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ABSTRACT Concatenated systems combining the error correcting power of trellis codes and Reed-Solomon (RS) codes have been used to achieve error rates between 10^{-10} and 10^{-20} on satelite communications links [1]. The need for spectral efficiency has motivated consideration of of non-binary signalling with Ungerboeck [2,3] codes or the Pragmatic standard recently proposed by Viterbi [4], in place of quaternary signalling with rate 1/2 coding. In this paper we investigates the use of 8-PSK trellis-coded modulation (TCM) in a concatenated system using a (255,233) RS code. The use of the 8-PSK codes doubles the spectral efficiency with a performance cost of approximately 2 dB.

1. Introduction

A concatenated system is shown in figure 1. User data is first encoded by the RS Block encoder. The RS code symbols are then interleaved and convolutionally encoded. At the receiving end of the channel, modulated signals are decoded using the Viterbi algorithm, preferably with soft decisions. The output from the Viterbi decoder is then deinterleaved and RS decoded, recovering the user data. The RS code block consists of 255 symbols, a symbol consisting of 8-bits. An error in one or more of the eight bits of a given symbol results in a symbol error. The RS block code can correct up to 11 symbol errors.

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The purpose of the interleaver is to aleviate the effect of burst noise during transmission. Given the structure of the RS code, we can see what the proper operation of the interleaver should be. A single bit error in RS code symbol counts as a symbol error and additional bit errors in the same symbol cause no further damage. An interleaver-deinterleaver operation which redistributes bit errors among multiple symbols in the same block will degrade, not improve, the performance of the system. Therefore, the interleaver should be designed to reduce the correlation between *symbol* errors, and to redistribute clustered symbol errors among several RS blocks. This fact will be important in the forgoing analysis.

Needless to say, it is not practical to measure bit error rates between 10^{-10} and 10^{-20} by computer simulation. When the concatenated system is operating in this range, the Viterbi decoder will be operating at a bit error rate between 10^{-3} and 10^{-5}. Therefore, it is feasible to characterize the Viterbi decoder output using simulations, and then algebraicly determine the resulting probability of error for the RS code. Such an approach has been used to predict the performance of concatenated systems with binary signalling [5], here we extend the technique to concatenated systems using TCM.

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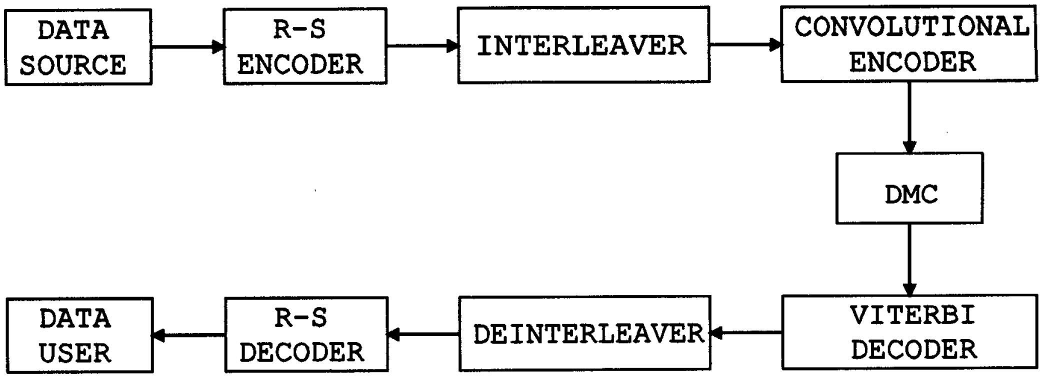

Figure 1. Concatenated Code System

2. Characterization of the Viterbi Decoder Output

The approach used in [5] suggests that the output of the Viterbi decoder be modelled as a two-state first-order Markov process. The process, illustrated by the state diagram of figure 2, consists of a burst state, in which the probability of error is very high, and a clear state, in which the probability of error is very low. The process is characterized by the probabilities of transition from either state to the other. The transitional probabilities reflect the intercorrelation between errors in the Viterbi decoder output.

