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Biegel, Benjamin; Stoustrup, Jakob; Andersen, Palle; Hansen, Lars Henrik

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Predictive Control of Demand Side Units Participating in the Primary Frequency Reserve Market

Benjamin Biegel   Jakob Stoustrup   Palle Andersen   Lars Henrik Hansen

Abstract— We consider an aggregator controlling a mixed portfolio of conventional power generators and demand side units. The generators are controllable within certain power and ramp limitations while the demand side units are characterized by flexible consumptions and therefore can be treated as energy storages of limited capacity. We address the problem of reducing the load on the conventional generators by letting the flexible consumers participate in the provision of primary frequency reserve. In particular, it is desired that the flexible consumers compensate for rapid grid frequency changes. In this work, we design an aggregator control strategy based on closed-loop model predictive control. The controller is able to mobilize the flexible consumers ahead of time such that we are able to reduce the load on the conventional generators by more extensive use of the demand side units.

I. INTRODUCTION

With an increasing focus on climate-related issues and rising fossil fuel prices, the penetration of renewable energy sources is likely to increase in the foreseeable future throughout the developed world [1]. Indeed many actions are taken from a political point to increase the penetration of renewables: in the US almost all states have renewable portfolio standards or goals ensuring a certain percentage of renewables [2]. Similarly, the commission of the European Countries has set targets of 20 % renewables by 2020 [3] while China has doubled the wind power production every year since 2004 [4].

As a consequence of this increase of renewables, the power system is moving from a system with fewer centralized conventional power plants to a system with a large number of distributed smaller production units [5]. As an example of this evolution, Denmark has moved from a situation with a total of 16 central power plants in 1980, to a system which today consists of 16 central power plants, 1000 local combined heat and power plants and around 6000 wind turbines [6].

A number of challenges are associated when replacing central power plants with distributed generating units: the central power plants not only deliver power but also provide ancillary services to ensure reliable delivery of electricity and secure operation of the transmission system. This includes frequency stability support, power balancing, voltage control etc. When these power plants are replaced with renewables

such as wind turbines and photovoltaics, the ability to provide ancillary services in the classical sense disappears; the renewable energy sources will typically maximize the power production thus not provide ancillary services. Though recent works suggest that renewable production units can take part in the balancing effort in certain conditions (see, e.g., [7], [8]), it remains impossible for wind power plants and photovoltaics to provide ancillary services when there is no or little wind or solar irradiation.

Another benefit of conventional fossil fuel power plant generators is that they are synchronous with the grid and therefore provide rotating inertia supporting the grid frequency against changes [9]. As renewable energy sources typically interface with the grid via power electronics, they will not be able to provide this inertia [10].

Moreover, renewables are often times intermittent sources characterized by highly fluctuating power generation: they can suddenly increase or decrease the production depending on the weather conditions. These sudden production changes are not always predictable and can therefore be severe for the grid stability [11].

It is therefore evident, that in a grid of high penetration of renewables, the need for balancing ancillary services will increase [12], [13]. As the conventional power plants are phased out, alternative sources of ancillary services must be found. One of the approaches towards alternative ancillary services is the *smart grid* concept, where consumers take part in the balancing effort [14], [15]. The idea is to utilize the demand side in a way beneficial for the grid stability by moving loads in time, e.g. by allowing local devices with large time constants to store more or less energy at convenient times, thereby adjusting the momentary consumption. One obvious method to do so is by exploiting large thermal time constants in deep freezers, refrigerators, local heat pumps etc. See, e.g., [16].

A lot of effort is put into research in the context of demand side flexibility utilization to support the electrical grid. In [17], a hierarchical Model Predictive Control (MPC) design is introduced to utilize flexible consumers to counteract quickly fluctuating imbalances. This idea is extended in [18] and [19], where the ability to handle grid congestion is included in the controller design. But while the works [17], [18], [19] illustrate that flexible consumers are able to contribute to the balancing effort, they do not describe how this can be accomplished in a liberalized market setting. Further, the cases are idealized such that the controller possesses almost perfect predictions of the future fluctuations.

In this work, we examine the possibilities of using a mixed portfolio of demand side units and productions units to participate in the ancillary service market by providing primary frequency reserve. Following, we design a controller that is able to mobilize the portfolio of generators and consumers to provide primary frequency reserve at minimum cost. The controller achieves this by utilizing the demand side units with hardly any ramp constraints to compensate for the fast frequency changes while using the slow and inexpensive conventional power generators to release the demand side units. Hereby the load on the conventional generators is kept at a minimum. This control behavior is achieved based on a closed-loop model predictive control strategy, which is able to prepare the storages and generators ahead of time for the future unknown frequency changes.

