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Time-Varying FOPDT Modeling and On-line Parameter Identification

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Abstract:
A type of Time-Varying First-Order Plus Dead-Time (TV-FOPDT) model is extended from SISO format into a MISO version by explicitly taking the disturbance input into consideration. Correspondingly, a set of on-line parameter identification algorithms oriented to MISO TV-FOPDT model are proposed based on the Mixed-Integer-Nonlinear Programming, Least-Mean-Square and sliding window techniques. The proposed approaches can simultaneously estimate the time-dependent system parameters, as well as the unknown disturbance input if it is the case, in an on-line manner. The proposed concepts and algorithms are firstly illustrated through a numerical example, and then applied to investigate transient superheat dynamic modeling in a supermarket refrigeration system.

Keywords: System identification, FOPDT, time-delay system

1. INTRODUCTION

Modeling a large scale complex system for the control purpose is often a challenging task. By following the first-principle strategy, this modeling task often needs to extensively explore diverse physical domains and knowledge. It can be very time consuming and also bear the risk that the developed model could be too complicated to be used for supporting control design. A Alternatively, the data-driven modeling strategy, sometimes also referred to as *black-box* modeling, could avoid extensive working loads and meanwhile control the complexity of the intended model, but the accuracy of the model developed through this strategy heavily depend on how the experiment is designed, the selection of model structure and the quality of the measured data (Ljung (1999)). Thereby the estimated model usually is not flexible and has strong limitation of applicable range. In general, the modeling issue of a large-scaled complex system needs to balance all above mentioned perspectives. In this paper, we will investigate an online (data-driven) system identification approach for a class of time-varying linear time-delay systems, which is basically extended from conventional First-Order Plus Dead-Time (FOPDT) models (Åström and Hägglund (1995)).

The FOPDT model has been extensively used in the process modeling and control due to its simplicity. However, sometimes this type of model is not sufficient and flexible enough to catch the key dynamic characteristics of complex system under diverse operating conditions. For instance, Gralda and MacArthur (1992); Li *et al* (2008); Rasmussen and Larsen (2009) have discovered that system parameters, such as the system gain, time-constant and time-delay, in a standard FOPDT formulation of the transient superheat dynamic can be time-varying. In order to keep the FOPDT model's merit, but, meanwhile be able to correctly reflect system's key transient dynamics, Sun and Yang (2011); Yang and Sun (2011) proposed a type of nonlinear FOPDT model, named *Time-Varying FOPDT (TV-FOPDT)*, to model the transient superheat dynamic in a refrigeration system. The TV-FOPDT model is an extension of the standard FOPDT by allowing the system parameters, i.e., the system gain, the time constant and the time-delay, to be time dependent. Of course, this time dependency could also be due to the situation that some system parameters are correlated with some system variables, such as the input dependent time-delay which is studied in (Sun and Yang (2011)).

The TV-FOPDT model exhibits the system's dynamic characteristics from both the frequency and time perspectives within one complete formulation. Some similar TV-FOPDT models have also been observed in a number of nonlinear control applications, such as Brazauskas and Levisauskas (2007); Lee *et al* (1997); Kwok *et al* (1997); Richard (2003). For instance, the similar model used in Brazauskas and Levisauskas (2007) is called an *adaptive transfer function*. This adaptive model is obtained by linearizing the nonlinear dynamic description at each sampling time, so that the system parameters are (sampling) time dependent. Lee *et al* (1997) proposed a nonlinear FOPDT model by linearizing the nonlinear system at a number of different operating points, so that the system parameters of this type of FOPDT are operating-point dependent. No matter through which way to linearize the considered nonlinear system, both models and modeling approaches request a precise nonlinear system description beforehand. An on-line identification approach of a time dependent FOPDT is proposed in Kwok *et al* (1997), by using the so-called long-range predictive identification method. However, due to the technical limitation, the considered unknown time-delay in Lee *et al* (1997) is classified into four different potential scenarios before converting a nonlinear identification optimization problem into a Least-Square (LS) problem using the spectral factorization technique.

