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Publication date:
1998

Document Version
Early version, also known as pre-print

EUROPEAN COOPERATION IN
THE FIELD OF SCIENTIFIC
AND TECHNICAL RESEARCH

EURO-COST

COST 259 TD (98) 11
Bern, February 1998

Transmit-Receive Diversity in a Scattering Environment

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Abstract

Two antenna arrays, one transmit and one receive, illuminate a scattering environment of random scatterers between the two antennas. It is assumed that the channel is reciprocal for uplink and downlink. The maximum power transfer between the two antennas is studied as a function of the angular spread seen from the antennas. For small angular spreads the maximum power transfer corresponds to the product of the two antenna gains, while some reduced gain occurs for wider angular spreads. For wide angular spreads both antennas contribute to the diversity gain.

Introduction

The most common form of diversity is receive diversity, where a number of antenna signals are combined in such a way as to enhance a quality factor, which could be the power or signal quality in a certain sense. However, in a more general situation there is also a possibility of using transmit diversity or both, although there are some problems with transmit diversity in the sense that often the propagation channel is unknown, making it impossible in principle to choose the proper antenna weights. This problem will not be discussed in detail here, it will just be assumed that the channel is reciprocal, which will be the case for stationary or slowly moving terminals in a TDD (time-division-duplex) system, where the same carrier is used for uplink and downlink. The application could be a link between two computer terminals, where an area at each terminal would be available for a planar array of antenna elements. The main topic of the paper is a discussion of the possible advantages of having transmit diversity as well as receive diversity in the link. For simplicity only the narrow band case will be treated, so in an environment with large delay spreads some equalisation may be needed. The implication for wideband communication between two antenna apertures is a much reduced path loss, which decreases with frequency, instead of the normal free space path loss quadratically increasing with frequency. The cost is an additional complexity.

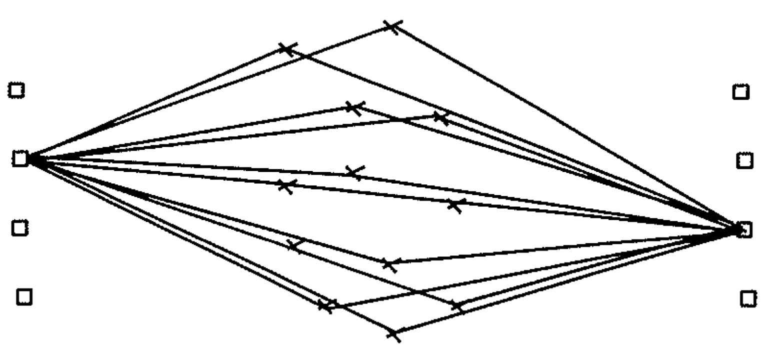

Figure 1 *Two antenna arrays connected via a random medium*

Description of arrays and environment

Consider the situation in Fig. 1 where an array of NT elements is transmitting towards an array of NR receive elements through a collection of random scatterers. No line-of-sight exists. Each element illuminates all the scatterers as illustrated in the connection between two arbitrary elements. A transmit current vector $\boldsymbol{I} = (I_1\ , I_2\ , I_3 ...)$ excites the transmit array, and at the receive array a receiving factor $\boldsymbol{V} = (V_1\ , V_2\ , V_3 .....)$ is used to combine the signals into one output port. The scatterers are chosen to be random complex isotropic scatterers, and multiple scattering is neglected. The question addressed is the following. Given a collection of scatterers of a given angular extent, how can $\boldsymbol{I}$ and $\boldsymbol{V}$ be chosen, such that maximum gain in the transfer of power is achieved. First, a simple iterative strategy is applied.

1. The transmit array is transmitting a maximum gain free space beam towards the scatterers, i.e. in $\boldsymbol{I}^{(1)}$ all the elements I_i are identical. The magnitudes are normalised such that the total radiated power is constant, independent also of the number of elements.

