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Optimal Decision Fusion in Multiple Sensor Systems

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The problem of optimal data fusion in the sense of the Neyman-Pearson (N-P) test in a centralized fusion center is considered. The fusion center receives data from various distributed sensors. Each sensor implements a N-P test individually and independently of the other sensors. Due to limitations in channel capacity, the sensors transmit their decision instead of raw data. In addition to their decisions, the sensors may transmit one or more bits of quality information. The optimal, in the N-P sense, decision scheme at the fusion center is derived and it is seen that an improvement in the performance of the system beyond that of the most reliable sensor is feasible, even without quality information, for a system of three or more sensors. If quality information bits are also available at the fusion center, the performance of the distributed decision scheme is comparable to that of the centralized N-P test. Several examples are provided and an algorithm for adjusting the threshold level at the fusion center is provided.

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I. INTRODUCTION

The problem of data fusion in a central decision center has attracted the attention of several investigators due to the increasing interest in the deployment of multiple sensors for communication and surveillance purposes. Because of a limited transmission capacity, the sensors are required to transmit their decision (with or without quality information bits) instead of the raw data the decisions are based upon. A centralized fusion center is responsible for combining the received information from the various sensors into a final decision.

Tenney and Sandell [1] have treated the Bayesian detection problem with distributed sensors. However, they did not consider the design of data fusion algorithms. Sadjadi [2] has considered the problem of general hypothesis testing in a distributed environment and has provided a solution in terms of a number of coupled equations. The decentralized sequential detection problem has been investigated in [3–5]. Chair and Varshney [6] have considered the problem of data fusion in a central center when the data that the fusion center receives consist of the decisions made by each sensor individually and independently from each other. They derive the optimal fusion rule for the likelihood ratio (LR) test. It turns out that the sufficient statistics for the LR test is a weighted average of the decisions of the various sensors with weights that are functions of the individual probabilities of false alarm P_F and the probabilities of detection P_D. However, the maximum a-posteriori (MAP) test or the LR test require either exact knowledge of the a-priori probabilities of the tested hypotheses or the assumption that all hypotheses are equally likely. However, if the Neyman-Pearson (NP) test is employed at each sensor, the same test must be used to fuse the data at the fusion center, in order to maximize the probability of detection for fixed probability of false alarm.

We derive the optimal decision scheme when the N-P test is used at the fusion center. The optimal decision scheme, in the N-P sense, is derived: 1) for cases where the various sensors transmit exclusively their decisions to the fusion center, and 2) for cases where the various sensors transmit quality bits along with their decisions indicating the degree of their confidence in their decision.

II. DECISION FUSION WITH THE NEYMAN-PEARSON TEST

Consider the problem of two hypotheses testing with H_1 designating one hypothesis and H_0 the alternative. Assume that the prior probabilities on the two hypotheses are not known. A number of sensors N receive observations and independently implement the N-P test. Let u_j designate the decision of the jth sensor having taken into account all the observations available to this sensor at the time of the decision. If the decision of the jth sensor favors hypothesis H_1, the sensor sets $u_j =$

$+1$, otherwise it sets $u_j = -1$. Every sensor transmits its decision to the fusion center, so that the fusion center has all N decisions available for processing at the time of the decision making. Let (P_{F_j}, P_{D_j}) designate the pair of the probability of false alarm and the probability of detection at which the jth sensor operates and implements the N-P test. The fusion center implements the N-P test using all the decisions that the individual sensors have communicated, i.e., it formulates the LR test:

$$\Lambda(u) = \frac{P(u_1, u_2, \ldots, u_N | \mathrm{H}_1)}{P(u_1, u_2, \ldots, u_N | \mathrm{H}_0)} \underset{\mathrm{H}_0}{\overset{\mathrm{H}_1}{\gtrless}} t \tag{1}$$

where $u = (u_1, u_2, \ldots, u_N)$ is a $1 \times N$ row vector with entries the decisions of the individual sensors, and t the threshold to be determined by the desirable probability of false alarm at the fusion center P_F^f, i.e.,

$$\sum_{\Lambda(u) > t^*} P(\Lambda(u) | \mathrm{H}_0) = P_F^f \tag{2}$$

Since the decisions of each sensor are independent from each other, the LR test (1) gives

$$\Lambda(u) = \prod_{i=1}^{N} \frac{P(u_i | \mathrm{H}_1)}{P(u_i | \mathrm{H}_0)} \underset{\mathrm{H}_0}{\overset{\mathrm{H}_2}{\gtrless}} t \tag{3}$$

from which the result in [6] is readily obtained. In order to implement the N-P test we need to compute $P(\Lambda(u)|\mathrm{H}_0)$. However, due to the independence assumption, it is easier to obtain the distribution $P(\log \Lambda(u)|\mathrm{H}_0)$ which can be expressed as the convolution of the individual $P(\log \Lambda(u_i)|\mathrm{H}_0)$. Thus, it follows from (3):

$$P(\log \Lambda(u) | \mathrm{H}_0) = P(\log \Lambda(u_1)|\mathrm{H}_0) * \cdots * P(\log \Lambda(u_N)|\mathrm{H}_0). \tag{4}$$