As what we noticed so far that all these TV-FOPDT relevant studies are limited to Single-Input Single-Output (SISO) consideration, which is not realistic in many complicated systems. Thereby, in this paper we propose a type of Multiple-Input Single-Output (MISO) TV-FOPDT model, and correspondingly, two sets of parameter identification algorithms are proposed to optimally estimate the time dependent unknown system coefficients, as well as the unknown input, in an on-line manner. The proposed concepts and algorithms are firstly tested through a number of numerical examples, and then are applied to model and identify the superheat dynamic in a supermarket refrigeration system based on the experimental field data.

The rest of the paper is organized as the following: Section 2 formulates the proposed MISO TV-FOPDT model; Section 3 proposes a set of on-line MISO TV-FOPDT identification algorithms depending on different disturbance input situations; Section 4 illustrates the proposed concepts and methods through a numerical example and the superheat dynamic modeling in a supermarket refrigeration system; finally, we conclude the paper in Section 5.

2. MISO TV-FOPDT FORMULATION

The output of a MISO TV-FOPDT model can be described as

$$X(s) = G_1^t(s)U_1(s) + G_2^t(s)U_2(s), \tag{1}$$

where $u_1(t)$ is a scalar known (control) input. $u_2(t)$ is a scalar disturbance input, and it can be measurable/known or unmeasurable/unknown, depending on a specific application. $x(t)$ is the "noise-free" output, and $X(s)$, $\{U_i(s)\}_{i=1,2}$ are the corresponding Laplace-transforms of the system output and inputs, respectively. The two transfer functions in (1) are type of TV-FOPDT models (Yang and Sun (2011)), i.e.,

$$G_1^t(s) \hat{=} \frac{K_1^t}{\tau_1^t s + 1} e^{-T_1^t s}, \tag{2}$$

and

$$G_2^t(s) \hat{=} \frac{K_2^t}{\tau_2^t s + 1} e^{-T_2^t s}, \tag{3}$$

where τ_i^t, K_i^t, T_i^t for $i = 1, 2$ are corresponding system time constants, gains and time-delays, respectively. The superscript t indicates that these system parameter can be time dependent.

The measurement of a MISO TV-FOPDT model can be described as

$$y(t) = C^t x(t) + \omega(t), \tag{4}$$

where C^t is a scalar coefficient and it can be time dependent as well. The measurement noise $\omega(t)$ is assumed as a white Gaussian noise with zero mean and variance $Q(t)$. The combination of (1) and (4) is referred to as a MISO TV-FOPDT model in the following.

Compared with the state space description of time-delay systems (Richard (2003)), the TV-FOPDT description can be possibly converted into a time-varying state space model with input time-delay. Vise versa, a state space description of a linear time-delay system can also be possibly converted into an equivalent TV-FOPDT model. Thereby, the TV-FOPDT identification methods proposed here can also be possibly used to identify system coefficients of a time-delay system described by a state space formulation. Of course, some extra knowledge or information may be needed in order to achieve a unique solution (identifiability issue Orlov *et al* (2003)).

3. ON-LINE MISO TV-FOPDT IDENTIFICATION

The estimation of system time-delay is always a challenging issue for time-delay systems, and the on-line estimation of time-varying time-delay is even more open at this stage (Ljung (1999); Richard (2003)). We also observed that the estimation of varying time-delay can result in a non-convex optimization problem (Yang and Sun (2011)). The simplest and straightforward way to estimate the input-output delay of a LTI system is to use the cross-correlation analysis (Richard (2003)), or by some experimental approaches (Åström and Hägglund (1995)). These signal-based approaches are heavily limited by the measured signals' quality and signal-to-noise-ratio. From the time-domain model-based point of view, because the time-delay exhibits inside the time index of state/input/output variables, some mathematical operation is often needed so as to be able to have the explicit expression of this parameter out of the time index, before any identification algorithm proceeds further. This mathematical operation can be realized through applying an integrator or derivative filter on both side of the system equation (Ahmed *et al* (2006); Young (2002)). These methods are only suitable for time-invariant parameters and off-line identification, i.e., the input signal should be available beforehand.

