Efficient Operational Control of Conjunctive-Use Systems

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INTRODUCTION

Increasing demands on limited water resources are leading managers to consider innovative system designs and control methods. "Conjunctive Use"Ñthe coordinated management of groundwater and surface waterÑis an affordable and environmentally sound method for enhancing the reliability of water supply systems (Fisher et al. 1995; Lettenmaier and Burges 1979). However, the potential for using the subsurface as a natural storage facility has not been fully recognized, and most large water supply systems continue to depend exclusively on surface water supplies (van der Leeden et al. 1990). Managers of many systems view groundwater as providing only a back-up supply used only in times of shortage (Lettenmaier and Burges 1979).

A barrier to conjunctive-use methods is that it is not clear how best to operate conjunctive-use systems. Current understanding and practice of water management are the result of many years of experience in regulating water systems. Based on this experience, system managers have been able to develop efficient operating rules that minimize risk and expected costs of system operations. However, few conjunctive-use projects have been undertaken and existing literature on the management of conjunctive-use systems is limited. As a result, we do not have established guidelines to aid planners in the evaluation of conjunctive-use benefits.

This paper is an initial effort to identify methods and results that may help managers of conjunctive-use systems determine appropriate policies for system operations and project evaluation. We will attempt to answer three questions: (1) How should conjunctive-use systems be operated to regulate fluctuating and uncertain water supplies? (2) How should water be allocated in storage between surface and subsurface storage? (3) How should the value of proposed system designs be evaluated? This paper will demonstrate how agencies may determine the monetary value of system operations and plans. Consequently, managers can evaluate alternatives and assess their tradeoffs with greater objectivity.

POLICY IDENTIFICATION

When uncertain inputs affect control decisions, appropriate control decisions cannot be identified by a pre-determined control schedule. Instead, we make "real time" decisions, delaying as long as possible to gather information about streamflow and other stochastic inputs.

Real-time operation of reservoir systems involves identification of appropriate supply and allocation decisions. By developing mathematical models that simulate the structure and dynamics of reservoir systems, we may simulate management options conveniently, reducing the need to rely on a trial-and-error approach. Also, by applying optimization methods, we may efficiently identify the best options for control when a range of options exists.

Together, modeling and optimization are sometimes called "systems analysis." Systems analysis involves (1) the development of a mathematical model that simulates the essential elements of an actual system, (2) the development of a value model that quantifies management goals and system performance, and (3) an optimization procedure that identifies efficient management policies.

Unfortunately, optimization procedures have been limited in their ability to realistically solve many practical problems. Because of this limitation, many systems-analysis problems have been simplified excessively. Burges and Maknoon (1975) note that, "A feature of nearly all the literature is the assumption that one or several parameters or variables dominate the problem at hand." Consequently, many efforts to solve conjunctive-use problems have ignored important physical characteristics as well as important non-physical characteristics arising from economic and legal factors (Burges and Maknoon 1975).

This paper focuses on the effect that differences in transfer rates and storage capacity have on optimal control of systems containing both surface and subsurface storage. On one hand, surface storage can be filled and drained rapidly, while rates of aquifer recharge and pumping are limited. On the other hand, subsurface storage capacity considerably exceeds available surface storage in many watersheds (Buras 1963).

Because of these differences, efficient conjunctive-management practices should be different from management practices for either surface or subsurface storage used alone. Surface reservoirs have a greater ability to respond to needs during short-term severe droughts, while aquifers have a greater ability to meet water needs during long-term moderate droughts. If we are to operate systems efficiently and reliably, it is important to recognize the capabilities of conjunctive-use systems and to not view subsurface storage as only a back-up supply.

PROBLEM DESCRIPTION

The conjunctive-use problem uses a system model based on the system of the East Bay Municipal Utilities District (EBMUD) of California that supplies water to communities along the eastern edge of central and southern San Francisco Bay.

