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Abstract: This paper analyses few schemes for combining base station antenna array signals in wireless DS/CDMA. The performances of equal gain combining (EGC), likelihood rank test (LRT) and a modified rank test (MRT) are evaluated using simulation studies. The results indicate that, under certain assumptions on multiple access interference statistics, the probability of error of MRT is lower than that of EGC, if a few high power interfering users are present along with a low power user of interest. If there are a moderately large number of users and if the received power of all the users are nearly the same, then EGC outperforms MRT. In fact, under this condition, the performance of EGC is close to that of the optimal likelihood ratio test.

I. Introduction

Direct sequence code division multiple access (DS/CDMA) is an alternative to frequency division or time division multiple access scheme based cellular networks. In [1], for the IS-95 cellular standard, an antenna array 2D non-coherent RAKE receiver with equal gain combining (EGC) as the decision rule was considered. Further details of this receiver can be found in [1].

The total received signal at the base station in the IS-95 mobile radio environment is given by [1]

$$x(t) = \sum_{i=1}^{N} \sum_{l=1}^{L_i} \rho_i \sqrt{P_i}\psi_i \Big[W^{(h)}(t-\tau_{l,i}) a_i^I\left(t-\tau_{l,i}\right) + jW^{(h)}(t-T_0-\tau_{l,i}) a_i^Q(t-T_0 -\tau_{l,i})\Big] \cdot \Big[\cos\left(\theta_{l,i}\right) + j\sin\left(\theta_{l,i}\right)\Big] \boldsymbol{a}_{l,i} + \boldsymbol{n}(t) \quad (1)$$

where N is the number of users in the system, L_i is the number of paths received from the i^{th} user, ρ_i models the effects of path loss and log-normal shadowing, P_i is the transmitted power per symbol, ψ_i is a Bernoulli random variable with probability of success v that models the voice activity of the user, $W^h(t)$ is the h^{th} orthogonal Walsh function, T_0 is the time offset between the I and Q channels, $\boldsymbol{a}_{l,i}$ is the S X 1 response vector of the cell site antenna array to signals in the l^{th} path from the i^{th} user, S denotes the number of elements in the array, $\tau_{l,i}$ is the time delay of the l^{th} multipath component and $\theta_{l,i} = \omega_c\tau_{l,i}$. ω_c is the carrier angular frequency. The product of the user pseudo noise (PN) code and the I or Q channel PN code is denoted as a_i^I and a_i^Q respectively. $\boldsymbol{n}(t)$ is the additive complex Gaussian noise vector with zero mean and covariance $\sigma_n^2\mathbf{I}\delta(t_1 - t_2)$, where σ_n^2 is the height of the power spectral density of $\boldsymbol{n}(t)$. T_w/T_c is the processing gain of the system where T_w is the symbol period and T_c is the chip period.

Suppose we are interested in the signal sent by the first user. Eqn. (1) can be rewritten as

$$x(t) = \sum_{l=1}^{L_1} \rho_1 \sqrt{P_1}\psi_1 \Big[W^{(h)}(t-\tau_{l,1}) a_i^I\left(t-\tau_{l,1}\right) + jW^{(h)}(t-T_0-\tau_{l,1}) a_1^Q\left(t-T_0-\tau_{l,1}\right)\Big] \cdot \Big[\cos\left(\theta_{l,1}\right) + j\sin\left(\theta_{l,1}\right)\Big] \boldsymbol{a}_{l,1} + \boldsymbol{m}(t) + \boldsymbol{n}(t) \quad (2)$$

where

$$\boldsymbol{m}(t) = \sum_{i=2}^{N} \sum_{l=1}^{L_i} \rho_i \sqrt{P_i}\psi_i \Big[W^{(h)}(t-\tau_{l,i}) a_i^I\left(t-\tau_{l,i}\right) + jW^{(h)}(t-T_0-\tau_{l,i}) \cdot a_i^Q\left(t-T_0-\tau_{l,i}\right)\Big]\Big[\cos\left(\theta_{l,i}\right) + j\sin\left(\theta_{l,i}\right)\Big] a_{l,i} \quad (3)$$

is the multiple access interference (MAI).