The general on-line MISO TV-FOPDT identification problem can be formulated as *to optimally estimate the time-dependent parameters* $\{K_i^t\}_{i=1,2}$, $\{\tau_i^t\}_{i=1,2}, \{T_i^t\}_{i=1,2}$ *and* C^t, *based on measurements of system input(s) and output in an on-line optimal manner*. If the disturbance input $u_2(t)$ is available, then it will also be used in the identification procedure. Otherwise, it may need to be estimated as well.

In order to make the identification procedure proposed in the following feasible, we make the following assumptions:

- The subsystem (2) and (3) have common time constants, i.e., $\tau_2^t \equiv \tau_1^t \hat{=} \tau^t$ for any $t \geq 0$;
- In case that $u_2(t)$ is unknown and it needs to be estimated as well, we assume $K_2^t \equiv 1$, $T_2^t \equiv 0$; and
- The system output gain C^t is always constant, without losing of generality, we assume $C^t \equiv 1$.

The second assumption is reasonable w.r.t. the fact that any deviations of K_2^t and T_2^t from assumed values can be accounted as the influence from the unknown input $u_2(t)$.

3.1 Discretization

The considered continuous-time system (1) and (4) can be approximated by its discrete-time equivalence once a proper sampling frequency is selected. Here we denote the sampling period as T_s, then there is

$$X(z) = G_1^k(z)U_1(z) + G_2^k(z)U_2(z), \tag{5}$$

with

$$G_1^k(z) = \frac{K_1^k(1-\alpha^k)}{z^{l_1^k}(z-\alpha^k)}, \tag{6}$$

and

$$G_2^k(z) = \frac{K_2^k(1-\alpha^k)}{z^{l_2^k}(z-\alpha^k)}, \tag{7}$$

where $\alpha^k \hat{=} \exp^{-\frac{T_s}{\tau^k}}$. $\{K_i^k\}_{i=1,2}$ and τ^k are called as the kth-step sampled system gains and time constant, respectively (Yang and Sun (2011)). It should be noticed that $\{K_i^k\}_{i=1,2}$ and τ^k are not as same as $\{K_i^t\}_{i=1,2}$ and τ^t in (2) and (3), correspondingly. The former ones are piecewise-constant (i.e., constant during every sampling period) time functions, while the latter ones are continuously-varying time functions. The relationship of them can be expressed as:

$$K_i^k = K_i^t, \text{ for } i = 1,2 \text{ and } \tau^k = \tau^t \text{ when } t = kT_s$$

for any integer k.

The coefficients $\{l_i^k\}_{i=1,2}$ in (6) and (7) are some integer numbers and they are called the corresponding discrete approximations of the kth-step sampled time-delays (Yang and Sun (2011)), which are denoted as $\{T_i^k\}_{i=1,2}$ with the property: $T_i^k = T_i^t$ when $t = kT_s$ for any integer k. Coefficients $\{l_i^k\}_{i=1,2}$ satisfy the property:

$$l_i^k T_s \le T_i^k < (l_i^k + 1)T_s, \text{ for } i = 1,2. \tag{8}$$

Define $\beta_i^k \hat{=} K_i^k(1-\alpha^k)$ for $i = 1,2$, then, the system (5) can be converted into a difference equation description as:

$$x(k) = \alpha^k x(k-1) + \beta_1^k u_1(k - l_1^k - 1) + \beta_2^k u_2(k - l_2^k - 1), \tag{9}$$

for $k = l_m + 1, l_m + 2, \cdots \infty$, where $l_m \hat{=} max\{l_1^k, l_2^k\}$. The output signal will be measured at each sampling time, i.e.,

$$y(k) = x(k) + \omega(k). \tag{10}$$

By combining (9) and (10), we have

$$\begin{aligned} y(k) = &\ \alpha^k y(k-1) + \beta_1^k u_1(k - l_1^k - 1) \\ &+ \beta_2^k u_2(k - l_2^k - 1) + \hat{\omega}(k), \end{aligned} \tag{11}$$