The EBMUD System

The EBMUD system serves the residential needs of approximately 1.2 million people, as well as the industrial, commercial, and institutional needs in the East Bay region of the San Francisco Bay area. About 95 percent of its water supply is from the Mokelumne River's 575-square mile watershed on the western slope of the Sierra Nevada Mountains.

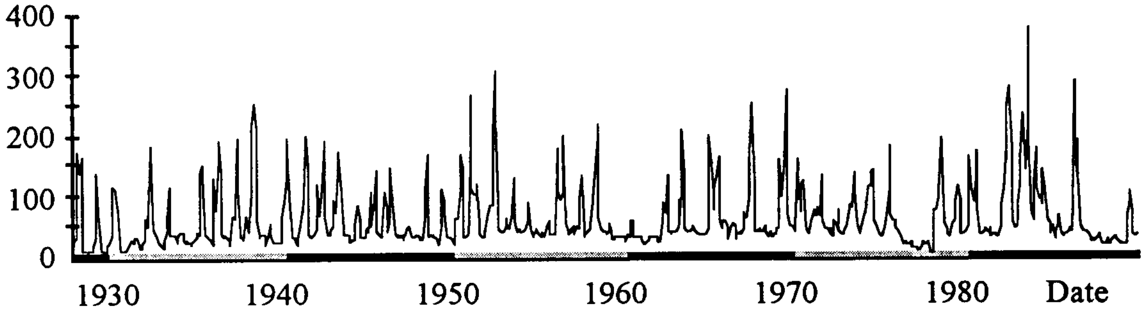
Mokelumne River streamflow supplies are seasonal and uncertain (Figure 1). Average monthly flow varies from a high of over 100 thousand acre-feet (TAF) in May to a low of about 30 TAF through the fall months. Quite typically, a year will have a month of high flow approaching 200 TAF and a month of low flow approaching 10 TAF. At times, the natural flow of the river has ceased. Flow averages 720 TAF annually but has varied between 130 TAF and 1,595 TAF.

Future streamflow can be predicted with some accuracy because (1) flow is highly seasonal, (2) monthly flow is highly autocorrelated, with a correlation coefficient of 0.8 to 0.9, and (3) much of the late spring and summer runoff is from melting snowpack that is measured throughout the winter. Therefore, consideration of the season, prior streamflow, and measurements of snowpack can contribute to streamflow prediction.

EBMUD manages two surface reservoirs having a combined capacity of 641 thousand acre-feet (TAF) on the Mokelumne River. Up to 200 TAF of this storage is reserved for flood control (under agreements with the U. S. Army Corps of Engineers), and an additional 21 TAF is dead storage. Besides water supply and flood control functions, these reservoirs also have a combined hydropower generating capacity of 39 megawatts.

An 82 mile aqueduct transports water to the service area for use or for storage in five terminal surface reservoirs. The terminal reservoirs provide an additional 155 TAF of storage capacity (with 17 TAF of dead storage). The aqueduct has a delivery capacity of 200 million gallons per day (mgd) by gravity flow or 325 mgd with pumping. This maximum delivery capacity coincides with EBMUD's permits to divert up to 325 mgd (364 TAF per year) for use in its service district. EBMUD must also manage the system to meet water rights of senior users in the Mokelumne basin. These requirements are currently on the order of 100 TAF annually, but these may significantly increase in the future.

Figure 1: Hydrograph of Mokelumne River monthly streamflow (1000's acre-feet)



Proposed Aquifer Storage

Due to increasing needs for water in the district and in the Mokelumne River basin, EBMUD has been considering a number of options to prevent deterioration of its water supply reliability. Among their options are proposals that would add subsurface storage and/or increase surface storage.