The outline of the rest of the paper is as follows. First, in Sec. II, we briefly describe the various forms of balancing services. Next, in Sec. III, we present a general model for the generators and consumers. In Sec. IV, we design a closed-loop predictive controller that utilizes the portfolio of production and consumption units to provide primary frequency reserve at minimum operational cost. Sec. V illustrates the methods with a numerical example and finally, Sec. VI sums up the work.

II. PRIMARY FREQUENCY RESERVES

In the following, we briefly describe primary frequency reserve and how a mixed portfolio of consumers and generators are able to provide this ancillary service.

A. Primary Frequency Reserve Specifications

In the electrical grid, Transmission System Operators (TSOs) are responsible for enabling a secure and reliable power system by keeping balance between production and consumption as well as maintaining power quality and ensuring a stable transmission system. In general, the TSOs do not possess production units, and therefore procure ancillary services from suppliers [20].

To ensure balance, the TSOs must maintain the system frequency at its target value. In order to do this, a certain amount of active power must be kept in reserve and available for control such that frequency deviations can be restored. For this purpose, three types of frequency reserve services exist: primary, secondary and tertiary frequency reserves [9], where we concentrate about the fastest reserve, namely the primary frequency reserve.

The primary frequency reserve is an automatic control which is used in frequency control. A main target for the primary control is to stabilize the frequency in the case of major outages of either loads or suppliers. The primary control reserve is required to sustain at least a certain amount of time, as it is then relieved by the secondary control [21]. The time scale for activation primary frequency reserve is in the area of 10-30 seconds.

B. Consumers Providing Primary Frequency Reserve

In the context of ancillary services, two main consumer properties are important. The fist property is that the consumers will have very high ramp limits as they are determined by the time it takes to switch the devices on/off, which is very fast compared to adjusting the power production of e.g. a combined heat and power plant. The second property is that flexible consumers only are able to store a limited amount of energy. This is evident from the fact that the flexible consumers in general only are able to move consumption in time, not actually use more or less energy. If we as an example consider an electrically heated house, a cold storage, or an electric vehicle battery, we observe that they indeed are flexible and thus able to store energy, but that they over time will use the same amount of energy.

Due to the high ramp limits of the demand side units, they are well suited for primary frequency control where a fast response is needed. But as they are limited in energy capacity, we can not rely solely on demand side units; we will therefore consider a portfolio consisting of both demand side units and conventional generators. The idea is to use the demand side units to compensate for the fast changes in frequency while using slow and inexpensive generators to relieve the demand side units. The consumers will then allow us to reduce the actuation of the conventional power plants, in particular the fast generators which are also most expensive to operate. In the following, we consider such a mixed portfolio.

III. MODELING

We consider a portfolio of a total of n power production and demand side units interconnected in a star topology consisting of n_l lines, see Fig. 1. We limit the work to star topology girds as this corresponds to the topology of low voltage grids; however, the methods in the paper can easily be extended to meshed grids.

The n units are under the jurisdiction of an aggregator who is able to control their power consumption/production within given limits. The aggregator utilizes the portfolio to participate in the primary frequency reserve market and must control the units accordingly depending on their characteristics and on the amount frequency reserve sold to the TSO. Throughout the modeling of the system, we describe the dynamics with discrete time equations and use subscript t to indicate the sample number.

A. Generators and Demand Side Units

We describe both the generators and the demand side units using the same model. The n units in the portfolio are characterized by power consumptions/productions $u \in \mathbf{R}^n$ subject to power constraints

$$u^{\min} \preceq u_t \preceq u^{\max} \tag{1}$$

where $u^{\min}, u^{\max} \in \mathbf{R}^n$ are lower and upper limits, respectively. Here $\preceq$ represents componentwise inequality. Note that the power consumption/production u is a small signal value; hence the lower power limits $u^{\min}$ can be negative.
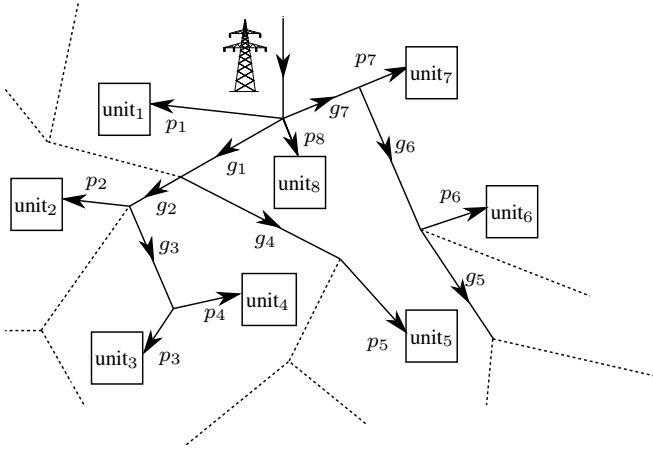
Fig. 1. Interconnected producers and consumers in a grid of limited capacity (figure adapted from [22]).