for $k = l_m + 1, l_m + 2, \cdots \infty$. Here $\hat{\omega}(k)$ is the filtered noise through the relationship $\hat{\omega}(k) = \omega(k) - \alpha^k \omega(k-1)$. If the disturbance input $u_2(t)$ is measurable along with the system operation, then, the original system identification problem based on (1) and (4) is transferred to be an identification problem based on (11): *to develop a procedure which can optimally estimate sample-dependent coefficients* α^k, $\{\beta_i^k\}_{i=1,2}$, *and* $\{l_i^k\}_{i=1,2}$ *based on a number of sampled inputs and output in an on-line manner.* We refer to this problem as *Case-with-Known-*$u_2(t)$ *(abbr. Case-Known-u2).*

If the disturbance $u_2(t)$ is not measurable or unknown along with the system operation, we will simplify the considered identification problem using the assumption $K_2^k \equiv 1$, $T_2^k \equiv 0$. Then, system (11) is simplified as

$$y(k) = \alpha^k y(k-1) + \beta_1^k u_1(k - l_1^k - 1) + \hat{u}_2^k + \hat{\omega}(k), \tag{12}$$

where $\hat{u}_2^k \hat{=} (1-\alpha^k)u_2(t-1)$. In the following, we regards $\hat{u}_2^k$ as one unknown sample-dependent system parameter, instead of an input signal. Thereby, the original system identification problem based on (1) and (4) is transferred to be an identification problem based on (12): *to develop a procedure which can optimally estimate sample-dependent coefficients* α^k, β_1^k, l_1^k *and* $\hat{u}_2^k$, *based on a number of sampled inputs and output in an on-line manner.* We refer to this problem as *Case-with-Unknown-*$u_2(t)$ *(abbr. Case-Unknown-u2).* We recommend readers to check our previous work (Yang and Sun (2011); Sun and Yang (2011)) for the case that $u_2(t)$ is always zero.

3.2 Iterative LMSP Method for Case-Known-u2

Assume the considered system (11) currently is at kth sampling step, and we take a moving window with the length of N for selecting latest sampled data at each step. Define $\Upsilon^k \hat{=} [\alpha^k\ \beta_1^k\ \beta_2^k]^T$, then the identification of (11) at the kth sampling step can be formulated as a Stochastic Mixed Integer Non-Linear Programming (SMINLP) problem defined as

$$\min_{\substack{l_1^k, l_2^k\,:\,\textit{positive integers} \\ \Upsilon^k \in \Omega^k}} \mathbf{E}\{\| B_N^k - A_N^k(l_1^k, l_2^k)\Upsilon^k \|_2^2\}, \tag{13}$$

where B_N^k is a stack of N number of latest measured outputs up the current kth step, i.e.,

$$B_N^k \hat{=} [y(k)\ y(k-1)\ \cdots\ y(k-N+1)]^T. \tag{14}$$

$A_N^k(l_1^k, l_2^k)$ is a stack of N pair of measured inputs and outputs, depending on parameters l_1^k and l_2^k, i.e.,

$$A_N^k(l_1^k, l_2^k) \hat{=} \begin{pmatrix} y(k-1) & u_1(k-l_1^k-1) & u_2(k-l_2^k-1) \\ y(k-2) & u_1(k-l_1^k-2) & u_2(k-l_2^k-2) \\ \vdots & \vdots & \vdots \\ y(k-N) & u_1(k-l_1^k-N) & u_2(k-l_2^k-N) \end{pmatrix}. \tag{15}$$

Ω^k represents the possible range of Υ^k, which is determined by limits of the original system gains $\{K_i^t\}_{i=1,2}$ and time constant τ^t in (2) and (3) at the current sampling time kT_s.

