0000			0.0440	0.0440
Hotelling-Lawley	1.0000			0.0498	0.0498
Roy's	1.0000			0.0494	0.0494

Table 2.5. Type I error rates: etypeI, p=2, m=2, n=220

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000			0.0516	0.0516
Pillai's Trace	1.0000			0.0510	0.0510
Hotelling-Lawley	1.0000			0.0556	0.0556
Roy's	1.0000			0.0542	0.0542

Table 2.6. Type I error rates: etypeI, p=3, m=3, n=30

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	0.9994	0.0626		0.0674	0.0610
Pillai's Trace	0.9986	0.0396		0.0438	0.0222
Hotelling-Lawley	0.9996	0.1116		0.1200	0.1438
Roy's	0.9996	0.0974		0.1070	0.2858

Table 2.7. Type I error rates: etypeI, p=3, m=3, n=80

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0802		0.0718	0.0886
Pillai's Trace	1.0000	0.0746		0.0658	0.0742
Hotelling-Lawley	1.0000	0.0952		0.0862	0.1094
Roy's	1.0000	0.0914		0.0814	0.2224

Table 2.8. Type I error rates: etypeI, p=3, m=3, n=130

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0478		0.0538	0.0518
Pillai's Trace	1.0000	0.0444		0.0510	0.0452
Hotelling-Lawley	1.0000	0.0572		0.0614	0.0602
Roy's	1.0000	0.0548		0.0594	0.1718

Table 2.9. Type I error rates: etypeI, p=3, m=3, n=180

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0438		0.0474	0.0486
Pillai's Trace	1.0000	0.0414		0.0446	0.0436
Hotelling-Lawley	1.0000	0.0484		0.0524	0.0558
Roy's	1.0000	0.0478		0.0504	0.1568

Table 2.10. Type I error rates: etypeI, p=3, m=3, n=230

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0456		0.0486	0.0414
Pillai's Trace	1.0000	0.0436		0.0458	0.0392
Hotelling-Lawley	1.0000	0.0506		0.0540	0.0466
Roy's	1.0000	0.0486		0.0514	0.1628

Table 2.11. Type I error rates: etypeI, p=3, m=3, n=280

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0510		0.0442	0.0464
Pillai's Trace	1.0000	0.0494		0.0422	0.0426
Hotelling-Lawley	1.0000	0.0540		0.0480	0.0516
Roy's	1.0000	0.0530		0.0464	0.1548

Table 2.12. Type I error rates: etypeI, p=3, m=3, n=330

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0426		0.0524	0.0418
Pillai's Trace	1.0000	0.0420		0.0506	0.0386
Hotelling-Lawley	1.0000	0.0464		0.0552	0.0456
Roy's	1.0000	0.0452		0.0548	0.1538

Table 2.13. Type I error rates: etypeI, p=3, m=3, n=380

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0448		0.0484	0.0420
Pillai's Trace	1.0000	0.0436		0.0476	0.0402
Hotelling-Lawley	1.0000	0.0464		0.0512	0.0462
Roy's	1.0000	0.0462		0.0502	0.1526

Table 2.14. Type I error rates: etypeI, p=3, m=3, n=430

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0540		0.0466	0.0496
Pillai's Trace	1.0000	0.0532		0.0462	0.0482
Hotelling-Lawley	1.0000	0.0562		0.0482	0.0526
Roy's	1.0000	0.0550		0.0478	0.1620

Table 2.15. Type I error rates: etypeI, p=4, m=4, n=40

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0688	0.0598	0.0630	0.0548
Pillai's Trace	1.0000	0.0424	0.0384	0.0392	0.0130
Hotelling-Lawley	1.0000	0.1220	0.1140	0.1108	0.1462
Roy's	1.0000	0.1090	0.1012	0.0984	0.5086

Table 2.16. Type I error rates: etypeI, p=4, m=4, n=90

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0852	0.0874	0.0902	0.1158
Pillai's Trace	1.0000	0.0776	0.0780	0.0840	0.0844
Hotelling-Lawley	1.0000	0.1058	0.1068	0.1094	0.1514
Roy's	1.0000	0.0984	0.0994	0.1040	0.4472

Table 2.17. Type I error rates: etypeI, p=4, m=4, n=140

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0514	0.0606	0.0562	0.0612
Pillai's Trace	1.0000	0.0480	0.0562	0.0524	0.0514
Hotelling-Lawley	1.0000	0.0622	0.0696	0.0680	0.0746
Roy's	1.0000	0.0574	0.0672	0.0622	0.3510

Table 2.18. Type I error rates: etypeI, p=4, m=4, n=190

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0412	0.0486	0.0450	0.0388
Pillai's Trace	1.0000	0.0378	0.0462	0.0408	0.0324
Hotelling-Lawley	1.0000	0.0488	0.0546	0.0510	0.0492
Roy's	1.0000	0.0450	0.0514	0.0480	0.3178

Table 2.19. Type I error rates: etypeI, p=4, m=4, n=240

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0492	0.0470	0.0416	0.0440
Pillai's Trace	1.0000	0.0472	0.0442	0.0400	0.0382
Hotelling-Lawley	1.0000	0.0546	0.0536	0.0472	0.0524
Roy's	1.0000	0.0530	0.0498	0.0458	0.3238

Table 2.20. Type I error rates: etypeI, p=4, m=4, n=290

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0462	0.0430	0.0534	0.0454
Pillai's Trace	1.0000	0.0432	0.0422	0.0514	0.0416
Hotelling-Lawley	1.0000	0.0500	0.0474	0.0578	0.0526
Roy's	1.0000	0.0482	0.0452	0.0556	0.3280

Table 2.21. Type I error rates: etypeI, p=4, m=4, n=340

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0502	0.0474	0.0524	0.0498
Pillai's Trace	1.0000	0.0486	0.0464	0.0510	0.0464
Hotelling-Lawley	1.0000	0.0564	0.0516	0.0552	0.0540
Roy's	1.0000	0.0536	0.0500	0.0542	0.3316

Table 2.22. Type I error rates: etypeI, p=4, m=4, n=390

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0476	0.0438	0.0522	0.0438
Pillai's Trace	1.0000	0.0464	0.0426	0.0506	0.0422
Hotelling-Lawley	1.0000	0.0502	0.0458	0.0560	0.0492
Roy's	1.0000	0.0488	0.0446	0.0548	0.3278

Table 2.23. Type I error rates: etypeI, p=4, m=4, n=440

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0492	0.0484	0.0542	0.0514
Pillai's Trace	1.0000	0.0486	0.0478	0.0536	0.0484
Hotelling-Lawley	1.0000	0.0506	0.0510	0.0574	0.0558
Roy's	1.0000	0.0498	0.0498	0.0554	0.3182

Table 2.24. Type I error rates: etypeI, p=4, m=4, n=490

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0502	0.0454	0.0470	0.0452
Pillai's Trace	1.0000	0.0498	0.0432	0.0462	0.0414
Hotelling-Lawley	1.0000	0.0524	0.0490	0.0500	0.0500
Roy's	1.0000	0.0516	0.0470	0.0486	0.3284

Table 2.25. Type I error rates: etypeI, p=4, m=4, n=540

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0502	0.0498	0.0520	0.0490
Pillai's Trace	1.0000	0.0492	0.0486	0.0504	0.0470
Hotelling-Lawley	1.0000	0.0522	0.0524	0.0554	0.0514
Roy's	1.0000	0.0510	0.0508	0.0542	0.3244

Table 2.26. Type I error rates: etypeI, p=4, m=4, n=590

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0516	0.0502	0.0504	0.0472
Pillai's Trace	1.0000	0.0506	0.0494	0.0492	0.0450
Hotelling-Lawley	1.0000	0.0538	0.0520	0.0518	0.0506
Roy's	1.0000	0.0528	0.0516	0.0514	0.3206

Table 2.27. Type I error rates: etypeI, p=5, m=5, n=50

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0570	0.0490	0.0538	0.0306
Pillai's Trace	1.0000	0.0386	0.0294	0.0338	0.0038
Hotelling-Lawley	1.0000	0.1122	0.1064	0.1084	0.1258
Roy's	1.0000	0.1024	0.0980	0.1000	0.7242

Table 2.28. Type I error rates: etypeI, p=5, m=5, n=100

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0986	0.0894	0.0932	0.1290
Pillai's Trace	1.0000	0.0892	0.0774	0.0842	0.0860
Hotelling-Lawley	1.0000	0.1228	0.1156	0.1200	0.1920
Roy's	1.0000	0.1150	0.1050	0.1106	0.6734

Table 2.29. Type I error rates: etypeI, p=5, m=5, n=150

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0576	0.0592	0.0642	0.0720
Pillai's Trace	1.0000	0.0530	0.0548	0.0596	0.0586
Hotelling-Lawley	1.0000	0.0712	0.0734	0.0770	0.0968
Roy's	1.0000	0.0658	0.0682	0.0708	0.5526

Table 2.30. Type I error rates: etypeI, p=5, m=5, n=200

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0478	0.0476	0.0468	0.0434
Pillai's Trace	1.0000	0.0446	0.0450	0.0438	0.0356
Hotelling-Lawley	1.0000	0.0586	0.0554	0.0564	0.0574
Roy's	1.0000	0.0538	0.0508	0.0512	0.5222

Table 2.31. Type I error rates: etypeI, p=5, m=5, n=250

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0448	0.0428	0.0446	0.0390
Pillai's Trace	1.0000	0.0414	0.0404	0.0398	0.0312
Hotelling-Lawley	1.0000	0.0498	0.0482	0.0514	0.0490
Roy's	1.0000	0.0474	0.0446	0.0482	0.5124

Table 2.32. Type I error rates: etypeI, p=5, m=5, n=300

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0484	0.0488	0.0486	0.0380
Pillai's Trace	1.0000	0.0460	0.0472	0.0460	0.0320
Hotelling-Lawley	1.0000	0.0536	0.0558	0.0532	0.0474
Roy's	1.0000	0.0520	0.0524	0.0510	0.5210

Table 2.33. Type I error rates: etypeI, p=5, m=5, n=350

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0490	0.0476	0.0500	0.0414
Pillai's Trace	1.0000	0.0476	0.0454	0.0468	0.0372
Hotelling-Lawley	1.0000	0.0532	0.0530	0.0550	0.0512
Roy's	1.0000	0.0518	0.0508	0.0524	0.5222

Table 2.34. Type I error rates: etypeI, p=5, m=5, n=400

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0490	0.0460	0.0454	0.0452
Pillai's Trace	1.0000	0.0476	0.0448	0.0444	0.0398
Hotelling-Lawley	1.0000	0.0528	0.0510	0.0502	0.0522
Roy's	1.0000	0.0502	0.0474	0.0482	0.5180

Table 2.35. Type I error rates: etypeI, p=5, m=5, n=450

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0510	0.0452	0.0522	0.0494
Pillai's Trace	1.0000	0.0490	0.0438	0.0504	0.0434
Hotelling-Lawley	1.0000	0.0550	0.0482	0.0550	0.0564
Roy's	1.0000	0.0532	0.0470	0.0538	0.5096

Table 2.36. Type I error rates: etypeI, p=5, m=5, n=500

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0476	0.0458	0.0434	0.0414
Pillai's Trace	1.0000	0.0458	0.0440	0.0418	0.0376
Hotelling-Lawley	1.0000	0.0508	0.0510	0.0464	0.0468
Roy's	1.0000	0.0494	0.0490	0.0448	0.5172

Table 2.37. Type I error rates: etypeI, p=5, m=5, n=550

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0494	0.0474	0.0478	0.0488
Pillai's Trace	1.0000	0.0478	0.0462	0.0464	0.0446
Hotelling-Lawley	1.0000	0.0528	0.0510	0.0522	0.0548
Roy's	1.0000	0.0518	0.0496	0.0504	0.5220

Table 2.38. Type I error rates: etypeI, p=5, m=5, n=600

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0486	0.0504	0.0486	0.0458
Pillai's Trace	1.0000	0.0478	0.0492	0.0476	0.0422
Hotelling-Lawley	1.0000	0.0504	0.0532	0.0504	0.0510
Roy's	1.0000	0.0496	0.0520	0.0498	0.5154

Table 2.39. Type I error rates: etypeI, p=6, m=6, n=60

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0476	0.0532	0.0478	0.0190
Pillai's Trace	1.0000	0.0286	0.0326	0.0290	0.0010
Hotelling-Lawley	1.0000	0.1034	0.1154	0.1004	0.0996
Roy's	1.0000	0.1008	0.1136	0.0996	0.8888

Table 2.40. Type I error rates: etypeI, p=6, m=6, n=110

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.1044	0.1058	0.1028	0.1478
Pillai's Trace	1.0000	0.0922	0.0936	0.0910	0.0916
Hotelling-Lawley	1.0000	0.1356	0.1340	0.1314	0.2360
Roy's	1.0000	0.1226	0.1222	0.1210	0.8472

Table 2.41. Type I error rates: etypeI, p=6, m=6, n=160

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0658	0.0644	0.0656	0.0854
Pillai's Trace	1.0000	0.0616	0.0586	0.0620	0.0622
Hotelling-Lawley	1.0000	0.0822	0.0816	0.0814	0.1136
Roy's	1.0000	0.0744	0.0734	0.0750	0.7386

Table 2.42. Type I error rates: etypeI, p=6, m=6, n=210

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0524	0.0526	0.0502	0.0516
Pillai's Trace	1.0000	0.0498	0.0482	0.0464	0.0390
Hotelling-Lawley	1.0000	0.0618	0.0626	0.0616	0.0690
Roy's	1.0000	0.0566	0.0584	0.0542	0.7148

Table 2.43. Type I error rates: etypeI, p=6, m=6, n=260

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0418	0.0490	0.0446	0.0352
Pillai's Trace	1.0000	0.0396	0.0450	0.0426	0.0280
Hotelling-Lawley	1.0000	0.0502	0.0568	0.0526	0.0500
Roy's	1.0000	0.0442	0.0528	0.0488	0.7180

Table 2.44. Type I error rates: etypeI, p=6, m=6, n=310

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0424	0.0506	0.0438	0.0360
Pillai's Trace	1.0000	0.0408	0.0470	0.0410	0.0294
Hotelling-Lawley	1.0000	0.0506	0.0596	0.0502	0.0476
Roy's	1.0000	0.0472	0.0548	0.0470	0.7020

Table 2.45. Type I error rates: etypeI, p=6, m=6, n=360

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0454	0.0454	0.0466	0.0400
Pillai's Trace	1.0000	0.0432	0.0438	0.0448	0.0334
Hotelling-Lawley	1.0000	0.0504	0.0500	0.0532	0.0520
Roy's	1.0000	0.0480	0.0476	0.0494	0.6966

Table 2.46. Type I error rates: etypeI, p=6, m=6, n=410

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0432	0.0422	0.0464	0.0400
Pillai's Trace	1.0000	0.0420	0.0412	0.0448	0.0354
Hotelling-Lawley	1.0000	0.0496	0.0480	0.0510	0.0496
Roy's	1.0000	0.0460	0.0454	0.0490	0.6966

Table 2.47. Type I error rates: etypeI, p=6, m=6, n=460

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0570	0.0464	0.0488	0.0480
Pillai's Trace	1.0000	0.0550	0.0436	0.0466	0.0424
Hotelling-Lawley	1.0000	0.0626	0.0510	0.0528	0.0576
Roy's	1.0000	0.0590	0.0482	0.0504	0.7200

Table 2.48. Type I error rates: etypeI, p=6, m=6, n=510

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0458	0.0506	0.0458	0.0406
Pillai's Trace	1.0000	0.0442	0.0494	0.0440	0.0352
Hotelling-Lawley	1.0000	0.0514	0.0552	0.0502	0.0478
Roy's	1.0000	0.0490	0.0536	0.0470	0.7026

Table 2.49. Type I error rates: etypeI, p=6, m=6, n=560

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0484	0.0548	0.0500	0.0414
Pillai's Trace	1.0000	0.0472	0.0526	0.0482	0.0346
Hotelling-Lawley	1.0000	0.0510	0.0582	0.0528	0.0484
Roy's	1.0000	0.0498	0.0562	0.0512	0.7060