2. At the receive array a maximum ratio combining is performed, i.e the weights are chosen equal to the complex conjugate of the received signal. This maximises the output power.

3. Since the channel is reciprocal the same weights are used in retransmission back to the transmitter, where now a vector proportional to $\boldsymbol{I}^{(2)}$ is received. A weighting factor $\boldsymbol{I}^{(2)*}$ is now used to transmit from the transmit array again, and the iterations repeat.

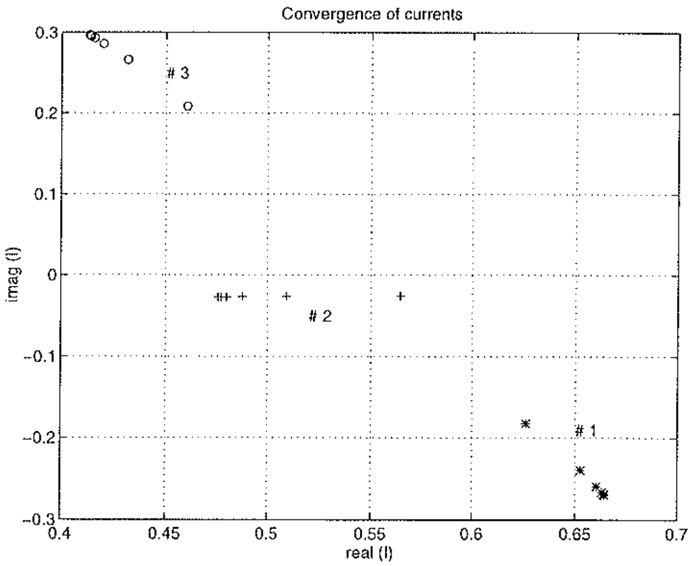

***Figure 2** Convergence of the three currents in the transmit array of a (3,3) link through 4-5 iterations.*

Numerical experiments have shown the process to converge rapidly (Figure 2), so the final result maximises the power transfer from the transmitter to the receiver.

Gain optimization by matrix methods.

Although it is possible in practice to use the iterative technique above for finding the relative currents and the maximum gain, it is also possible to find the results directly using matrix theory. For simplicity an equal number of elements, *N*, are used on both sides.

For given positions of the arrays the coupling between the elements is given by

$$\begin{aligned} V_1 &= Z_{11}\, I_1 + Z_{12}\, I_2 \\ V_2 &= Z_{21}\, I_2 + Z_{22}\, I_2 \end{aligned} \quad \textbf{(1)}$$

in agreement with Figure 3. Note that it is not a standard impedance matrix, the V's are on the right side, the I's on the left side of the scatterers, so in general Z_{12} is different from Z_{21}.

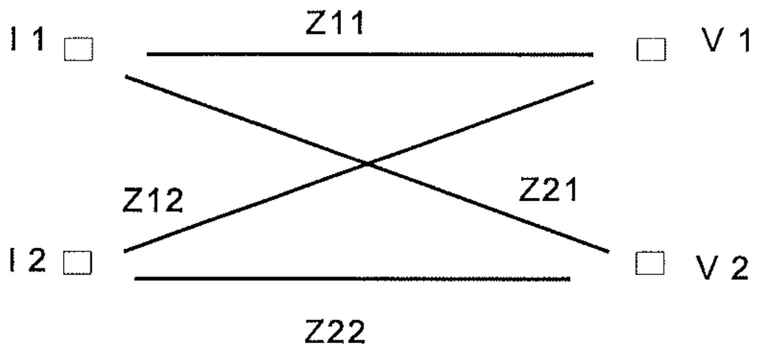

***Figure 3** Matrix description of the coupling between the antennas on the two sides.*

The elements of the Z-matrix are the sum of all the scattering interactions shown in Figure 1. In matrix notation

$$\begin{aligned} \boldsymbol{V}_r &= \boldsymbol{Z}\cdot\boldsymbol{I}_t \\ \boldsymbol{V}_t = \boldsymbol{V}_r^* &= \boldsymbol{Z}^*\cdot\boldsymbol{I}_t^* \end{aligned} \quad \textbf{(2)}$$