For a power producer, the power constraints represent the maximum and minimum deviation from the nominal production, while for a consumer it describes the maximum and minimum deviation in power consumption. We define u in consumption terms such that $(u_t)_i < 0$ corresponds to a decrease in consumption compared to the nominal consumption for device i and vice versa for $(u_t)_i > 0$. Further, the units are subject to ramp limit given by

$$\Delta u^{\min} \preceq \Delta u_t \preceq \Delta u^{\max} \tag{2}$$

where $\Delta u_t = u_t - u_{t-1}$ and where $\Delta u^{\min}, \Delta u^{\max} \in \mathbf{R}^n$ describe the ramp limits.

With each unit, we associate an amount of stored energy $x \in \mathbf{R}^n$. The relation between the power consumption u and the stored energy x is described by the difference equation (see, e.g., [23])

$$x_{t+1} = Ax_t - Bu_t \tag{3}$$

where $A, B \in \mathbf{R}^{n \times n}$ are diagonal matrices where the diagonal elements of A and B describe the first order dynamics of the energy storages. The model only represents the flexible part of the units and therefore does not contain any base load. The storage limits are given by

$$x^{\min} \preceq x_t \preceq x^{\max}, \tag{4}$$

where $x^{\min}, x^{\max} \in \mathbf{R}^n$ describe the lower and upper limits, respectively. These power constraints could be extended to be time-varying which for example would allow us to specify a specific time where a battery must be fully charged etc., but in this work we keep the limits time-invariant for simplicity. For a house with electrical heating, the limits could represent the lowest and highest allowed temperature in the house. Similarly for an electrical vehicle, the limits could represent an empty and a full battery. Note that for generators, we simply let the corresponding entries in the matrices A, B equal zero, as they do not possess the ability to store energy.

The consumed or produced power of the units flow through the links of the grid, as illustrated in Fig. 1. The partial flows

$g \in \mathbf{R}^{n_l}$ through the links caused by the generators and consumers are given by

$$g_t = Gu_t \tag{5}$$

where $G \in \mathbf{R}^{n_l \times n}$ has the structure

$$G_{ij} = \begin{cases} 1 & \text{if unit } j \text{ is supplied through link } i, \\ 0 & \text{otherwise.} \end{cases}$$

In Fig. 1 this is illustrated: the different consumers with power consumption $p_1, \ldots, p_8$ will load the different lines with loads $g_1, \ldots, g_7$ depending on the grid structure, which is described by G.

The grid is protected from overcurrents by electrical fuses; hence, the partial line flows are subject to given partial flow constraints

$$g_t \preceq g^{\max} \tag{6}$$

where $g^{\max} \in \mathbf{R}^{n_l}$ describes the limits. Note that such limits are not currently an issue, but it is expected to be an issue in the future when large numbers of heat pumps and electrical vehicles will be put into operation. Therefore it is possible that legislations or markets will enforce such partial flow limits. See, e.g., [19]. Further note that voltage issues also are expected in the coming years on long thin distribution lines that are subject to large loads. By including a more sophisticated model, voltage limits could also be included as constraints to the problem but this is not done in this work.

Finally note, that the total power delivery of the portfolio is given by $\mathbf{1}^T u_t$, where $\mathbf{1}$ is a vector of all ones. The total power $\mathbf{1}^T u_t$ is positive for a net production and negative for a net consumption.

B. Primary Frequency Reserve

The aggregated generators and consumers participate in the primary frequency reserve market by placing a symmetric bid of p MW for a certain time span (for instance 4 hours in some systems [24]). If the bid is accepted, the aggregator must provide the sold primary frequency reserve. The specifications of the delivery of primary frequency control depend on the system. Typical specifications are that primary control must be provided linearly with the frequency deviation in the frequency deviation interval ±200 mHz; further, the activation time of the full reserve must be no more than 30 seconds.

Let $\Delta f_t \in \mathbf{R}$ describe the frequency deviation from the nominal frequency at sample t. Then the aggregator must track the reference r_t at sample t given by

$$r_t = \max\left(\min\left(p\Delta f_{t-t_0}/\Delta\overline{f}, p\right), -p\right). \tag{7}$$

Here $\Delta\overline{f}$ is the frequency deviation at which the full bid must be activated, e.g., $\Delta\overline{f} = 200$ mHz as described above. The scalar t_0 is the number of samples before the full reserve should be activated, e.g., $t_0 = 3$ if the activation time is 30 seconds as described above and the sampling time is 10 seconds.