Table 2.50. Type I error rates: etypeI, p=6, m=6, n=610

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0502	0.0436	0.0494	0.0390
Pillai's Trace	1.0000	0.0482	0.0432	0.0476	0.0340
Hotelling-Lawley	1.0000	0.0546	0.0474	0.0532	0.0464
Roy's	1.0000	0.0530	0.0452	0.0508	0.6972

Table 2.51. Type I error rates: etypeI, p=6, m=6, n=660

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0500	0.0482	0.0444	0.0410
Pillai's Trace	1.0000	0.0484	0.0466	0.0444	0.0374
Hotelling-Lawley	1.0000	0.0526	0.0506	0.0488	0.0460
Roy's	1.0000	0.0510	0.0492	0.0460	0.7018

Table 2.52. Type I error rates: etypeI, p=7, m=7, n=70

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0438	0.0416	0.0384	0.0074
Pillai's Trace	1.0000	0.0240	0.0210	0.0204	0.0002
Hotelling-Lawley	1.0000	0.0972	0.0968	0.0976	0.0676
Roy's	1.0000	0.1116	0.1112	0.1110	0.9640

Table 2.53. Type I error rates: etypeI, p=7, m=7, n=120

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.1142	0.1070	0.1086	0.1558
Pillai's Trace	1.0000	0.1008	0.0934	0.0958	0.0802
Hotelling-Lawley	1.0000	0.1486	0.1446	0.1448	0.2620
Roy's	1.0000	0.1380	0.1326	0.1336	0.9370

Table 2.54. Type I error rates: etypeI, p=7, m=7, n=170

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0668	0.0752	0.0698	0.0980
Pillai's Trace	1.0000	0.0622	0.0680	0.0640	0.0676
Hotelling-Lawley	1.0000	0.0870	0.0936	0.0920	0.1424
Roy's	1.0000	0.0780	0.0848	0.0816	0.8766

Table 2.55. Type I error rates: etypeI, p=7, m=7, n=220

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0498	0.0514	0.0530	0.0432
Pillai's Trace	1.0000	0.0472	0.0458	0.0490	0.0334
Hotelling-Lawley	1.0000	0.0622	0.0636	0.0632	0.0670
Roy's	1.0000	0.0560	0.0566	0.0572	0.8526

Table 2.56. Type I error rates: etypeI, p=7, m=7, n=270

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0436	0.0442	0.0474	0.0304
Pillai's Trace	1.0000	0.0404	0.0420	0.0440	0.0224
Hotelling-Lawley	1.0000	0.0510	0.0538	0.0572	0.0484
Roy's	1.0000	0.0470	0.0502	0.0520	0.8548

Table 2.57. Type I error rates: etypeI, p=7, m=7, n=320

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0408	0.0496	0.0468	0.0414
Pillai's Trace	1.0000	0.0390	0.0468	0.0440	0.0298
Hotelling-Lawley	1.0000	0.0484	0.0602	0.0556	0.0562
Roy's	1.0000	0.0448	0.0550	0.0506	0.8450

Table 2.58. Type I error rates: etypeI, p=7, m=7, n=370

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0472	0.0462	0.0450	0.0362
Pillai's Trace	1.0000	0.0454	0.0442	0.0430	0.0262
Hotelling-Lawley	1.0000	0.0546	0.0540	0.0520	0.0484
Roy's	1.0000	0.0504	0.0494	0.0478	0.8548

Table 2.59. Type I error rates: etypeI, p=7, m=7, n=420

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0466	0.0454	0.0520	0.0380
Pillai's Trace	1.0000	0.0446	0.0428	0.0500	0.0292
Hotelling-Lawley	1.0000	0.0528	0.0506	0.0558	0.0484
Roy's	1.0000	0.0492	0.0484	0.0544	0.8512

Table 2.60. Type I error rates: etypeI, p=7, m=7, n=470

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0500	0.0512	0.0452	0.0394
Pillai's Trace	1.0000	0.0486	0.0488	0.0432	0.0308
Hotelling-Lawley	1.0000	0.0566	0.0574	0.0494	0.0498
Roy's	1.0000	0.0542	0.0536	0.0474	0.8544

Table 2.61. Type I error rates: etypeI, p=7, m=7, n=520

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0472	0.0514	0.0468	0.0412
Pillai's Trace	1.0000	0.0456	0.0498	0.0456	0.0340
Hotelling-Lawley	1.0000	0.0510	0.0552	0.0534	0.0510
Roy's	1.0000	0.0494	0.0542	0.0500	0.8438

Table 2.62. Type I error rates: etypeI, p=7, m=7, n=570

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0446	0.0470	0.0410	0.0442
Pillai's Trace	1.0000	0.0434	0.0452	0.0394	0.0380
Hotelling-Lawley	1.0000	0.0482	0.0512	0.0468	0.0522
Roy's	1.0000	0.0462	0.0486	0.0436	0.8476

Table 2.63. Type I error rates: etypeI, p=7, m=7, n=620

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0506	0.0464	0.0514	0.0406
Pillai's Trace	1.0000	0.0492	0.0452	0.0504	0.0350
Hotelling-Lawley	1.0000	0.0536	0.0512	0.0560	0.0498
Roy's	1.0000	0.0514	0.0486	0.0540	0.8450

Table 2.64. Type I error rates: etypeI, p=7, m=7, n=670

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0452	0.0486	0.0506	0.0454
Pillai's Trace	1.0000	0.0440	0.0472	0.0494	0.0402
Hotelling-Lawley	1.0000	0.0492	0.0530	0.0542	0.0538
Roy's	1.0000	0.0474	0.0506	0.0516	0.8508

Table 2.65. Type I error rates: etypeI, p=7, m=7, n=720

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0474	0.0506	0.0518	0.0462
Pillai's Trace	1.0000	0.0462	0.0498	0.0510	0.0392
Hotelling-Lawley	1.0000	0.0514	0.0554	0.0562	0.0514
Roy's	1.0000	0.0482	0.0518	0.0536	0.8594

Table 2.66. Type I error rates: etypeI, p=7, m=7, n=770

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0470	0.0480	0.0498	0.0432
Pillai's Trace	1.0000	0.0464	0.0472	0.0486	0.0394
Hotelling-Lawley	1.0000	0.0506	0.0530	0.0540	0.0504
Roy's	1.0000	0.0480	0.0504	0.0516	0.8462

Table 2.67. Type I error rates: etypeI, p=7, m=7, n=820

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0512	0.0464	0.0526	0.0408
Pillai's Trace	1.0000	0.0496	0.0458	0.0514	0.0362
Hotelling-Lawley	1.0000	0.0542	0.0506	0.0546	0.0462
Roy's	1.0000	0.0524	0.0480	0.0540	0.8440

Table 2.68. Type I error rates: etypeI, p=7, m=7, n=870

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0484	0.0480	0.0490	0.0482
Pillai's Trace	1.0000	0.0468	0.0474	0.0490	0.0426
Hotelling-Lawley	1.0000	0.0516	0.0512	0.0514	0.0538
Roy's	1.0000	0.0498	0.0496	0.0500	0.8462

Table 2.69. Type I error rates: etypeI, p=7, m=7, n=920

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0478	0.0494	0.0496	0.0442
Pillai's Trace	1.0000	0.0474	0.0480	0.0492	0.0406
Hotelling-Lawley	1.0000	0.0516	0.0522	0.0524	0.0498
Roy's	1.0000	0.0492	0.0504	0.0504	0.8412

Table 2.70. Type I error rates: etypeI, p=8, m=8, n=80

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0262	0.0286	0.0250	0.0006
Pillai's Trace	1.0000	0.0146	0.0144	0.0144	0.0000
Hotelling-Lawley	1.0000	0.0746	0.0800	0.0716	0.0242
Roy's	1.0000	0.1154	0.1262	0.1136	0.9918

Table 2.71. Type I error rates: etypeI, p=8, m=8, n=130

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.1072	0.1168	0.1176	0.1684
Pillai's Trace	1.0000	0.0926	0.1012	0.1016	0.0846
Hotelling-Lawley	1.0000	0.1500	0.1652	0.1596	0.3022
Roy's	1.0000	0.1384	0.1508	0.1454	0.9844

Table 2.72. Type I error rates: etypeI, p=8, m=8, n=180

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0858	0.0810	0.0814	0.1206
Pillai's Trace	1.0000	0.0800	0.0748	0.0746	0.0772
Hotelling-Lawley	1.0000	0.1102	0.1032	0.1026	0.1794
Roy's	1.0000	0.1014	0.0926	0.0936	0.9594

Table 2.73. Type I error rates: etypeI, p=8, m=8, n=230

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0566	0.0542	0.0546	0.0574
Pillai's Trace	1.0000	0.0516	0.0496	0.0498	0.0422
Hotelling-Lawley	1.0000	0.0704	0.0700	0.0678	0.0880
Roy's	1.0000	0.0630	0.0616	0.0612	0.9450

Table 2.74. Type I error rates: etypeI, p=8, m=8, n=280

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0420	0.0460	0.0500	0.0384
Pillai's Trace	1.0000	0.0390	0.0424	0.0468	0.0258
Hotelling-Lawley	1.0000	0.0504	0.0560	0.0610	0.0592
Roy's	1.0000	0.0454	0.0512	0.0548	0.9382

Table 2.75. Type I error rates: etypeI, p=8, m=8, n=330

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0504	0.0448	0.0474	0.0286
Pillai's Trace	1.0000	0.0474	0.0426	0.0456	0.0202
Hotelling-Lawley	1.0000	0.0594	0.0548	0.0578	0.0490
Roy's	1.0000	0.0554	0.0492	0.0522	0.9332

Table 2.76. Type I error rates: etypeI, p=8, m=8, n=380

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0434	0.0512	0.0444	0.0378
Pillai's Trace	1.0000	0.0412	0.0478	0.0420	0.0248
Hotelling-Lawley	1.0000	0.0512	0.0602	0.0532	0.0526
Roy's	1.0000	0.0466	0.0556	0.0486	0.9328

Table 2.77. Type I error rates: etypeI, p=8, m=8, n=430

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0476	0.0430	0.0480	0.0346
Pillai's Trace	1.0000	0.0454	0.0420	0.0440	0.0252
Hotelling-Lawley	1.0000	0.0536	0.0488	0.0530	0.0478
Roy's	1.0000	0.0502	0.0454	0.0500	0.9382

Table 2.78. Type I error rates: etypeI, p=8, m=8, n=480

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0492	0.0480	0.0462	0.0358
Pillai's Trace	1.0000	0.0476	0.0470	0.0442	0.0270
Hotelling-Lawley	1.0000	0.0554	0.0556	0.0524	0.0474
Roy's	1.0000	0.0516	0.0512	0.0494	0.9278

Table 2.79. Type I error rates: etypeI, p=8, m=8, n=530

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0474	0.0420	0.0490	0.0368
Pillai's Trace	1.0000	0.0466	0.0408	0.0462	0.0276
Hotelling-Lawley	1.0000	0.0532	0.0496	0.0556	0.0480
Roy's	1.0000	0.0500	0.0452	0.0514	0.9386

Table 2.80. Type I error rates: etypeI, p=8, m=8, n=580

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0440	0.0440	0.0430	0.0378
Pillai's Trace	1.0000	0.0428	0.0422	0.0420	0.0310
Hotelling-Lawley	1.0000	0.0504	0.0492	0.0488	0.0454
Roy's	1.0000	0.0468	0.0464	0.0462	0.9366

Table 2.81. Type I error rates: etypeI, p=8, m=8, n=630

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0524	0.0460	0.0472	0.0384
Pillai's Trace	1.0000	0.0504	0.0438	0.0440	0.0318
Hotelling-Lawley	1.0000	0.0588	0.0524	0.0522	0.0496
Roy's	1.0000	0.0552	0.0488	0.0496	0.9418

Table 2.82. Type I error rates: etypeI, p=8, m=8, n=680

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0492	0.0496	0.0416	0.0398
Pillai's Trace	1.0000	0.0480	0.0470	0.0408	0.0330
Hotelling-Lawley	1.0000	0.0528	0.0554	0.0460	0.0504
Roy's	1.0000	0.0502	0.0512	0.0436	0.9404

Table 2.83. Type I error rates: etypeI, p=8, m=8, n=730

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0484	0.0448	0.0496	0.0402
Pillai's Trace	1.0000	0.0472	0.0436	0.0494	0.0342
Hotelling-Lawley	1.0000	0.0522	0.0496	0.0528	0.0484
Roy's	1.0000	0.0494	0.0464	0.0512	0.9264

Table 2.84. Type I error rates: etypeI, p=8, m=8, n=780

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0498	0.0502	0.0494	0.0388
Pillai's Trace	1.0000	0.0488	0.0490	0.0476	0.0354
Hotelling-Lawley	1.0000	0.0528	0.0542	0.0546	0.0474
Roy's	1.0000	0.0510	0.0520	0.0520	0.9280

Table 2.85. Type I error rates: etypeI, p=8, m=8, n=830

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0498	0.0514	0.0490	0.0500
Pillai's Trace	1.0000	0.0488	0.0498	0.0480	0.0448
Hotelling-Lawley	1.0000	0.0544	0.0536	0.0532	0.0588
Roy's	1.0000	0.0524	0.0520	0.0514	0.9298

Table 2.86. Type I error rates: etypeI, p=8, m=8, n=880

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0438	0.0530	0.0482	0.0422
Pillai's Trace	1.0000	0.0430	0.0514	0.0454	0.0368
Hotelling-Lawley	1.0000	0.0474	0.0552	0.0520	0.0474
Roy's	1.0000	0.0458	0.0544	0.0490	0.9308

Table 2.87. Type I error rates: etypeI, p=8, m=8, n=930

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0508	0.0442	0.0440	0.0360
Pillai's Trace	1.0000	0.0498	0.0438	0.0430	0.0324
Hotelling-Lawley	1.0000	0.0528	0.0472	0.0490	0.0414
Roy's	1.0000	0.0512	0.0452	0.0470	0.9304

Table 2.88. Type I error rates: etypeI, p=8, m=8, n=980

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0500	0.0514	0.0476	0.0462
Pillai's Trace	1.0000	0.0496	0.0500	0.0470	0.0406
Hotelling-Lawley	1.0000	0.0528	0.0540	0.0502	0.0544
Roy's	1.0000	0.0518	0.0526	0.0492	0.9336

Table 2.89. Type I error rates: etypeI, p=9, m=9, n=100

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0456	0.0510	0.0454	0.0034
Pillai's Trace	1.0000	0.0294	0.0330	0.0272	0.0000
Hotelling-Lawley	1.0000	0.1074	0.1128	0.1066	0.0568
Roy's	1.0000	0.1594	0.1632	0.1542	0.9984

Table 2.90. Type I error rates: etypeI, p=9, m=9, n=150

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.1184	0.1198	0.1190	0.1786
Pillai's Trace	1.0000	0.1052	0.1052	0.1028	0.0800
Hotelling-Lawley	1.0000	0.1642	0.1660	0.1652	0.3180
Roy's	1.0000	0.1524	0.1552	0.1534	0.9946

Table 2.91. Type I error rates: etypeI, p=9, m=9, n=200

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0804	0.0816	0.0724	0.1098
Pillai's Trace	1.0000	0.0730	0.0724	0.0666	0.0632
Hotelling-Lawley	1.0000	0.1010	0.1036	0.0960	0.1678
Roy's	1.0000	0.0930	0.0920	0.0868	0.9806

Table 2.92. Type I error rates: etypeI, p=9, m=9, n=250

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0564	0.0534	0.0512	0.051
Pillai's Trace	1.0000	0.0520	0.0490	0.0468	0.0340
Hotelling-Lawley	1.0000	0.0730	0.0694	0.0676	0.0786
Roy's	1.0000	0.0660	0.0622	0.0602	0.9812