The LR $\Lambda(u_i)$ assumes two values. Either $(1 - P_{D_i})/(1 - P_{F_i})$ when $u_i = 0$ with probability $1 - P_{F_i}$ under hypothesis H_0 and probability $1 - P_{D_i}$ under hypothesis H_1, or, P_{D_i}/P_{F_i} when $u_i = 1$ with, probability P_{F_i} under hypothesis H_0 and probability P_{D_i} under hypothesis H_1. Hence, we can write

$$P(\log \Lambda(u_i)|\mathrm{H}_0) = (1 - P_{F_i})\, \delta\left(\log \Lambda(u_i) - \log \frac{1 - P_{D_i}}{1 - P_{F_i}}\right) + P_{F_i}\, \delta\left(\log \Lambda(u_i) - \log \frac{P_{D_i}}{P_{F_i}}\right) \tag{5}$$

and

$$P(\log \Lambda(u_i)|\mathrm{H}_1) = (1 - P_{D_i})\, \delta\left(\log \Lambda(u_i) - \log \frac{1 - P_{D_i}}{1 - P_{F_i}}\right) + P_{D_i}\, \delta\left(\log \Lambda(u_i) - \log \frac{P_{D_i}}{P_{F_i}}\right) \tag{6}$$

where

$$\delta(x) = \begin{cases} 1 \text{ for } x = 0 \\ 0 \text{ for } x \neq 0. \end{cases}$$

At the fusion center, the probability of false alarm

$$P_F^f = \sum_{\Lambda(u) > t^*} P(\Lambda(u)|\mathrm{H}_0) \tag{7}$$

where t^* is a threshold chosen to satisfy (7) for a given P_F^f. Similarly, the probability of detection at the fusion center

$$P_D^f = \sum_{\Lambda(u) > t^*} P(\Lambda(u)|\mathrm{H}_1). \tag{8}$$

A. Similar Sensors

When all the sensors are similar and operate at the same level of probability of false alarm and probability of detection, i.e., $P_{F_i} = P_{F_j} = P_F$ and $P_{D_i} = P_{D_j} = P_D$ for every i and j, all the probability distributions in (3) are the same and the N-P test leads to the following scheme at the fusion center. (Expression similar to (9) and (10) were obtained in [6] for the LR test.)

$$\sum_{i=1}^{N} a_i u_i \underset{\mathrm{H}_0}{\overset{\mathrm{H}_1}{\gtrless}} t \tag{9}$$

where

$$a_i = \begin{cases} \log\left(\dfrac{P_D}{P_F}\right), & \text{if } u_i = +1,\ i = 1, \ldots, N \\[2ex] \log\left(\dfrac{1 - P_F}{1 - P_D}\right), & \text{if } u_i = -1,\ i = 1, \ldots, N. \end{cases} \tag{10}$$

If k out of the N decisions favor hypothesis H_1, (9) can be rewritten as

$$k\left(\log\left[\frac{P_D(1 - P_F)}{P_F(1 - P_D)}\right]\right) \underset{\mathrm{H}_0}{\overset{\mathrm{H}_1}{\gtrless}} t + N \log\left(\frac{1 - P_F}{1 - P_D}\right). \tag{11}$$

For all sensible tests, though, $P_F < P_D$. Hence, $\log \dfrac{P_D(1 - P_F)}{P_F(1 - P_D)} > 0$ and the N-P test becomes

$$k \underset{\mathrm{H}_0}{\overset{\mathrm{H}_1}{\gtrless}} t^* \tag{12}$$

where t^* is some threshold to be determined so that a certain overall false alarm probability P_F^f is attained at the fusion center.

The random variable k has a binomial distribution with parameters N and $\mathrm{P_F}$ under H_0 and parameters N and $\mathrm{P_D}$ under H_1. Hence, P_F^f and the overall probability of detection P_D^f are given by

$$P_F^f = \sum_{i=[t_f^*]}^{N} \binom{N}{i} P_F^i (1-P_F)^{N-i} \tag{13}$$

$$P_D^f = \sum_{i=[t_f^*]}^{N} \binom{N}{i} P_D^i (1-P_D)^{N-i} \tag{14}$$

where $[t_f^*]$ indicates the smallest integer exceeding t^*. The threshold t^* must be determined so that (12) gives an acceptable overall probability of false alarm.

For the configuration of N sensors, we are interested to know whether the N-P test can provide a (P_F^f, P_D^f) pair such that

$$P_F^f \leqq \min_{i \in N}\{P_{F_i}\} \quad \text{and} \quad P_D^f > \max_{i \in N}\{P_{D_i}\} \tag{15}$$

where (P_{D_i}, P_{F_i}) is the N-P test level for sensor i, $i = 1, \ldots, N$.

The next Theorem shows that condition (15) can be satisfied if the randomized N-P test is used at the fusion center, the number of sensors N is greater than two, and all the sensors are characterized by the same (P_F, P_D) pair.

Theorem. In a configuration of N similar sensors, all operating at the same $(P_F, P_D) = (p, q)$, *the randomized N-P test at the fusion center can provide a* (P_F^f, P_D^f) *satisfying (15) if* $N \geqq 3$.

More precisely, for $N \geqq 3$, *the randomized N-P test can be fixed so that*

$$P_F^f = P_F = p \quad \text{and} \quad P_D^f > P_D = q \tag{16}$$

where P_F *and* P_D *are the probability of false alarm and probability of detection at the individual sensors.*

Proof. First we show that for $N = 2$, condition (15) cannot be satisfied with the second inequality as a strict one. Then we prove that for $N = 3$, the randomized N-P test satisfies condition (15). By using the fact that for fixed probability of false alarm, the probability of detection at the fusion center is maximized by the N-P test among all mappings from the observation space into the decision space, we prove by induction that condition (15) is satisfied for all $N \geqq 3$.