We model the grid frequency as a first order system

$$\Delta f_{t+1} = a\Delta f_t + w_t \tag{8}$$

where $w_t \in \mathcal{W} = [\underline{w}, \overline{w}]$ is the change in frequency at every sample which is assumed bounded, white and zero mean. The reason for this model is that we assume a large system where we do not affect the system frequency; however, the accumulated primary control will drive the frequency towards the nominal frequency. The bounds reflect that the frequency in the grid can not jump arbitrarily from sample to sample. The parameter $a \in \mathbf{R}$ describes how fast the grid restores to the nominal frequency. Note that a linear model of any order can be chosen, but for the sake of simplicity it is chosen to use a first order model.

IV. CONTROLLER SYNTHESIS

The basis of the controller is that the n generators and consumers are aggregated and utilized to bid into the primary frequency reserve market with a bid of p MW. The goal of the controller is to provide primary frequency reserve according to the given specifications at the lowest possible price while honoring the limits of the generators, consumers, and the links in the grid. We emphasize that the provision of primary frequency reserve is based on a portfolio of units with various characteristics, ranging from storages to small and large generators – this is in contrast to conventional reserve provision done by a single power plant. In order to optimize cost, the controller must exploit this diversity of the units, especially the fast ramp limits of the demand side units.

A. Problem Formulation

Based on the overall model of generators, consumers, and the the grid, we construct a problem formulation which is later used to design a controller.

1) Constraints: The aggregator must provide a certain amount of frequency reserve depending on the deviation from the nominal grid frequency Δf and on the amount of sold primary reserve p. The amount of primary reserve, that the aggregator must provide, is described by (7) and gives the following constraint to the aggregator

$$\mathbf{1}^T u_\tau = r_\tau \qquad (9)$$

for $\tau \geq 0$. Further, the aggregator must honor the rate-, power- and energy storage limitations of grid, generators, and consumers, which can be described as follows:

$$x_\tau \in \mathcal{X}, \quad u_\tau \in \mathcal{U}, \quad \Delta u_\tau \in \Delta\mathcal{U} \qquad (10)$$

for $\tau \geq 0$ where

$$\mathcal{U} = \{u | u^{\min} \preceq u \preceq u^{\max}, \ Gu \preceq g^{\max}\}$$
$$\mathcal{X} = \{x | x^{\min} \preceq x \preceq x^{\max}\}$$
$$\Delta\mathcal{U} = \{\Delta u | \Delta u^{\min} \preceq \Delta u \preceq \Delta u^{\max}\}.$$

2) Objective: The objective of the aggregator is to minimize the average production cost of delivering the sold frequency reserve.

The cost of operating the portfolio is a function of u and x. We assume a convex stage cost function $\ell : \mathbf{R}^n \times \mathbf{R}^n \to \mathbf{R}$

and define the average cost J_∞ as

$$J_T(x, u) = \frac{1}{T} \sum_{\tau=0}^{T-1} \ell(x_{\tau+1}, u_\tau) \qquad (11)$$

$$J_\infty(x, u) = \limsup_{T \to \infty} J_T(x, u). \qquad (12)$$

If we consider an operating production unit, the cost of providing frequency reserve will reflect the cost of deviating from the nominal operation point and is thus a function of u. For a flexible consumer, the cost of providing frequency reserve will reflect the discomfort associated with storing energy and is therefore a function of x. For a house with electrical heating, the discomfort cost would represent the cost of deviating from the desired temperature set-point.

B. Closed-loop Model Predictive Control

The problem formulation states that the controller must ensure the provision of the required primary frequency reserve while minimizing the average production cost. In other words: the objective J_∞ is to be minimized under the constraints (9) and (10). In the following, we design a receding horizon control strategy which approximately solves this problem. The receding horizon controller minimizes J_T over a the finite horizon of T samples and applies first control input; at next sample this optimization is redone (hence the name *receding horizon*). This results in an economic finite horizon model predictive controller, as the objective is a minimization of an economical cost and not a distance to a certain reference, as is the case in stabilization problems.

A main question in the controller design concerns tracking the reference r_t, as this reference is driven by the unpredictable disturbance w, see (7). One obvious way to deal with the disturbance is to use the expected value, i.e., let $w_{\tau|t} = \mathbf{E}(w) = 0, \tau \geq t$ at sample t. The benefit of this strategy is that it leads to the design of a simple certainty equivalent MPC strategy but on the other hand, such simple disturbance model may lead to poor performance [25]. In particular, a certainty equivalent strategy will not prepare the storages in the power portfolio for possible future up- and down-regulation needs as it assumes no future disturbance.

Another way to handle the unpredictable disturbance is to design a robust model predictive controller that optimizes a single control signal to minimize the worst case cost under all possible disturbance realizations. While this formulation takes the future disturbances into account in the optimization, it suffers from often being conservative [26]. The reason for this conservatism is that this strategy is open-loop within the horizon, in the sense that the controller does not take into account that at the next time sample, more information will be available and the optimization will be redone including this new information.