Table 2.93. Type I error rates: etypeI, p=9, m=9, n=300

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0452	0.0498	0.0462	0.0354
Pillai's Trace	1.0000	0.0414	0.0454	0.0428	0.019
Hotelling-Lawley	1.0000	0.0554	0.0606	0.0612	0.0608
Roy's	1.0000	0.0498	0.0548	0.0536	0.9742

Table 2.94. Type I error rates: etypeI, p=9, m=9, n=350

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0470	0.0418	0.0434	0.0276
Pillai's Trace	1.0000	0.0438	0.0390	0.0392	0.0180
Hotelling-Lawley	1.0000	0.0568	0.0498	0.0546	0.0460
Roy's	1.0000	0.0526	0.0460	0.0484	0.9788

CHAPTER 3

TYPE I ERROR SIMULATIONS, ETYPE II

Table 3.1. Type I error rates: etypeII, p=2, m=2, n=20

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	0.3882			0.0194	0.0194
Pillai's Trace	0.2942			0.0096	0.0096
Hotelling-Lawley	0.4980			0.0358	0.0358
Roy's	0.4794			0.0334	0.0334

Table 3.2. Type I error rates: etypeII, p=2, m=2, n=70

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	0.9488			0.0336	0.0336
Pillai's Trace	0.9434			0.0288	0.0288
Hotelling-Lawley	0.9554			0.0392	0.0392
Roy's	0.9542			0.0382	0.0382

Table 3.3. Type I error rates: etypeII, p=2, m=2, n=120

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	0.9968			0.0420	0.0420
Pillai's Trace	0.9968			0.0398	0.0398
Hotelling-Lawley	0.9970			0.0476	0.0476
Roy's	0.9970			0.0458	0.0458

Table 3.4. Type I error rates: etypeII, p=2, m=2, n=170

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000			0.0494	0.0494
Pillai's Trace	1.0000			0.0484	0.0484
Hotelling-Lawley	1.0000			0.0534	0.0534
Roy's	1.0000			0.0524	0.0524

Table 3.5. Type I error rates: etypeII, p=2, m=2, n=220

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	0.9998			0.0466	0.0466
Pillai's Trace	0.9998			0.0452	0.0452
Hotelling-Lawley	0.9998			0.0490	0.0490
Roy's	0.9998			0.0488	0.0488

Table 3.6. Type I error rates: etypeII, p=2, m=2, n=270

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1			0.0516	0.0516
Pillai's Trace	1			0.0506	0.0506
Hotelling-Lawley	1			0.0536	0.0536
Roy's	1			0.0532	0.0532

Table 3.7. Type I error rates: etypeII, p=2, m=2, n=320

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1			0.0494	0.0494
Pillai's Trace	1			0.0484	0.0484
Hotelling-Lawley	1			0.052	0.052
Roy's	1			0.0514	0.0514

Table 3.8. Type I error rates: etypeII, p=2, m=2, n=370

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1			0.046	0.046
Pillai's Trace	1			0.0452	0.0452
Hotelling-Lawley	1			0.0478	0.0478
Roy's	1			0.0466	0.0466

Table 3.9. Type I error rates: etypeII, p=2, m=2, n=420

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1			0.049	0.049
Pillai's Trace	1			0.0488	0.0488
Hotelling-Lawley	1			0.0496	0.0496
Roy's	1			0.0492	0.0492

Table 3.10. Type I error rates: etypeII, p=2, m=2, n=470

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1			0.0478	0.0478
Pillai's Trace	1			0.047	0.047
Hotelling-Lawley	1			0.0494	0.0494
Roy's	1			0.0492	0.0492

Table 3.11. Type I error rates: etypeII, p=3, m=3, n=30

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	0.5592	0.0132		0.0146	0.0074
Pillai's Trace	0.464	0.008		0.0082	0.0014
Hotelling-Lawley	0.6808	0.0322		0.0294	0.0274
Roy's	0.6554	0.0268		0.0268	0.0842

Table 3.12. Type I error rates: etypeII, p=3, m=3, n=80

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	0.986	0.0274		0.031	0.0258
Pillai's Trace	0.9846	0.023		0.0274	0.0188
Hotelling-Lawley	0.9884	0.037		0.0406	0.036
Roy's	0.9878	0.0344		0.0374	0.1138

Table 3.13. Type I error rates: etypeII, p=3, m=3, n=130

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	0.9998	0.0398		0.0398	0.0364
Pillai's Trace	0.9998	0.0358		0.0376	0.0306
Hotelling-Lawley	1	0.0464		0.046	0.0452
Roy's	0.9998	0.044		0.0446	0.1404

Table 3.14. Type I error rates: etypeII, p=3, m=3, n=180

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1	0.044		0.0448	0.04
Pillai's Trace	1	0.0414		0.0426	0.0364
Hotelling-Lawley	1	0.0512		0.05	0.046
Roy's	1	0.0488		0.0484	0.1596

Table 3.15. Type I error rates: etypeII, p=3, m=3, n=230

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1	0.0488		0.0496	0.0458
Pillai's Trace	1	0.0456		0.0472	0.0428
Hotelling-Lawley	1	0.0528		0.0528	0.052
Roy's	1	0.051		0.051	0.1636

Table 3.16. Type I error rates: etypeII, p=3, m=3, n=280

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1	0.0476		0.0516	0.0456
Pillai's Trace	1	0.046		0.0502	0.0436
Hotelling-Lawley	1	0.0506		0.0544	0.0488
Roy's	1	0.0496		0.054	0.1654

Table 3.17. Type I error rates: etypeII, p=3, m=3, n=330

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1	0.045		0.0472	0.0448
Pillai's Trace	1	0.044		0.0458	0.0414
Hotelling-Lawley	1	0.0476		0.0502	0.0486
Roy's	1	0.0468		0.0496	0.1566

Table 3.18. Type I error rates: etypeII, p=3, m=3, n=380

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1	0.0454		0.0422	0.0414
Pillai's Trace	1	0.0448		0.0412	0.0392
Hotelling-Lawley	1	0.048		0.0448	0.046
Roy's	1	0.0468		0.0434	0.1528

Table 3.19. Type I error rates: etypeII, p=3, m=3, n=430

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1	0.0526		0.0472	0.046
Pillai's Trace	1	0.052		0.0452	0.044
Hotelling-Lawley	1	0.0542		0.0498	0.049
Roy's	1	0.0534		0.049	0.1568

Table 3.20. Type I error rates: etypeII, p=4, m=4, n=40

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	0.7104	0.011	0.0118	0.0098	0.002
Pillai's Trace	0.6282	0.0066	0.0054	0.0058	2.00E-04
Hotelling-Lawley	0.8196	0.0214	0.0272	0.0194	0.015
Roy's	0.802	0.0182	0.0216	0.0172	0.1528

Table 3.21. Type I error rates: etypeII, p=4, m=4, n=90

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	0.9956	0.0256	0.0252	0.0284	0.015
Pillai's Trace	0.9946	0.0218	0.0216	0.0248	0.0076
Hotelling-Lawley	0.9974	0.0366	0.0358	0.0396	0.028
Roy's	0.9966	0.0336	0.0328	0.0358	0.2366

Table 3.22. Type I error rates: etypeII, p=4, m=4, n=140

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1	0.038	0.04	0.0388	0.03
Pillai's Trace	1	0.035	0.0366	0.0352	0.0218
Hotelling-Lawley	1	0.0458	0.0496	0.0456	0.0422
Roy's	1	0.0426	0.0462	0.0424	0.303

Table 3.23. Type I error rates: etypeII, p=4, m=4, n=190

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1	0.0458	0.0418	0.0412	0.0342
Pillai's Trace	1	0.0434	0.0396	0.0368	0.0272
Hotelling-Lawley	1	0.0534	0.0504	0.0472	0.0462
Roy's	1	0.049	0.0474	0.0448	0.3232

Table 3.24. Type I error rates: etypeII, p=4, m=4, n=240

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1	0.039	0.046	0.0448	0.0426
Pillai's Trace	1	0.0378	0.0428	0.043	0.0346
Hotelling-Lawley	1	0.0428	0.0524	0.0512	0.0484
Roy's	1	0.0406	0.05	0.049	0.3188

Table 3.25. Type I error rates: etypeII, p=4, m=4, n=290

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1	0.048	0.0438	0.047	0.0456
Pillai's Trace	1	0.0464	0.0422	0.044	0.0416
Hotelling-Lawley	1	0.0524	0.049	0.0506	0.0534
Roy's	1	0.0504	0.0468	0.0492	0.3198

Table 3.26. Type I error rates: etypeII, p=4, m=4, n=340

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1	0.0498	0.0496	0.0488	0.0464
Pillai's Trace	1	0.047	0.0486	0.0468	0.042
Hotelling-Lawley	1	0.055	0.0522	0.0544	0.0526
Roy's	1	0.0526	0.051	0.0524	0.3352

Table 3.27. Type I error rates: etypeII, p=4, m=4, n=390

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1	0.0496	0.0448	0.0426	0.0492
Pillai's Trace	1	0.049	0.0438	0.041	0.0456
Hotelling-Lawley	1	0.0532	0.0468	0.0468	0.0558
Roy's	1	0.0522	0.0466	0.0446	0.317

Table 3.28. Type I error rates: etypeII, p=4, m=4, n=440

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1	0.0484	0.0474	0.0434	0.0408
Pillai's Trace	1	0.0464	0.046	0.042	0.0384
Hotelling-Lawley	1	0.0502	0.051	0.0458	0.0452
Roy's	1	0.0496	0.05	0.045	0.319

Table 3.29. Type I error rates: etypeII, p=4, m=4, n=490

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1	0.0466	0.0468	0.0514	0.0502
Pillai's Trace	1	0.0458	0.0458	0.0506	0.0464
Hotelling-Lawley	1	0.0484	0.0486	0.0546	0.0548
Roy's	1	0.048	0.0474	0.0528	0.3248

Table 3.30. Type I error rates: etypeII, p=5, m=5, n=50

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	0.814	0.007	0.0058	0.0076	2.00E-04
Pillai's Trace	0.7416	0.0042	0.0026	0.0044	0
Hotelling-Lawley	0.8978	0.0186	0.0174	0.019	0.0064
Roy's	0.8888	0.0164	0.0144	0.017	0.2336

Table 3.31. Type I error rates: etypeII, p=5, m=5, n=100

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	0.9978	0.0236	0.0276	0.022	0.0094
Pillai's Trace	0.9978	0.0194	0.022	0.0178	0.0042
Hotelling-Lawley	0.9988	0.035	0.0392	0.0352	0.023
Roy's	0.9984	0.0304	0.0346	0.0298	0.3898

Table 3.32. Type I error rates: etypeII, p=5, m=5, n=150

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1	0.0402	0.0438	0.0356	0.0244
Pillai's Trace	1	0.0362	0.0402	0.0326	0.0146
Hotelling-Lawley	1	0.0526	0.053	0.0446	0.0416
Roy's	1	0.0476	0.0492	0.041	0.497

Table 3.33. Type I error rates: etypeII, p=5, m=5, n=200

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1	0.0408	0.047	0.0434	0.036
Pillai's Trace	1	0.038	0.0436	0.0404	0.0278
Hotelling-Lawley	1	0.0502	0.0524	0.0516	0.0512
Roy's	1	0.047	0.0502	0.049	0.5112

Table 3.34. Type I error rates: etypeII, p=5, m=5, n=250

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1	0.0402	0.0446	0.0434	0.0388
Pillai's Trace	1	0.0382	0.0424	0.041	0.0304
Hotelling-Lawley	1	0.0486	0.0522	0.0504	0.0488
Roy's	1	0.0448	0.0494	0.0474	0.518

Table 3.35. Type I error rates: etypeII, p=5, m=5, n=300

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1	0.049	0.0488	0.05	0.0474
Pillai's Trace	1	0.047	0.0464	0.0472	0.0376
Hotelling-Lawley	1	0.056	0.0546	0.055	0.0594
Roy's	1	0.0534	0.0514	0.0522	0.5274

Table 3.36. Type I error rates: etypeII, p=5, m=5, n=350

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1	0.0464	0.0422	0.0486	0.0392
Pillai's Trace	1	0.0448	0.041	0.0476	0.0334
Hotelling-Lawley	1	0.0506	0.0464	0.0544	0.048
Roy's	1	0.0482	0.0436	0.0518	0.5146

Table 3.37. Type I error rates: etypeII, p=5, m=5, n=400

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1	0.052	0.0526	0.0504	0.0488
Pillai's Trace	1	0.0506	0.0516	0.0488	0.0432
Hotelling-Lawley	1	0.056	0.0574	0.0542	0.0554
Roy's	1	0.0536	0.0552	0.052	0.5246

Table 3.38. Type I error rates: etypeII, p=5, m=5, n=450

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1	0.0462	0.0518	0.0456	0.048
Pillai's Trace	1	0.0446	0.0504	0.0446	0.0416
Hotelling-Lawley	1	0.051	0.0566	0.0502	0.0544
Roy's	1	0.0482	0.0548	0.0478	0.53

Table 3.39. Type I error rates: etypeII, p=5, m=5, n=500

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1	0.0502	0.0486	0.0508	0.05
Pillai's Trace	1	0.0492	0.0482	0.0496	0.0464
Hotelling-Lawley	1	0.0552	0.0538	0.0542	0.0572
Roy's	1	0.053	0.0518	0.0524	0.5188

Table 3.40. Type I error rates: etypeII, p=5, m=5, n=550

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1	0.0424	0.0516	0.0502	0.0462
Pillai's Trace	1	0.0418	0.05	0.0496	0.0422
Hotelling-Lawley	1	0.0448	0.054	0.0544	0.054
Roy's	1	0.0442	0.0532	0.0516	0.5266

Table 3.41. Type I error rates: etypeII, p=5, m=5, n=600

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1	0.0486	0.0512	0.0512	0.0476
Pillai's Trace	1	0.048	0.0506	0.0502	0.0428
Hotelling-Lawley	1	0.0522	0.055	0.054	0.0532
Roy's	1	0.0504	0.0528	0.0532	0.5044

Table 3.42. Type I error rates: etypeII, p=5, m=5, n=650

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1	0.0496	0.0528	0.0474	0.0424
Pillai's Trace	1	0.0486	0.0522	0.0464	0.0394
Hotelling-Lawley	1	0.0518	0.0556	0.0514	0.047
Roy's	1	0.0506	0.0538	0.0492	0.5202

Table 3.43. Type I error rates: etypeII, p=6, m=6, n=60

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	0.8834	0.0056	0.0066	0.0068	2.00E-04
Pillai's Trace	0.8274	0.0028	0.0034	0.0028	0
Hotelling-Lawley	0.9458	0.0164	0.0144	0.0152	0.0016
Roy's	0.9438	0.0158	0.0142	0.015	0.3324

Table 3.44. Type I error rates: etypeII, p=6, m=6, n=110

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	0.9988	0.0224	0.0268	0.0232	0.0048
Pillai's Trace	0.9986	0.018	0.022	0.0196	0.0016
Hotelling-Lawley	0.9996	0.0336	0.0392	0.0358	0.0142
Roy's	0.9996	0.0292	0.0344	0.0306	0.569

Table 3.45. Type I error rates: etypeII, p=6, m=6, n=160

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1	0.036	0.0338	0.0374	0.0168
Pillai's Trace	1	0.032	0.03	0.0324	0.0092
Hotelling-Lawley	1	0.048	0.0452	0.0502	0.0336
Roy's	1	0.0428	0.04	0.0448	0.6628

Table 3.46. Type I error rates: etypeII, p=6, m=6, n=210

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1	0.0452	0.0382	0.0446	0.028
Pillai's Trace	1	0.042	0.034	0.042	0.0178
Hotelling-Lawley	1	0.057	0.0456	0.0546	0.0434
Roy's	1	0.0516	0.0422	0.0506	0.693

Table 3.47. Type I error rates: etypeII, p=6, m=6, n=260

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1	0.0412	0.0512	0.0398	0.0298
Pillai's Trace	1	0.0394	0.0472	0.0374	0.021
Hotelling-Lawley	1	0.0512	0.0584	0.0474	0.047
Roy's	1	0.0466	0.0542	0.0432	0.7104