Block diagrams for the receiver structure are given in [1, Figure 2-Figure 4]. The receiver has a 2D-RAKE structure to track the multipath components in both time and space. A 2D-RAKE receiver is a conventional RAKE with a beamforming processor in the front-end [2]. Using the optimum beamforming weights, the output of the beamformer for the k^{th} multipath component of the 1st

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user (assumed as the desired user), assuming that the first Walsh symbol is transmitted, is given by [1]

$$\left|U_{k,1}^{(n)}\right|^2, \qquad n \in (1,2,\ldots,M) \tag{4}$$

where

$$U_{k,1}^{(n)} = 2A_1\sqrt{T_w}\,e^{j\theta_{k,1}}\boldsymbol{a}_{k,1} + \boldsymbol{m}_{k,1}^{(n)} + \boldsymbol{n}_{k,1}^{(n)}, \tag{5}$$

$A_i = \rho_i\sqrt{P_i}\psi_i$, $\boldsymbol{m}_{k,1}^{(n)}$ is the multiple access interference signal vector and $\boldsymbol{n}_{k,1}^{(n)}$ is due to the additive white Gaussian noise (AWGN). Expressions for $\boldsymbol{m}_{k,1}^{(n)}$ and $\boldsymbol{n}_{k,1}^{(n)}$ can be found in [1]. M equals 64 in the IS-95 CDMA standard. In [1], an equal gain combining of the path variables $\{\left|U_{k,1}^{(n)}\right|^2, k = 1,2,\ldots,L_1\}$ was carried out. In this paper we consider a new rank based algorithm for combining these variables. The paper is organized as follows. In section II, we present equal gain combining and the rank based diversity combining schemes. Section III provides some simulation studies that verify the results in [1], with regard to the assumption of MAI being Gaussian distributed for a large number of users. We have extended this study to situations involving a very small number of users. Section IV provides performance comparisons of the schemes discussed in section II. In this section we also provide probability of error results for likelihood ratio test (LRT), under the assumption of Gaussian MAI. We conclude this paper in section V.

II. Diversity Combining Schemes

In this section we describe two diversity combining algorithms. Assuming the number of paths received from each user to be the same (i.e., $L = L_i$, $i = 1,\ldots,N$), the received samples can be grouped into M groups of L samples each. Now, the signal detection problem can be visualized as arranging the ML samples in a matrix with M rows and L columns and then identifying the unique row of samples that corresponds to the transmitted signal of user 1.

For equal gain combining (EGC), the decision variables for the 1st user are given by

$$Z_1^{(n)} = \sum_{l=1}^{L} |U_{l,1}^{(n)}|^2 \qquad n \in (1,\ldots,M) \tag{6}$$

The EGC then decides l as the signal row where

$$l = \arg\max_{n\in\{1,\ldots M\}} Z_1^{(n)} \tag{7}$$

In [3], a specific rank order type of test, called the Modified Rank Test (MRT), was considered. First, a reduced rank matrix is created by ranking the elements in each column. Thus, the (i,j) element of the reduced rank matrix is given by $R_{ij} = k$ where k is the k^{th} rank of $|U_{j,1}^{(i)}|^2$ among $|U_{j,1}^{(i)}|^2, i = 1,\ldots M$, $(k \in 1,\ldots M)$. For the Modified Rank Test a value matrix is created where the (i,j) element of the value matrix is given by

$$V_{ij} = \begin{cases} R_{ij} & \text{if } R_{ij} \geq M - P + 1 \\ 0 & \text{Otherwise} \end{cases} \tag{8}$$

where P is an appropriate threshold of the MRT.

In MRT, the decision variables for the 1st user are given by

$$S_i = \sum_{j=1}^{L} V_{ij} \qquad i \in (1,\ldots,M) \tag{9}$$

The MRT decides l as the signal row where

$$l = \arg\max_{i\in\{1,\ldots M\}} S_i \tag{10}$$

Note that when P equals 1(M), the MRT reduces to Majority Logic Combining (MLC) (Reduced Rank Sum Test (RRST)), [3]. If the joint density of $\{U_{l,1}^{(n)}, l = 1,\ldots,L, n = 1,\ldots,M\}$ is known, it may be possible to implement a likelihood ratio test. The performance of LRT, when $\{U_{l,1}^{(n)}\}$ are jointly Gaussian, was evaluated in [4] by simulation studies.