The above described certainty equivalent controller and robust MPC controller are both open-loop MPC strategies, where the next sample of the control signal is chosen from optimization of a *single* control sequence. In order to design a controller that is able to prepare the power portfolio for future frequency changes in a non-conservative fashion, we instead

consider closed-loop MPC. In contrast to open-loop MPC where we optimize a single control sequence, closed-loop MPC optimizes a sequence of *control policies*. This means that we do not commit to a certain control input sequence for the whole control horizon; instead we choose a control policy which will allow different control sequences depending on the realizations of the future disturbances. Hereby the controller will achieve a closed-loop behavior, where we allow recourse as more information becomes available (see, e.g., [26], [22], [27]). Note that the terminology of open-loop MPC vs. closed-loop MPC is adopted from the literature, e.g., the references above. Further, note that *both* open-loop and closed-loop MPC strategies indeed are receding horizon control strategies where reoptimization is performed at each sample when new measurements are available; however, only the closed-loop control strategy considers the various possible disturbance outcomes within each optimization.

Such closed-loop MPC strategy is considered in the following. The motivation is that this strategy will enable us to act preemptively against future disturbance realizations, even though they are unpredictable. By considering all possible disturbance realizations, instead of just the expected value of the disturbance, we obtain a controller that is able to mobilize the storages such that they are ready to provide both up- and down-regulation, depending on the future unpredicted frequency behavior. In a sense, closed-loop MPC is a systematic way of implementing a mid-ranging strategy on the energy storages [28], however where we avoid being conservative due to the closed-loop fashion where recourse is allowed.

1) Min-Max Feedback Predictive Control: One way to implement closed-loop MPC is a min-max approach. In this approach, all possible disturbance realizations within a finite horizon are considered and the maximum cost is minimized over a sequence of control policies. As the disturbance w is bounded in a polytope $\mathcal{W}$ and as the model of the dynamics is linear and the objective is convex, we know that such min-max optimization can be performed by considering the vertices of the disturbance polytope alone [22].

The min-max method is chosen as this method clearly illustrates the main message of this paper: that performance is increased when our control strategy takes the possible future frequency deviation realizations into account and hereby plan for the future unknown events. Other strategies could have been chosen instead such as scenario based strategies. See, e.g., [29], [30].

For a finite horizon T, the controller must therefore consider the 2^T extreme disturbance realizations based on the vertices $\underline{w}$, $\overline{w}$ of $\mathcal{W}$. Following the notation of [22], we denote the extreme disturbance sequences and associated reference sequences

$$\{w_t^i, \ldots, w_{t+T-1}^i\}, \ i \in \mathcal{I} \qquad (13)$$
$$\{r_t^i, \ldots, r_{t+T-1}^i\}, \ i \in \mathcal{I}, \qquad (14)$$

respectively, where $i \in \mathcal{I} = \{1, 2, \ldots, 2^T\}$; i.e., $\mathcal{I}$ describes the number of extreme disturbance realizations. The reference sequences can be found based on previous frequency measurements and the disturbance sequence by (7). Similarly, we denote the control and state sequences

$$\{u_t^i, \ldots, u_{t+T-1}^i\}, \ i \in \mathcal{I} \qquad (15)$$
$$\{x_t^i, \ldots, x_{t+T}^i\}, \ i \in \mathcal{I}, \qquad (16)$$

respectively. The objective of the controller is to optimize the control sequences $\{u_t^i, \ldots, u_{t+T-1}^i\}$ such that the maximum cost of $\sum_{\tau=t}^{\tau+T} \ell(x_\tau^i, u_\tau^i)$ for $i \in \mathcal{I}$ is minimized. Based on the dynamics and constraints of the system and on the cost function, we are able to formulate this as a finite horizon optimization problem. At sample t the optimization problem is given as follows:

$$
\begin{aligned}
\text{minimize} \quad & \max_{i \in \mathcal{I}} \sum_{\tau=t}^{t+T-1} \ell(x_{\tau+1}^i, u_\tau^i) \\
\text{subject to} \quad & x_{\tau+1}^i = Ax_\tau^i + Bu_\tau^i \\
& x_{\tau+1}^i \in \mathcal{X}, \ u_\tau^i \in \mathcal{U}, \ \Delta u_\tau^i \in \Delta\mathcal{U} \\
& \mathbf{1}^T u_\tau^i = r_\tau^i \\
& x_\tau^{i_1} = x_\tau^{i_2} \Rightarrow u_\tau^{i_1} = u_\tau^{i_2}
\end{aligned}
\qquad (17)
$$

for $\tau = t, \ldots, t + T - 1$; $i, i_1, i_2 \in \mathcal{I}$ and where the variables are the control sequences $\{u_t^i, \ldots, u_{t+T-1}^i\}$ and associated states $\{x_{t+1}^i, \ldots, x_{t+T}^i\}$. The data is the reference sequences $\{r_t^i, \ldots, r_{t+T-1}^i\}$, the previous input $u_{t-1}^i = u_{t-1}$ and the current state $x_t^i = x_t$. Note that the grid frequency dynamics (8) are indirectly included in the optimization problem as the reference sequences r_τ^i are generated based on the possible extreme disturbance realization. This means that we must construct the reference sequences r_τ^i at each iteration as described in the algorithm later in this section.