Accessible subsurface storage appears substantial. Both the Mokelumne River and EBMUD Aqueduct run west from the Sierras across large, extensive aquifers of the California Central Valley. The area is underlain by fresh-water bearing formations of thick sand and gravel totaling a few hundred feet in the east and increasing to almost two thousand feet near the Delta. Well capacities are frequently 500 to 1500 gallons per minute (gpm), with specific capacities of 35 to 60 gpm/ft and transmissivities of 60,000 to 80,000 gal/day/ft (DWR 1967). Also, there is a great amount of available space for aquifer storage: pumping for local agricultural and municipal needs has depressed the water table by an average 50 to 100 feet below virgin levels.

System Model

The conjunctive-use model is a simplified representation of the EBMUD system with an added aquifer-storage component (Figure 2). This model does not allow us to solve specific operating or planning problems for EBMUD, but it does illustrate some of the general characteristics of conjunctive-use control and planning.

Variables used to quantify the conjunctive-use model consist of control variables, $\mathbf{u}$, state variables, $\mathbf{x}$, and stochastic variables, $\mathbf{s}$. Control variables identify the decisions that we apply to regulate the system. These include decisions to supply water to meet demands and to allocate water between surface and subsurface storage. State variables define important information that helps identify appropriate control decisions. In this model, the state is defined by the amount of water currently stored separately in surface and subsurface reservoirs and by inflow observed in the previous month. Stochastic variables define uncontrollable inputs that influence system control. In this model, a stochastic variable is used to represent inflow. Table 1 summarizes the variables of the conjunctive-use model and bounds on these variables. Units are in thousands of acre-feet (TAF).

The state of the system evolves under the influence of control decisions and inflow. These dynamics are summarized by the linear transition equation:

$$\mathbf{x}_{t+1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \mathbf{x}_t + \begin{bmatrix} -1 & 1 & -1 & -1 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \mathbf{u}_t + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \mathbf{s}_t$$

This equation describes the change in surface and subsurface storage levels during any stage t. The ending surface storage is the sum of beginning storage, inflow, and pumping, minus water supplied to users, recharge, and release downstream. The ending subsurface storage is the sum of beginning storage and recharge minus pumping. In addition, the state of the system evolves subject to the bounds shown in Table 1 that constrain feasible decisions and attainable states.

Figure 2: Conjunctive-use system model

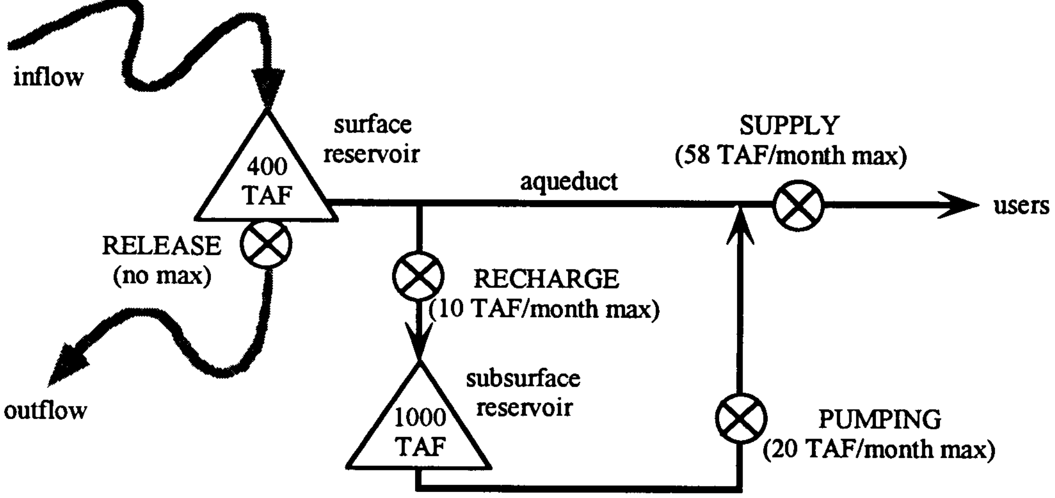


Table 1: Variables of the conjunctive-use model

Variable	Type	Definition	Min.	Max.
u_1	control	supplied water for use	0	58
u_2	control	subsurface pumping	0	20
u_3	control	subsurface recharge	0	10
u_4	control	release downstream	0	infinite
x_1	state	water in surface storage	0	400
x_2	state	water in subsurface storage	0	1000
x_3	state	prior stage inflow from streams	0	infinite
s_1	stochastic	inflow from streams	0	infinite

Streamflow Model

Streamflow is described by a discrete monthly model. Flow is autocorrelated, varies with the season, and has skewed distributions. This model does not include snowpack measurements that should be part of a practical model of the EBMUD system.