Table 3.48. Type I error rates: etypeII, p=6, m=6, n=310

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1	0.0448	0.0486	0.0488	0.036
Pillai's Trace	1	0.042	0.0454	0.0462	0.0298
Hotelling-Lawley	1	0.0506	0.0554	0.0566	0.0488
Roy's	1	0.0472	0.0518	0.0536	0.7098

Table 3.49. Type I error rates: etypeII, p=6, m=6, n=360

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1	0.0514	0.051	0.0432	0.0424
Pillai's Trace	1	0.0502	0.0488	0.0394	0.0356
Hotelling-Lawley	1	0.0556	0.0568	0.0492	0.0562
Roy's	1	0.0538	0.0536	0.0468	0.7118

Table 3.50. Type I error rates: etypeII, p=6, m=6, n=410

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1	0.05	0.0498	0.051	0.0464
Pillai's Trace	1	0.0476	0.0484	0.0496	0.0382
Hotelling-Lawley	1	0.0564	0.0566	0.056	0.0574
Roy's	1	0.0528	0.0536	0.0526	0.7232

Table 3.51. Type I error rates: etypeII, p=6, m=6, n=460

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1	0.0484	0.044	0.0492	0.0394
Pillai's Trace	1	0.046	0.042	0.0478	0.0344
Hotelling-Lawley	1	0.0524	0.048	0.0532	0.0476
Roy's	1	0.0494	0.0462	0.051	0.7158

Table 3.52. Type I error rates: etypeII, p=6, m=6, n=510

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1	0.0492	0.0422	0.0438	0.0426
Pillai's Trace	1	0.0474	0.0404	0.0424	0.0362
Hotelling-Lawley	1	0.0544	0.0468	0.0488	0.05
Roy's	1	0.0508	0.0444	0.0466	0.7026

Table 3.53. Type I error rates: etypeII, p=6, m=6, n=560

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1	0.0474	0.0468	0.0468	0.044
Pillai's Trace	1	0.0464	0.0452	0.0458	0.0384
Hotelling-Lawley	1	0.0524	0.0514	0.0494	0.0548
Roy's	1	0.05	0.049	0.0478	0.7048

Table 3.54. Type I error rates: etypeII, p=6, m=6, n=610

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1	0.0488	0.0476	0.049	0.0428
Pillai's Trace	1	0.0482	0.0458	0.0462	0.0386
Hotelling-Lawley	1	0.0524	0.0512	0.0536	0.0502
Roy's	1	0.0506	0.0496	0.0516	0.7014

Table 3.55. Type I error rates: etypeII, p=6, m=6, n=660

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1	0.0488	0.0558	0.0492	0.047
Pillai's Trace	1	0.047	0.055	0.0478	0.0438
Hotelling-Lawley	1	0.0534	0.06	0.0526	0.0524
Roy's	1	0.051	0.0574	0.0508	0.7068

Table 3.56. Type I error rates: etypeII, p=6, m=6, n=710

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1	0.0472	0.0516	0.0458	0.0462
Pillai's Trace	1	0.046	0.0498	0.0444	0.0424
Hotelling-Lawley	1	0.0494	0.0554	0.0484	0.052
Roy's	1	0.0482	0.0538	0.0472	0.7062

Table 3.57. Type I error rates: etypeII, p=7, m=7, n=70

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	0.9108	0.004	0.0044	0.0038	0
Pillai's Trace	0.8614	0.0022	0.002	0.002	0
Hotelling-Lawley	0.9662	0.0116	0.012	0.0098	0.001
Roy's	0.9718	0.0128	0.0144	0.012	0.4366

Table 3.58. Type I error rates: etypeII, p=7, m=7, n=120

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	0.9998	0.0238	0.02	0.0228	0.0028
Pillai's Trace	0.9998	0.0202	0.0154	0.0182	0
Hotelling-Lawley	1	0.0338	0.033	0.0346	0.0086
Roy's	1	0.0302	0.0276	0.031	0.7042

Table 3.59. Type I error rates: etypeII, p=7, m=7, n=170

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1	0.0314	0.0312	0.0328	0.0134
Pillai's Trace	1	0.0286	0.0276	0.028	0.006
Hotelling-Lawley	1	0.0446	0.0428	0.0466	0.0294
Roy's	1	0.0404	0.0378	0.0402	0.807

Table 3.60. Type I error rates: etypeII, p=7, m=7, n=220

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1	0.0426	0.0408	0.0466	0.0198
Pillai's Trace	1	0.0384	0.0366	0.0428	0.0126
Hotelling-Lawley	1	0.0512	0.054	0.057	0.0392
Roy's	1	0.046	0.0482	0.051	0.845

Table 3.61. Type I error rates: etypeII, p=7, m=7, n=270

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1	0.0494	0.042	0.0434	0.0264
Pillai's Trace	1	0.0456	0.0398	0.0408	0.0196
Hotelling-Lawley	1	0.0598	0.053	0.0522	0.0422
Roy's	1	0.0536	0.047	0.0474	0.8426

Table 3.62. Type I error rates: etypeII, p=7, m=7, n=320

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1	0.046	0.0486	0.0512	0.04
Pillai's Trace	1	0.0438	0.045	0.048	0.029
Hotelling-Lawley	1	0.0544	0.0584	0.0576	0.058
Roy's	1	0.0504	0.0532	0.0536	0.8496

Table 3.63. Type I error rates: etypeII, p=7, m=7, n=370

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1	0.0476	0.0478	0.053	0.0388
Pillai's Trace	1	0.0452	0.0466	0.0508	0.0298
Hotelling-Lawley	1	0.0538	0.056	0.0586	0.0512
Roy's	1	0.0504	0.051	0.0558	0.8556

Table 3.64. Type I error rates: etypeII, p=7, m=7, n=420

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1	0.0478	0.0448	0.047	0.0376
Pillai's Trace	1	0.0458	0.0428	0.0452	0.0286
Hotelling-Lawley	1	0.0528	0.0506	0.0524	0.05
Roy's	1	0.0506	0.047	0.0496	0.8534

Table 3.65. Type I error rates: etypeII, p=7, m=7, n=470

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1	0.0466	0.0448	0.0454	0.0454
Pillai's Trace	1	0.045	0.0428	0.044	0.035
Hotelling-Lawley	1	0.051	0.049	0.0512	0.0566
Roy's	1	0.0492	0.0466	0.0484	0.8504

Table 3.66. Type I error rates: etypeII, p=7, m=7, n=520

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1	0.0488	0.0486	0.0464	0.0428
Pillai's Trace	1	0.0472	0.0454	0.0446	0.0374
Hotelling-Lawley	1	0.055	0.0524	0.0504	0.0506
Roy's	1	0.0516	0.0504	0.0476	0.8468

Table 3.67. Type I error rates: etypeII, p=7, m=7, n=570

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1	0.052	0.0478	0.0488	0.0446
Pillai's Trace	1	0.05	0.0456	0.0466	0.0382
Hotelling-Lawley	1	0.0572	0.0528	0.0522	0.0522
Roy's	1	0.055	0.0504	0.0506	0.8358

Table 3.68. Type I error rates: etypeII, p=7, m=7, n=620

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1	0.0448	0.049	0.0516	0.0446
Pillai's Trace	1	0.0434	0.0472	0.0498	0.0392
Hotelling-Lawley	1	0.048	0.0546	0.0562	0.0516
Roy's	1	0.0464	0.0518	0.0536	0.8406

Table 3.69. Type I error rates: etypeII, p=7, m=7, n=670

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1	0.0554	0.0462	0.0442	0.0436
Pillai's Trace	1	0.0532	0.045	0.0438	0.0372
Hotelling-Lawley	1	0.0592	0.0502	0.049	0.0502
Roy's	1	0.0576	0.0484	0.0462	0.8448

Table 3.70. Type I error rates: etypeII, p=7, m=7, n=720

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1	0.0462	0.0484	0.0486	0.0496
Pillai's Trace	1	0.0452	0.0474	0.048	0.042
Hotelling-Lawley	1	0.049	0.0518	0.0526	0.057
Roy's	1	0.047	0.05	0.0512	0.8522

Table 3.71. Type I error rates: etypeII, p=7, m=7, n=770

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1	0.049	0.048	0.0478	0.0462
Pillai's Trace	1	0.0482	0.0468	0.047	0.0408
Hotelling-Lawley	1	0.0528	0.0522	0.0514	0.053
Roy's	1	0.051	0.0504	0.0492	0.8482

Table 3.72. Type I error rates: etypeII, p=7, m=7, n=820

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1	0.0458	0.0506	0.0452	0.0466
Pillai's Trace	1	0.0452	0.0496	0.0444	0.0424
Hotelling-Lawley	1	0.0486	0.0544	0.0478	0.055
Roy's	1	0.0466	0.0514	0.0464	0.8478

Table 3.73. Type I error rates: etypeII, p=8, m=8, n=80

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	0.9104	0.0012	0.002	0.0026	0
Pillai's Trace	0.8438	0.0006	0.0008	0.0014	0
Hotelling-Lawley	0.9704	0.005	0.0056	0.0084	0
Roy's	0.9842	0.0104	0.009	0.0116	0.5312

Table 3.74. Type I error rates: etypeII, p=8, m=8, n=130

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	0.9998	0.019	0.0202	0.019	0.0012
Pillai's Trace	0.9998	0.0148	0.0144	0.0156	0
Hotelling-Lawley	0.9998	0.03	0.0332	0.0326	0.0078
Roy's	0.9998	0.0274	0.0292	0.028	0.7994

Table 3.75. Type I error rates: etypeII, p=8, m=8, n=180

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1	0.0362	0.0342	0.0372	0.0088
Pillai's Trace	1	0.0316	0.0304	0.0332	0.0026
Hotelling-Lawley	1	0.049	0.0482	0.0512	0.027
Roy's	1	0.0432	0.043	0.0456	0.9112

Table 3.76. Type I error rates: etypeII, p=8, m=8, n=230

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1	0.0448	0.0352	0.0422	0.0186
Pillai's Trace	1	0.0392	0.0322	0.0382	0.008
Hotelling-Lawley	1	0.0586	0.0508	0.054	0.0346
Roy's	1	0.051	0.0412	0.0474	0.9288

Table 3.77. Type I error rates: etypeII, p=8, m=8, n=280

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1	0.0404	0.0422	0.046	0.0242
Pillai's Trace	1	0.0358	0.0386	0.0422	0.0134
Hotelling-Lawley	1	0.0488	0.0524	0.0558	0.0438
Roy's	1	0.0446	0.0468	0.0508	0.9312

Table 3.78. Type I error rates: etypeII, p=8, m=8, n=330

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1	0.046	0.0416	0.0442	0.0324
Pillai's Trace	1	0.043	0.0388	0.0416	0.0216
Hotelling-Lawley	1	0.0536	0.0492	0.0538	0.0488
Roy's	1	0.0492	0.044	0.0482	0.9328

Table 3.79. Type I error rates: etypeII, p=8, m=8, n=380

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1	0.0472	0.047	0.0426	0.032
Pillai's Trace	1	0.0434	0.044	0.0406	0.0218
Hotelling-Lawley	1	0.055	0.056	0.0508	0.047
Roy's	1	0.0514	0.0514	0.0464	0.9392

Table 3.80. Type I error rates: etypeII, p=8, m=8, n=430

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1	0.0412	0.0406	0.0456	0.0328
Pillai's Trace	1	0.038	0.0392	0.043	0.024
Hotelling-Lawley	1	0.049	0.05	0.0542	0.0462
Roy's	1	0.0452	0.0448	0.0494	0.9322

Table 3.81. Type I error rates: etypeII, p=8, m=8, n=480

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1	0.0446	0.048	0.0452	0.0374
Pillai's Trace	1	0.0422	0.0456	0.043	0.0294
Hotelling-Lawley	1	0.0498	0.0538	0.0514	0.0508
Roy's	1	0.0468	0.0508	0.0482	0.936

Table 3.82. Type I error rates: etypeII, p=8, m=8, n=530

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1	0.0486	0.0434	0.054	0.0402
Pillai's Trace	1	0.047	0.0422	0.0516	0.0328
Hotelling-Lawley	1	0.055	0.049	0.0608	0.053
Roy's	1	0.0528	0.0462	0.056	0.9356

Table 3.83. Type I error rates: etypeII, p=8, m=8, n=580

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1	0.0506	0.0482	0.0486	0.0416
Pillai's Trace	1	0.048	0.047	0.0474	0.0332
Hotelling-Lawley	1	0.0568	0.055	0.0538	0.0516
Roy's	1	0.0528	0.0506	0.0508	0.9312

Table 3.84. Type I error rates: etypeII, p=8, m=8, n=630

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1	0.0488	0.049	0.0502	0.037
Pillai's Trace	1	0.0474	0.0462	0.0482	0.0318
Hotelling-Lawley	1	0.0536	0.0552	0.0568	0.0454
Roy's	1	0.0514	0.052	0.0526	0.936

Table 3.85. Type I error rates: etypeII, p=8, m=8, n=680

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1	0.0462	0.0434	0.0516	0.0414
Pillai's Trace	1	0.0448	0.042	0.0494	0.0342
Hotelling-Lawley	1	0.049	0.0474	0.058	0.053
Roy's	1	0.047	0.0454	0.0544	0.9374

Table 3.86. Type I error rates: etypeII, p=8, m=8, n=730

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1	0.0486	0.044	0.0486	0.0402
Pillai's Trace	1	0.0472	0.0432	0.0468	0.0334
Hotelling-Lawley	1	0.0526	0.0486	0.0516	0.0496
Roy's	1	0.0504	0.0456	0.0496	0.9318

Table 3.87. Type I error rates: etypeII, p=8, m=8, n=780

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0494	0.0476	0.0476	0.0412
Pillai's Trace	1.0000	0.0478	0.0458	0.0462	0.037
Hotelling-Lawley	1.0000	0.0548	0.052	0.0522	0.0498
Roy's	1	0.052	0.0496	0.0512	0.9372

Table 3.88. Type I error rates: etypeII, p=8, m=8, n=830

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0476	0.0488	0.0482	0.0454
Pillai's Trace	1.0000	0.0468	0.0476	0.0464	0.0388
Hotelling-Lawley	1.0000	0.051	0.052	0.0516	0.0532
Roy's	1	0.0492	0.0506	0.0498	0.9374

Table 3.89. Type I error rates: etypeII, p=8, m=8, n=880

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0514	0.0536	0.0454	0.0406
Pillai's Trace	1.0000	0.0502	0.0528	0.0444	0.0364
Hotelling-Lawley	1.0000	0.0544	0.0578	0.049	0.046
Roy's	1	0.0528	0.055	0.0472	0.9334

Table 3.90. Type I error rates: etypeII, p=8, m=8, n=930

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0510	0.0466	0.0494	0.0426
Pillai's Trace	1.0000	0.0506	0.0456	0.0486	0.0370
Hotelling-Lawley	1.0000	0.0540	0.0508	0.0546	0.0480
Roy's	1	0.0516	0.0484	0.0510	0.9268

Table 3.91. Type I error rates: etypeII, p=8, m=8, n=980

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0444	0.0438	0.0502	0.0416
Pillai's Trace	1.0000	0.0434	0.0428	0.0480	0.0370
Hotelling-Lawley	1.0000	0.0486	0.0464	0.0536	0.0462
Roy's	1.0000	0.0460	0.0448	0.0516	0.9318

Table 3.92. Type I error rates: etypeII, p=8, m=8, n=1030

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0458	0.0430	0.0506	0.0444
Pillai's Trace	1.0000	0.0448	0.0422	0.0500	0.0392
Hotelling-Lawley	1.0000	0.0484	0.0464	0.0532	0.0508
Roy's	1.0000	0.0464	0.0440	0.0524	0.9272

Table 3.93. Type I error rates: etypeII, p=8, m=8, n=1080

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0500	0.0526	0.0462	0.0400
Pillai's Trace	1.0000	0.0494	0.0518	0.0458	0.0364
Hotelling-Lawley	1.0000	0.0526	0.0550	0.0498	0.0446
Roy's	1.0000	0.0506	0.0538	0.0484	0.9268