The first two constraints in (17) regard the system dynamics and the input and state constraints. The third constraint assures that the controller indeed provides the required primary reserve. The last constraint, $x_\tau^{i_1} = x_\tau^{i_2} \Rightarrow u_\tau^{i_1} = u_\tau^{i_2}$, is a causality constraint (see [22], [26]) which is described in the following.

The closed-loop min-max model predictive controller is illustrated in Fig. 2. The figure illustrates the extreme disturbance realizations with a horizon $T = 3$ when we are at time sample t; further, the figure shows the control- and state sequences for the given horizon. We can use the figure to describe the behavior of the controller: at sample t we observe the state x_t and determine the control sequences and associated state sequences $\{u_t^i, u_{t+1}^i, u_{t+2}^i\}$, $\{x_t^i, x_{t+1}^i, x_{t+2}^i, x_{t+3}^i\}$ such that the objective is minimized. Due to the causality constraint, we have that $u_t^i = u_t$ as $x_t^i = x_t$ which means that we settle on a single control signal u_t which is applied to the plant. We, however, do not settle on single future control signals u_{t+1}, u_{t+2}; instead we design a control sequence for each possible extreme disturbance realization and do not choose which control signal to apply until next sample when w_t is known. In this way, the controller takes into account our ability to perform recourse as more information becomes available.

Finally we note that Problem (17) is a convex optimization problem as the causality constraint can be reformulated to
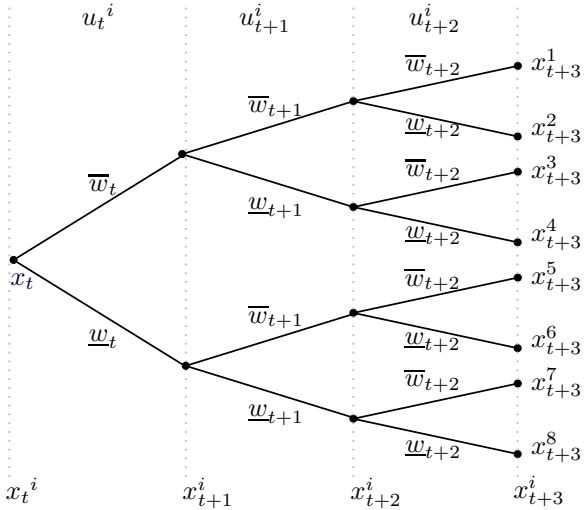
Fig. 2. The 8 different extreme disturbance realizations with a horizon $T = 3$.

linear equality constraints. This means that the problem can be solved globally and efficiently [31].

2) The Control Algorithm: Based on the above description of the closed-loop optimization, we are able to formulate the controller algorithm:

At sample t

1) Collect the current storage levels of the consumers x_t, the previously applied control input u_{t-1} and the current grid frequency f_t.
2) Construct the extreme disturbance sequences $\{w_t^i, \ldots, w_{t+T-1}^i\}$, $i \in \mathcal{I}$ based on the disturbance vertices $\underline{w}, \overline{w}$.
3) Construct the extreme reference sequences $\{r_t^i, \ldots, r_{t+T-1}^i\} \in \mathcal{I}$ based on the previous references $r_{t-t_0}, \ldots, r_{t-1}$, the disturbance sequences and the amount of sold primary reserve p using (7).
4) Solve Problem (17) and denote the optimal control sequences $\{u_t^{i\star}, \ldots, u_{t+T-1}^{i\star}\}$, $i \in \mathcal{I}$.
5) Apply the fist control input $u_t^\star = u_t^{i\star}$ to the generators and consumers.
6) Increase t by one and repeat from 1.

C. Scalability and Implementation

A major difficulty with the presented method is the scalability as the min-max MPC method scales exponentially with the control horizon. For larger number of devices and in particular for large horizons, the presented method therefore has its limitations. For practical implementation, it might therefore be necessary to alter the method for example to scenario based methods, see [29], [30], or methods that assume a certain class of policies, for example causal affine functions of the uncertainty as in [32], instead of dealing with each of the 2^T extreme disturbance realizations.

V. NUMERICAL EXAMPLE

We perform a number of numerical examples that illustrate the behavior of the closed-loop MPC algorithm. The examples are kept at a conceptual level with a small number of units to clearly visualize the behavior of the controller. We consider a portfolio of four units: two consumers and two generators. They have the following characteristics.