III. MAI Model

It has been shown in [1] that for a large number of users ($N \approx 40$), the MAI signal vector $\boldsymbol{m}_{k,1}^{(n)}$ can be modeled as spatially white complex Gaussian random vector. We have also verified that the individual components of $\boldsymbol{m}_{k,1}^{(n)}$ are Gaussian (see Fig. 1 and Fig. 2). However, this model no longer holds good for a small number of users. In fact, we observe that the individual components of $\boldsymbol{m}_{k,1}^{(n)}$ can be approximated to have a Laplace distribution when there are only a few simultaneous users present in the system (Fig. 3 - 4 show results for $N = 5$). The joint densities of these components cannot be assessed easily. Instead we calculate the covariance matrix of the MAI vector $\hat{\mathbf{R}}_{uu,k,1}^{(n),N}$. Also, the Forbenius norm [5] of the error $\|\mathbf{e}^N\|_F = \|\hat{\mathbf{R}}_{uu,k,1}^{(n),N} - \mathbf{I}\|_F$ is calculated. A low value of $\|\mathbf{e}^N\|_F$ indicates that the random variables can be assumed to be statistically uncorrelated.

The covariance matrix of the MAI for $N = 40$ was found to be

$\text{Real}(\hat{\mathbf{R}}_{uu,k,1}^{(n),40})=$

$$\begin{bmatrix} 1.0004 & 0.0377 & -0.0086 & 0.0012 & 0.0093 \\ 0.0377 & 1.0261 & 0.0163 & -0.0190 & -0.0122 \\ -0.0086 & 0.0163 & 0.9844 & 0.0210 & 0.0014 \\ 0.0012 & -0.0044 & 0.0210 & 1.0032 & 0.0391 \\ 0.0093 & -0.0122 & 0.0014 & 0.0391 & 0.9859 \end{bmatrix}$$

$\text{Imag}(\hat{\mathbf{R}}_{uu,k,1}^{(n),40})=$

$$\begin{bmatrix} 0.0000 & -0.0045 & -0.0001 & -0.0010 & -0.0203 \\ 0.0045 & 0.0000 & 0.0017 & 0.0017 & 0.0061 \\ 0.0001 & -0.0017 & 0.0000 & -0.0019 & 0.0074 \\ 0.0010 & -0.0017 & 0.0019 & 0.0000 & -0.0045 \\ 0.0203 & -0.0061 & -0.0074 & 0.0045 & 0.0000 \end{bmatrix}$$

Also, the covariance matrix of the MAI for $N = 5$ was calculated to be

$\text{Real}(\hat{\mathbf{R}}_{uu,k,1}^{(n),5})=$

$$\begin{bmatrix} 1.0056 & 0.0209 & 0.0087 & 0.0179 & -0.0025 \\ 0.0209 & 0.9760 & 0.0105 & 0.0190 & -0.0028 \\ 0.0087 & 0.0105 & 1.0395 & 0.0279 & 0.0091 \\ 0.0179 & 0.0190 & 0.0279 & 0.9832 & 0.0260 \\ -0.0025 & -0.0028 & 0.0091 & 0.0260 & 0.9957 \end{bmatrix}$$

$\text{Imag}(\hat{\mathbf{R}}_{uu,k,1}^{(n),5})=$

$$\begin{bmatrix} 0.0000 & 0.0044 & -0.0030 & -0.0032 & -0.0360 \\ -0.0044 & 0.0000 & -0.0003 & -0.0034 & 0.0091 \\ 0.0030 & 0.0003 & 0.0000 & 0.0073 & 0.0013 \\ 0.0032 & 0.0034 & -0.0073 & 0.0000 & -0.0028 \\ 0.0360 & -0.0091 & -0.0013 & 0.0028 & 0.0000 \end{bmatrix}$$

The covariance matrix $\hat{\mathbf{R}}_{uu,k,1}^{(n),40}$ is similar to the one obtained in [1]. The Forbenius norm of the error when $N = 5$ ($\|\mathbf{e}^5\|_F$) is estimated to be 0.0112 which is slightly higher than the estimated value for $N = 40$ ($\|\mathbf{e}^{40}\|_F = 0.0050$). Although we have only verified the MAI vector that for N = 5, the components of the MAI vector are uncorrelated Laplace variables, we assume them to be statistically independent in the following discussion.

