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Abstract—**Paralleled dc converter systems are widely used in distribution systems and uninterruptable power supplies. This paper implements a hierarchical control in a droop-controlled dc-dc conversion system with special focus on improving system efficiency which is dealt within the tertiary regulation. As the efficiency of each converter changes with output power, virtual resistances (VRs) are set as decision variables for adjusting power sharing proportion among converters. It is noteworthy that apart from restoring the voltage deviation, secondary control plays an important role to stabilize dc bus voltage when implementing tertiary regulation. Moreover, system dynamic is affected when shifting VRs. Therefore, the stability is considered in optimization by constraining the eigenvalues arising from dynamic state space model of the system. Genetic algorithm is used in searching for global efficiency optimum while keeping stable operation. Simulation results are shown to demonstrate the effectiveness of the method.**

Keywords—dc-dc conversion system, droop control, virtual resistance, genetic algorithm, efficiency optimization, stability

I. INTRODUCTION

Direct current (dc) electricity distribution is generally accepted in applications where high efficiency, high reliability and simple control are required [1]-[5]. During last decades, parallel dc-dc conversion systems (see Fig. 1) have been widely used in various applications, as the paralleling of dc-dc converters gives many advantages. Some of the notable ones among them are enhanced flexibility, reduced thermal and electric stress, improved reliability and so forth [1].

However, current sharing among converters is a challenging issue. Up to date, several kinds of current sharing approaches were proposed, among which master-slave and droop are the two most popular methods [2]. Additionally, since droop control is a decentralized strategy which does not have a single point of failure and offers higher reliability and flexibility, it is preferred [4] in multi-converter systems.

Although the dc voltage droop control facilitates autonomous power sharing among paralleled converters, in its basic from it does not guarantee an efficient operation of the system. Converter efficiency is related with its operation point which finally influences the system losses [6]. Operation points for converters can be optimized so as to achieve higher system efficiency.
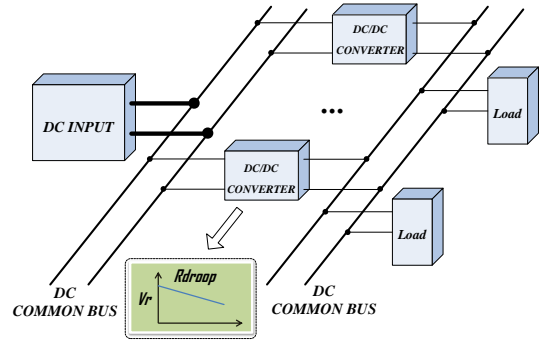

Fig. 1. Droop-controlled paralleled dc-dc converter system

However, stability issues may appear when droop parameters, also termed virtual resistances (VRs) [5], are altered [7]. As the bandwidth of optimization control is usually much lower than inner loops and droop controller, the optimization for adjusting VRs may be performed online.

In this paper, a droop-controlled buck converter based dc-dc conversion system is taken as an example. The structure of hierarchical control method is presented in section II, with droop, bus voltage restoration and optimization control being distinguished. In section III, the optimization problem is formulated by defining the objective function and respective constraints which include also system stability restrictions. Section IV proposes optimization algorithm and the testing process for parameter tuning. In section V, simulation results are presented and discussed. Finally, Section VI gives the conclusion.