Let $N = 2$ and $(P_F, P_D) = (p, q)$ for both sensors. Using (4), (5), (6), (9), and (10), the LR distributions at the fusion center under hypothesis H_0 and H_1 are plotted for the reader's convenience in Figs. 1 and 2, respectively. Since for all p in (0, 1)

$$p^2 < p < 2p(1-p) + p^2 \tag{17}$$

it follows that, in order to satisfy $P_F^f = p$, the randomized N-P test must be used at the fusion center with threshold $q(1-q)/p(1-p)$ and randomizing factor ω defined by

$$p^2 + \omega 2p(1-p) = p \tag{18}$$

where $0 < \omega < 1$. Solving (18) we obtain $\omega = 0.5$, independent of p. Since P_D^f is determined by an expression symmetric to (18) (see Figs. 1 and 2), $P_D^f = q$ for $\omega = 0.5$. Hence, neither condition (16) nor condition (15) (which is more restrictive) can be satisfied for $N = 2$.

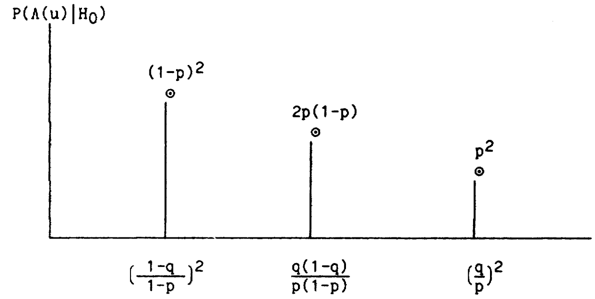

Fig. 1. Distribution of LR at fusion center under hypothesis H_0 for two similar sensor system, $N = 2$

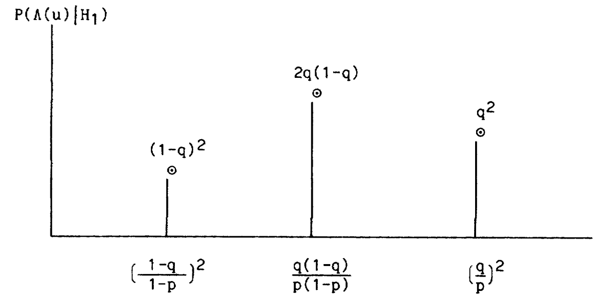

Fig. 2. Distribution of LR at fusion center under hypothesis H_1 for two similar sensor system, $N = 2$.

Let $N = 3$. The distributions of the LR under H_0 and H_1 are given in Figs. 3 and 4, respectively. From Fig. 3,

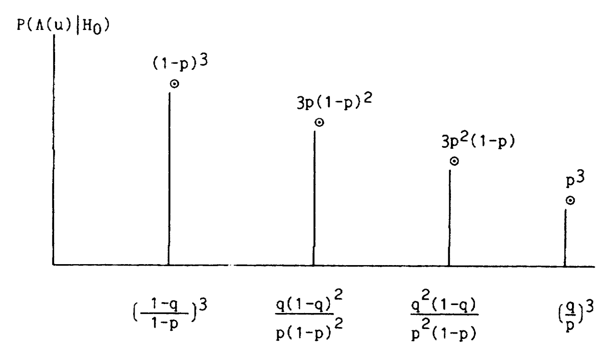

Fig. 3. Distribution of LR at fusion center under hypothesis H_0 for three similar sensor system, $N = 3$.

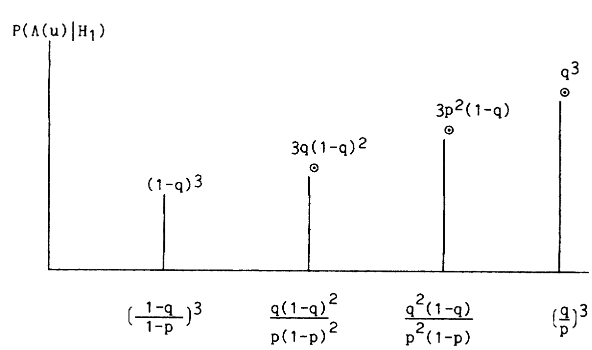

Fig. 4. Distribution of LR at fusion center under hypothesis H_1 for three similar sensor system, $N = 3$.

if the threshold at the fusion center is set at $q^2(1-q)/p^2(1-p)$,

$$P_F^f = p^3 + 3p^2(1-p) < p \tag{19}$$

for $0 < p < 0.5$. The left-hand side (LHS) of inequality (19) is greater than p for $p > 0.5$. Hence, since $P_F < 0.5$, the randomized N-P test that satisfies (15) at the fusion center is determined by

$$p^3 + 3p^2(1-p) + \omega 3p(1-p)^2 = p \tag{20}$$

from which

$$\omega = \tfrac{1}{3} - \frac{p}{3(1-p)}\,. \tag{21}$$

Hence, ω is a positive fraction for $0 < p < 0.5$.

Since P_D^f at the fusion center is given by an expression similar to (20) (see Fig. 4), with q in place of p, and $q > 0.5$, it follows from (20) that $P_D^f > q$, which proves the Theorem for $N = 3$.

Assume that the randomized N-P test satisfies condition (16) for an arbitrary number of sensors N. We show that it also satisfies the condition for $N + 1$, and thus complete the induction and the proof of the Theorem.

Let $U_N = \{u_1, u_2, \ldots, u_N\}$ designate the set of decisions from the N sensors that are available at the fusion center. All the sensors operate at the same level (p, q). Let $f_N(U_N)$ designate some decision rule at the fusion center operating at fixed probability of false alarm p. Let $f_N^{\text{N-P}}(U_N)$ designate the randomized N-P test at the fusion center at level p. For fixed probability of false alarm, the probability of detection at the fusion center (power of test) is maximized for the N-P decision rule among all possible decision rules.