The distribution of streamflow is described by the seasonal 3-parameter model

$$s_\tau = \chi_\tau + \exp(\varepsilon\sigma_\tau + \mu_\tau)$$

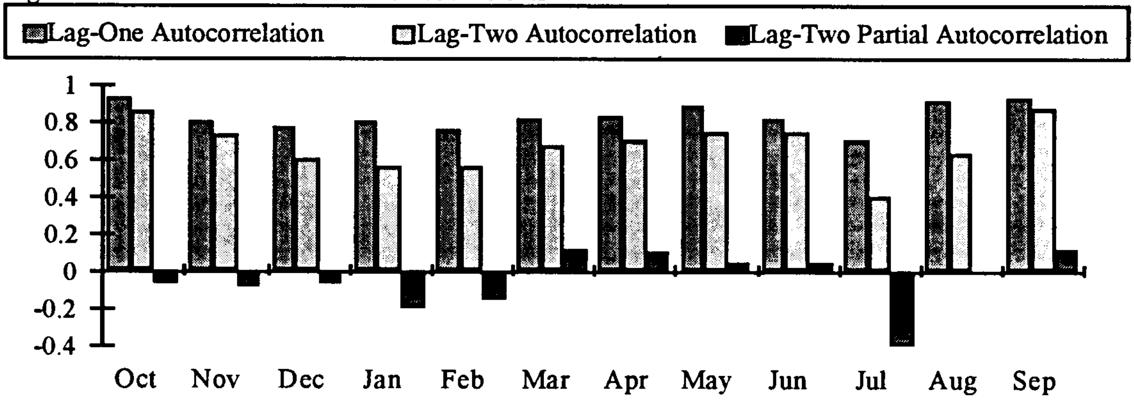
The month is identified by subscript τ. Parameter χ_τ shifts streamflow values to fit more closely a lognormal distribution modeled by log-mean μ_τ and log-standard deviation σ_τ. The stochastic variable ε has a standard-normal distribution.

Parameters are estimated from the historical data. The shift parameter, χ_τ, is estimated by

$$\chi_\tau \cong \frac{w_t(q)\, w_t(1\text{-}q) - w_t(0.5)^2}{w_t(q) + w_t(1\text{-}q) - 2w_t(0.5)}$$

as recommended by *Stedinger* (1980), where parameter $w_t(q)$ is the sample value for the q'th quantile. We follow the recommendation of *Stedinger*, choosing $w_t(q)$ and $w_t(1\text{-}q)$ to be the largest and smallest observed values. Correlation is modeled as first-order auto-regressive (Figure 3). Correlation coefficients are calculated for log-transformed flow values because this produces coefficients that are more significant than those calculated prior to transformation. Table 2 summarizes the parameters calculated for each month.

Figure 3: Streamflow autocorrelation coefficients



(partial autocorrelation is the correlation that remains after accounting for shorter lags)

Table 2: Streamflow parameters

Month (τ)	shift (χ)	log mean (μ)	log std. dev. (σ)	corr. coeff. (ρ)
Oct.	-600 *	6.448	.018	.928
Nov.	-6.806	3.617	.527	.812
Dec.	-1.106	3.627	.736	.776
Jan.	-3.512	3.824	.698	.807
Feb.	-3.029	3.875	.723	.763
Mar.	-10.101	4.227	.573	.821
Apr.	-24.363	4.569	.477	.824
May	-90.159	5.271	.377	.896
Jun.	-35.207	4.824	.537	.821
Jul.	-1 **	3.573	.612	.700
Aug.	-89.184	4.804	.094	.917
Sep.	-600 *	6.447	.016	.935