II. HIERARCHICAL CONTROL FOR DC SYSTEM

Hierarchical control [5],[8] was proposed for economical and stable operation in microgrid. The three control levels, primary control, secondary control and tertiary control, are integrated together to achieve control requirements with different significances and time scales. Primary control enables power sharing among converters and defines system stability. Secondary control deals with power quality issues and controls voltage and frequency deviation, harmonics and unbalances. Tertiary control acts on set-points within the primary and secondary control and achieves optimal operation while taking into consideration both safety and economic.
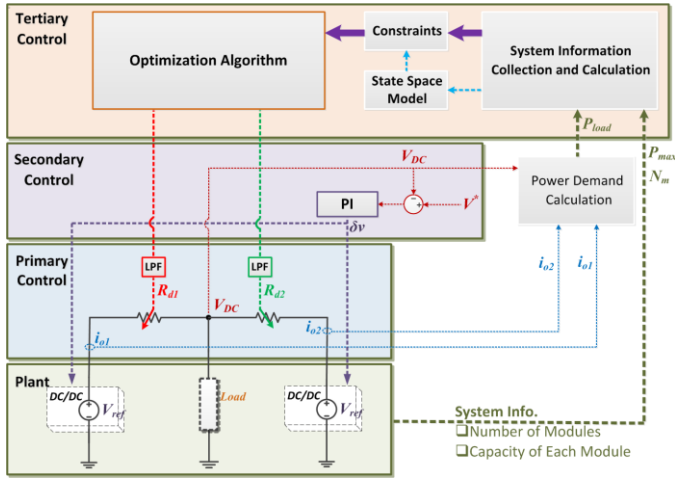
Fig. 2.   Hierarchical Control in DC System

The concept of hierarchical control can be mimicked into paralleled dc-dc converter system, as shown in Fig. 2. The plant block shows a simplified equivalent circuit of two dc supplies connected in parallel powering a common load bus. Droop controlled dc-dc converter acts as a voltage source in series with VR. In primary level, droop control method is implemented which include the VR expressed as follows:

$$v_{DC}^* = v_{ref} - R_d \cdot i_o \qquad (1)$$

where i_o is the output current, R_d is the VR, and v_{ref} is the output voltage reference at no load. Usually VR is fixed by the maximum allowed voltage deviation ε_v and maximum output current i_{max}:

$$R_d = \varepsilon_v / i_{max} \qquad (2)$$

Primary loop ensures power sharing and stable operation, however, according to (1), the voltage deviation is inherent and depends on load current. In order to solve this problem, secondary control is implemented. The dc bus voltage is sensed and compared with desired voltage v^*, with the difference being sent to a PI (Portional-Integral) controller to generate a compensating quantity δv for each converter reference:

$$\delta v = k_p (v^* - v_{DC}) + k_i \int (v^* - v_{DC}) dt \qquad (3)$$

$$v_{DC}^* = v_{ref} + \delta v - R_d \cdot i_o \qquad (4)$$

Ultimately, tertiary level receives system data including the number of operation modules, the conversion capability of each module and load demand calculation. Received information is processed to adjust VR of each converter so as to achieve better performance. Also, a low pass filter (LPF) is implemented between tertiary regulation and primary droop to smooth the shifting of VR.

It is noteworthy that secondary is important when considering higher level controls. Because of the low bandwidth of tertiary level regulation, without secondary control it may not be able to fast restore voltage deviation caused by droop control and stochastic load changing. In this sense, secondary control provides significant support to stabilize bus voltage. Low pass filters with different time constant are essential for differentiate regulation speed of secondary and tertiary control.

III.   OPTIMIZATION PROBLEM FORMULATION

Although modern power electronic system provides high efficiency conversion, losses are inevitable. The minimization of losses is always pursued. In a paralleling system, total losses are mostly related with conversion loss which is caused by switching, driver and filter parasitic elements in each converter. Paralleled converters normally have different efficiency curves due to different configurations and parameters. Even if constant input and output voltages are assumed, converter efficiency changes with load current, as shown in Fig. 3 [9][10]. As the highest efficiency is usually reached between 30% to 60% load, there exists a room for optimization, which is to find the power sharing proportion where the losses of the system are minimum.

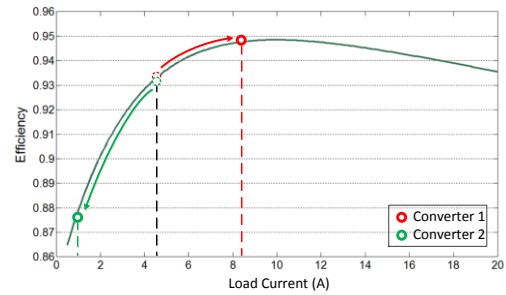#### A.   Converter Efficiency and Objective Function