If both time-delays are known, then the optimization problem (13) is simplified to a standard Least Mean Square (LMS) problem. We have observed that due to the time-varying delays, the considered SMINLP problem (13) can be non-convex. This observation results us into the usage of the Branch-and-Bound (BB) method from MINLP techniques (Grossman and Sahinidis (2002)) at the current stage. The BB method is basically an approach to enumerate all considered possibilities under the condition that some pre-knowledge about the system time-delays can be obtained, such as the potential upper and lower limits of the time-delays for the entire system or at each sampling step. An iterative numerical approach, which we refer to as an *Iterative LMS Prediction (LMSP) algorithm*, is proposed in the following, by combining BB method, LMS and sliding window techniques for solving SMINLP problem (13):

- *Pre-knowledge*: The upper and lower boundaries for time-delays in terms of some integer number multiplying with sampling period. Without losing of generality, we assume that $l^k_{i\,\min} \le l^k_i \le l^k_{i\,\max}$ and $l^k_{i\,\min}$, $l^k_{i\,\max}$ are known beforehand for $i = 1, 2$.

- *Initialization:* selection of a sliding window, which could consists of a specific type of window, the length (N) and the potential weighting etc., e.g., using the forgetting factor (Fortescue *et al* (1981)).
- *Data collection period*: Since the beginning of the operation, the algorithm only collect sampled data until the process reaches a specific step first time, which is denoted as k_{ini}, where $k_{ini} = N+max\{l^k_{1\,max}, l^k_{2\,max}\}$. It is to guarantee that there are enough data for constructing matrix (7).
- *Iteration period:* The iterative identification starts from k_{ini}th step. At each step, two while-loops need to be constructed w.r.t. l^k_i starting from $l^k_{i\,\min}$ and ending at $l^k_{i\,\max}$ for $i = 1, 2$ by taking unit increments.
  · For each iteration (t_1, t_2) of $\{l^{t_1}_1, l^{t_2}_2\}$ with $l^k_{i\,\min} \le l^{t_i}_i \le l^k_{i\,\max}$ for $i = 1, 2$, a LMS problem of (13) can be solved as
  $$\Upsilon^k(l^{t_1}_1, l^{t_2}_2) = ((A^k_N(l^{t_1}_1, l^{t_2}_2))^T A^k_N(l^{t_1}_1, l^{t_2}_2))^{-1} (A^k_N(l^{t_1}_1, l^{t_2}_2))^T B^k_N. \quad (16)$$
  and record the corresponding prediction error of (13).
  · The set of $(\Upsilon^k(l^{t_1}_1, l^{t_2}_2)$ and $(l^{t_1}_1, l^{t_2}_2)$ which result in the minimal prediction error among all iterations moving from $l^k_{i\,\min}$ to $l^k_{i\,\max}$ for $i = 1, 2$, denoted as $(\Upsilon^{k*}, \{l^{k*}_i\}_{i=1,2})$, is the optimal solution for (13) at the current step.
  · The optimal estimation of system parameters of (11), τ^k and $\{K^k_i\}_{i=1,2}$, can be obtained from $\Upsilon^{k*} = [\alpha^{k*} \;\; \beta^{k*}_1 \;\; \beta^{k*}_2]^T$ by
  $$\tau^{k*} = -\frac{T_s}{\ln \alpha^{k*}}, \text{and } K^{k*}_i = \frac{\beta^{k*}_i}{1-\alpha^{k*}} \text{ for } i = 1, 2. \quad (17)$$
  The optimally estimated time-delays of $\{T^k_i\}_{i=1,2}$ are $\{l^{k*}_i T_s\}_{i=1,2}$.
- Repeat the above steps when a set of new data of inputs and output are obtained.

3.3 Iterative LMSP Method for Case-Unknown-u2

For the situation of Case-Unknown-u2, the on-line system identification of (12) is to obtain the optimal estimations of α^k, β^k_1, l^k_1 and $\hat{u}^k_2$ at each sampling step based on the obtained measurements of $u_1(t)$ and $y(t)$. For the purpose of simplification, we denote l^k_1 as l^k in the following. Then, the proposed approach for Case-Known-u2 in last Subsection can also be used over here with some minor changes.