* distribution has negative skew, χ set to -600; ** positive value adjusted to -1

Value Model

Cost of system operations results from an inability to meet maximum demands ($u_1^{\max}$) of 58 TAF per month (600 TAF annually). Shortage cost is a quadratic function that produces a marginal cost of $1000 for the first acre-foot of rationing and a marginal cost of $9000 per acre-foot for complete disruption of supplies. Total cost in each stage is given by

$$C(\mathbf{u}) = (5 - 4\,u_1/u_1^{\max})\,(u_1^{\max} - u_1)$$

Cost is in millions of dollars and the supply decision is in thousands of acre-feet annually. This cost is standardized to produce a zero total cost for fully meeting demands.

Problem Solution

Our goal is to identify a control policy that minimizes the combined expected costs of water rationing and system operations. Total costs accumulated over a multi-year time horizon are

$$V = \sum_{t=t_1}^{t_N} C_t$$

for an N month horizon. If there were no stochastic inputs, we could identify the current and future best decisions as those that minimize total accumulated cost

$$F(\mathbf{x}_{t_0}) = \min_{\mathbf{u}_{(t_0)},\ldots,\mathbf{u}_{(t_N)}} \{ V(\mathbf{x}_{t_0},\mathbf{u}) \}$$

We could then identify a single control schedule, $[\mathbf{u}_{(t)},\ldots,\mathbf{u}_{(t_N)}]$ for initial state $\mathbf{x}_{t_0}$. However, the presence of stochastic inputs means that we do not know the future states of a system. Instead, we must identify a series of control policies, $\mathbf{U}$, for each stage

$$F(\mathbf{x}_{t_0}) = \min_{\mathbf{U}_t,\ldots,\mathbf{U}_{t_N}} \{ \mathrm{E}_{\mathbf{s}_{(t)},\ldots,\mathbf{s}_{(t_N)}} \{ V(\mathbf{x}_{t_0},\mathbf{u},\omega) \} \}$$

These policies provide control decisions for any possible state of the system.

To solve control policies of the conjunctive-use problem, we apply a modified gradient dynamic programming (GDP) method. GDP is a dynamic programming method developed by Foufoula-Georgiou and Kitanidis (1988) to solve dynamic control problems of greater complexity than previously possible. Results are presented for the first stage of a thirty-year time horizon.

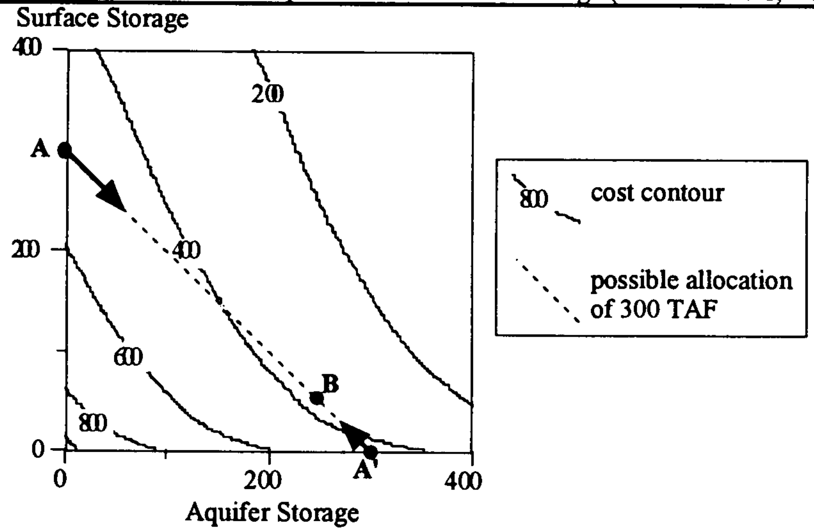
RESULTS

The solution of the simple conjunctive-use problem includes an expected-cost function and a set of functions that identify each control decision. These functions identify cost and controls for any initial storage levels and prior inflow.