For maximum ratio combining or maximization of signal-to-noise ratio the weights for reception and transmission are chosen as V^* (complex conjugate). The transmission matrix towards the left equals the transpose of **Z** due to the numbering, and the received signal on the left equals

$$\begin{aligned} k\, \boldsymbol{I}_r = \boldsymbol{Z}^T\cdot\boldsymbol{V}_t &= \boldsymbol{Z}^T\cdot\boldsymbol{Z}^*\cdot\boldsymbol{I}_t^* = \boldsymbol{Y}\cdot\boldsymbol{I}_r \\ \boldsymbol{Y} &= \boldsymbol{Z}^T\cdot\boldsymbol{Z}^* \end{aligned} \quad \textbf{(3)}$$

The current on the left should be proportional to the current on the right hand side in the steady state, so $\boldsymbol{Y}$ is the Hermitian matrix describing the round trip. The above equation is an eigenvalue equation with k as the eigenvalue and the corresponding current as the eigenvector. The power transfer is given by

$$P = \frac{\boldsymbol{I}^{*T} \cdot \boldsymbol{Y} \cdot \boldsymbol{I}}{\boldsymbol{I}^{*T} \cdot \boldsymbol{I}} \tag{4}$$

and it is easy to see, that when I is the solution to the eigenvalue equation, then P is an eigenvalue, and in order to maximize P, it should be chosen as the largest eigenvalue. Since $\boldsymbol{Y}$ is Hermitian the eigenvalues are positive and real. An analytical result for the (2,2) case is treated in the appendix.

In the following some numerical results will be presented, where the scatterers (M=25) will be given a random position in x and y midway between the two arrays, so the average spread is the same seen from each antenna. The center of the scatterers lie on a line connecting the two arrays. A variable parameter is the angular spread (standard deviation) of the scatterer distribution as seen from the arrays. The spacing between antenna elements is half a wavelength.

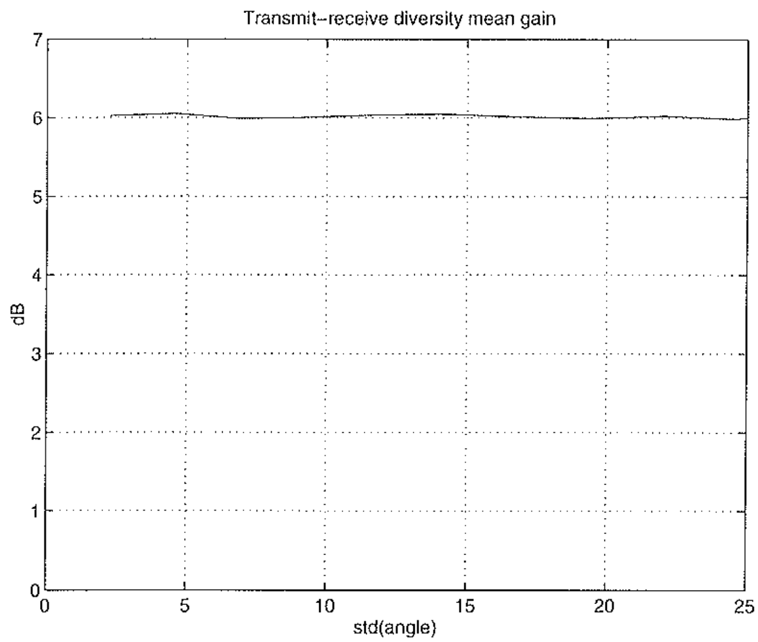

Figure 4 *Receive only diversity. NR=4, NT=1. Gain relative to one element as a function of angular spread.*