Fig. 3.   Typical Converter Efficiency Curve

A typical efficiency curve extracted from experimental setup is shown in Fig. 3. Matlab Curve Fitting Tool is used to transform the experimental data into function:

$$\eta(i) = 0.975 \cdot e^{-2 \times 10^{-3} \cdot i} - 0.1257 \cdot e^{-0.3 \cdot i} \qquad (5)$$

where η is converter efficiency and i is converter output current. Then the power conversion losses of a system with n paralleled converters may be calculated as follows:

$$P_{cvr\_loss} = \sum_{j=1}^{n} V_{DC} \cdot I_j \cdot \frac{1 - \eta_j}{\eta_j} \qquad (6)$$

where V_{DC} is dc bus voltage, I_j is the output current of jth converter and η_j is the efficiency of jth converter. Minimization of system total conversion losses, P_{cvr_loss}, is taken as the objective in the following optimization problem.

Assuming two converters with the same efficiency curve as shown in Fig. 3, the general approach for enhancing system efficiency is to use only one converter in low and medium power conditions instead of equal sharing load power.

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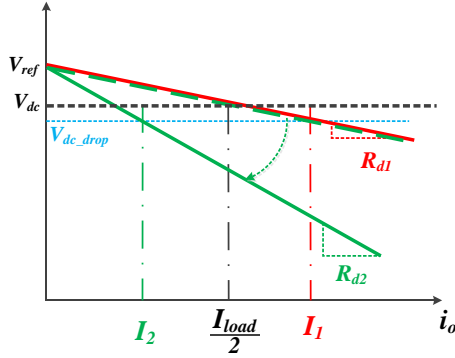## B. Effect of Droop Shifting and Decision Variable

Fig. 4. Sharing Proportion Adjusting by Droop Shifting

In order to change the current sharing proportion, a droop shifting method is proposed, as shown in Fig. 4. Two converters are given the same reference voltage v_{ref}. Originally, the two converters are sharing the load current equally ($I_1=I_2=I_{load}/2$). If the VR of one of the converters is shifted to another value (see green line in Fig. 4), the sharing proportion is changed. Then, from Eq. (1) one can get:

$$I_j = \frac{V_{ref} - V_{DC}}{R_{dj}} \qquad (7)$$

where I_j and R_{dj} is the jth converter output current and VR. In a 2-converter system, the load sharing ratio is:

$$\frac{I_1}{I_2} = \frac{R_{d2}}{R_{d1}} \qquad (8)$$

Accordingly, the optimization is to find an optimal proportion of load current sharing by changing VR. However, the VR shifting certainly has influence on dc bus voltage deviation and system dynamics. Also the power conversion capability of each converter should be taken into consideration.

C. Steady State Constraints

Although dc bus voltage is kept in 48V with secondary control, bus voltage constraint is still needed for limiting droop shifting region. If resistive load is considered, the dc bus voltage can be calculated as:

$$V_{DC} = \frac{V_{ref} \cdot R_{load}}{R_{load} + R_D} \qquad (9)$$

$$R_D = \frac{1}{\sum_{j}^{n} \frac{1}{R_{dj}}} \qquad (10)$$

where R_D is the equivalent value of total VR, R_{load} is the equivalent load resistance. Output current of each converter is also limited.

D. System Dynamic Constraints

The dynamic model of a paralleled buck converter system (2 modules) is shown in Fig. 5. Voltage and current loops can be accomplished by conventional PI controllers together with output *LC* filter. VR appears as a proportional current feedback (R_{d1} and R_{d2}) over inner control loops.
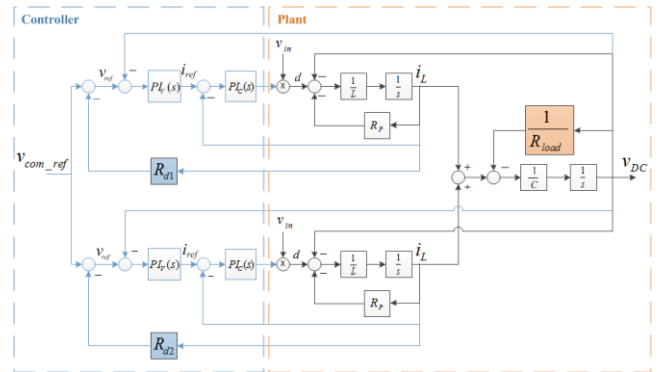