In reality, only very rarely the system will be servicing such a low number of simultaneous users. We can, however, assume that in a given time, a few number of users will have a high priority over the other large number of users. Such users' signals will have a relatively large power compared to other low priority users. Hence the MAI resulting from the high priority users can be modeled as Laplace, while the MAI due to the other users can be modeled as Gaussian. We studied the performances of the receivers mentioned above under this MAI assumption as well as their performances under the assumption of Gaussian MAI.

IV. Performance Comparison

We consider the case corresponding to low Doppler frequency and ideal power control [1], i.e. the signal to noise ratio is a fixed quantity and is given by $\gamma_s = \bar{\gamma} LS$. Here $\bar{\gamma}$ is the symbol energy to interference-plus-noise ratio per path per antenna and is given by

$$\bar{\gamma} = \frac{2A_1T_w}{T_c(\sigma_I^2 + \sigma_n^2)} \tag{11}$$

where

$$\sigma_I^2 = C\left[\sum_{i=2}^{N} vLE\{\rho_i^2 P_i\} + (L-1)E\{\rho_1^2 P_1\}\right]. \tag{12}$$

Here, the thermal noise power σ_n^2 is assumed to be equal to the desired user's signal power, i.e.,

$$\sigma_n^2 = T_c E\{\rho_1^2 P_1\}. \tag{13}$$

C is a constant equal to 2 for bandlimited channel which is assumed in our analysis. The performances of MRT and EGC are evaluated by finding the probability of bit error (P_b) in identifying the signal row. Assuming that the first Walsh symbol is transmitted, the probability of symbol error (P_M) for EGC is given by

$$P_M(\gamma_s) = P(Z_1^{(1)} < \max(Z_1^{(2)}, Z_1^{(3)}, .., Z_1^{(M)})) \tag{14}$$

The P_M for MRT is given by

$$P_M(\gamma_s) = P(S_1 < \max(S_2, S_3, .., S_M)) \tag{15}$$

The corresponding bit error probability (P_b) is given by

$$P_b(\gamma_s) = \frac{2^{J-1}}{2^J - 1} P_M(\gamma_s) \tag{16}$$

where $J = \log_2 M$ bits. In order to estimate the probability of errors given by (14) and (15), we compute the corresponding variables $\{Z_1^{(n)}\}$ and $\{S_n\}, n = 1,..,M$, by generating the random variables $\{U_{k,1}^{(n)}, k = 1,..,L,$ $n = 1,..,M\}$ using the appropriate IMSL [5] routines. Enough samples were simulated to obtain a confidence coefficient exceeding 0.95.

In [3], it was concluded that retaining a few rank values in the MRT can provide a reasonably good performance in several signal detection problems. In the present 64-ary detection scheme, MRT with P between 5 and 8 seems to give the best performance. Let the ratio of the signal power of a user with high priority to the signal power of a user with low priority be denoted as μ. For convenience, let ∂_i denote the ratio of the received power from the first path to the received power from the i^{th} path.

In Table 1, the P_b of user 1 (low power user) corresponding to ECG, RRST, MRT with $P = 6$ and MLC are given with the number of priority users N_h being 5, $N =$

6 and $\mu = 10$. The results are given for a voice activity factor (v) of 0.375 and a processing gain (T_w / T_c) of 256. The number of paths (L) are assumed to be three. Let R be the ratio of the P_b of EGC to the P_b of MRT with $P = 6$. It can be seen that when the path strengths of the 3 paths are equal, then $R = 3.095$. When the second and third path strengths are half the first path strength, $R = 6.199$, and when $\partial_2 = 0.7$ and $\partial_3 = 0.2$, then $R = 9.062$.