Receive only diversity

The traditional receive only diversity with $NR=4$ and $NT=1$ gives a constant gain of 4 (= 6 dB) independent of angular spread. Figure 4 shows the traditional receive only case for a 4 element receive array with some fluctuations, where the gain is the average gain over several thousand realisations of the random medium. If there had been only one scatterer the mean gain would also have been 4 = 6 dB. The result is maybe not so trivial as it looks. At the left end with small angular spreads, the collection of scatterers act as an effective point source, and the 6 dB is the normal antenna gain for a 4 element array illuminating a point source. The diversity effect is negligible, because the antenna signals are highly correlated. At the right hand side the spread is large, the signals are uncorrelated, and we get the diversity gain of 4 due to maximal ratio combination. There are no iterations, since there is only one transmit element. Thus the average gain is independent of the correlations, whereas the distribution of course is highly dependent.

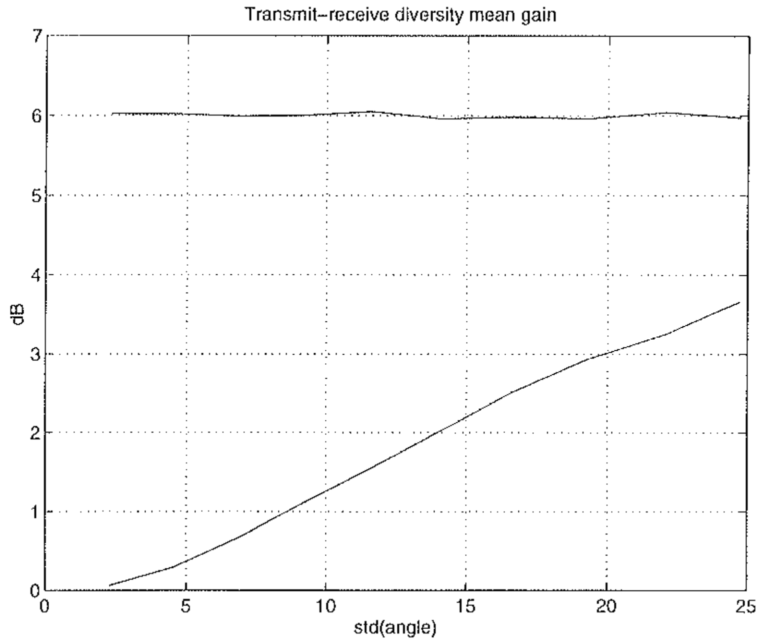

Figure 5 *Transmit only diversity. NR=1, NT=4. Upper curve: Gain relative to one element as a function of angular spread. Lower curve: Diversity improvement over fixed beam.*

Transmit only diversity

Figure 5 shows the opposite case with diversity at the transmit side only, and due to the reciprocity of the channel the gain relative to one element is 6 dB for the same reasons as before. The lower curve shows the improvement due to the iteration from a fixed beam situation (only one iteration is needed in this case), so it is the gain improvement due to the adjustment of the antenna currents. The difference between the two curves is the gain for a fixed beam with the same current in each element. The decreasing gain for this case is due to limited illumination of the scattering region.

Transmit receive diversity

The result of full transmit receive diversity with *NR=4* , *NT=4* is shown in Figure 6. For small angular spreads the full 12 dB is achieved as the sum of the two antenna gains. The gain falls about 2 dB for large angular spreads. It is worth noting that the iterations only add 2 dB, which we might interpret as follows. The transmit array mainly focuses the energy towards the collection of scatterers and the receive array adds the diversity gain.