Fig. 5. Dynamic model of a 2-converter system

Based on Fig. 5, the system can be described by the following dynamic model:

$$
\begin{aligned}
&\textit{Plant Part}:
\begin{cases}
L_{(k)} \dfrac{di_{L(k)}}{dt} = v_{in(k)} \cdot d_{(k)} - v_{DC} \\[2mm]
C \dfrac{dv_{DC}}{dt} = \sum_{k=1}^{N} i_{L(k)} - \dfrac{1}{R_{load}} \cdot v_{DC}
\end{cases} \\[6mm]
&\textit{Controller Part}:
\begin{cases}
d_{(k)} = (\dfrac{I_{c(k)}}{s} + P_{c(k)}) \cdot (i_{ref(k)} - i_{L(k)}) \\[2mm]
i_{ref(k)} = (\dfrac{I_{v(k)}}{s} + P_{v(k)}) \cdot (v_{ref(k)} - v_{DC}) \\[2mm]
v_{ref(k)} = v_{com\_ref} - R_{d(k)} \cdot i_{L(k)}
\end{cases}
\end{aligned}
\qquad (11)
$$

where subscript k denotes the kth converter parameters, N is the total number of converters, L and C are inductance and capacitance of the converter output filter, R_{load} is the equivalent resistance of the connected load, v_{in} is the source voltage, d is the duty ratio, i_L and v_{DC} are the converter inductor current and capacitor voltage respectively. P_v, P_c, I_v and I_c are the control parameters of voltage and current loop PI controllers, i_{ref} and v_{ref} are the reference for current and voltage loops, v_{com_ref} is the common voltage reference for all the converters.
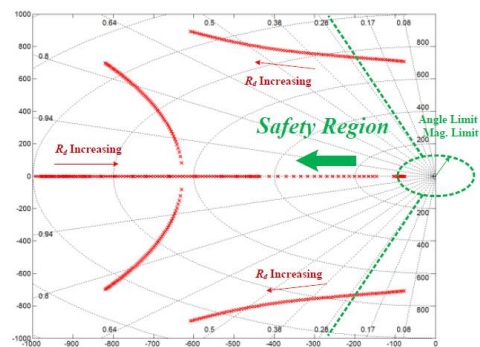

Fig. 6. Root locus analysis for system stabiltiy

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In order to analyze a general multi-module system, Eq. (11) are transferred into a complete state space model:

$$\dot{x}_s = A_s \cdot x_s + B_s \cdot u \tag{12}$$

where x_s is the complete state vector of dimension $[(3N+1)*1]$, u is the input vector with dimension $[(N+1)*1]$, matrices A_s is of dimension $[(3N+1)*(3N+1)]$, while B_s matrix is of dimension $[(3N+1)*(N+1)]$. A_s is also named state matrix whose eigenvalues represent system dynamics.

Based on the system state space model, root locus may be used to examine the system dynamics. Root locus for a 2-converter system where, VRs from 0.1 to 1, is shown in Fig. 6. With all the eigenvalues located in the left-half plane (negative real part), the system is stable. However, the eigenvalues may be too near to the unstable region. Accordingly, in order to ensure that system operates with acceptable dynamic properties, a safety region is outlined with limitation on apparant value and angle of each eigenvalue (see green dashed line in Fig. 6).

E. Optimization Problem Formulation

Based on the analysis above, the optimization problem can be described as:

Objective Function: $Min\{P_{cvr\_loss}\}$ (13)

Decision Variables: $\{R_{d1}, R_{d2}, ..., R_{dn}\}$ (14)