Expected Cost Function

Increasing storage levels increase the systems ability to meet water demands and to save water for future use. Figure 4 plots the expected cost of system operations for the month of November when October's inflow was above average (60 TAF). As we can see, increasing shortage levels reduce the expected cost of shortages. Costs, in millions of dollars, are plotted against storage levels.

Figure 4: Contours of Expected cost of water shortage (million dollars, November with October inflow = 60 TAF).

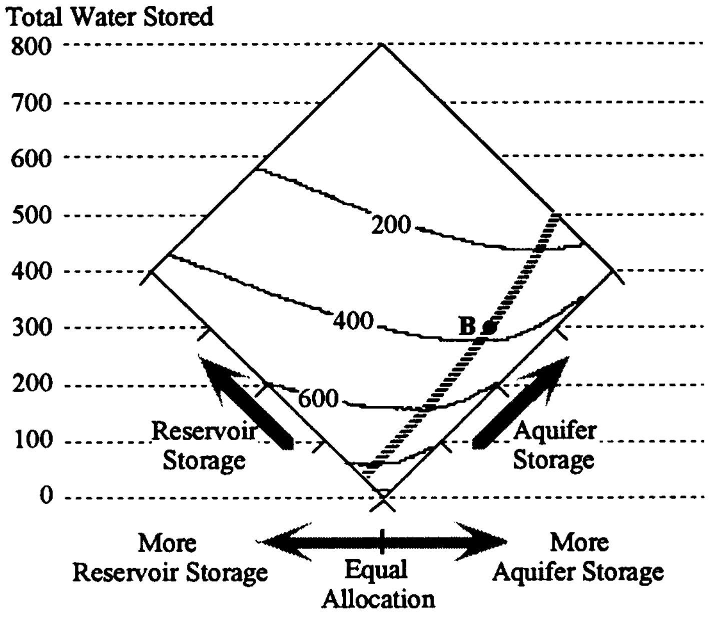


We can use the expected cost function to anticipate allocation decisions by identifying the least cost allocation of stored water. If the benefit of one additional unit of groundwater is greater than the benefit of one additional unit of water in the surface reservoir, we would prefer to recharge groundwater. For example, we can allocate 300 TAF anywhere along the line A--A' in Figure 4, identifying the minimum cost allocation at point B.

Figure 5 reorients Figure 4 to display more clearly the least cost allocation for any total amount of stored water. The least cost allocations occur at the trough in the cost contours.

We observe that the least cost allocation strongly favors storing water in the subsurface, except when storage levels are very low. We can understand this result by recognizing that these contours specify costs expected in November following a month of above average October streamflow. The least cost allocation favors groundwater because (1) the likelihood of near-term rationing is low at the beginning of the winter rainy season, and (2) the likelihood that the surface reservoir will spill is large if surface storage is high. Since the model assumes that current demand for water can be met only from storage and not from current streamflow, it is necessary to store a small amount of water in the surface reservoir to meet short-term demands.