Table 3.94. Type I error rates: etypeII, p=8, m=8, n=1130

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0504	0.0500	0.0486	0.0464
Pillai's Trace	1.0000	0.0494	0.0496	0.0482	0.0404
Hotelling-Lawley	1.0000	0.0534	0.0520	0.0506	0.0528
Roy's	1.0000	0.0516	0.0506	0.0500	0.9318

Table 3.95. Type I error rates: etypeII, p=9, m=9, n=100

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	0.9790	0.0040	0.0016	0.0028	0.0000
Pillai's Trace	0.9596	0.0016	0.0008	0.0012	0.0000
Hotelling-Lawley	0.9944	0.0090	0.0052	0.0066	0.0000
Roy's	0.9970	0.0140	0.0104	0.0118	0.6924

Table 3.96. Type I error rates: etypeII, p=9, m=9, n=150

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0158	0.0150	0.0186	0.0010
Pillai's Trace	1.0000	0.0136	0.0116	0.0158	0.0000
Hotelling-Lawley	1.0000	0.0302	0.0290	0.0324	0.0030
Roy's	1.0000	0.0274	0.0260	0.0286	0.9086

Table 3.97. Type I error rates: etypeII, p=9, m=9, n=200

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0344	0.0362	0.0354	0.0076
Pillai's Trace	1.0000	0.0290	0.0322	0.0320	0.0020
Hotelling-Lawley	1.0000	0.0508	0.0520	0.0520	0.0238
Roy's	1.0000	0.0440	0.0438	0.0454	0.9668

Table 3.98. Type I error rates: etypeII, p=9, m=9, n=250

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0444	0.0364	0.0408	0.0122
Pillai's Trace	1.0000	0.0390	0.0332	0.0344	0.0064
Hotelling-Lawley	1.0000	0.0554	0.0478	0.0592	0.0312
Roy's	1.0000	0.0506	0.0424	0.0496	0.9708

Table 3.99. Type I error rates: etypeII, p=9, m=9, n=300

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0464	0.0466	0.0448	0.0246
Pillai's Trace	1.0000	0.0418	0.0412	0.0404	0.0136
Hotelling-Lawley	1.0000	0.0582	0.0590	0.0552	0.0416
Roy's	1.0000	0.0528	0.0510	0.0484	0.9772

Table 3.100. Type I error rates: etypeII, p=9, m=9, n=350

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0460	0.0380	0.0482	0.0266
Pillai's Trace	1.0000	0.0418	0.0360	0.0446	0.0160
Hotelling-Lawley	1.0000	0.0548	0.0478	0.0574	0.0494
Roy's	1.0000	0.0488	0.0400	0.0526	0.9784

Table 3.101. Type I error rates: etypeII, p=9, m=9, n=400

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0468	0.0450	0.0450	0.0314
Pillai's Trace	1.0000	0.0446	0.0422	0.0422	0.0200
Hotelling-Lawley	1.0000	0.0568	0.0542	0.0564	0.0450
Roy's	1.0000	0.0512	0.0492	0.0498	0.9782

Table 3.102. Type I error rates: etypeII, p=9, m=9, n=450

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0468	0.0446	0.0444	0.0318
Pillai's Trace	1.0000	0.0446	0.0424	0.0420	0.0222
Hotelling-Lawley	1.0000	0.0536	0.0506	0.0516	0.0470
Roy's	1.0000	0.0504	0.0470	0.0468	0.9740

Table 3.103. Type I error rates: etypeII, p=9, m=9, n=500

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0424	0.0450	0.0504	0.0334
Pillai's Trace	1.0000	0.0406	0.0436	0.0476	0.0242
Hotelling-Lawley	1.0000	0.0476	0.0530	0.0576	0.0458
Roy's	1.0000	0.0446	0.0490	0.0534	0.9778

Table 3.104. Type I error rates: etypeII, p=9, m=9, n=550

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0464	0.0440	0.0482	0.0414
Pillai's Trace	1.0000	0.0434	0.0430	0.0472	0.0308
Hotelling-Lawley	1.0000	0.0538	0.0496	0.0526	0.0548
Roy's	1.0000	0.0500	0.0466	0.0500	0.9794

CHAPTER 4

TYPE I ERROR SIMULATIONS, ETYPE III

Table 4.1. Type I error rates: etypeIII, p=2, m=2, n=20

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	0.9286			0.0580	0.0580
Pillai's Trace	0.8822			0.0314	0.0314
Hotelling-Lawley	0.9606			0.0962	0.0962
Roy's	0.9572			0.0902	0.0902

Table 4.2. Type I error rates: etypeIII, p=2, m=2, n=70

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000			0.0602	0.0602
Pillai's Trace	1.0000			0.0568	0.0568
Hotelling-Lawley	1.0000			0.0718	0.0718
Roy's	1.0000			0.0692	0.0692

Table 4.3. Type I error rates: etypeIII, p=2, m=2, n=120

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000			0.0442	0.0442
Pillai's Trace	1.0000			0.0422	0.0422
Hotelling-Lawley	1.0000			0.0494	0.0494
Roy's	1.0000			0.0482	0.0482

Table 4.4. Type I error rates: etypeIII, p=2, m=2, n=170

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000			0.0502	0.0502
Pillai's Trace	1.0000			0.0482	0.0482
Hotelling-Lawley	1.0000			0.0530	0.0530
Roy's	1.0000			0.0524	0.0524

Table 4.5. Type I error rates: etypeIII, p=2, m=2, n=220

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000			0.0410	0.0410
Pillai's Trace	1.0000			0.0392	0.0392
Hotelling-Lawley	1.0000			0.0458	0.0458
Roy's	1.0000			0.0448	0.0448

Table 4.6. Type I error rates: etypeIII, p=2, m=2, n=270

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000			0.0428	0.0428
Pillai's Trace	1.0000			0.0426	0.0426
Hotelling-Lawley	1.0000			0.0444	0.0444
Roy's	1.0000			0.0436	0.0436

Table 4.7. Type I error rates: etypeIII, p=2, m=2, n=320

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000			0.0480	0.0480
Pillai's Trace	1.0000			0.0476	0.0476
Hotelling-Lawley	1.0000			0.0508	0.0508
Roy's	1.0000			0.0492	0.0492

Table 4.8. Type I error rates: etypeIII, p=3, m=3, n=30

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	0.9962	0.0560		0.0494	0.0460
Pillai's Trace	0.9912	0.0366		0.0292	0.0116
Hotelling-Lawley	0.9976	0.0954		0.0908	0.1042
Roy's	0.9976	0.0872		0.0804	0.2366

Table 4.9. Type I error rates: etypeIII, p=3, m=3, n=80

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0602		0.0616	0.0660
Pillai's Trace	1.0000	0.0540		0.0550	0.0572
Hotelling-Lawley	1.0000	0.0770		0.0768	0.0864
Roy's	1.0000	0.0714		0.0722	0.2054

Table 4.10. Type I error rates: etypeIII, p=3, m=3, n=130

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0530		0.0492	0.0500
Pillai's Trace	1.0000	0.0498		0.0464	0.0436
Hotelling-Lawley	1.0000	0.0594		0.0564	0.0608
Roy's	1.0000	0.0566		0.0538	0.1652

Table 4.11. Type I error rates: etypeIII, p=3, m=3, n=180

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0440		0.0454	0.0418
Pillai's Trace	1.0000	0.0422		0.0432	0.0382
Hotelling-Lawley	1.0000	0.0502		0.0496	0.0492
Roy's	1.0000	0.0478		0.0480	0.1620

Table 4.12. Type I error rates: etypeIII, p=3, m=3, n=230

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0480		0.0440	0.0428
Pillai's Trace	1.0000	0.0454		0.0406	0.0384
Hotelling-Lawley	1.0000	0.0508		0.0484	0.0482
Roy's	1.0000	0.0498		0.0466	0.1506

Table 4.13. Type I error rates: etypeIII, p=3, m=3, n=280

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0482		0.0470	0.0488
Pillai's Trace	1.0000	0.0460		0.0458	0.0456
Hotelling-Lawley	1.0000	0.0528		0.0504	0.0562
Roy's	1.0000	0.0514		0.0486	0.1620

Table 4.14. Type I error rates: etypeIII, p=3, m=3, n=330

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0452		0.0496	0.0478
Pillai's Trace	1.0000	0.0444		0.0478	0.0458
Hotelling-Lawley	1.0000	0.0476		0.0544	0.0504
Roy's	1.0000	0.0466		0.0532	0.1590

Table 4.15. Type I error rates: etypeIII, p=3, m=3, n=380

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0492		0.0460	0.0470
Pillai's Trace	1.0000	0.0476		0.0448	0.0440
Hotelling-Lawley	1.0000	0.0530		0.0488	0.0494
Roy's	1.0000	0.0520		0.0480	0.1616

Table 4.16. Type I error rates: etypeIII, p=4, m=4, n=40

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0500	0.0420	0.0466	0.0324
Pillai's Trace	1.0000	0.0314	0.0238	0.0282	0.0050
Hotelling-Lawley	1.0000	0.0908	0.0810	0.0864	0.0988
Roy's	1.0000	0.0826	0.0728	0.0770	0.4144

Table 4.17. Type I error rates: etypeIII, p=4, m=4, n=90

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0632	0.0660	0.0720	0.0766
Pillai's Trace	1.0000	0.0538	0.0572	0.0668	0.0560
Hotelling-Lawley	1.0000	0.0798	0.0856	0.0914	0.1150
Roy's	1.0000	0.0736	0.0790	0.0836	0.4056

Table 4.18. Type I error rates: etypeIII, p=4, m=4, n=140

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0478	0.0502	0.0570	0.0500
Pillai's Trace	1.0000	0.0450	0.0476	0.0532	0.0400
Hotelling-Lawley	1.0000	0.0618	0.0610	0.0658	0.0674
Roy's	1.0000	0.0554	0.0572	0.0624	0.3560

Table 4.19. Type I error rates: etypeIII, p=4, m=4, n=190

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0428	0.0464	0.0456	0.0440
Pillai's Trace	1.0000	0.0408	0.0444	0.0436	0.0360
Hotelling-Lawley	1.0000	0.0486	0.0536	0.0550	0.0542
Roy's	1.0000	0.0468	0.0504	0.0506	0.3198

Table 4.20. Type I error rates: etypeIII, p=4, m=4, n=240

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0502	0.0474	0.0466	0.0432
Pillai's Trace	1.0000	0.0482	0.0438	0.0448	0.0380
Hotelling-Lawley	1.0000	0.0562	0.0528	0.0524	0.0500
Roy's	1.0000	0.0542	0.0510	0.0500	0.3296

Table 4.21. Type I error rates: etypeIII, p=4, m=4, n=290

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0546	0.0506	0.0478	0.0492
Pillai's Trace	1.0000	0.0526	0.0482	0.0468	0.0444
Hotelling-Lawley	1.0000	0.0596	0.0536	0.0506	0.0596
Roy's	1.0000	0.0572	0.0522	0.0492	0.3492

Table 4.22. Type I error rates: etypeIII, p=4, m=4, n=340

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0486	0.0430	0.0442	0.0398
Pillai's Trace	1.0000	0.0476	0.0416	0.0428	0.0358
Hotelling-Lawley	1.0000	0.0524	0.0480	0.0472	0.0448
Roy's	1.0000	0.0504	0.0462	0.0458	0.3136

Table 4.23. Type I error rates: etypeIII, p=4, m=4, n=390

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0488	0.0456	0.0502	0.0442
Pillai's Trace	1.0000	0.0474	0.0448	0.0488	0.0404
Hotelling-Lawley	1.0000	0.0514	0.0500	0.0532	0.0480
Roy's	1.0000	0.0498	0.0480	0.0516	0.3252

Table 4.24. Type I error rates: etypeIII, p=4, m=4, n=440

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0522	0.0494	0.0504	0.0480
Pillai's Trace	1.0000	0.0510	0.0482	0.0496	0.0446
Hotelling-Lawley	1.0000	0.0548	0.0518	0.0536	0.0528
Roy's	1.0000	0.0538	0.0510	0.0522	0.3322

Table 4.25. Type I error rates: etypeIII, p=4, m=4, n=490

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0506	0.0448	0.0494	0.0502
Pillai's Trace	1.0000	0.0496	0.0438	0.0484	0.0464
Hotelling-Lawley	1.0000	0.0534	0.0486	0.0508	0.0552
Roy's	1.0000	0.0524	0.0468	0.0502	0.3212

Table 4.26. Type I error rates: etypeIII, p=5, m=5, n=50

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0400	0.0386	0.0412	0.0164
Pillai's Trace	1.0000	0.0238	0.0222	0.0250	0.0016
Hotelling-Lawley	1.0000	0.0810	0.0818	0.0812	0.0784
Roy's	1.0000	0.0740	0.0754	0.0740	0.6428

Table 4.27. Type I error rates: etypeIII, p=5, m=5, n=100

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0752	0.0742	0.0784	0.0900
Pillai's Trace	1.0000	0.0634	0.0676	0.0706	0.0548
Hotelling-Lawley	1.0000	0.0940	0.0984	0.1010	0.1386
Roy's	1.0000	0.0856	0.0900	0.0918	0.6368

Table 4.28. Type I error rates: etypeIII, p=5, m=5, n=150

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0566	0.0538	0.0506	0.0534
Pillai's Trace	1.0000	0.0518	0.0494	0.0442	0.0424
Hotelling-Lawley	1.0000	0.0696	0.0650	0.0628	0.0734
Roy's	1.0000	0.0640	0.0610	0.0572	0.5548

Table 4.29. Type I error rates: etypeIII, p=5, m=5, n=200

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0482	0.0430	0.0448	0.0396
Pillai's Trace	1.0000	0.0452	0.0396	0.0416	0.0300
Hotelling-Lawley	1.0000	0.0564	0.0520	0.0564	0.0532
Roy's	1.0000	0.0526	0.0472	0.0504	0.5248

Table 4.30. Type I error rates: etypeIII, p=5, m=5, n=250

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0474	0.0426	0.0484	0.0406
Pillai's Trace	1.0000	0.0450	0.0390	0.0456	0.0334
Hotelling-Lawley	1.0000	0.0538	0.0498	0.0572	0.0516
Roy's	1.0000	0.0512	0.0458	0.0536	0.5184

Table 4.31. Type I error rates: etypeIII, p=5, m=5, n=300

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0472	0.0408	0.0460	0.0354
Pillai's Trace	1.0000	0.0454	0.0384	0.0426	0.0302
Hotelling-Lawley	1.0000	0.0520	0.0464	0.0514	0.0428
Roy's	1.0000	0.0492	0.0442	0.0486	0.5154

Table 4.32. Type I error rates: etypeIII, p=5, m=5, n=350

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0414	0.0518	0.0474	0.0412
Pillai's Trace	1.0000	0.0390	0.0488	0.0464	0.0376
Hotelling-Lawley	1.0000	0.0474	0.0568	0.0512	0.0504
Roy's	1.0000	0.0446	0.0548	0.0486	0.5240

Table 4.33. Type I error rates: etypeIII, p=5, m=5, n=400

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0508	0.0504	0.0434	0.0438
Pillai's Trace	1.0000	0.0496	0.0490	0.0424	0.0388
Hotelling-Lawley	1.0000	0.0554	0.0534	0.0486	0.0516
Roy's	1.0000	0.0534	0.0524	0.0464	0.5250

Table 4.34. Type I error rates: etypeIII, p=5, m=5, n=450

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0504	0.0468	0.0498	0.0440
Pillai's Trace	1.0000	0.0490	0.0462	0.0484	0.0402
Hotelling-Lawley	1.0000	0.0536	0.0510	0.0546	0.0506
Roy's	1.0000	0.0522	0.0486	0.0532	0.5286

Table 4.35. Type I error rates: etypeIII, p=5, m=5, n=500

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0478	0.0458	0.0418	0.0448
Pillai's Trace	1.0000	0.0456	0.0448	0.0418	0.0412
Hotelling-Lawley	1.0000	0.0502	0.0482	0.0454	0.0502
Roy's	1.0000	0.0486	0.0470	0.0440	0.5184