Subject to:
$$\begin{cases} V_{DC\_MIN} \leq V_{DC} \leq V_{DC\_MAX} \\ \{I_1, I_2, ..., I_n\} \leq I_{MAX} \\ Re(\lambda_{s1}, \lambda_{s2}, ..., \lambda_{sm}) \leq 0 \\ Mag(\lambda_{s1}, \lambda_{s2}, ..., \lambda_{sm}) \geq Mag_{MIN} \\ ABS[Angle(\lambda_{s1}, \lambda_{s2}, ..., \lambda_{sm})] \geq Angle_{MIN} \end{cases} \tag{15}$$

where P_{cvr_loss} is the total power conversion loss, R_d is the VR of each converter, V_{DC_MIN} and V_{DC_MAX} are allowed bus voltage deviation, I_{MAX} is maximal conversion current limit of each converter, λ_{si} is the eigenvalue of state matrix A_s, Mag_{MIN} and $Angle_{MIN}$ are allowed apparant value and angle value of eigenvalue. It should be noted that although with secondary control dc bus voltage is stabilized to reference value, dc bus voltage constraints are still needed in optimization. If V_{DC} is not constrained, optimization will always go to low voltage which results in impractical solutions.

IV. OPTIMIZAITON ALGORITHM

For solving the optimization model formulated in Section IV, a proper algorithm should be implemented. The selection of algorithm is based on the analysis of objective function. Global and local optimization methods are taken into option. The fastest optimization algorithms only seek local optimum point which is called local optimization, such as simplex method and gradient based algorithms. However, local optimization does not guarantee global optimal solution. On the other hand, global optimization algorithms, such as genetic algorithm (GA) and Particle Swarm Optimization (PSO), are able to find global optimum. However, they may require more computational time and space. Consequently, preliminary tests are necessary for improving algorithm efficiency.

A. Objective Function Analysis

According to Eq.(5)-(10), in a system with two droop-controlled buck converters, under certain load resistance R_{load} and reference voltage V_{ref}, objective P_{cvr_loss} can be plotted with respect to VRs (R_{d1}, R_{d2}), as shown in Fig. 7 (a)-(c). In high load condition (c), the objective function is convex and may be solved by local optimization methods. However, in low and medium load conditions (a) and (b), by using local optimization, the final solution depends on the initial point as shown in Fig. 7 (d). In Fig. 7 (d), the red dashed line is dc bus voltage constraint and blue dashed lines show the ridge between two minimum sides. Different solutions are obtained with different initial points (red and blue points). Local optimization is not capable of climbing over the 'ridge'. Consequently, global optimization methods are preferred in this problem. In this paper, genetic algorithm solver in Matlab is used to solve the nonlinear-constrained continuous-variable optimization problem formulated in Section III.

(a) Objective Function under R_{load} = 10 Ohm

(b) Objective Function under R_{load} = 4.4 Ohm

(c) Objective Function under R_{load} = 2 Ohm

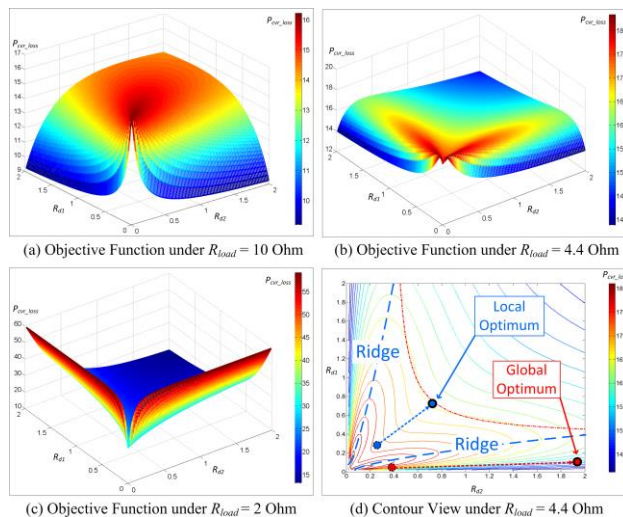(d) Contour View under R_{load} = 4.4 Ohm