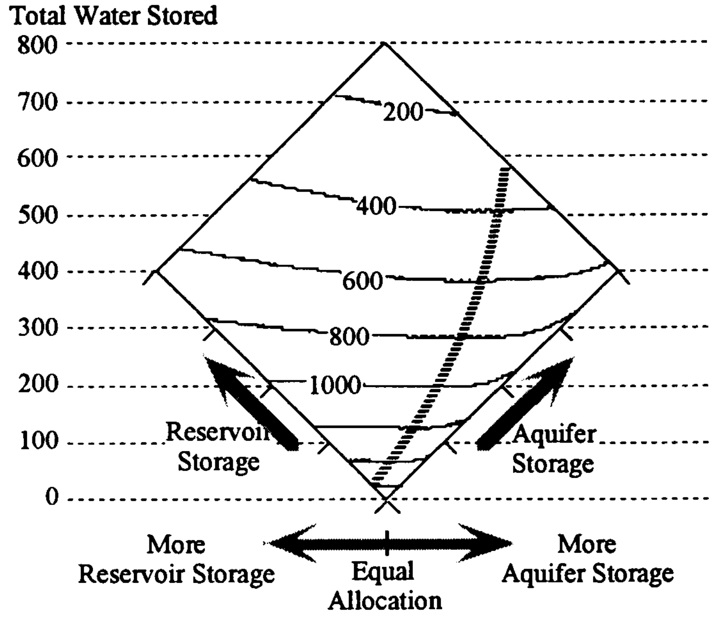
Figure 5: Contours of Expected cost of water shortage and least cost allocation of total stored water (million dollars, November with October inflow = 60 TAF).



Effect of Autocorrelation

When streamflow time series are autocorrelated, low streamflow produces expected future flows that are below average. As a result, below-average flow increases the likelihood of future water shortage. Figures 6 displays expected cost contours for November when October streamflow is 20 TAF instead of the 60 TAF shown in Figure 5. As a result, expected costs are greater for all storage levels.

Figure 6: Contours of expected cost of water shortage and least cost allocation of total stored water (million dollars, November with October inflow = 20 TAF).

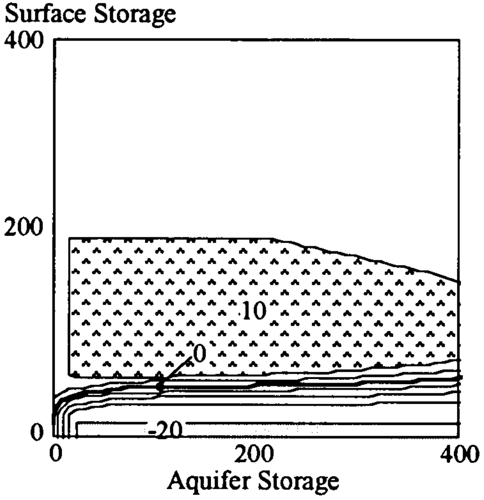


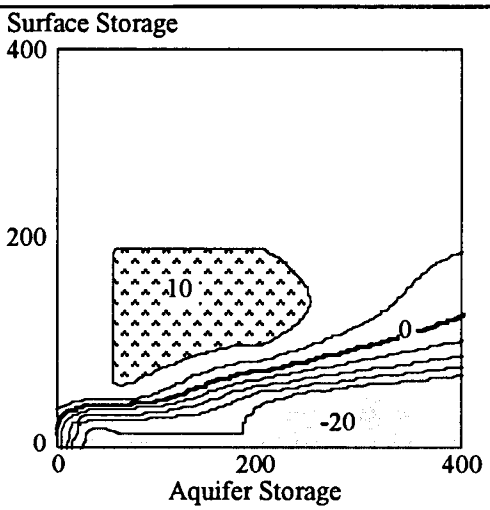
Changes in prior streamflow impact not only the expected cost function but also control decisions. We can observe how allocation decisions change by comparing the minimum cost allocations of Figure 5 and 6. As prior streamflow decreases, the likelihood of near-term water shortages increases. Figure 6 indicates that we have a lower preference for aquifer storage when prior inflow is low. The risk of near-term shortage is greater, and it is less likely that we will fill the surface reservoir. As a result, the minimum expected cost is less sensitive to allocation and the cost contours are flatter. As the risk of near-term shortage becomes high, it would be a bad policy to have all water stored in the aquifer because we can meet only about one-third of maximum demand through pumping.

Allocation Policy

Figures 7 and 8 display the allocation policy for November following above average and below average streamflow in October. The policy gives the net change in subsurface storage due to recharge (positive values) and pumping (negative values). Irregular contours are an artifact of the state-space grid used in the dynamic programming solution. Storage and decisions are in thousands of acre-feet.