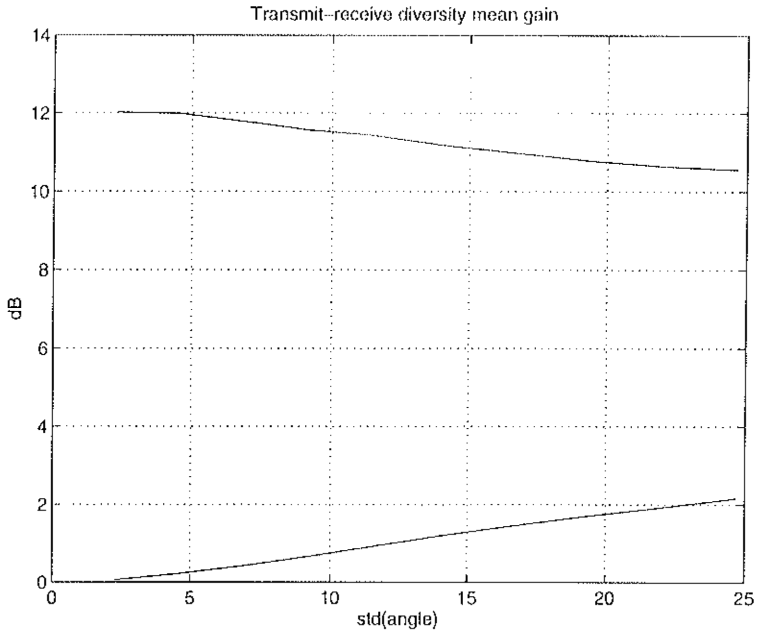

***Figure 6** Full transmit receive diversity. NR=4, NT=4. Upper curve: Gain relative to one element as a function of angular spread. Lower curve: Diversity improvement over fixed beam.*

Further understanding of the mechanisms may be achieved by calculating the gain for different sizes of arrays for completely decorrelated signals (*N*=*NT*=*NR* for this case). The result is shown in Figure 7, where also the completely correlated case is shown, corresponding to a very small angular spread (gain equals N^2).

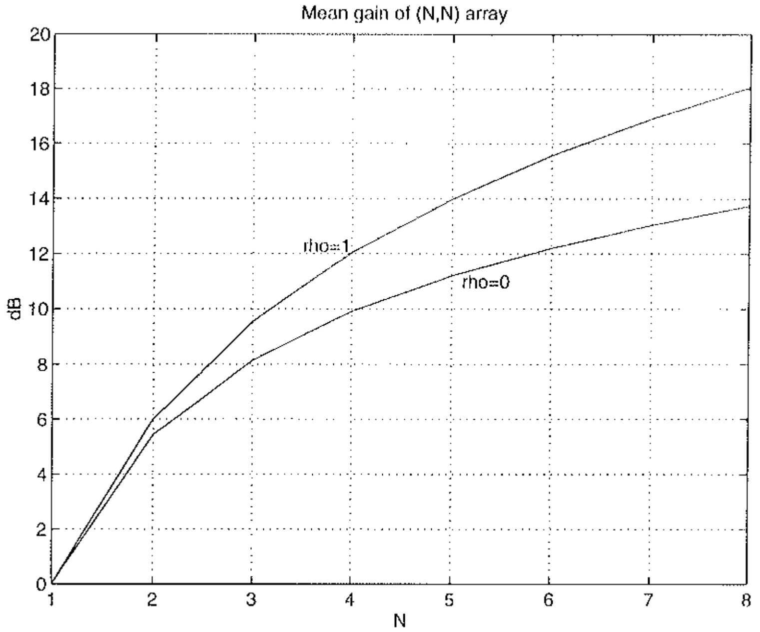

***Figure** 7 Maximum gain of two linear arrays transmitting through a random medium giving rise to uncorrelated and correlated antenna signals.*