Let $U_{N+1} = \{U_N, u_{N+1}\}$ designate the decision ensemble of $N + 1$ similar sensors all operating at the same level (p, q). Then by choosing $f_{N+1}(U_{N+1}) = f_N^{\text{N-P}}(U_N)$,

$$\begin{aligned} P_D(f_{N+1}^{\text{N-P}}(U_{N+1})) &= \max_{f_{N+1}(U_{N+1})} P_D(f_{N+1}(U_{N+1})) \\ &\geqq P_D(f_N^{\text{N-P}}(U_N)) \end{aligned} \tag{22}$$

from which it follows that

$$P_{D,\,N+1}^f \geqq P_{D,N}^f > q. \tag{23}$$

Thus the induction is complete and so is the proof of the Theorem.

Consider a system of four sensors $N = 4$ all operating at $P_F = 0.05$ and $P_D = 0.95$. If $t_f^* = 2$, from the binomial cumulative table we get $P_F^f = 0.014$ and $P_D^f = 0.9995$ at the fusion center, i.e., a considerable improvement in the performance of the overall system. From the binomial cumulative table it can be seen that at least three sensors are required for the decision fusion scheme to improve the performance of the system, as the Theorem suggests.

To assess the performance of the fusion scheme further, we compare it with the best centralized scheme, the N-P test which utilizes raw data, not decisions, from the different sensors. The loss associated with the use of decisions instead of raw data at the fusion center, is assessed by means of a simple example. Let a single observation from each of the four ($N = 4$) sensors be distributed normally (see Fig. 5) as

$$\begin{aligned} r_i &\sim G(0, 1), \qquad \text{under } H_0 \\ &\sim G(S, 1), \qquad \text{under } H_1. \end{aligned}$$

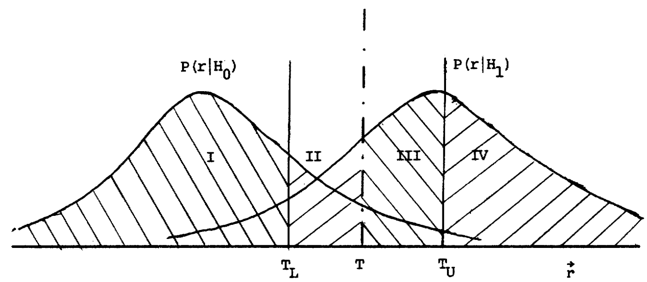

Fig. 5. Data distribution at each sensor under hypotheses H_1 and H_0, and confidence regions. Threshold is indicated by T. The intervals $(-\infty, T_L)$ and (T_U, ∞) are designated "confidence" regions. Interval (T_L, T_U) is designated "no confidence" region.

The N-P test utilizing all the r_is will have the form

$$\sum_{i=1}^{N} r_i > t_b. \tag{24}$$

To achieve a false alarm P_F^b, a threshold of

$$t_b = \sqrt{N}\, Q^{-1}(P_F^b) \tag{25}$$

is needed at the fusion center, where $Q(\;) = 1 - \Phi(\;)$, with $\Phi(\;)$ the cumulative distribution function (cdf) of the standard normal, and Q^{-1} is the inverse function of Q. Moreover,

$$P_D^b = Q\left(\frac{t_b - NS}{\sqrt{N}}\right). \tag{26}$$

To obtain a $P_F = 0.05$ and $P_D = 0.95$ at each sensor, a signal satisfying $t_i = Q^{-1}(0.05)$ is required, from which $t_i = 1.64$, and $0.05 = 1 - Q(t_i - S)$ from which $S = 3.29$.

Consider achieving a $P_F^b = 0.001$ at the fusion center with the four sensors. This requires a threshold $t_b = 2\, Q^{-1}(0.001) = 6.18$, from which $P_D^b = 0.9998$ (see (25) and (26)).

This example shows that the best decentralized fusion scheme achieves a $(P_F^f, P_D^f) = (0.014, 0.9995)$, whereas the best centralized fusion scheme achieves a $(P_F^b, P_D^b) = (0.001, 0.9998)$ for the same sensors. Clearly the loss in power associated with transmitting highly condensed information from the sensors to the fusion center is causing the degradation in the performance of the fusion scheme. As a compromise, a multibit information could be transmitted to the fusion center containing quality information related to the degree of confidence that a sensor has about its decision along with the decision itself. This situation is examined in Section III.

Table I gives the different N-P test thresholds that the fusion center can operate so that condition (15) is satisfied. The thresholds were found using the interactive fusion algorithm (IFA) that we developed (see the Appendix).

TABLE I

Decision Fusion : 5		Sensor System
Sensors PF : Equal x		Unequal _
Sensors PD : Equal x		Unequal _
Threshold @ Fusion Center	Probability of Detection @ Fusion Center	Probability of False Alarm @ Fusion Center
PDMAX = .95000		PFMIN = .50000E-01
t*	PD	PF
6859.0	.977407	.300000E-04
19.000	.998842	.115812E-02
.52631E-01	.999970	.225925E-01
	1 SENSOR OFF	
PDMAX = .95000		PFMIN = .50000E-01
t*	PD	PF
361.00	.985981	.481250E-03
1.0000	.999519	.140187E-01
	2 SENSORS OFF	
PDMAX = .95000		PFMIN = .50000E-01
t*	PD	PF
19.000	.992750	.725000E-02

