THE EVEN BALANCE.

BY JOHN NEWTON LYLE.

HOW is the balance kept even? By taking as much out of one scale pan as out of the other, or by putting as much in one pan as in the other.

These two principles are so glaringly self-evident that few consider them worthy of a second thought. They are given a place, however, among the axioms of a very remarkable scientific work published at Alexandria in Egypt several centuries before the Christian Era. They were called "common notions" by their clear-headed, common sense Greek author and were stated in the following intelligible language: "If equals be taken from equals, the remainders are equal; if equals be added to equals the wholes are equal."

Has the truth of these two axioms ever been called in question? Yes, by an entire school of mathematical aeronauts who for two centuries past have been attempting aviation above the atmosphere in which alone it is possible.

This school demands "that we can take indifferently the one for the other two quantities which differ from each other only by an infinitely small quantity or (which is the same thing) that a quantity which is increased or diminished only by another quantity infinitely less than itself can be considered as remaining the same."

The demand is represented as being used to increase or diminish one member of an equation, the other member remaining untouched, while at the same time the resulting equation is said to be absolutely accurate.

Of course this procedure is in conflict with the two Euclidean axioms quoted above.

The apology offered for this discourtesy to Euclid is that the phrase "infinitely small" is used.

With deep regret the apology is herein declined for the reason that the "infinitely small" quantities of the hypothesis are retained in the first member of the equation as dividend and divisor whilst absolutely rejected from the second member. Are the properties of quantity different in one member of an equation from what they are in the other?

Remember that we are dealing now with mathematical symbols, not with fortune telling charms; with self-evident truths, not with statements neither self-evident nor true.

Has the phrase "infinitely small" as applied to mathematical quantity magical virtue?

Remember we are mathematicians and not magicians.

Is the modern calculus a species of occultism or is it a demonstrable science? Are its professors conjurers or scientific geometers?

The two Euclidean axioms to which reference has been made are either true all of the time or false all of the time. They can not be true a part of the time and a part of the time false.

A question of far-reaching importance in mathematics and philosophy here arises.

Can the first differential coefficient be obtained by the use of finite increments only, and without antagonizing the Euclidean axioms? This question was answered in the affirmative in the volume of the *American Mathematical Monthly* for the year 1894.

The subject was discussed in two articles, the one entitled "Are Differentials Finite Quantities?" the other, "The First Differential Coefficient of the Circle."

There is unity in mathematical science. The modern should not discredit the ancient but harmonize therewith.

The hypotheses introduced by both the German and the English mathematicians to explain the processes of the modern calculus were criticised relentlessly by Bishop Berkeley, who was himself a mathematician.

The English mathematicians lost their tempers on account of Berkeley's criticisms and stormed around in genuine John Bull fashion. The phlegmatic school of Leibnitz, however, ignored what Berkeley had to say respecting their transcendental, anti-Euclidean hypotheses, and instead of meeting Berkeley's objections candidly, honestly and bravely they did not meet them at all, but contented themselves with disparaging his idealistic philosophy and tar-water remedies which had nothing on earth to do with the modern calculus. This surely is a phenomenon. A land dominated by the idealism of Kant, Fichte, Schelling and Hegel refuses to consider the objections to the demands of De L'Hopital because of the idealistic philosophy of the objector!

What Berkeley really did in a speculative way was to carry the assumptions of the current philosophy of his day to their logical conclusion. This conclusion was absurd from the viewpoint of common sense and proves the falsity of the premises from which he argued. Berkeley, however, accepted both the false assumptions and their logical corollaries and gave to the world his idealistic philosophy. Whatever induced him to give to mankind his tar-water healing system remains an unsolved mystery. He alleges in quaint language that a patient once took an overdose—a quart of the potent stuff—and "was wrought all manner of ways." The same objection that lies against Berkeley's idealism applies to that of later writers. From the viewpoint of common sense, Borden P. Bowne's conclusions are as absurd as those of Berkeley. Consequently, his premises, which are of Kantian origin, are equally in need of revision.

Abundant industry and conscientiousness must be accorded to both Berkeley and Bowne. They undoubtedly stuck to their job and laboriously evolved what was wrapped up in their initial hypotheses.

Their service to mankind was that of labor-saving machines. The duty left to their successors is that of rectifying their premises.

Lobatchevsky, also, set out from false premises, reached absurd conclusions, but whilst on the journey his premises underwent a process of evolution so that at the end of the trip they could not be recognized as the ones from which he started. The trouble with the transcendental non-Euclidean is that he does not understand the principles of even balance.

EDITORIAL COMMENT.

John Newton Lyle, of Bentonville, Arkansas, protests in the name of common sense against non-Euclidean geometry, and quotes literally some of the paradoxical statements of the advocates of this theory. The problem is too complicated to discuss here, and there is no need of entering into it, because a statement of it has been made in the editor's little book *Foundation of Mathematics,* and the gist of it has been recapitulated in his summary of the philosophy of science, entitled *Philosophy of Form.* The significance of metageometry does not lie in the refutation of Euclid. Euclid remains as reliable as ever before. It merely proves that Euclid is not the only possible system of geometry, and that other systems can be constructed which do not rest on the principle that parallel lines will never meet except in infinity. One of the difficulties of mathematics is the conception of zero, and also in modern mathematics the conception of the infinitely small, which latter has been not justly identified with naught, because for practical purposes the infinitely small is a negligible factor. Our correspondent, Mr. Lyle, is quite right that no amount of reasoning or suppression of reasoning can identify the infinitely small with zero, but many paradoxes are based upon this identification.

Our correspondent is the author of a brief manual entitled "The Euclidean or Common Sense Theory of Space," and presumably because he found it hard to have a hearing, being, as he himself states, "76 years young," dares *The Open Court* by assuming that it is a shut court to him, but we gladly give him space for his article because we believe that his views are typical of large numbers of thinkers who stand up for common sense even in the face of the learned authority of such original geniuses as Lobatchevsky, Bolyai, and their host of followers.