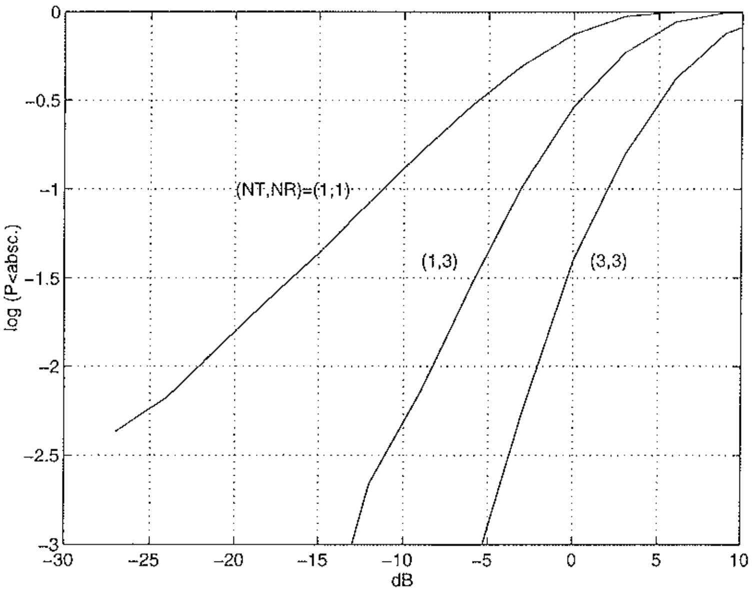

***Figure** 8 Cumulative distribution of gain relative to one element for angular spread = 12.5°.*

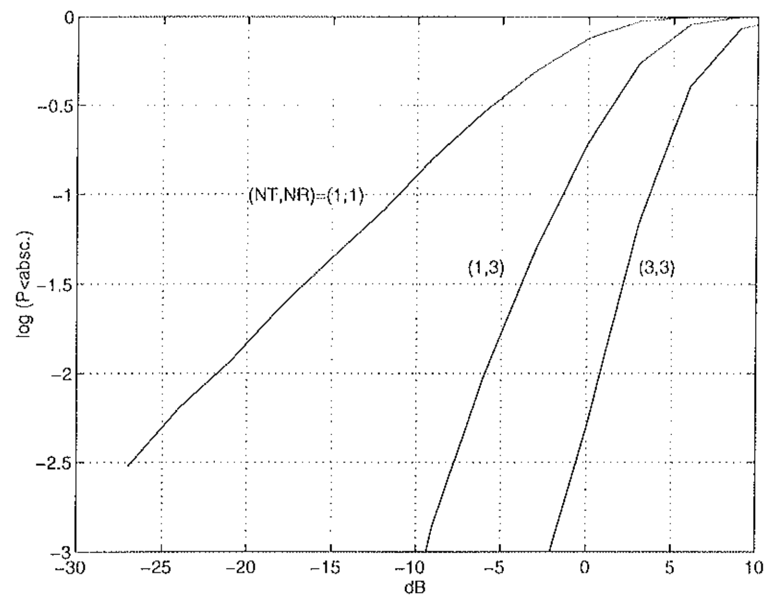

Figure 9 *Cumulative distribution of gain relative to one element for angular spread = 25°.*

Diversity distributions

The distribution of gain as a function of the Rayleigh fading in the environment is shown in Figures 8 and 9 for two different angular spreads, 12.5° and 25° respectively. The spreads are large enough for a considerable diversity gain, although it can be noted that the antenna signals are not quite uncorrelated for the spread equal to 12.5°. It is interesting to note that the distribution function for the full transmit-receive situation (3,3) has a steeper slope than the normal (1,3) case, which means that the transmit array adds additional diversity to that achieved by the receive array alone. At the 1 % level the additional gain is about 7 dB. In fact, simulations show that the (3,3) case is very close to the (1,6) case, so 6-times diversity is obtained whether the six antennas are all on one side, or whether they are distributed on both sides.

Figure 10 indicates the distribution for the (6,6) case with a 30 dB diversity gain at the 1% level for completely uncorrelated signals.