B. Disimilar Sensors

Case 1. All the sensors operate at the same probability of false alarm level P_F, but different levels of probability of detection from each other, i.e., $P_{D_i} \neq P_{D_j}$, $i \neq j$. Without loss of generality we assume the ranking $P_{D_1} > P_{D_2} > \cdots > P_{D_N}$, from which the following ordering in the abscissae of the conditional distribution of the individual LRs results:

$$\frac{1 - P_{D_1}}{1 - P_F} < \frac{1 - P_{D_2}}{1 - P_F} < \cdots < \frac{1 - P_{D_N}}{1 - P_F} < \frac{P_{D_N}}{P_F} < \cdots < \frac{P_{D_1}}{P_F}.$$

The conditional distribution of the compound LR at the fusion center is obtained by convolving the individual distributions, using the IFA. Convolution of the distributions $P(\log \Lambda(u_i)|\mathrm{H}_k)$ corresponds to linear shifts of their logarithmic abscissae, which is translated into addition of logarithms. Hence, the distribution of the LR $P(\Lambda(u)|\mathrm{H}_k)$ at the fusion center can be obtained directly by multiplication of the abscissae of the $P(\Lambda(u)_i)|\mathrm{H}_k)$. Hence the point of the distribution $P(\Lambda(u)|\mathrm{H}_k)$ which is closest to the origin has abscissa $\frac{(1-P_{D_1})\cdots(1-P_{D_N})}{(1-P_F)^N}$ and ordinate $(1-P_{D_1})\cdots(1-P_{D_N})$ under H_1 or $(1-P_F)^N$ under H_0. On the other hand, the point farthest apart from the origin has abscissa $\frac{P_{D_1}P_{D_2}\cdots P_{D_N}}{P_F^N}$ and ordinate $P_{D_1}\cdots P_{D_N}$ under H_1 or P_F^N under H_0. In between these two extreme points, the abscissae of the distribution of the compound LR have the form $\prod_{i\in S}\frac{P_{D_i}}{P_F}\prod_{j\in \bar{S}}\frac{P_{D_j}}{1-P_F}$ where S is a subset of integers from $\{1, 2, \ldots, N\}$ and $\bar{S}$ its complement with respect to this set. The corresponding ordinates are $\prod_{i\in S}P_{D_i}\prod_{j\in\bar{S}}(1-P_{D_j})$ under H_1 or $P_F^{|S|}(1-P_F)^{|\bar{S}|}$ under H_0, where $|\Omega|$ designates the cardinality of the set Ω. Once the distribution of the compound LR is determined, the threshold at the fusion center can be determined to satisfy a given probability of false alarm P_F^f from which the probability of detection P_D^f is determined. At the fusion center we want to set-up the threshold so that $P_F^f \leq P_F$ while $P_D^f > \max_i \{P_{D_i}\}$. This is achieved by the IFA as the following example illustrates.

Consider a five-sensor system. All the sensors operate at the same level $P_F = 0.05$. However, due to different noise environments or quality of the sensors, they yield different P_Ds as Table II indicates.

TABLE II
Probability Of Detection At The Individual Sensors For The Same Probability Of False Alarm In A Five Sensor System

i	1	2	3	4	5
P_{D_i}	0.95	0.94	0.93	0.92	0.91

Table III summarizes all the choices of thresholds at the fusion center that satisfy condition (15) as given by the IFA. A significant improvement in the system performance is achieved by fusing the individual decisions.

Case 2. The different sensors operate at different probabilities of false alarm and probabilities of detection, i.e., $P_{F_i} \neq P_{F_j}$ and $P_{D_i} \neq P_{D_j}$, $i \neq j$. The distribution of the cumulative LR of the fusion center is obtained numerically as in case 2, and the threshold t_f^* is found to satisfy a given P_F^f. Ideally, the threshold t_f^* must be chosen so that condition (15) is satisfied. However, this may not always be feasible. The following examples illustrate the procedure.

We consider three different systems with five, four, and three sensors. Each system results by eliminating the sensor with the lowest P_D from the system that has one more sensor. For the five-sensor system, the (P_D, P_F) of the sensors are given in Table IV.

Table V summarizes the results as obtained by IFA.

TABLE III

Decision Fusion :	5	Sensor System
Sensors PF : Equal	x	Unequal _
Sensors PD : Equal	_	Unequal x

Threshold @ Fusion Center	Probability of Detection @ Fusion Center	Probability of False Alarm @ Fusion Center
PDMAX = .95000		PFMIN = .50000E-01
t*	PD	PF
6163.2	.957817	.300000E-04
53.004	.963797	.142812E-03
45.880	.968973	.255625E-03
40.339	.973523	.368437E-03
38.907	.977913	.481250E-03
34.208	.981772	.594062E-03
32.081	.985391	.706874E-03
29.610	.988731	.819687E-03
28.207	.991913	.932499E-03
24.416	.994668	.104531E-02
20.705	.997003	.115812E-02
.20998	.997454	.330156E-02
.17806	.997835	.544500E-02
.15413	.998165	.758843E-02
.14683	.998480	.973187E-02
.13552	.998771	.118753E-01
.12709	.999043	.140187E-01
.11174	.999282	.161622E-01
.10778	.999513	.183056E-01
.94760E-01	.999717	.204490E-01
.82023E-01	.999892	.225925E-01

TABLE IV
Probability Of False Alarm And Detection For A Five-Sensor System With Disimilar Sensors

i	1	2	3	4	5
P_{F_i}	0.05	0.04	0.03	0.02	0.01
P_{D_i}	0.95	0.94	0.93	0.92	0.91

In all cases, a significant improvement in the performance of the system is achieved from fusing the decisions.