In order to consider the influences of different data, especially from time evolutional point of view, we employ a constant forgetting factor in the following. The forgetting factor, denoted as ρ, has the property $0 < \rho < 1$. The output data vector corresponding to (6) becomes

$$B^k_N \hat{=} [y(k) \;\; \rho y(k-1) \;\; \cdots \;\; \rho^{N-1} y(k-N+1)]^T, \quad (18)$$

and the input-output data matrix corresponding to (7) becomes

$$A^k_N(l^k) \hat{=} \begin{bmatrix} y(k-1) & u_1(k-l^k-1) & 1 \\ \rho y(k-2) & \rho u_1(k-l^k-2) & \rho \\ \vdots & \vdots & \vdots \\ \rho^{N-1} y(k-N) & \rho^{N-1} u_1(k-l^k-N) & \rho^{N-1} \end{bmatrix} \quad (19)$$

Define $\theta^k \hat{=} [\alpha^k \;\; \beta^k \;\; \hat{u}^k_2]^T$, the optimal identification problem of (12) at the kth sampling step can be formulated as:

$$\min_{\substack{l^k \in L \\ \theta^k \in \Theta^k}} \quad \mathbf{E}\{\| B^k_N - A^k_N(l^k)\theta^k \|^2_2\}, \quad (20)$$

Here Θ^k represents the possible range of θ^k, L stands for the boundaries of time-delay. ρ is a forgetting factor, which is used to decrease the effect of old data to the estimation at the current sampling time. It is quite useful especially under the circumstance that some of system characteristics may vary according to time (Fortescue *et al* (1981)).

By using the BB method, there is only one while-loop is requested, due to the fact that only l^k needs to be estimated. The LMS solution and its corresponding covariance at the tth iteration can be obtained as

$$\begin{aligned} \hat{\theta}^k(l^t) &= ((A^k_N(l^t))^T \hat{Q}(k)^{-1} A^k_N(l^t))^{-1} (A^k_N(l^t))^T \hat{Q}(k)^{-1} B^k_N, \\ Cov(\hat{\theta}^k) &= ((A^k_N(l^t))^T \hat{Q}(k)^{-1} A^k_N(l^t))^{-1}, \end{aligned} \quad (21)$$

for $l^k_{min} \ge l^t \ge l^k_{max}$. Here $\hat{Q}(k)$ is the variance of filtered noise $\hat{\omega}(k)$ at the kth sampling step. It is estimated through

$$\hat{Q}(k) = Q(k) - [(\alpha^{k-1})^\star]^2 Q(k-1).$$

The optimal solutions of τ^k and K^k_1 at the kth step will have the same expressions as in (17), and the optimal estimation of sampled unknown input $u_2(t)$ has the formulation as

$$(u^{k-1}_2)^\star = \frac{(\hat{u}^k_2)^*}{(1-\alpha^{k*})^2}.$$

It can be noticed that the proposed approaches can guarantee the global optimal solution as long as the priori knowledge of time-delay boundaries is reasonable and the considered situation has a good SNR. The estimation accuracy here has two perspective meanings: (i) The accuracy of estimated parameters w.r.t. true values at each sampling time; (ii) The accuracy of the time varying features captured by estimated parameters w.r.t. those of true parameters. Both issues are relevant to the window selection. The proposed algorithms are iterative but not recursive, i.e., the estimation result at each step is independent from other step's results. Thereby, the proposed algorithms won't be disturbed by the diverging problem, while it is often an big issue for recursive computation algorithms (Orlov *et al* (2003)).