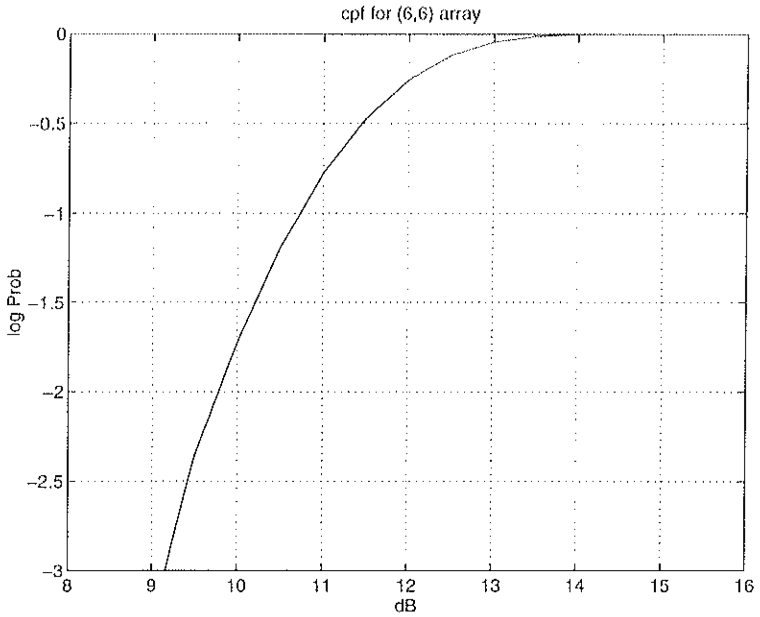

Figure 10 *Probability distribution for a (6,6) array indicating a 30 dB diversity gain at the 1% level.*

Conclusion

Applying maximal ratio combining for both the transmit and receive array consecutively in an iterative manner leads to a maximum value of the transfer of power from the transmit to the receive antenna. Approximately full free space gain of the two arrays independent of number of elements is achieved, although there is some reduction for wide angular spreads. Cumulative distribution functions indicate that the combined transmit receive situation approximately equals an $NR+NT$ type diversity independent of the distribution on the two sides.

These conclusions rely on the existence of a known channel seen from the transmitter, or alternatively a reciprocal channel.

The implications for wireless links are that the use of apertures (area or line arrays) will lead to much reduced pathloss between two antennas, and that this pathloss will decrease with frequency. The disadvantage is of course that adaptive arrays must be used, and that cost may be the ultimate limiting factor.

Appendix

An example for N=2

For $N=2$ it is possible to find the eigenvalue analytically, thus getting a better understanding of the transmit-receive diversity situation. Changing the notation slightly for convenience $\mathbf{Z}$ is now written as

$$\mathbf{Z} = \begin{pmatrix} Z_1 & Z_2 \\ Z_3 & Z_4 \end{pmatrix} \tag{A1}$$

which gives a $\mathbf{Y}$ as

$$\mathbf{Y} = \begin{pmatrix} |Z_1|^2+|Z_3|^2 & Z_1Z_2^*+Z_3Z_4^* \\ Z_2Z_1^*+Z_4Z_3^* & |Z_2|^2+|Z_4|^2 \end{pmatrix} = \begin{pmatrix} a & c \\ c^* & b \end{pmatrix} \tag{A2}$$

The maximum gain or the maximum eigenvalue is given by

$$P_{\max} = \frac{1}{2}\left(a+b+\sqrt{(a-b)^2+4|c|^2}\right) \tag{A3}$$

Transmit-only or receive-only corresponds to $Z_2 = Z_4 = 0$ giving $b=c=0$, and

$$P_{\max} = a = |Z_1|^2+|Z_3|^2 \tag{A4}$$

i.e. the normal two element diversity. In order to have the full four element diversity corresponding to

$$P_{\max} = a+b = |Z_1|^2+|Z_3|^2+|Z_2|^2+|Z_4|^2 \tag{A5}$$

an additional condition of $Z_1=Z_3$ and $Z_2=Z_4$ must be fulfilled. This explains why in general the gain is less than for the completely coherent case. On the other hand, P_{max} can only be zero (a deep fade) when a and b both are zero at the same time, i.e.

$$|Z_1|^2+|Z_3|^2+|Z_2|^2+|Z_4|^2 = 0 \tag{A6}$$

which explains why the diversity distribution is close to the one for four element diversity.