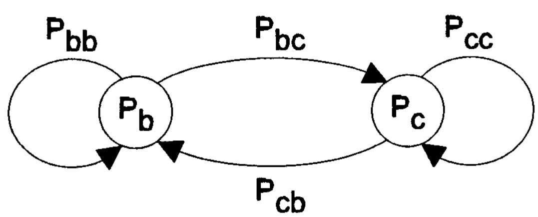

Figure 2. State Diagram for Markov Process Model

The Markov process is completely specified by four parameters:

P_{1b} = *probability of error when in burst state* (1)

P_{1c} = *probability of error when in clear state* (2)

P_{bc} = *probability of transition from burst state to clear state* (3)

P_{cb} = *probability of transitionfrom clear state to burst state* (4)

From these four, additional useful parameters may be derived:

$P_{0b} = 1-P_{1b}$, $P_{0c} = 1-P_{1c}$,
$P_{bb} = 1-P_{bc}$, $P_{cc} = 1-P_{cb}$,

$P_b = \frac{P_{cb}}{P_{cb}+P_{bc}}$, *the probability that the process will be in the burst state at any given time* (5)

$P_c = \frac{P_{bc}}{P_{cb}+P_{bc}}$, *the probability that the process will be in the clear state at any given time.* (6)

To determine these parameters, which depend on channel signal-to-noise ratio, it is necessary to observe statistics of bit error patterns in the Viterbi decoder output. We define the following observable characteristics:

P_1 = *the probability of a single bit error* (7)

P_{11} = *the probability of two consecutive bit errors* (8)

P_{111} = *the probability of three consecutive bit errors* (9)

P_{101} = *the probability of a bit-error, a non-bit-error, and a bit-error, in succession* (10)

The observable characteristics are algebraicly related to the process parameters as follows:

$$P_1 = P_b P_{1b} + P_c P_{1c} \quad (11)$$

$$P_{11} = P_b P_{1b} P_{bb} P_{1b} + P_b P_{1b} P_{bc} P_{1c} + P_c P_{1c} P_{cb} P_{1b} + P_c P_{1c} P_{cc} P_{1c} \quad (12)$$

$$P_{111} = P_b P_{1b} P_{bb} P_{1b} P_{bb} P_{1b} + P_b P_{1b} P_{bc} P_{1c} P_{cb} P_{1b} + \ldots + P_c P_{1c} P_{cc} P_{1c} P_{cc} P_{1c} \quad (13)$$

$$P_{101} = P_b P_{1b} P_{bb} P_{0b} P_{bb} P_{1b} + P_b P_{1b} P_{bc} P_{0c} P_{cb} P_{1b} + \ldots + P_c P_{1c} P_{cc} P_{0c} P_{cc} P_{1c} \quad (14)$$

The expressions for P_{111} and P_{101} are similar to equations (11) and (12), except that they consist of eight terms of six factors each.

If the two-state process is a valid model of the Viterbi decoder output, then the expressions for P_1, P_{11}, P_{111} and P_{101} (11 through 14) are four equations from which the four unknowns P_{1b}, P_{1c}, P_{bc}, and P_{cb} may be determined. The system is nonlinear, and there will in fact be two solutions, one of which exactly reverses the roles of the burst state and the clear state. Also, if $P_{bc} = P_{cc}$ and $P_{cb} = P_{bb}$, or if $P_{1b} = P_{1c}$, the model will generate a process in which bit errors are independent, and the solution will not be unique.

Given a set of numbers P_1, P_{11}, P_{111} and P_{101} not actually generated by the two-state process, a real solution may or may not exist. The algebraic solution of the system (11) through (14) is an exceedingly difficult exercise, and a reasonable approach to finding the model parameters is to use a numerical search to find the process parameters which match the observed characteristics. This approach was used to determine the parameters which charaterize the output of a TCM decoder.

3. Calculation of the Probability of Symbol Error

Given the parameters of the Markov process, it is possible to calculate P(B), the probability that the process will be in the burst state for B of the K bits of a symbol; given K $\geq$ 3. A symbol error is defined to be the event that one or more of the bits of the symbol are in error. Therefore, the probability of symbol error is given by

$$P_S = \sum_{B=0}^{K} (1-(P_{0b})^B (P_{0c})^{K-B}) P(B) \quad (15)$$

P(B) is found by considering the K-bit symbol to be divided into segments of consecutive bits in burst state or in clear state. If we let R_b be the number of segments in burst state, and R_c be the number of segments in clear state, then it is clear that $0 \leq Rb \leq B$ and $R_c = R_b - 1$, Rb or Rb + 1. Now,