TABLE V

Decision Fusion :	5	Sensor System
Sensors PF : Equal	_	Unequal x
Sensors PD : Equal	_	Unequal x

Threshold @ Fusion Center	Probability of Detection @ Fusion Center	Probability of False Alarm @ Fusion Center
PDMAX = .95000		PFMIN = .10000E-01
t*	PD	PF
57882.	.957817	.269200E-05
426.86	.960153	.816400E-05
373.63	.962908	.155360E-04
358.72	.966248	.248480E-04
284.83	.969430	.360200E-04
273.46	.973289	.501320E-04
239.36	.977840	.691439E-04
160.34	.981459	.917159E-04
153.94	.985848	.120228E-03
134.74	.991024	.158640E-03
102.72	.997003	.216852E-03
.99369	.997179	.393780E-03
.75752	.997382	.661908E-03
.66305	.997622	.102314E-02
.63660	.997912	.147942E-02
.42643	.998143	.202115E-02
.37325	.998416	.275098E-02
.35836	.998746	.367287E-02
.28454	.999061	.477889E-02
.27319	.999442	.617598E-02
.23912	.999892	.805816E-02
	1 SENSOR OFF	
PDMAX = .95000		PFMIN = .20000E-01
t*	PD	PF
1129.9	.976981	.150400E-03
4.6908	.979548	.697600E-03
4.1058	.982575	.143480E-02
3.9420	.986246	.236600E-02
3.1300	.989742	.348320E-02
3.0051	.993983	.489440E-02
2.6303	.998984	.679560E-02
	2 SENSORS OFF	
PDMAX = .95000		PFMIN = .30000E-01
t*	PD	PF
32.222	.989720	.458000E-02

III. TRANSMISSION OF DECISIONS ALONG WITH QUALITY INFORMATION

Consider the case where the jth sensor transmits quality information bits to the fusion center about its decision along with the decision itself. The simplest case corresponds to the transmission of binary $\{0, 1\}$ quality information indicating the degree of confidence that the sensor has on the decision that it transmits. Under the scenario, a bit one indicates ''confidence'', whereas a bit zero indicates ''no confidence''. Fig. 5 illustrates how the binary quality bit c is defined. A strip (T_L, T_U) about the threshold T of an individual sensor is designated as region of no confidence and the bit $c = 0$ is transmitted along with the decision when the observation r falls into this region. The two regions forming the compliment of the (T_L, T_u) region are considered confidence regions and the bit $c = 1$ is transmitted along with the decision when the observations fall into one of the two regions.

The joint probability distribution of (u, c) (skipping the sensor index for simplicity) can be easily obtained from

$$P(u,c|\mathrm{H}_k) = P(c|u,\mathrm{H}_k)\, P(u|\mathrm{H}_k), \qquad k = 0, 1 \tag{27}$$

where $P(u|\mathrm{H}_k)$, $u = \pm 1$ and $k = 0, 1$ is specified by P_F and P_D, and referring to Fig. 5,

$$
P(c|u,\mathrm{H}_k) = \begin{cases}
P(c=1|u=1,\mathrm{H}_k) = \int_{IV} dP(r|\mathrm{H}_k) / \int_{III \cup IV} dP(r|\mathrm{H}_k) = C_{11}^k \\
P(c=0|u=1,\mathrm{H}_k) = \int_{III} dP(r|\mathrm{H}_k) / \int_{III \cup IV} dP(r|\mathrm{H}_k) = C_{01}^k \\
P(c=0|u=-1,\mathrm{H}_k) = \int_{II} dP(r|\mathrm{H}_k) / \int_{I \cup II} dP(r|\mathrm{H}_k) = C_{00}^k \\
P(c=1|u=-1,\mathrm{H}_k) = \int_{I} dP(r|\mathrm{H}_k) / \int_{I \cup II} dP(r|\mathrm{H}_k) = C_{10}^k
\end{cases} \quad (28)
$$

for $k = 0, 1$.

Hence, for every sensor

$$
P(u=i,\ c=j|\mathrm{H}_k) = C_{ji}^k\, P(u=i|\mathrm{H}_k), \quad i = -1, 1, \text{ and } j = 0, 1 \quad (29)
$$

and

$$
\Lambda(u=i,\ c=j) = \frac{P(u=i,\ c=j|\mathrm{H}_1)}{P(u=i,\ c=j|\mathrm{H}_0)} = \frac{C_{ji}^1\, P(u=i|\mathrm{H}_1)}{C_{ji}^0\, P(u=i|\mathrm{H}_0)}, \quad i = -1, 1, \text{ and } j = 0, 1. \quad (30)
$$

Combining (6) and (22) we obtain

$$
\begin{aligned}
P(\Lambda(u,c)|\mathrm{H}_1) = {} & C_{11}^1\, P_D\, \delta\left(\Lambda(u,c) - \frac{C_{11}^1\, P_D}{C_{11}^0\, P_F}\right) \\
& + C_{01}^1\, P_D\, \delta\left(\Lambda(u,c) - \frac{C_{01}^1\, P_D}{C_{01}^0\, P_F}\right) \\
& + C_{00}^1(1-P_D)\, \delta\left(\Lambda(u,c) - \frac{C_{00}^1\,(1-P_D)}{C_{00}^0\,(1-P_F)}\right) \\
& + C_{10}^1(1-P_D)\, \delta\left(\Lambda(u,c) - \frac{C_{10}^1\,(1-P_D)}{C_{10}^1\,(1-P_F)}\right)
\end{aligned} \quad (31)
$$