Fig. 7. Droop-controlled paralleled dc-dc converter system

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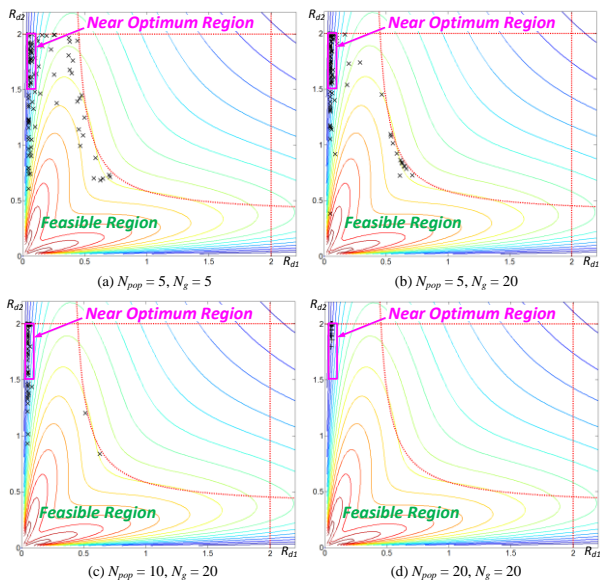## B. Genetic Algorithm Settings and Testing

Fig. 8.   GA parameters tuning

The basic parameters of GA significantly influence the performance of the program [11][12]. For different sorts of problems, good parameter settings of GA can be significantly different. When selecting parameters, such as population size (N_{pop}) and maximum number of generations (N_g), there is usually a tradeoff between computational time and quality of final solutions. In addition, as these parameters cannot be treated separately, a rational matching is also important.

In this paper, crossover rate is set to 0.8 (default setting), N_{pop} and N_g are tuned to achieve better performance. Case R_{load}=4.4Ω is used to adjust parameters because of the representativeness under this load condition, the algorithm is conducted 100 times to gather the final solutions (see Fig. 8). In order to use the least computational time while ensuring acceptable quality of final solutions, the tuning process starts from N_{pop} =5, N_g =5(see Fig. 8 (a)).With this setting algorithm is not able to always put solutions into near-optimum region. To improve the performance, both N_{pop} and N_g are increased from 5 to 20 (see Fig. 8 (a)-(d)). Final settings (N_{pop} =20, N_g =20) are able to enforce the objective function to converge to a near-optimum region.

V.   SIMULATION RESULTS

Simulation is conducted in Simulink to validate the method. The parameters of the study system configuration are shown in TABLE I. Conventionally VR (R_{d_fix}) is set according to Eq. (2). Mag_{MIN} and $Angle_{MIN}$ values are set according to root locus analysis results under 90% load condition with fixed droop. Optimized results are compared with fixed droop system.

In simulation, two converters in a system are assumed to have same efficiency curve (as shown in Fig. 3). As the total load current is changed from 2.5A to 20A, the efficiency of optimized system and non-optimized system are compared and the results are shown in Fig. 9.

As can be seen from Fig. 9, in low load current conditions (load current less than 12.5A), the efficiency improvement is impressive, while in high load conditions, the system efficiency can hardly be improved especially when converters have same efficiency features. This result is in accordance with Fig. 3 showing that when total load current is around 12.5A, there is a

little difference in system efficiency of when employing single converter or making equal sharing of two converters.

TABLE I.   SIMULATION STUDY SYSTEM PARAMETER SETTINGS

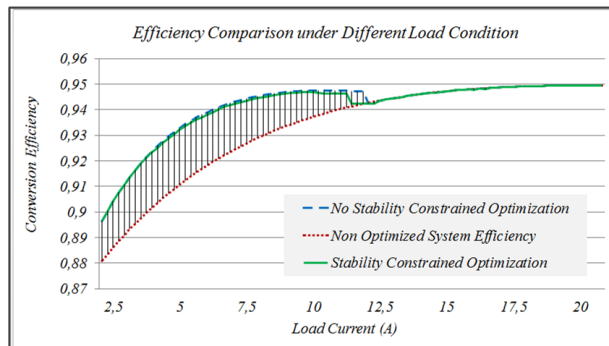| DC Bus	Converter (Average Model)			Stability	
Voltage Range	Type	Current (I_{MAX})	Fixed Droop (R_{d_fix})	Mag_{MIN}	$Angle_{MIN}$
48 ± 2.4V (±5%)	100/48V buck	20A	0.24Ω	90	1.85rad

Fig. 9.   Efficiency Improvement

An example case (load current = 11A) is shown in Fig. 10 to illustrate the effect of stability constraints. Fig. 10 (a) indicates the power loss sacrifice of stability constrained optimization while Fig. 10 (b) shows that the system stability is ensured by constraining eigenvalues of the system state matrix.