Table 4.36. Type I error rates: etypeIII, p=5, m=5, n=550

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0532	0.0486	0.0512	0.0456
Pillai's Trace	1.0000	0.0524	0.0476	0.0490	0.0424
Hotelling-Lawley	1.0000	0.0558	0.0522	0.0538	0.0490
Roy's	1.0000	0.0554	0.0502	0.0526	0.5184

Table 4.37. Type I error rates: etypeIII, p=5, m=5, n=600

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0520	0.0502	0.0520	0.0498
Pillai's Trace	1.0000	0.0510	0.0500	0.0512	0.0458
Hotelling-Lawley	1.0000	0.0558	0.0538	0.0544	0.0552
Roy's	1.0000	0.0542	0.0516	0.0534	0.5178

Table 4.38. Type I error rates: etypeIII, p=6, m=6, n=60

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0342	0.0332	0.0394	0.0076
Pillai's Trace	1.0000	0.0206	0.0176	0.0236	0.0004
Hotelling-Lawley	1.0000	0.0786	0.0770	0.0836	0.0598
Roy's	1.0000	0.0774	0.0746	0.0822	0.8314

Table 4.39. Type I error rates: etypeIII, p=6, m=6, n=110

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0716	0.0752	0.0706	0.0826
Pillai's Trace	1.0000	0.0640	0.0650	0.0606	0.0448
Hotelling-Lawley	1.0000	0.1018	0.1012	0.0974	0.1518
Roy's	1.0000	0.0902	0.0912	0.0868	0.8156

Table 4.40. Type I error rates: etypeIII, p=6, m=6, n=160

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0654	0.0594	0.0542	0.0614
Pillai's Trace	1.0000	0.0576	0.0542	0.0478	0.0436
Hotelling-Lawley	1.0000	0.0784	0.0742	0.0720	0.0960
Roy's	1.0000	0.0718	0.0660	0.0650	0.7434

Table 4.41. Type I error rates: etypeIII, p=6, m=6, n=210

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0450	0.0480	0.0516	0.0374
Pillai's Trace	1.0000	0.0402	0.0452	0.0474	0.0264
Hotelling-Lawley	1.0000	0.0556	0.0578	0.0594	0.0560
Roy's	1.0000	0.0506	0.0542	0.0554	0.7262

Table 4.42. Type I error rates: etypeIII, p=6, m=6, n=260

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0444	0.0456	0.0390	0.0360
Pillai's Trace	1.0000	0.0410	0.0436	0.0364	0.0288
Hotelling-Lawley	1.0000	0.0532	0.0558	0.0480	0.0504
Roy's	1.0000	0.0484	0.0506	0.0438	0.7126

Table 4.43. Type I error rates: etypeIII, p=6, m=6, n=310

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0436	0.0434	0.0428	0.0356
Pillai's Trace	1.0000	0.0410	0.0410	0.0412	0.0266
Hotelling-Lawley	1.0000	0.0510	0.0500	0.0498	0.0488
Roy's	1.0000	0.0474	0.0454	0.0464	0.7136

Table 4.44. Type I error rates: etypeIII, p=6, m=6, n=360

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0480	0.0530	0.0506	0.0402
Pillai's Trace	1.0000	0.0458	0.0498	0.0484	0.0314
Hotelling-Lawley	1.0000	0.0538	0.0586	0.0554	0.0506
Roy's	1.0000	0.0502	0.0546	0.0538	0.7142

Table 4.45. Type I error rates: etypeIII, p=6, m=6, n=410

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0504	0.0524	0.0492	0.0416
Pillai's Trace	1.0000	0.0488	0.0504	0.0476	0.0358
Hotelling-Lawley	1.0000	0.0568	0.0566	0.0546	0.0484
Roy's	1.0000	0.0536	0.0546	0.0518	0.7022

Table 4.46. Type I error rates: etypeIII, p=6, m=6, n=460

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0500	0.0448	0.0480	0.0434
Pillai's Trace	1.0000	0.0476	0.0438	0.0466	0.0368
Hotelling-Lawley	1.0000	0.0538	0.0500	0.0532	0.0518
Roy's	1.0000	0.0528	0.0478	0.0502	0.7118

Table 4.47. Type I error rates: etypeIII, p=6, m=6, n=510

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0456	0.0468	0.0440	0.0448
Pillai's Trace	1.0000	0.0432	0.0444	0.0430	0.0396
Hotelling-Lawley	1.0000	0.0504	0.0528	0.0496	0.0530
Roy's	1.0000	0.0478	0.0486	0.0462	0.6960

Table 4.48. Type I error rates: etypeIII, p=6, m=6, n=560

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0440	0.0470	0.0456	0.0442
Pillai's Trace	1.0000	0.0434	0.0452	0.0438	0.0398
Hotelling-Lawley	1.0000	0.0474	0.0516	0.0490	0.0514
Roy's	1.0000	0.0448	0.0480	0.0462	0.7012

Table 4.49. Type I error rates: etypeIII, p=6, m=6, n=610

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0508	0.0500	0.0494	0.0468
Pillai's Trace	1.0000	0.0502	0.0486	0.0488	0.0412
Hotelling-Lawley	1.0000	0.0546	0.0530	0.0536	0.0554
Roy's	1.0000	0.0524	0.0512	0.0512	0.7134

Table 4.50. Type I error rates: etypeIII, p=6, m=6, n=660

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0484	0.0504	0.0452	0.0478
Pillai's Trace	1.0000	0.0474	0.0490	0.0440	0.0426
Hotelling-Lawley	1.0000	0.0518	0.0548	0.0492	0.0544
Roy's	1.0000	0.0502	0.0516	0.0474	0.7080

Table 4.51. Type I error rates: etypeIII, p=7, m=7, n=70

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0304	0.0280	0.0302	0.0024
Pillai's Trace	1.0000	0.0166	0.0154	0.0168	0.0000
Hotelling-Lawley	1.0000	0.0702	0.0670	0.0732	0.0274
Roy's	1.0000	0.0816	0.0774	0.0836	0.9324

Table 4.52. Type I error rates: etypeIII, p=7, m=7, n=120

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0810	0.0832	0.0818	0.0872
Pillai's Trace	1.0000	0.0726	0.0710	0.0700	0.0386
Hotelling-Lawley	1.0000	0.1136	0.1160	0.1116	0.1774
Roy's	1.0000	0.1032	0.1050	0.1002	0.9330

Table 4.53. Type I error rates: etypeIII, p=7, m=7, n=170

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0534	0.0582	0.0610	0.0714
Pillai's Trace	1.0000	0.0472	0.0532	0.0550	0.0436
Hotelling-Lawley	1.0000	0.0726	0.0810	0.0836	0.1102
Roy's	1.0000	0.0628	0.0730	0.0744	0.8750

Table 4.54. Type I error rates: etypeIII, p=7, m=7, n=220

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0528	0.0508	0.0486	0.0416
Pillai's Trace	1.0000	0.0492	0.0460	0.0462	0.0264
Hotelling-Lawley	1.0000	0.0658	0.0646	0.0608	0.0628
Roy's	1.0000	0.0594	0.0590	0.0554	0.8524

Table 4.55. Type I error rates: etypeIII, p=7, m=7, n=270

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0444	0.0474	0.0456	0.0334
Pillai's Trace	1.0000	0.0420	0.0438	0.0426	0.0210
Hotelling-Lawley	1.0000	0.0568	0.0552	0.0530	0.0516
Roy's	1.0000	0.0492	0.0522	0.0488	0.8602

Table 4.56. Type I error rates: etypeIII, p=7, m=7, n=320

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0462	0.0494	0.0438	0.0318
Pillai's Trace	1.0000	0.0428	0.0468	0.0412	0.0246
Hotelling-Lawley	1.0000	0.0536	0.0592	0.0522	0.0468
Roy's	1.0000	0.0502	0.0550	0.0472	0.8574

Table 4.57. Type I error rates: etypeIII, p=7, m=7, n=370

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0400	0.0484	0.0490	0.0402
Pillai's Trace	1.0000	0.0384	0.0458	0.0466	0.0308
Hotelling-Lawley	1.0000	0.0494	0.0560	0.0578	0.0550
Roy's	1.0000	0.0444	0.0522	0.0540	0.8590

Table 4.58. Type I error rates: etypeIII, p=7, m=7, n=420

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0548	0.0472	0.0448	0.0406
Pillai's Trace	1.0000	0.0526	0.0450	0.0426	0.0310
Hotelling-Lawley	1.0000	0.0622	0.0532	0.0518	0.0534
Roy's	1.0000	0.0592	0.0508	0.0478	0.8428

Table 4.59. Type I error rates: etypeIII, p=7, m=7, n=470

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0490	0.0434	0.0498	0.0406
Pillai's Trace	1.0000	0.0478	0.0414	0.0478	0.0324
Hotelling-Lawley	1.0000	0.0554	0.0478	0.0556	0.0546
Roy's	1.0000	0.0518	0.0456	0.0528	0.8494

Table 4.60. Type I error rates: etypeIII, p=7, m=7, n=520

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0456	0.0474	0.0488	0.0398
Pillai's Trace	1.0000	0.0442	0.0464	0.0468	0.0326
Hotelling-Lawley	1.0000	0.0508	0.0536	0.0526	0.0474
Roy's	1.0000	0.0484	0.0510	0.0512	0.8460

Table 4.61. Type I error rates: etypeIII, p=7, m=7, n=570

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0460	0.0486	0.0452	0.0386
Pillai's Trace	1.0000	0.0438	0.0482	0.0446	0.0314
Hotelling-Lawley	1.0000	0.0506	0.0546	0.0488	0.0488
Roy's	1.0000	0.0484	0.0518	0.0468	0.8438

Table 4.62. Type I error rates: etypeIII, p=7, m=7, n=620

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0528	0.0428	0.0458	0.0416
Pillai's Trace	1.0000	0.0512	0.0414	0.0450	0.0344
Hotelling-Lawley	1.0000	0.0576	0.0470	0.0510	0.0494
Roy's	1.0000	0.0546	0.0446	0.0484	0.8468

Table 4.63. Type I error rates: etypeIII, p=7, m=7, n=670

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0480	0.0466	0.0494	0.0448
Pillai's Trace	1.0000	0.0466	0.0452	0.0488	0.0388
Hotelling-Lawley	1.0000	0.0522	0.0494	0.0516	0.0530
Roy's	1.0000	0.0494	0.0480	0.0506	0.8466

Table 4.64. Type I error rates: etypeIII, p=7, m=7, n=720

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0474	0.0510	0.0472	0.0464
Pillai's Trace	1.0000	0.0462	0.0502	0.0460	0.0420
Hotelling-Lawley	1.0000	0.0508	0.0538	0.0526	0.0544
Roy's	1.0000	0.0494	0.0518	0.0502	0.8514

Table 4.65. Type I error rates: etypeIII, p=7, m=7, n=770

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0506	0.0494	0.0512	0.0484
Pillai's Trace	1.0000	0.0500	0.0470	0.0504	0.0414
Hotelling-Lawley	1.0000	0.0548	0.0532	0.0542	0.0576
Roy's	1.0000	0.0524	0.0514	0.0526	0.8476

Table 4.66. Type I error rates: etypeIII, p=7, m=7, n=820

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0478	0.0490	0.0520	0.0464
Pillai's Trace	1.0000	0.0470	0.0476	0.0512	0.0420
Hotelling-Lawley	1.0000	0.0492	0.0514	0.0552	0.0510
Roy's	1.0000	0.0484	0.0500	0.0538	0.8516

Table 4.67. Type I error rates: etypeIII, p=8, m=8, n=80

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0186	0.0170	0.0178	0.0000
Pillai's Trace	1.0000	0.0076	0.0074	0.0086	0.0000
Hotelling-Lawley	1.0000	0.0566	0.0530	0.0548	0.0072
Roy's	1.0000	0.0898	0.0844	0.0920	0.9788

Table 4.68. Type I error rates: etypeIII, p=8, m=8, n=130

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0916	0.0830	0.0898	0.0878
Pillai's Trace	1.0000	0.0800	0.0692	0.0758	0.0374
Hotelling-Lawley	1.0000	0.1290	0.1238	0.1266	0.1946
Roy's	1.0000	0.1162	0.1124	0.1144	0.9758

Table 4.69. Type I error rates: etypeIII, p=8, m=8, n=180

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0660	0.0690	0.0652	0.0748
Pillai's Trace	1.0000	0.0586	0.0636	0.0578	0.0418
Hotelling-Lawley	1.0000	0.0888	0.0968	0.0852	0.1234
Roy's	1.0000	0.0794	0.0852	0.0744	0.9518

Table 4.70. Type I error rates: etypeIII, p=8, m=8, n=230

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0496	0.0478	0.0492	0.0422
Pillai's Trace	1.0000	0.0446	0.0422	0.0452	0.0262
Hotelling-Lawley	1.0000	0.0634	0.0610	0.0638	0.0720
Roy's	1.0000	0.0564	0.0548	0.0562	0.9446

Table 4.71. Type I error rates: etypeIII, p=8, m=8, n=280

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0414	0.0472	0.0434	0.0308
Pillai's Trace	1.0000	0.0392	0.0442	0.0398	0.0192
Hotelling-Lawley	1.0000	0.0530	0.0588	0.0570	0.0530
Roy's	1.0000	0.0466	0.0528	0.0500	0.9340

Table 4.72. Type I error rates: etypeIII, p=8, m=8, n=330

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0442	0.0482	0.0456	0.0326
Pillai's Trace	1.0000	0.0410	0.0446	0.0432	0.0220
Hotelling-Lawley	1.0000	0.0526	0.0570	0.0568	0.0516
Roy's	1.0000	0.0484	0.0522	0.0500	0.9400

Table 4.73. Type I error rates: etypeIII, p=8, m=8, n=380

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0526	0.0496	0.0502	0.0344
Pillai's Trace	1.0000	0.0478	0.0468	0.0466	0.0216
Hotelling-Lawley	1.0000	0.0598	0.0586	0.0586	0.0496
Roy's	1.0000	0.0566	0.0532	0.0540	0.9366

Table 4.74. Type I error rates: etypeIII, p=8, m=8, n=430

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0478	0.0482	0.0448	0.0344
Pillai's Trace	1.0000	0.0456	0.0460	0.0432	0.0250
Hotelling-Lawley	1.0000	0.0540	0.0544	0.0530	0.0504
Roy's	1.0000	0.0504	0.0512	0.0478	0.9338

Table 4.75. Type I error rates: etypeIII, p=8, m=8, n=480

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0474	0.0466	0.0482	0.0390
Pillai's Trace	1.0000	0.0460	0.0444	0.0458	0.0298
Hotelling-Lawley	1.0000	0.0516	0.0528	0.0552	0.0524
Roy's	1.0000	0.0498	0.0494	0.0522	0.9322

Table 4.76. Type I error rates: etypeIII, p=8, m=8, n=530

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0550	0.0492	0.0454	0.0346
Pillai's Trace	1.0000	0.0514	0.0482	0.0432	0.0280
Hotelling-Lawley	1.0000	0.0600	0.0562	0.0506	0.0486
Roy's	1.0000	0.0574	0.0522	0.0478	0.9334

Table 4.77. Type I error rates: etypeIII, p=8, m=8, n=580

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0466	0.0446	0.0448	0.0390
Pillai's Trace	1.0000	0.0450	0.0426	0.0438	0.0318
Hotelling-Lawley	1.0000	0.0504	0.0484	0.0500	0.0518
Roy's	1.0000	0.0488	0.0468	0.0466	0.9356

Table 4.78. Type I error rates: etypeIII, p=8, m=8, n=630

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0456	0.0476	0.0512	0.0452
Pillai's Trace	1.0000	0.0438	0.0468	0.0498	0.0368
Hotelling-Lawley	1.0000	0.0514	0.0540	0.0568	0.0550
Roy's	1.0000	0.0484	0.0508	0.0534	0.9352