Table I
Probability of bit error of a low power user for different tests with $\mu = 10$, $L = 3$, $S = 3$, $N_h = 5$, $N = 6$

Path Strength	EGC	RRST	MRT ($P = 6$)	MLC	Ratio R
$\partial_2 = 1.0$, $\partial_3 = 1.0$	0.00238	0.00132	0.00076	0.0236	3.095
$\partial_2 = 0.5$, $\partial_3 = 0.5$	0.00055	0.00024	0.00008	0.0019	6.199
$\partial_2 = 0.7$, $\partial_3 = 0.2$	0.00063	0.00019	0.00007	0.0014	9.062

The above results indicate that when the path strengths are equal, the performances of MRT is slightly better than that of EGC. However, under varying path strengths, the MRT achieves significant performance gain over EGC. When the MAI is Gaussian, as happens with a moderate to large number of users of same power, the EGC outperforms MRT. In fact in such situations, the performance of EGC is close to that of the optimal likelihood ratio test (see Table II, III).

Table II
Bit Error Rate (BER) for $S = 1$, $N = 45$, $M = 64$, $\partial_l = 1$, $l \in (2,3,..,L)$

	Analytical/ Simulated BER	Simulated BER	
L	EGC	RRST	LRT
2	1.20×10^{-2}	2.44×10^{-2}	9.44×10^{-3}
4	3.45×10^{-2}	7.12×10^{-2}	3.08×10^{-2}

Table III
Bit Error Rate (BER) for $S = 3$, $L = 2$, $M = 64$, $\partial_2 = 1$

	Analytical / Simulated BER	Simulated BER	
N	EGC	RRST	LRT
132	9.54×10^{-3}	2.16×10^{-2}	8.22×10^{-3}
153	2.20×10^{-2}	3.82×10^{-2}	1.94×10^{-2}
192	5.53×10^{-2}	8.29×10^{-2}	5.13×10^{-2}

V. Conclusion

We considered the equal gain combining (EGC) and modified rank test (MRT) for combining antenna array signals in wireless DS/CDMA under a specific user environment. The environment considered consists of a few high power interfering users along with a low power user of interest. Simulation results show that, for a small number of users, the multiple access interference (MAI) can be modeled as a Laplace density. Also, results indicate that under this condition, the MRT does better than EGC for varying path strengths. Under the condition of a large number of simultaneous users having equal power, the EGC performs better than the MRT.

References

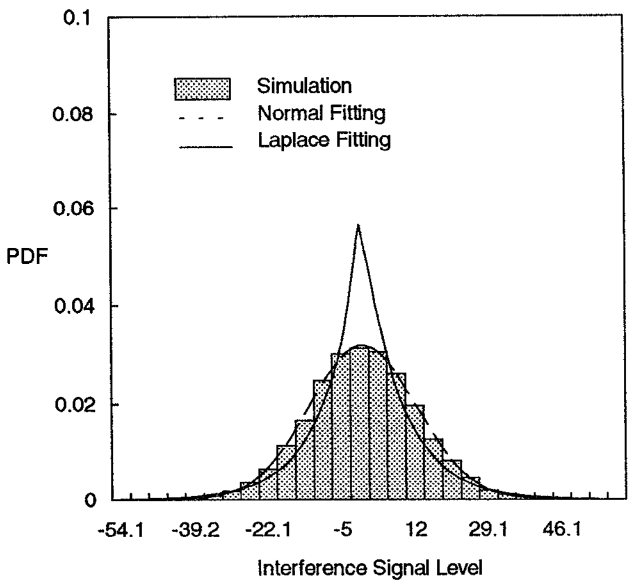
[1] A. F. Naguib and A. Paulraj, "Performance of DS/CDMA with M-ary Orthogonal Modulation Cell Site Antenna Arrays," *IEEE Journal on Selected Areas in Communications*, vol. 14, Dec. 1996, pp. 1770-1783.

[2] A. F. Naguib and A. Paulraj, "Performance of CDMA Cellular Networks with Base-Station Antenna Arrays," *Proc. International Zurich Seminar on Digital Communication*, March 1994, pp. 87-100.

[3] V. Annampedu and R. Viswanathan, "A Rank Order Test for M-ary Detection," *Proceedings of the Thirty First Annual Conference on Information Sciences and Systems*, John Hopkins University, Baltimore, March 1997, pp. 819-824.

[4] V. Annampedu, "Decision Combining in M-ary Communication Problems: A Rank Based Test," M.S. Thesis, Department of Electrical Engineering, Southern Illinois University at Carbondale, IL 62901, Feb. 1997.