Figure 7: Aquifer recharge and pumping (TAF for November with October inflow = 60 TAF), left.
Figure 8: Aquifer recharge and pumping (TAF for November with October inflow = 20 TAF), right.



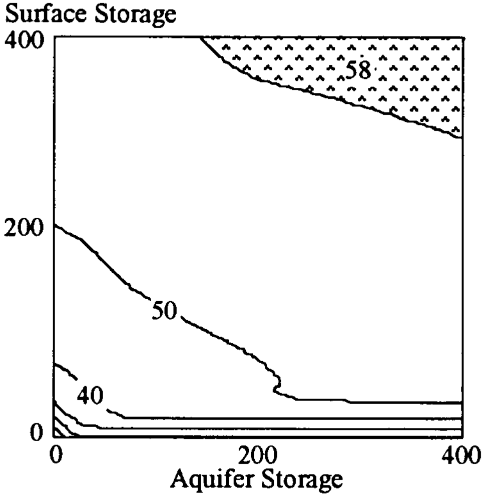


According to Figures 7 and 8, we should recharge groundwater when surface storage levels are not high or extremely low. Comparing Figure 7 and 8 to Figures 5 and 6, we see that the allocation decisions are appropriate to achieve the least-cost allocation (failure to recharge when surface storage is high may be the result of the minor pumping and recharge costs applied to the objective function to prevent simultaneous pumping and recharge).

Supply Policy

Figure 9 displays the supply decision for November when streamflow in October is below average.

Figure 9: Water supply (TAF for November with October inflow = 20 TAF).



The likelihood of shortage is greater when storage levels are low. As a result, there is a greater incentive to hedge when levels are low. Figure 9 shows that less water is released as storage levels in both reservoirs decrease. Even if sufficient water is available to meet all demands, we should ration to hedge against future shortages.

CONCLUSIONS

This paper has illustrated the optimal control of a simple conjunctive-use system with uncertain and autocorrelated inputs. The results demonstrate that differences in the characteristics of surface and subsurface storage influence optimal control.

These results demonstrate that optimal management of conjunctive-use systems takes advantage of the strengths of surface and subsurface storage, achieving greater system efficiency and reliability than alternate systems using either surface or subsurface storage in isolation. Water supply systems should be designed to employ the strengths of both mechanisms, resulting in greater reliability and lower cost than a system using either method in isolation. As Burges and Maknoon (1975) point out, "Whenever multiple sources of water with different characteristics, as is the case with groundwater and surface water systems, are available, it may be possible to develop an operating strategy which exploits the different characteristics of the sources." Willis and Yeh (1987) recognized that, "By controlling the total water resources of a region, conjunctive use planning can increase the efficiency, reliability, and cost-effectiveness of water use, particularly in river basins with spatial or temporal imbalances in water demands and natural supplies."

Systems analysis methods, such as presented here, can provide practical information to planners and managers of water supply systems. Managers can efficiently obtain operating rules. Planners can evaluate the impact of changes in system configuration on expected costs.

Initial efforts to employ aquifer storage may be much cheaper than expansion of surface storage. Because many of the largest water supply systems have relied entirely on surface storage (van der Leeden et al. 1990) , there may be many opportunities to use aquifer storage. Lettenmaier and Burges (1979) found that, for their model, developing aquifer storage to buffer against variations in streamflow was about an order of magnitude cheaper than developing surface storage.

The problem presented in this paper is an example of systems analysis applied to conjunctive-use systems. The example problem does not provide a practical solution as we use a simple model to focus more clearly on differences between surface and subsurface storage. Additional characteristics that are important include groundwater pumping and recharge costs, a more sophisticated streamflow model that incorporates snowpack measurements, and other objectives of system operation such as hydropower generation and, perhaps, flood control. Nevertheless, the results do provide insight into the operation of conjunctive-use systems, and the potential value of systems analysis in evaluating system operations and plans.

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