Table 4.79. Type I error rates: etypeIII, p=8, m=8, n=680

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0482	0.0482	0.0398	0.0398
Pillai's Trace	1.0000	0.0480	0.0468	0.0392	0.0340
Hotelling-Lawley	1.0000	0.0514	0.0534	0.0448	0.0494
Roy's	1.0000	0.0504	0.0502	0.0416	0.9282

Table 4.80. Type I error rates: etypeIII, p=8, m=8, n=730

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0470	0.0492	0.0492	0.0478
Pillai's Trace	1.0000	0.0454	0.0480	0.0474	0.0410
Hotelling-Lawley	1.0000	0.0502	0.0540	0.0548	0.0560
Roy's	1.0000	0.0484	0.0518	0.0516	0.9326

Table 4.81. Type I error rates: etypeIII, p=8, m=8, n=780

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0506	0.0434	0.0488	0.0422
Pillai's Trace	1.0000	0.0486	0.0422	0.0480	0.0356
Hotelling-Lawley	1.0000	0.0536	0.0488	0.0530	0.0494
Roy's	1.0000	0.0522	0.0450	0.0494	0.9312

Table 4.82. Type I error rates: etypeIII, p=8, m=8, n=830

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0484	0.0514	0.0512	0.0412
Pillai's Trace	1.0000	0.0474	0.0504	0.0500	0.0356
Hotelling-Lawley	1.0000	0.0526	0.0554	0.0558	0.0498
Roy's	1.0000	0.0502	0.0534	0.0534	0.9370

Table 4.83. Type I error rates: etypeIII, p=8, m=8, n=880

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0522	0.0500	0.0432	0.0436
Pillai's Trace	1.0000	0.0514	0.0498	0.0428	0.0384
Hotelling-Lawley	1.0000	0.0552	0.0536	0.0470	0.0522
Roy's	1.0000	0.0540	0.0512	0.0444	0.9280

Table 4.84. Type I error rates: etypeIII, p=8, m=8, n=930

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0436	0.0504	0.0484	0.0438
Pillai's Trace	1.0000	0.0432	0.0502	0.0470	0.0386
Hotelling-Lawley	1.0000	0.0478	0.0536	0.0516	0.0496
Roy's	1.0000	0.0456	0.0524	0.0500	0.9334

Table 4.85. Type I error rates: etypeIII, p=8, m=8, n=980

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0458	0.0520	0.0472	0.0440
Pillai's Trace	1.0000	0.0454	0.0508	0.0464	0.0396
Hotelling-Lawley	1.0000	0.0486	0.0546	0.0520	0.0494
Roy's	1.0000	0.0470	0.0532	0.0494	0.9330

Table 4.86. Type I error rates: etypeIII, p=8, m=8, n=1030

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0512	0.0462	0.0546	0.0442
Pillai's Trace	1.0000	0.0508	0.0456	0.0526	0.0388
Hotelling-Lawley	1.0000	0.0538	0.0502	0.0570	0.0498
Roy's	1.0000	0.0522	0.0482	0.0548	0.9356

Table 4.87. Type I error rates: etypeIII, p=8, m=8, n=1080

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0478	0.0526	0.0456	0.0394
Pillai's Trace	1.0000	0.0464	0.0520	0.0448	0.0354
Hotelling-Lawley	1.0000	0.0496	0.0562	0.0498	0.0454
Roy's	1.0000	0.0486	0.0538	0.0474	0.9326

Table 4.88. Type I error rates: etypeIII, p=8, m=8, n=1130

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0494	0.0492	0.0468	0.0440
Pillai's Trace	1.0000	0.0488	0.0484	0.0460	0.0402
Hotelling-Lawley	1.0000	0.0530	0.0520	0.0502	0.0494
Roy's	1.0000	0.0510	0.0506	0.0476	0.9274

Table 4.89. Type I error rates: etypeIII, p=9, m=9, n=100

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0300	0.0252	0.0316	0.0002
Pillai's Trace	1.0000	0.0178	0.0146	0.0182	0.0000
Hotelling-Lawley	1.0000	0.0718	0.0648	0.0720	0.0120
Roy's	1.0000	0.1120	0.1064	0.1076	0.9964

Table 4.90. Type I error rates: etypeIII, p=9, m=9, n=150

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0820	0.0846	0.0904	0.0914
Pillai's Trace	1.0000	0.0696	0.0740	0.0782	0.0308
Hotelling-Lawley	1.0000	0.1222	0.1260	0.1278	0.1988
Roy's	1.0000	0.1106	0.1164	0.1176	0.9944

Table 4.91. Type I error rates: etypeIII, p=9, m=9, n=200

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0678	0.0628	0.0620	0.0724
Pillai's Trace	1.0000	0.0604	0.0566	0.0544	0.0344
Hotelling-Lawley	1.0000	0.0916	0.0844	0.0850	0.1260
Roy's	1.0000	0.0824	0.0740	0.0754	0.9816

Table 4.92. Type I error rates: etypeIII, p=9, m=9, n=250

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0548	0.0520	0.0506	0.0416
Pillai's Trace	1.0000	0.0508	0.0482	0.0470	0.0226
Hotelling-Lawley	1.0000	0.0702	0.0676	0.0686	0.0678
Roy's	1.0000	0.0642	0.0596	0.0600	0.9806

Table 4.93. Type I error rates: etypeIII, p=9, m=9, n=300

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0452	0.0462	0.0458	0.0258
Pillai's Trace	1.0000	0.0420	0.0422	0.0422	0.0126
Hotelling-Lawley	1.0000	0.0578	0.0588	0.0582	0.0474
Roy's	1.0000	0.0518	0.0512	0.0522	0.9794

Table 4.94. Type I error rates: etypeIII, p=9, m=9, n=350

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0466	0.0444	0.0430	0.0300
Pillai's Trace	1.0000	0.0440	0.0416	0.0396	0.0180
Hotelling-Lawley	1.0000	0.0578	0.0554	0.0546	0.0494
Roy's	1.0000	0.0522	0.0512	0.0488	0.9766

Table 4.95. Type I error rates: etypeIII, p=9, m=9, n=400

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0472	0.0442	0.0440	0.0276
Pillai's Trace	1.0000	0.0442	0.0430	0.0426	0.0174
Hotelling-Lawley	1.0000	0.0560	0.0530	0.0526	0.0460
Roy's	1.0000	0.0526	0.0492	0.0476	0.9758

Table 4.96. Type I error rates: etypeIII, p=9, m=9, n=450

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0426	0.0478	0.0488	0.0298
Pillai's Trace	1.0000	0.0406	0.0460	0.0460	0.0190
Hotelling-Lawley	1.0000	0.0500	0.0574	0.0574	0.0446
Roy's	1.0000	0.0456	0.0530	0.0520	0.9744

Table 4.97. Type I error rates: etypeIII, p=9, m=9, n=500

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0484	0.0400	0.0486	0.0368
Pillai's Trace	1.0000	0.0468	0.0366	0.0466	0.0276
Hotelling-Lawley	1.0000	0.0562	0.0460	0.0550	0.0496
Roy's	1.0000	0.0520	0.0420	0.0510	0.9802

Table 4.98. Type I error rates: etypeIII, p=9, m=9, n=550

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0480	0.0490	0.0440	0.0404
Pillai's Trace	1.0000	0.0468	0.0468	0.0424	0.0308
Hotelling-Lawley	1.0000	0.0558	0.0568	0.0508	0.0546
Roy's	1.0000	0.0514	0.0512	0.0468	0.9704

Table 4.99. Type I error rates: etypeIII, p=9, m=9, n=600

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.0514	0.0484	0.0514	0.0378
Pillai's Trace	1.0000	0.0504	0.0464	0.0486	0.0272
Hotelling-Lawley	1.0000	0.0570	0.0544	0.0580	0.0492
Roy's	1.0000	0.0528	0.0500	0.0534	0.9748

CHAPTER 5

POWER SIMULATIONS, ETYPE I

Table 5.1. Power simulations: etypeI, p=3, m=3, n=30

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	0.9998	0.8220		0.0690	0.7648
Pillai's Trace	0.9988	0.7906		0.0422	0.6026
Hotelling-Lawlay	0.9998	0.8546		0.1174	0.8344
Roy's	0.9998	0.8482		0.1084	0.8934

Table 5.2. Power simulations: etypeI, p=3, m=3, n=80

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.9968		0.0802	0.9970
Pillai's Trace	1.0000	0.9966		0.0722	0.9964
Hotelling-Lawlay	1.0000	0.9968		0.0970	0.9972
Roy's	1.0000	0.9968		0.0922	0.9980

Table 5.3. Power simulations: etypeI, p=3, m=3, n=130

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	1.0000		0.0530	1.0000
Pillai's Trace	1.0000	1.0000		0.0484	1.0000
Hotelling-Lawlay	1.0000	1.0000		0.0582	1.0000
Roy's	1.0000	1.0000		0.0566	1.0000

Table 5.4. Power simulations: etypeI, p=4, m=4, n=40

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.9826	0.8958	0.0674	0.9834
Pillai's Trace	1.0000	0.9774	0.8752	0.0442	0.9504
Hotelling-Lawlay	1.0000	0.9896	0.9206	0.1242	0.9928
Roy's	1.0000	0.9874	0.9152	0.1080	0.9984

Table 5.5. Power simulations: etypeI, p=4, m=4, n=90

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	1.0000	0.9980	0.0794	1.0000
Pillai's Trace	1.0000	1.0000	0.9978	0.0706	1.0000
Hotelling-Lawlay	1.0000	1.0000	0.9982	0.0958	1.0000
Roy's	1.0000	1.0000	0.9982	0.0892	1.0000

Table 5.6. Power simulations: etypeI, p=5, m=5, n=50

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.9982	0.9274	0.0640	1.0000
Pillai's Trace	1.0000	0.9972	0.9126	0.0392	0.9976
Hotelling-Lawlay	1.0000	0.9990	0.9478	0.1284	1.0000
Roy's	1.0000	0.9988	0.9446	0.1176	1.0000

Table 5.7. Power simulations: etypeI, p=5, m=5, n=100

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	1.0000	0.9988	0.0838	1.0000
Pillai's Trace	1.0000	1.0000	0.9988	0.0728	1.0000
Hotelling-Lawlay	1.0000	1.0000	0.9988	0.1070	1.0000
Roy's	1.0000	1.0000	0.9988	0.0990	1.0000

Table 5.8. Power simulations: etypeI, p=6, m=6, n=60

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	1.0000	0.9450	0.0546	1.0000
Pillai's Trace	1.0000	1.0000	0.9334	0.0316	1.0000
Hotelling-Lawlay	1.0000	1.0000	0.9624	0.1114	1.0000
Roy's	1.0000	1.0000	0.9616	0.1094	1.0000

Table 5.9. Power simulations: etypeI, p=7, m=7, n=70

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	1.0000	0.9656	0.0446	1.0000
Pillai's Trace	1.0000	1.0000	0.9570	0.0242	1.0000
Hotelling-Lawlay	1.0000	1.0000	0.9780	0.0982	1.0000
Roy's	1.0000	1.0000	0.9790	0.1104	1.0000

Table 5.10. Power simulations: etypeI, p=8, m=8, n=80

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	1.0000	0.9680	0.0328	1.0000
Pillai's Trace	1.0000	1.0000	0.9570	0.0208	1.0000
Hotelling-Lawlay	1.0000	1.0000	0.9802	0.0768	1.0000
Roy's	1.0000	1.0000	0.9840	0.1160	1.0000

Table 5.11. Power simulations: etypeI, p=9, m=9, n=100

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	1.0000	0.9848	0.0412	1.0000
Pillai's Trace	1.0000	1.0000	0.9808	0.0256	1.0000
Hotelling-Lawlay	1.0000	1.0000	0.9898	0.0858	1.0000
Roy's	1.0000	1.0000	0.9920	0.1286	1.0000

Table 5.12. Power simulations: etypeI, p=10, m=10, n=110

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	1.0000	0.9788	0.0098	1.0000
Pillai's Trace	1.0000	1.0000	0.9694	0.0036	1.0000
Hotelling-Lawlay	1.0000	1.0000	0.9874	0.0324	1.0000
Roy's	1.0000	1.0000	0.9958	0.1432	1.0000

CHAPTER 6

POWER SIMULATIONS, ETYPE II

Table 6.1. Power simulations: etypeII, p=3, m=3, n=30

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	0.5544	0.2182		0.0150	0.1190
Pillai's Trace	0.4614	0.1706		0.0076	0.0492
Hotelling-Lawley	0.6706	0.2972		0.0298	0.2098
Roy's	0.6504	0.2794		0.0266	0.3706

Table 6.2. Power simulations: etypeII, p=3, m=3, n=80

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	0.9836	0.7456		0.0306	0.6084
Pillai's Trace	0.9820	0.7304		0.0260	0.5614
Hotelling-Lawley	0.9878	0.7748		0.0408	0.6648
Roy's	0.9872	0.7664		0.0370	0.8300

Table 6.3. Power simulations: etypeII, p=3, m=3, n=130

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	0.9998	0.9468		0.0384	0.8932
Pillai's Trace	0.9998	0.9438		0.0354	0.8800
Hotelling-Lawley	0.9998	0.9544		0.0454	0.9084
Roy's	0.9998	0.9514		0.0422	0.9698

Table 6.4. Power simulations: etypeII, p=3, m=3, n=180

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.9880		0.0428	0.9734
Pillai's Trace	1.0000	0.9878		0.0410	0.9698
Hotelling-Lawley	1.0000	0.9888		0.0480	0.9760
Roy's	1.0000	0.9886		0.0462	0.9932

Table 6.5. Power simulations: etypeII, p=3, m=3, n=230

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.9974		0.0478	0.9940
Pillai's Trace	1.0000	0.9974		0.0458	0.9938
Hotelling-Lawley	1.0000	0.9974		0.0524	0.9950
Roy's	1.0000	0.9974		0.0504	0.9984

Table 6.6. Power simulations: etypeII, p=3, m=3, n=280

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.9996		0.0460	0.9986
Pillai's Trace	1.0000	0.9996		0.0448	0.9986
Hotelling-Lawley	1.0000	0.9998		0.0488	0.9986
Roy's	1.0000	0.9998		0.0476	1.0000

Table 6.7. Power simulations: etypeII, p=3, m=3, n=330

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	1.0000		0.0488	0.9998
Pillai's Trace	1.0000	1.0000		0.0472	0.9998
Hotelling-Lawley	1.0000	1.0000		0.0510	0.9998
Roy's	1.0000	1.0000		0.0500	1.0000

Table 6.8. Power simulations: etypeII, p=3, m=3, n=380

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.9998		0.0460	1.0000
Pillai's Trace	1.0000	0.9998		0.0444	1.0000
Hotelling-Lawley	1.0000	1.0000		0.0484	1.0000
Roy's	1.0000	0.9998		0.0478	1.0000

Table 6.9. Power simulations: etypeII, p=4, m=4, n=40

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	0.7074	0.4352	0.2466	0.0136	0.3102
Pillai's Trace	0.6294	0.3656	0.1920	0.0080	0.1252
Hotelling-Lawley	0.8148	0.5326	0.3382	0.0308	0.4794
Roy's	0.7938	0.5118	0.3190	0.0266	0.7984

Table 6.10. Power simulations: etypeII, p=4, m=4, n=90

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	0.9918	0.9396	0.7676	0.0292	0.9474
Pillai's Trace	0.9906	0.9316	0.7492	0.0226	0.9258
Hotelling-Lawley	0.9952	0.9502	0.8010	0.0396	0.9670
Roy's	0.9940	0.9472	0.7872	0.0360	0.9952

Table 6.11. Power simulations: etypeII, p=4, m=4, n=140

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.9960	0.9518	0.0368	0.9994
Pillai's Trace	1.0000	0.9958	0.9492	0.0338	0.9994
Hotelling-Lawley	1.0000	0.9966	0.9600	0.0424	0.9996
Roy's	1.0000	0.9964	0.9578	0.0396	1.0000