[5] G. H. Golub and C. F. V. Loan, *Matrix Computations*, 2nd ed. Baltimore, MD: John Hopkins University Press, 1989.



ig. 1. I-Channel: first antenna interference distribution for $N = 40$.

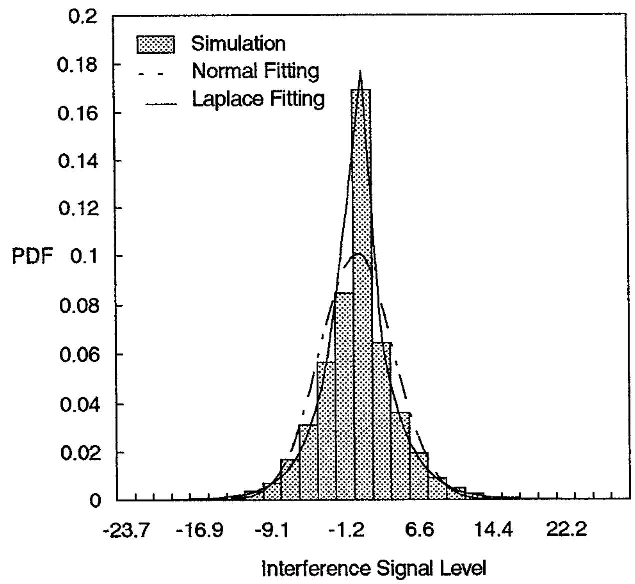


Fig. 3. I-Channel: first antenna interference distribution for $N = 5$.

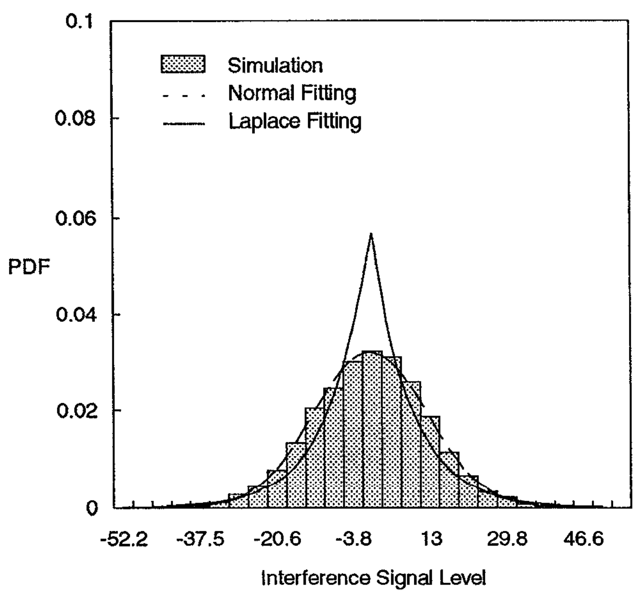


Fig. 2. Q-Channel: first antenna interference distribution for $N = 40$.

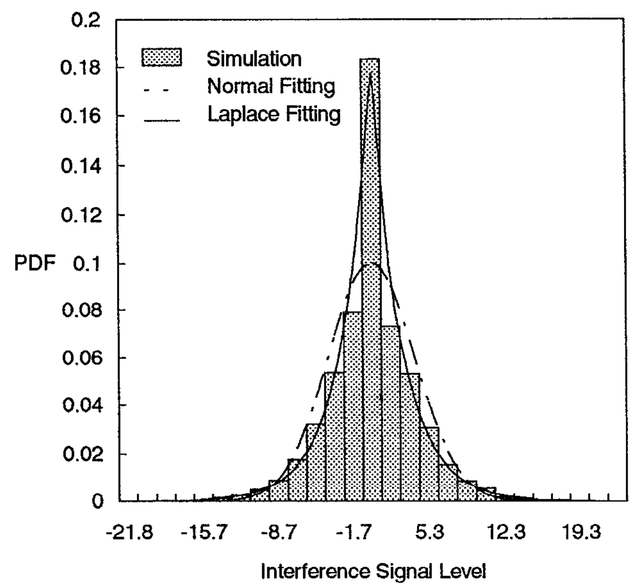


Fig. 4. Q-Channel: first interference distribution for $N = 5$.