- unit$_1$ and unit$_2$: ideal storages with no ramp limits but limited storage capacity; unit$_1$ is on line close to congestion.
- unit$_3$: slow generator with low operational cost.
- unit$_4$: fast generator with high operational cost.

Throughout the examples, we will use an open-loop certainty equivalent MPC controller as reference. This reference controller is implemented with same objective and constraints but use the expected value of the disturbance as prediction, i.e., $w_{t|\tau} = \mathbf{E}(w) = 0$, $\tau \geq 0$.

A cost function on the form

$$\ell(x_t, u_t) = x_t^T Q x_t + \|R u_t\|_1 \tag{18}$$

is used. The cost of utilizing the storages is assumed quadratic; this could reflect temperature comfort limits of an electrically heated house where a small deviation has close to no cost, while larger deviations are expensive. The cost of the generating is are chosen to be a weighted one-norm; this illustrates that even small changes in the operation of the generators have a significant cost. Note that we are operating with small-scale values and that u_t corresponds to deviations from the nominal power consumption/generation.

The aggregator managing the portfolio has sold $p = 5$ MW primary frequency reserve and we assume that the power reference must be met in 15 s and use a sampling rate of 15 s for simplicity. Finally, we assume that the frequency never changes faster than 40 mHz/sample and we use a prediction horizon of $T = 8$ samples. We can specify the properties of the optimization problem as follows:

$$x^{\min} = (0, \ 0, \ -, \ -)^T, \ x^{\max} = (80, \ 80, \ -, \ -)^T \text{ kWh}$$
$$\Delta u^{\max} = -\Delta u^{\min} = (100, \ 100, \ 25, \ 100)^T \text{ kW/s}$$
$$Q = \mathbf{diag}(1, \ 1, \ 0, \ 0), \ R = (0, \ 0, \ 10, \ 1)^T,$$

which simply state two consumers with limited capacity but no ramp limits, a slow inexpensive generator and a fast and expensive generator. We do not consider power limits. Further, unit$_1$, unit$_3$, unit$_4$ are on lines with no congestion while unit$_2$ is on a line which allows only 0.3 MW.

The desired behavior of the controller is to use the storages unit$_1$, unit$_2$ to provide fast regulation then use the slow inexpensive generator unit$_3$ to relieve the storages hereby avoiding using the expensive generator unit$_4$. But as utilizing the storages is also associated with a cost, the controller must ensure that the storage level in unit$_1$ and unit$_2$ are minimized while still being able to provide both up- and down-regulation.

In the following, we will look at two examples. The first example is constructed such that the ability of the closed-loop MPC controller to take preemptive action against future

frequency changes is made obvious. The second example is meant to be an example of normal operation for the controller.

A. Preemptive Action

In this example we consider an example where the frequency suddenly drops more than 0.2 Hz, see top plot of Fig. 3. The frequency drop causes the aggregator to provide the full 5 MW of up-regulation. The behavior of the closed-
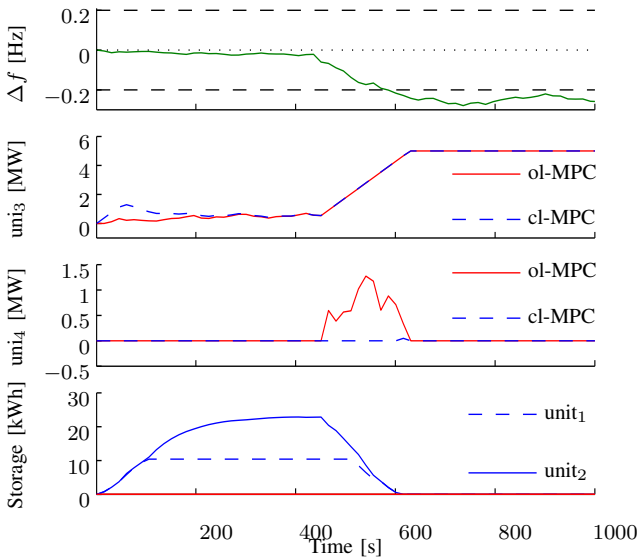

Fig. 3. Plot 1: grid frequency deviation Δf and limits $\pm \Delta \overline{f}$. Plot 2,3: power productions of $unit_3$ (slow, inexpensive) and $unit_4$ (fast, expensive) for the open-loop (red, solid) and the closed-loop controller (blue, dashed). Plot 4: energy levels of $unit_1$ on congested line (blue, dashed) and $unit_2$ on the non-congested line (blue, solid) in the closed-loop case; the open-loop storage levels are not visible as they remain zero.

loop MPC controller is seen in Fig. 3. In the first minutes where the frequency is stable, the controller uses the slow and inexpensive generator to fill up the energy storages of $unit_1$ and $unit_2$, mainly the storage of $unit_2$ where there is no congestion problem. The controller fills up the storages as it knows this will be beneficial in case of a sudden frequency drop.