Table 6.12. Power simulations: etypeII, p=4, m=4, n=190

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.9998	0.9920	0.0444	1.0000
Pillai's Trace	1.0000	0.9998	0.9916	0.0414	1.0000
Hotelling-Lawley	1.0000	0.9998	0.9930	0.0522	1.0000
Roy's	1.0000	0.9998	0.9924	0.0492	1.0000

Table 6.13. Power simulations: etypeII, p=5, m=5, n=50

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	0.8126	0.5924	0.2784	0.0114	0.5210
Pillai's Trace	0.7454	0.5230	0.2288	0.0072	0.2406
Hotelling-Lawley	0.8940	0.7030	0.3782	0.0246	0.7246
Roy's	0.8846	0.6862	0.3628	0.0220	0.9700

Table 6.14. Power simulations: etypeII, p=5, m=5, n=100

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	0.9974	0.9848	0.7776	0.0302	0.9972
Pillai's Trace	0.9962	0.9828	0.7568	0.0262	0.9908
Hotelling-Lawley	0.9988	0.9892	0.8194	0.0426	0.9990
Roy's	0.9984	0.9880	0.8058	0.0376	1.0000

Table 6.15. Power simulations: etypeII, p=5, m=5, n=150

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	0.9998	1.0000	0.9514	0.0396	1.0000
Pillai's Trace	0.9998	1.0000	0.9476	0.0362	1.0000
Hotelling-Lawley	0.9998	1.0000	0.9606	0.0500	1.0000
Roy's	0.9998	1.0000	0.9574	0.0452	1.0000

Table 6.16. Power simulations: etypeII, p=6, m=6, n=60

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	0.8828	0.7160	0.2994	0.0120	0.6922
Pillai's Trace	0.8248	0.6482	0.2456	0.0076	0.3338
Hotelling-Lawley	0.9432	0.8176	0.4044	0.0278	0.8812
Roy's	0.9410	0.8144	0.4016	0.0272	0.9988

Table 6.17. Power simulations: etypeII, p=6, m=6, n=110

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	0.9992	0.9940	0.7862	0.0284	1.0000
Pillai's Trace	0.9988	0.9928	0.7668	0.0244	0.9996
Hotelling-Lawley	0.9994	0.9962	0.8282	0.0430	1.0000
Roy's	0.9992	0.9958	0.8138	0.0372	1.0000

Table 6.18. Power simulations: etypeII, p=6, m=6, n=160

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	1.0000	0.9590	0.0384	1.0000
Pillai's Trace	1.0000	1.0000	0.9530	0.0346	1.0000
Hotelling-Lawley	1.0000	1.0000	0.9686	0.0496	1.0000
Roy's	1.0000	1.0000	0.9640	0.0440	1.0000

Table 6.19. Power simulations: etypeII, p=7, m=7, n=70

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	0.9058	0.7928	0.3022	0.0088	0.7924
Pillai's Trace	0.8578	0.7330	0.2422	0.0050	0.3660
Hotelling-Lawley	0.9620	0.8754	0.4140	0.0232	0.9476
Roy's	0.9678	0.8876	0.4334	0.0278	1.0000

Table 6.20. Power simulations: etypeII, p=7, m=7, n=120

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	0.9994	0.9988	0.7934	0.0272	1.0000
Pillai's Trace	0.9992	0.9980	0.7710	0.0214	1.0000
Hotelling-Lawley	0.9998	0.9996	0.8376	0.0458	1.0000
Roy's	0.9998	0.9994	0.8228	0.0380	1.0000

Table 6.21. Power simulations: etypeII, p=8, m=8, n=80

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	0.9192	0.8440	0.2952	0.0056	0.8278
Pillai's Trace	0.8702	0.7862	0.2226	0.0024	0.2612
Hotelling-Lawley	0.9710	0.9200	0.4248	0.0156	0.9714
Roy's	0.9856	0.9434	0.4868	0.0270	1.0000

Table 6.22. Power simulations: etypeII, p=8, m=8, n=130

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.9998	0.7912	0.0284	1.0000
Pillai's Trace	1.0000	0.9996	0.7666	0.0234	1.0000
Hotelling-Lawley	1.0000	0.9998	0.8424	0.0482	1.0000
Roy's	1.0000	0.9998	0.8286	0.0430	1.0000

Table 6.23. Power simulations: etypeII, p=9, m=9, n=100

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	0.9770	0.9506	0.4192	0.0062	1.0000
Pillai's Trace	0.9604	0.9264	0.3522	0.0038	1.0000
Hotelling-Lawley	0.9926	0.9772	0.5456	0.0194	1.0000
Roy's	0.9972	0.9874	0.6058	0.0306	1.0000

CHAPTER 7

POWER SIMULATIONS, ETYPE III

Table 7.1. Power simulations: etypeIII, p=3, m=3, n=30

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	0.9946	0.7566		0.0510	0.6530
Pillai's Trace	0.9902	0.7064		0.0294	0.4452
Hotelling-Lawley	0.9982	0.8102		0.0934	0.7658
Roy's	0.9980	0.8012		0.0844	0.8620

Table 7.2. Power simulations: etypeIII, p=3, m=3, n=80

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.9944		0.0688	0.9944
Pillai's Trace	1.0000	0.9944		0.0618	0.9938
Hotelling-Lawley	1.0000	0.9946		0.0834	0.9954
Roy's	1.0000	0.9944		0.0770	0.9984

Table 7.3. Power simulations: etypeIII, p=3, m=3, n=130

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	1.0000		0.0538	1.0000
Pillai's Trace	1.0000	1.0000		0.0502	1.0000
Hotelling-Lawley	1.0000	1.0000		0.0612	1.0000
Roy's	1.0000	1.0000		0.0580	1.0000

Table 7.4. Power simulations: etypeIII, p=4, m=4, n=40

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.9692	0.8394	0.0486	0.9664
Pillai's Trace	0.9996	0.9568	0.8028	0.0290	0.8948
Hotelling-Lawley	1.0000	0.9800	0.8862	0.0990	0.9852
Roy's	1.0000	0.9778	0.8794	0.0868	0.9972

Table 7.5. Power simulations: etypeIII, p=4, m=4, n=90

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	1.0000	0.9968	0.0682	1.0000
Pillai's Trace	1.0000	1.0000	0.9968	0.0594	1.0000
Hotelling-Lawley	1.0000	1.0000	0.9970	0.0844	1.0000
Roy's	1.0000	1.0000	0.9970	0.0766	1.0000

Table 7.6. Power simulations: etypeIII, p=5, m=5, n=50

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.9962	0.8904	0.0448	0.9984
Pillai's Trace	1.0000	0.9956	0.8646	0.0270	0.9878
Hotelling-Lawley	1.0000	0.9972	0.9226	0.0916	0.9998
Roy's	1.0000	0.9970	0.9190	0.0844	0.9998

Table 7.7. Power simulations: etypeIII, p=5, m=5, n=100

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	1.0000	0.9992	0.0652	1.0000
Pillai's Trace	1.0000	1.0000	0.9990	0.0588	1.0000
Hotelling-Lawley	1.0000	1.0000	0.9994	0.0874	1.0000
Roy's	1.0000	1.0000	0.9994	0.0776	1.0000

Table 7.8. Power simulations: etypeIII, p=6, m=6, n=60

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	0.8828	0.7160	0.2994	0.0120	0.6922
Pillai's Trace	0.8248	0.6482	0.2456	0.0076	0.3338
Hotelling-Lawley	0.9432	0.8176	0.4044	0.0278	0.8812
Roy's	0.9410	0.8144	0.4016	0.0272	0.9988

Table 7.9. Power simulations: etypeIII, p=6, m=6, n=110

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	0.9992	0.9940	0.7862	0.0284	1.0000
Pillai's Trace	0.9988	0.9928	0.7668	0.0244	0.9996
Hotelling-Lawley	0.9994	0.9962	0.8282	0.0430	1.0000
Roy's	0.9992	0.9958	0.8138	0.0372	1.0000

Table 7.10. Power simulations: etypeIII, p=6, m=6, n=110

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	1.0000	0.9590	0.0384	1.0000
Pillai's Trace	1.0000	1.0000	0.9530	0.0346	1.0000
Hotelling-Lawley	1.0000	1.0000	0.9686	0.0496	1.0000
Roy's	1.0000	1.0000	0.9640	0.0440	1.0000

Table 7.11. Power simulations: etypeIII, p=7, m=7, n=70

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	0.9058	0.7928	0.3022	0.0088	0.7924
Pillai's Trace	0.8578	0.7330	0.2422	0.0050	0.3660
Hotelling-Lawley	0.9620	0.8754	0.4140	0.0232	0.9476
Roy's	0.9678	0.8876	0.4334	0.0278	1.0000

Table 7.12. Power simulations: etypeIII, p=7, m=7, n=120

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	0.9994	0.9988	0.7934	0.0272	1.0000
Pillai's Trace	0.9992	0.9980	0.7710	0.0214	1.0000
Hotelling-Lawley	0.9998	0.9996	0.8376	0.0458	1.0000
Roy's	0.9998	0.9994	0.8228	0.0380	1.0000

Table 7.13. Power simulations: etypeIII, p=8, m=8, n=80

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	0.9192	0.8440	0.2952	0.0056	0.8278
Pillai's Trace	0.8702	0.7862	0.2226	0.0024	0.2612
Hotelling-Lawley	0.9710	0.9200	0.4248	0.0156	0.9714
Roy's	0.9856	0.9434	0.4868	0.0270	1.0000

Table 7.14. Power simulations: etypeIII, p=8, m=8, n=130

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	1.0000	0.9998	0.7912	0.0284	1.0000
Pillai's Trace	1.0000	0.9996	0.7666	0.0234	1.0000
Hotelling-Lawley	1.0000	0.9998	0.8424	0.0482	1.0000
Roy's	1.0000	0.9998	0.8286	0.0430	1.0000

Table 7.15. Power simulations: etypeIII, p=9, m=9, n=100

	b_1	b_2	b_{p-1}	b_p	MANOVA F
Wilk's Lambda	0.9770	0.9506	0.4192	0.0062	1.0000
Pillai's Trace	0.9604	0.9264	0.3522	0.0038	1.0000
Hotelling-Lawley	0.9926	0.9772	0.5456	0.0194	1.0000
Roy's	0.9972	0.9874	0.6058	0.0306	1.0000

CHAPTER 8

CONCLUSIONS

Multivariate linear regression is a semiparametric method that is nearly as easy to use as multiple linear regression if m is small. The m response and residual plots should be made as well as the DD plot. For the classical estimator, response and residual plots can look good for $n \geq 10p$, but for testing may need n as listed in table 8.1,and for highly skewed data, the robust tests may fail. If the plotted points in the DD plot cluster tightly about a line through the origin, then an elliptically contoured error distribution may be reasonable, and then the first row of $\hat{\boldsymbol{B}}$ corresponding to the intercepts should be similar for both the robust and classical estimators.

Table 8.1. n for a better type I error rates when H_0 is true

Test	$etypeI$	$etypeII$	$etypeIII$
Hotelling-Lawley	$141 + 0.64\,(m+p)^2$	$89 + 1.0\,(m+p)^2$	$150 + 0.65\,(m+p)^2$
Wilk's Lambda	$25 + 3\,(m+p)^2$	$141 + 2.3\,(m+p)^2$	$11 + 2.2\,(m+p)^2$
Pillai's Trace	$64 + 3.4\,(m+p)^2$	$90 + 3.6\,(m+p)^2$	$118 + 3.6\,(m+p)^2$

The *R* software was used to make plots and software. See R Development Core Team (2011). The programs in the collection of functions *mpack.txt* are available at (www.math.siu.edu/olive/mpack.txt). The function `rmregsim` was used to simulate the tests of hypotheses, and `rmregddsim` simulated the DD plots for various distributions. The function `rmltreg` makes the response and residual plots and computes the F_j, MANOVA F and MANOVA partial F test pvalues while the function `ddplot4` makes the DD plots. Similar functions for the classical estimator delete the initial "r."

REFERENCES

[1] Anderson, T.W. (1984), *An Introduction to Multivariate Statistical Analysis,* 2nd ed., Wiley, New York, NY.

[2] Buxton, L.H.D. (1920), "The Anthropology of Cyprus," *The Journal of the Royal Anthropological Institute of Great Britain and Ireland,* 50, 183-235.

[3] Cook, R.D., and Weisberg, S. (1999), *Applied Regression Including Computing and Graphics,* Wiley, New York, NY.

[4] Hampel, F.R. (1975), "Beyond Location Parameters: Robust Concepts and Methods," *Bulletin of the International Statistical Institute,* 46, 375-382.

[5] Henderson, H.V., and Searle, S.R. (1977), "Vec and Vech Operators for Matrices, with Some Uses in Jacobians and Multivariate Statistics," *The Canadian Journal of Statistics,* 7, 65-81.

[6] Hössjer, O. (1991), *Rank-Based Estimates in the Linear Model with High Breakdown Point*, Ph.D. Thesis, Report 1991:5, Department of Mathematics, Uppsala University, Uppsala, Sweden.

[7] Johnson, M.E. (1987), *Multivariate Statistical Simulation,* Wiley, New York, NY.

[8] Kakizawa, Y. (2009), "Third-order Power Comparisons for a Class of Tests for Multivariate Linear Hypothesis Under General Distributions," *Journal of Multivariate Analysis*, 100, 473-496.

[9] Khattree, R., and Naik, D.N. (1999), *Applied Multivariate Statistics with SAS Software,* 2nd ed., SAS Institute, Cary, NC.

[10] Kshirsagar, A.M. (1972), *Multivariate Analysis*, Marcel Dekker, New York, NY.

[11] Olive, D.J. (2002), "Applications of Robust Distances for Regression," *Technometrics,* 44, 64-71.

[12] Olive, D.J. (2005), "Two Simple Resistant Regression Estimators," *Computational Statistics and Data Analysis*, 49, 809-819.

[13] Olive, D.J. (2013a), *Robust Multivariate Linear Regression* Preprint available in on (www.math.siu.edu/olive)

[14] Olive, D.J. (2013b), "Plots, Prediction and Testing for the Multivariate Linear Model," preprint, see (www.math.siu.edu/olive/ppmultreg.pdf).

[15] Olive, D.J., and Hawkins, D.M. (2005), "Variable Selection for 1D Regression Models," *Technometrics*, 47, 43-50.

[16] Olive, D.J., and Hawkins, D.M. (2010), "Robust Multivariate Location and Dispersion," Preprint, see (www.math.siu.edu/olive/preprints.htm).

[17] Olive, D.J., and Hawkins, D.M. (2011), "Practical High Breakdown Regression," Preprint, see (www.math.siu.edu/olive/preprints.htm).

[18] R Development Core Team (2011), "R: a Language and Environment for Statistical Computing," R Foundation for Statistical Computing, Vienna, Austria, (www.R-project.org).

[19] Rousseeuw, P.J. (1984), "Least Median of Squares Regression," *Journal of the American Statistical Association,* 79, 871-880.

[20] Rousseeuw, P.J., and Van Driessen, K. (1999), "A Fast Algorithm for the Minimum Covariance Determinant Estimator," *Technometrics,* 41, 212-223.

[21] Seber, G.A.F., and Lee, A.J. (2003), *Linear Regression Analysis,* 2nd ed., Wiley, New York, NY.

[22] Su, Z., and Cook, R.D. (2012), "Inner Envelopes: Efficient Estimation in Multivariate Linear Regression," *Biometrika*, 99, 687-702.

[23] Zhang, J., Olive, D.J., and Ye, P. (2012), "Robust Covariance Matrix Estimation With Canonical Correlation Analysis," *International Journal of Statistics and Probability*, 1, 119-136.

VITA

Graduate School
Southern Illinois University

Hasthika S. Rupasinghe Date of Birth: August 15, 1985

509 S. Wall Street, Apt. 10, Carbondale, Illinois 62901

hasthika@siu.edu

University of Sri Jayewardenepura, Sri Lanka
Bachelor of Science (Special Degree), Mathematics, July 2010

Research Paper Title:
Robust Multivariate Linear Regression

Major Professor: Dr. D. Olive