Similarly, $P(\Lambda(u,c)|\mathrm{H}_0)$ is obtained from (29) by substituting P_D with P_F in the product-weights of the delta functions. Therefore, the probability distribution of the LR at the fusion center is given by the convolution

$$
P(\log\,\Lambda(u,c)|\mathrm{H}_k) = P(\log\,\Lambda(u_1,c_1)|\mathrm{H}_k) * \cdots * P(\log\,\Lambda(u_N,c_N)|\mathrm{H}_k). \quad (32)
$$

In the case where all the sensors operate at the same level $(P_F,\ P_D)$ the mathematics simplify somewhat, since

$$
\begin{aligned}
P(\Lambda(u,c)|\mathrm{H}_1) = {} & \Pr[k \text{ out of } N \text{ decisions favor } \mathrm{H}_1 \text{ and, } n \text{ out of these } k \text{ decisions have confidence index 1 and, } m \text{ out of the } N-k \text{ decisions that favor } \mathrm{H}_0 \text{ have confidence index } 1|\mathrm{H}_1] \\
= {} & \binom{k}{n}\, [C_{11}^1]^n\, [1-C_{11}^1]^{k-n} \binom{N-k}{m} \\
& \cdot [C_{10}^1]^m\, [1-C_{10}^1]^{N-k-m} \\
& \cdot \binom{N}{k}\, P_D^k\, (1-P_D)^{N-k}.
\end{aligned} \quad (33)
$$

Similarly,

$$
\begin{aligned}
P(\Lambda(u,c)|\mathrm{H}_0) = {} & \binom{k}{n} [C_{11}^0]^n\, [1-C_{11}^0]^{k-n} \\
& \cdot \binom{N-k}{m} [C_{10}^0]^m\, [1-C_{10}^0]^{N-k-m} \\
& \cdot \binom{N}{k}\, P_F^k\, (1-P_F)^{N-k}
\end{aligned} \quad (34)
$$

from which

$$
\begin{aligned}
P_F^f = \sum_{k=t_1^*}^{N} \sum_{n=t_2^*}^{k} \sum_{m=t_3^*}^{N-k} \Bigg[& \binom{k}{n} [C_{11}^0]^n \\
& \cdot [1-C_{11}^0]^{k-n} \binom{N-k}{m} [C_{10}^0]^m \\
& \cdot [1-C_{10}^0]^{N-k-m} \binom{N}{k}\, P_F^k (1-P_F)^{N-k} \Bigg].
\end{aligned} \quad (35)
$$

The P_D^f is obtained by an expression similar to (35) with P_D in place of P_F and the index 1 instead of 0 above C_{ij}. The thresholds t_1^*, t_2^*, and t_3^* are to be determined to satisfy a given probability of false alarm at the fusion center. Notice that more than one set of thresholds can yield the same P_F^f. Clearly, the set that results in the highest P_D^f must be selected.

From (35) it can be seen that a superior performance in regards to $(P_F^f,\ P_D^f)$ can be achieved when quality information is transmitted along with the decisions. The improvement in performance of the fusion center when quality information bits are transmitted comes from the fact that the summation over $P(\Lambda(u,c)|\mathrm{H}_k)$ can be made finer with the three different thresholds. To show that, consider the example of Section IIA. In this example four similar sensors $N=4$, operate at $P_F = 0.05$ and $P_D = 0.95$ from received data $r_i \sim N\ (0,\ 1)$ under H_0 and $r_i \sim N\ (S=3.29,\ 1)$ under H_1. The threshold at each sensor is set to $t_i = 1.64$ to satisfy P_F. Using Fig. 5 and the previous equations, we obtain for $t_{L,i} = 0.8t_i = 1.312$ and $t_{u,i} = 1.2$, and $t_i = 1.968$ the C_{ij}^ks that are given in Table VI.

Using the IFA, it follows that there is a choice of 33 different thresholds that the fusion center can operate so that (15) is satisfied as shown in Table VII. It can be seen from this table that there is a significant improvement in the performance of the overall system

TABLE VI
Quality Bit Coefficients For Gaussian Distributed Data

H_k / C_{ij}	H_1	H_0
C_{11}	0.948	0.46
C_{01}	0.052	0.54
C_{00}	0.52	0.047
C_{10}	0.48	0.953

TABLE VII

Decision Fusion :	4	Sensor System with Quality Bits
Sensors PF : Equal	x	Unequal _
Sensors PD : Equal	x	Unequal _

Threshold @ Fusion Center	Probability of Detection @ Fusion Center	Probability of False Alarm @ Fusion Center
PDMAX = .95000		PFMIN = .50000E-01
t*	PD	PF
62318.	.956002	.175551E-05
20357.	.960940	.199808E-05
9390.7	.961918	.210220E-05
2988.7	.963462	.261876E-05
2911.9	.980782	.856706E-05
951.21	.981595	.942131E-05
926.74	.990711	.192580E-04
438.79	.990738	.193191E-04
302.74	.990880	.197900E-04
139.65	.990937	.201943E-04
136.06	.992362	.306685E-04
44.446	.992406	.316713E-04
43.303	.993906	.663133E-04
42.189	.998114	.166041E-03
20.503	.998114	.166055E-03
14.146	.998129	.167161E-03
13.782	.998524	.195805E-03
6.5253	.998525	.195924E-03
6.3575	.998577	.204121E-03
4.5021	.998579	.204578E-03
2.0768	.998580	.204970E-03
2.0234	.998662	.245637E-03
1.9713	.999354	.596850E-03
.66097	.999355	.597499E-03
.64397	.999398	.664750E-03
.62741	.999762	.124555E-02
.29706	.999763	.124796E-02
.21036	.999763	.124850E-02
.20495	.999771	.128557E-02
.94544E-01	.999772	.130148E-02
.92113E-01	.999810	.171378E-02
.66951E-01	.999810	.171395E-02
.30090E-01	.999811	.175343E-02
.29316E-01	.999851	.311705E-02