4. ILLUSTRATIVE EXAMPLES

Consider a switching system (1) with properties as:

- When $t < 30$ second, there are $\tau^t = 2,\ K^t_1 = 3,\ K^t_2 = 3$ and $T^t_1 = 3.05$;
- When $t \ge 30$ second, system parameters change to be $\tau^t = 3,\ K^t_1 = 4,\ K^t_2 = 4,\ T^t_1 = 2.05$;
- The measurement noise follows $\mathcal{N}(0,\ 0.001)$;
- The sampling period is set as $T_s = 0.1$ second;

- A sliding rectangular window is selected with $N = 50$;
- A constant forgetting factor is selected as $\rho = 0.95$;
- The boundaries of delay are $l^k_{\max} = 40,\ l^k_{\min} = 0$;
- A sweeping signal is used as the control input;
- A piecewise constant signal is used as the unknown input, i.e., $u_2(t) = \begin{cases} 1, & t < 40 \\ 1.2, & 40 \le t < 60 \\ 2, & t \ge 60. \end{cases}$

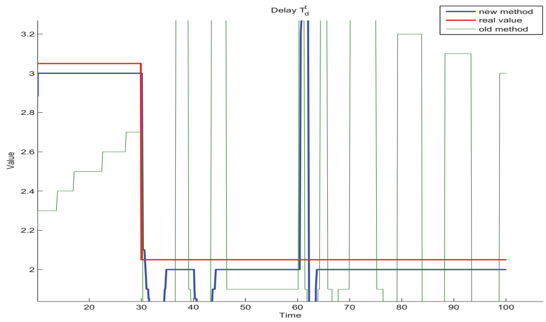

Fig. 1. Comparison of estimated time-delay T^t_1

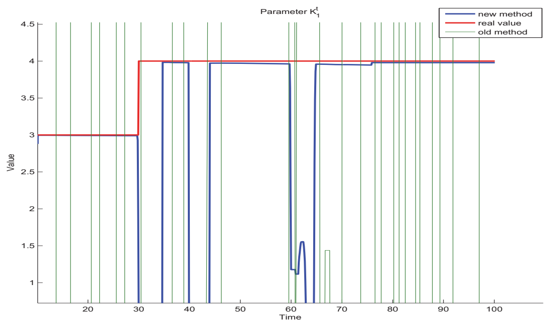

Fig. 2. Comparison of estimated system gain K^t_1

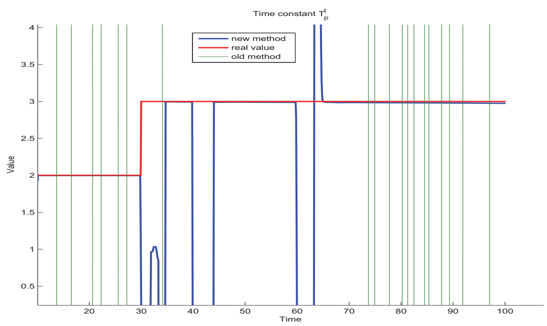

Fig. 3. Comparison of estimated time constant τ^t

Fig. 1, Fig. 2, Fig. 3 and Fig. 4 illustrate estimation results, respectively. Here the red line indicates the true value, the blue one shows the estimated result using the proposed approach here and the green one is the result using the original approach in Yang and Sun (2011). It is obvious that the proposed method exhibited much better results than the previous one did, which is originally proposed to cope with the SISO TV-FOPDT case. Some fluctuations can be clearly observed during a short period just after the switch occurred, as well as when the unknown input

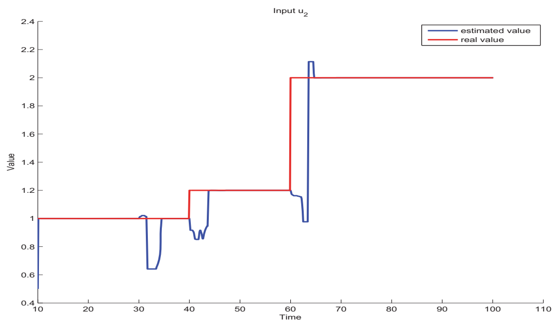

Fig. 4. Comparison of estimated unknown input with true values

jumped at 40th and 60th second, respectively. All steady-state estimated errors are bounded within 2%.