Exactly because of this preemptive action, the closed-loop MPC algorithm is able to provide the necessary up-regulation at the time of the frequency drop without utilizing the expensive generator $unit_4$; instead the storages compensate for the fast frequency drop while the slow generator $unit_3$ relieves the storages (see Fig. 3). This is exactly the desired behavior for the controller and is achieved as the controller minimizes the worst case future cost in a closed-loop manner.

Further we note, that the closed-loop MPC algorithm does not refill the storages $unit_1$ and $unit_2$ after they have been relieved; the controller knows that the reference never will exceed 5 MW even if the frequency drops further. Thereby no unnecessary storage actuation is performed.

As comparison we observe the behavior of the open-loop MPC reference controller. This controller does not

consider the effects of future frequency changes and therefore minimizes its cost function by keeping all storages empty. When the frequency drops, it is forced to use the expensive generator to provide up-regulation at a high cost. The comparison is presented in Fig. 3.

Note that we start the simulation with the storages empty, hereby the action of the closed-loop control becomes clear as it can be seen that it fills the storages. If we had started with the storages filled up, we would see the closed-loop control decrease the storage level to the same levels as in the presented example; on the contrary, the open-loop control would decrease the storage levels to zero as it does not expect future disturbances and therefore does not expect to benefit from non-empty storages.

B. Normal Operation

We now consider an example of what could be normal operation for the controller. It is assumed that the change in frequency is band-limited Gaussian noise with standard deviation 40 mHz per sample and limits ± 40 mHz per sample. An example of this is illustrated in Fig. 4 for a 50 minute sequence. The performance of the open-loop and
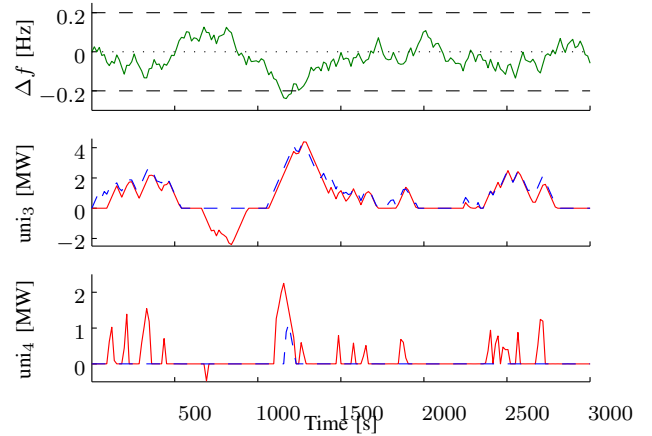

Fig. 4. Plot 1: grid frequency f and limits $\pm \Delta \overline{f}$. Plot 2,3: power productions of $unit_3$ (slow, inexpensive) and $unit_4$ (fast, expensive) for the open-loop (red, solid) and the closed-loop controller (blue, dashed).

the closed-loop MPC strategy is presented, illustrating that the closed-loop controller is able to almost completely avoid using the fast and expensive generator by more extensive and intelligent utilization of the storages. The example shows that the closed-loop MPC controller is able to let the storages act as a fast generator, thereby reducing the operational costs significantly.

To enhance the reliability of the results, 5 such 50 minute simulations are completed with different system frequency realizations, all revealing similar results: a significantly lower cost when utilizing the closed-loop MPC control law. The normalized costs for the 5 simulations are presented in Table I.

As previously mentioned, and ad-hoc control strategy could be to implement mid-ranging on the energy storages. This was done on the example presented here and by

Disturbance sequence	1	2	3	4	5
$J_{\text{ol-MPC}}$	0.79	0.70	1.00	0.34	0.56
$J_{\text{cl-MPC}}$	0.32	0.30	0.49	0.04	0.16

TABLE I

PERFORMANCE COMPARISON.

extensive tuning it was possible to achieve a performance that indeed was better than the certainty equivalent scheme, however still significantly worse than the closed-loop MPC control strategy. These results are not presented here.

VI. CONCLUSION

In this paper we have described how a mixed portfolio of power generators and flexible demand side units can be aggregated and used to provide primary frequency reserve. Hereby we are able to reduce the load on conventional generators. Further, we have shown how a simple model of the grid frequency and bounds on the change in frequency can be used in the design of a closed-loop model predictive controller. The controller assures that the frequency reserve obligation is met and that the grid constraints are honored, while minimizing the operational cost of the portfolio. Further, the closed-loop controller enables the energy storages to act preemptively against future rapid grid frequency changes, which significantly reduces the load on the conventional generators in the portfolio.

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