compared with the individual sensors and the fusion system without quality information. For a comparable $P_D^f = 0.9998$, the $P_F^f = 0.0013$ when quality bit information is transmitted as opposed to $(P_F^f, P_D^f) = (0.014, 0.9995)$ without quality information. The performance of the fusion center when one quality information bit is transmitted approaches that of the best centralized N-P test, as Table VIII suggests. It is interesting to notice that fusion of the decisions improves the performance of the overall system even in the case of two sensors when quality information bits are transmitted along with the decisions, as Table IX indicates. Table X shows the performance of a three sensor system with quality bits.

TABLE VIII
Comparative Results From 3 Different Fusion Systems With Four ($N=4$) Sensors, All Operating At Level $(P_F, P_D) = (0.05, 0.95)$ When The Individual Sensors Transmit:

	P_F^f	P_D^f
Only decisions	0.014	0.9995
Decision with one quality bit	0.0013	0.9998
Raw data (Best centralized N-P test)	0.001	0.9998

TABLE IX

Decision Fusion :	2	Sensor System with Quality Bits
Sensors PF : Equal	x	Unequal _
Sensors PD : Equal	x	Unequal _

Threshold @ Fusion Center	Probability of Detection @ Fusion Center	Probability of False Alarm @ Fusion Center
PDMAX = .95000		PFMIN = .50000E-01
t*	PD	PF
1.0654	.951900	.696499E-02
1.0380	.995129	.486111E-01

IV. CONCLUSIONS

The problem of fusing decisions from N independent sensors in a fusion center was considered. We assumed that each sensor transmits its decision to the fusion center. The decision of each individual sensor is based on the N-P test. The fusion center formulates the LR using all the received decisions and decides on which hypothesis is true using the N-P test also. The pdf of the

TABLE X

Decision Fusion :	3	Sensor System with Quality Bits
Sensors PF : Equal	x	Unequal _
Sensors PD : Equal	x	Unequal _

Threshold @ Fusion Center	Probability of Detection @ Fusion Center	Probability of False Alarm @ Fusion Center
PDMAX = .95000		PFMIN = .50000E-01
t*	PD	PF
40.645	.985857	.177933E-02
13.277	.987683	.191689E-02
6.1248	.987804	.193657E-02
1.9493	.987994	.203422E-02
1.8992	.994400	.540756E-02
.62039	.994500	.556904E-02
.60444	.997872	.111475E-01
.19745	.997890	.112365E-01
.88740E-01	.998065	.132165E-01
.28243E-01	.998250	.197652E-01

log LR at the fusion center was obtained as the convolution of the pdfs of the log LRs of the individual sensors. Once the pdf of the LR is obtained, the threshold at the fusion center is determined by a desired probability of false alarm.

For a fusion system with three or more sensors, all the sensors operating at the same (P_F, P_D) level, it was proved that if the N-P test is used to fuse the decisions, the probability of detection at the fusion center exceeds that of the individual sensor for the same probability of false alarm. However, if the sensors operate at arbitrary (P_F, P_D) levels, no general assessment can be made about the performance of the fusion center since the performance depends on how far the operating points of the sensors are from each other.

The problem of decision fusion when the sensors transmit quality information bits indicating their confidence on the decisions was also considered and the N-P test at the fusion center was derived. Several numerical examples showed that use of quality information can improve the performance of the fusion center considerably.

An IFA was developed to solve the fusion problem numerically. Once one of the three parameters (threshold, probability of false alarm, or probability of detection) is specified, the IFA determines the other two, given the probabilities of false alarm and detection of each individual sensor.

APPENDIX

The IFA receives as data the number of sensors, their (P_F, P_D) levels, and the C_{ij}^k quality information parameters if the sensors transmit quality information bits along with their decisions. It then computes the LR pdf at the fusion center conditioned on each hypothesis. After it computes the pdf, it asks the user which option he/she prefers. The alternative options are the following.

1) Display of the entire pdf.
2) Threshold computation for a given P_F^f and display of the corresponding P_D^f.
3) Determination of the thresholds that satisfy (15).
4) Threshold computation for a given P_D^f and display of the corresponding P_F^f.
5) Elimination of one or more sensors and repetition of the algorithm.
6) Quit.

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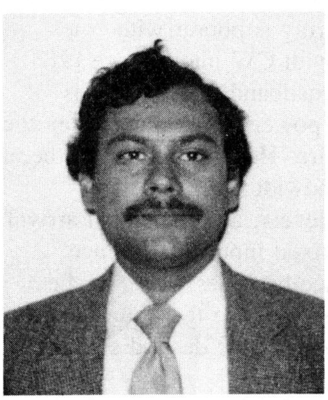

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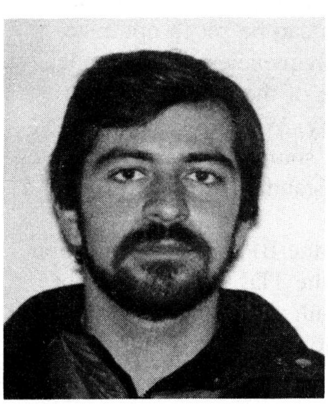

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