The data generated from a real-sized supermarket refrigeration system is used to estimate a MISO TV-FOPDT model of the superheat dynamic inside the considered system. We refer to Yang *et al* (2011) for a detailed description of this testing facility. A set of relay-type of control input (openness degree of the expansion valve) and the corresponding measured system output (superheat temp) are illustrated in Fig.5.

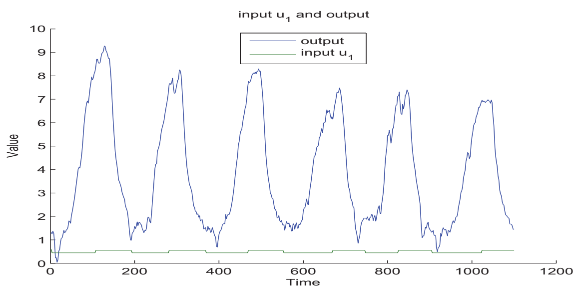

Fig. 5. The experimental input and output data

One set of estimated system parameters are illustrated in Fig. 6, Fig. 7 and Fig. 8, respectively. With respect to the expectation that a model is capable to model the transient superheat dynamic in a large operating region, there is no doubt that a MISO TV-FOPDT model should be the best candidate for this task at the current stage. The validation of the practical system study is still under going, and we expect to report those results in the near future.

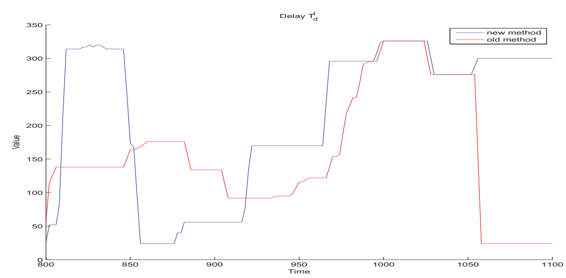

Fig. 6. Comparison of estimated time-delays

5. CONCLUSIONS

A type of MISO TV-FOPDT model is proposed based on the SISO TV-FOPDT model by introducing an extra

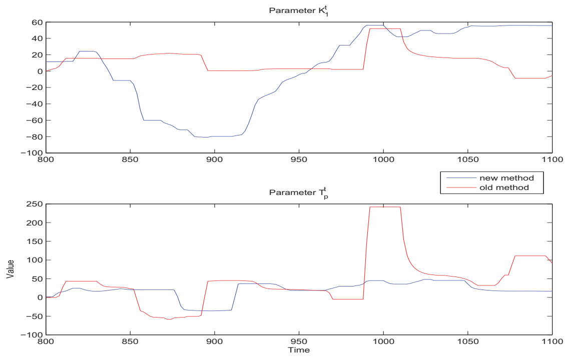

Fig. 7. Comparison of estimated system gains and time constants

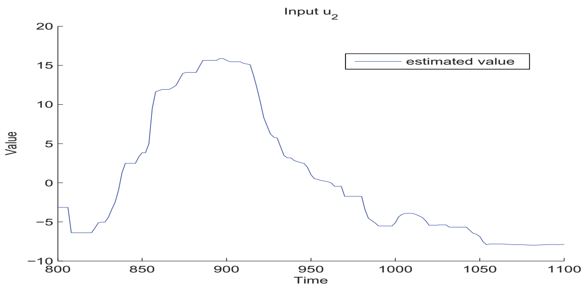

Fig. 8. Estimated the unknown disturbance input

input, named disturbance input, which is used to represent all key external influences to the system performance except the control input. A set of on-line parameter identification approaches are proposed to estimate the time-varying parameters as well as the unknown input if it is the case. It is noticed that the proper selection of a window type, window length, the forgetting factor and the input signal, can result into significantly different estimation results. Even though the proposed approaches are not computationally efficient yet, which is mainly due to the fact that we wish to avoid the non-convex optimization problems of (13)/(20) by sacrificing some computation efficiency, there is no doubt that the proposed model and approaches can provide a more flexible and realistic capability in modeling complicated dynamic systems.

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