$$P(B,R_b) = P(B,R_c=R_b-1) + P(B,R_c=R_b) + P(B,R_c=R_b+1) \quad (16)$$

$$P(B,R_c=R_b-1) = \binom{B-1}{R_b-1}\binom{K-B-1}{R_c-1} P_b P_{cb}{}^{R_b-1} P_{bc}{}^{R_c} P_{bb}{}^{B-R_b} P_{cc}{}^{K-B-R_c} \quad (17)$$

$$P(B,R_b=R_c) = \binom{B-1}{R_b-1}\binom{K-B-1}{R_c-1} P_b P_{cb}{}^{R_b-1} P_{bc}{}^{R_c} P_{bb}{}^{B-R_b} P_{cc}{}^{K-B-R_c} \quad (18)$$

$$+ \binom{B-1}{R_b-1}\binom{K-B-1}{R_c-1} P_c P_{cb}^{R_b} P_{bc}^{R_c-1} P_{bb}^{B-R_b} P_{cc}^{K-B-R_c} \quad (19)$$

and $$P(B,R_b+1) = \binom{K-1}{R_b-1}\binom{K-1}{R_c-1} P_c P_{cb}^{R_b} P_{bc}^{R_c-1} P_{bb}^{B-R_b} P_{cc}^{K-B-R_c} \quad (20)$$

The reasoning behind this is as follows. The combinatorial $\binom{B-1}{R_b-1}$ is the number of ways that B periods in the burst state can be divided among R_b segments in burst state, and likewise

$\binom{K-B-1}{R_c-1}$ is the number of ways that

K-B periods in clear state can be divided among R_c segments in clear state. Every segment except the first segment begins with a transition, hence P_{cb} or P_{bc}. The first segment begins with P_b or P_c. The remaining K periods must be non-transitions, hence P_{bb} or P_{cc}. Equations (16) through (20) are valid for $1 \leq B < K$, $R_b \geq 1$ and $R_c \geq 1$. If $R_b=0$ then B=0 and if $R_c=0$, B=K.

$$P(B=0) = P_c P_{cc}^{K-1} \quad (21)$$

$$P(B=K) = P_b P_{bb}^{K-1} \quad (22)$$

For 0<B<K:

$$P(B) = \sum_{R_b=1}^{K-1} P(B,R_b) \quad (23)$$

Where $P(B,R_b)$ is as specified by equations (16) through (20). Once P(B) is calculated using equation (23), the probability of symbol error is calculated using equation (15).

4. Calculation of the Reed-Solomon Block Error Probability

The (255,233) Reed-Solomon code consists of a block of 255 symbols of 8-bits each. Up to 11 symbol errors are correctable, therefore the probability of block error is given by

$$P_e = 1-\sum_{i=0}^{11} \binom{255}{i} P_s^{\,i}(1-P_s)^{255-i} \quad (24)$$

For completely independent bit errors:

$$P_s = 1 - P_1^{\,8} \quad (25)$$

For bit errors described by the model of section 2, P_s is calculated as described in section 3, using K=8.

If the RS codebits are transmitted using a rate 2/3 8-PSK Ungerboeck code, each state transition of the Markov process must correspond to a period of operation of the trellis code, which corresonds to two decoded bits. Therefore, we introduce the term Viterbi decoded symbol to refer to a pair of bits which are decoded in one stage of the Viterbi decoder trellis, and a RS code symbol then consists of four Viterbi decoded symbols. In determining the parameters from the Markov process model, a Viterbi decoded symbol is conidered to be in error if either of its two constituent bits are in error, and this probability becomes the statistic P_1. Likewise, P_{11} denotes the probability of two consecutive Viterbi decoded symbol errors, and P_{111} denotes the probability of three such errors. The parameters of the Markov process are then evaluated using the method of section 2, and P_s is evaluated as described in section 3, using K=4. The validity of equation (29) depends on the independence of RS code symbol errors, which is effected by the interleaver, which is designed to interleave blocks of four 8-PSK symbols.