Finally, a load profile is given to conversion system to validate tertiary optimization (see Fig. 11). Load is changed every 2 seconds while optimization is executed every 2 seconds. With secondary control, bus voltage is stabilized at 48V. For low load power conditions, system efficiency is improved as expected and system stability is ensured. Also can be seen from converter output current curves, instead of equally sharing load current, the optimization tends to change the current sharing ratio between the two converters.

The above simulation results indicate the feasibility of efficiency improvement for paralleled power conversion system. With increase in the number of converters, the room for optimization becomes larger. In addition, in actual system, converter efficiency cannot be exactly the same and more enhancements on system efficiency may be expected.
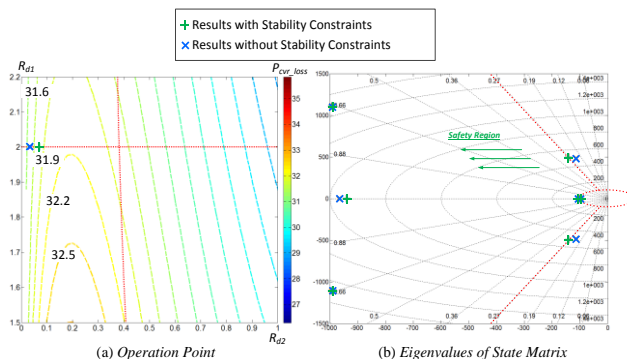

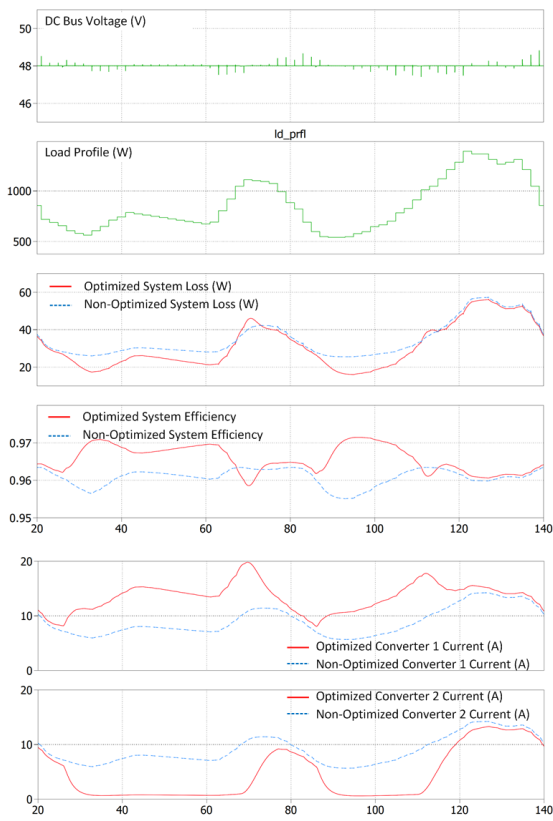Fig. 10. Example case for comparison on stability constraints

Fig. 11. Continuous load change test

Simulations are conducted in a system of two converters with same efficiency. The results indicate the potential of efficiency improvement for parallel converter system. Also the method proposed is demonstrated to be capable of improving system efficiency while keeping stable operation.

VI. CONCLUSION

In this paper, an optimization procedure for paralleled power converter system is proposed. The objective is to improve the system efficiency. Usually load current is equally shared by converters in a conversion system which causes low system efficiency especially in low and medium load conditions. This paper proposes a VR shifting method to adjust converter operation points. Hierarchical control conception is adopted so that droop method is employed on top of primary control level, secondary control takes charge of voltage deviation restoration while smoothing higher level regulation, GA is implemented in tertiary level for VR optimal shifting so as to improve system efficiency. GA parameters are tuned to achieve better processing. In order to ensure that the droop shifting does not affect system stability, small signal analysis is used for establishing stability constraints for optimization.

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