5. Results and Conclusion

Simulations were written in the C progaramming language to determine the parameters of the Markov Process Model and the symbol error rate at the input to the Reed-Solomon decoder. Calculating the symbol error rate from the model agrees with the measured symbol rate to within 10%. The symbol error rate obtained by simulation was used to calculate the probability of RS block decoding error error which is plotted in figure 3. For comparison, the RS decoding error rate was also determined using the symbol error rate from a rate 1/2 64-state convolutional code with QPSK modulation. The use of the 8-PSK code doubles the spectral efficiency. At decoding error rates between 10-10 and 10-20, the performance cost is approximately 2 dB.

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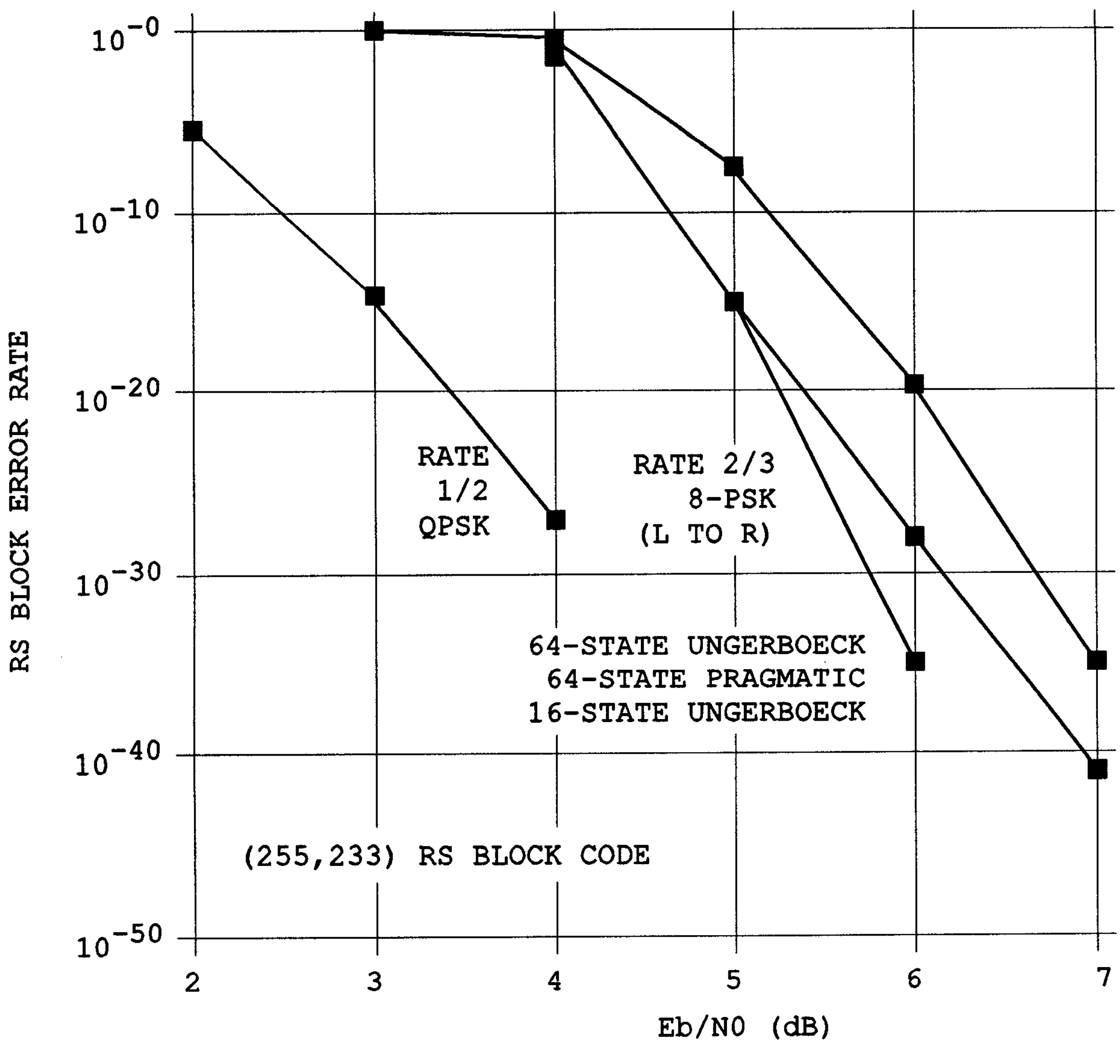

Figure 3. RS Block decoding error probability for Concatenated Systems