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Distributed Resource Allocation and Scheduling in OFDMA Wireless Networks

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Abstract— **In this paper we develop distributed resource allocation and scheduling algorithms for the uplink of an orthogonal frequency division multiple access (OFDMA) wireless network. We consider a time-slotted model, where in each time-slot the users are assigned to subchannels consisting of groups of OFDM tones. Each user can also allocate its transmission power among the subchannels it is assigned. We consider distributed algorithms for accomplishing this, where each user's actions depend only on knowledge of their own channel gains. Assuming a collision model for each subchannel, we characterize an optimal policy which maximizes the system throughput and also give a simpler sub-optimal policy. We study the scaling behavior of these policies in several asymptotic regimes for a broad class of fading distributions.**

I. Introduction

It is well established that dynamically allocating transmission resources can improve the performance of wireless networks. In this paper, we consider these approaches for the uplink in a wireless access network which uses orthogonal frequency division multiple access (OFDMA), such as in the IEEE 802.16 (WiMAX) standard. In OFDMA networks the primary resources are the assignment of tones or subcarriers to users and the allocation of a user's power across her assigned tones. Such resource allocation problems have been widely studied, e.g. see [1]–[4]. Most of this prior work focuses on the case in which resource allocation decisions are made by a centralized controller with knowledge of every user's channel state. Because of the required overhead and delays involved, it may not be feasible to acquire this information in a fast-fading environment or a system with a large number of users and/or subcarriers. Here, we instead consider approaches where each transmitter allocates its transmission rate and power based only on knowledge of its own channel conditions. This can be obtained, for example, via a single pilot signal broadcast by the receiver in a time-division duplex system [5]. This requires much less overhead, but since each user has incomplete information, a distributed approach for resource allocation is required.

In prior work [5], [6], we have considered a distributed scheduling approach based on the Aloha protocol for the case where all users communicate over a single flat-fading channel. In this approach each user randomly transmits with a probability based on its own channel gain. It is shown that as the number of users increases, the throughput of such a system scales at the same rate as that obtained by an optimal centralized controller. In [7], we extended this approach to an OFDMA-type of system, where each user can transmit over multiple subchannels, and can allocate transmission power across these subchannels. In [7], the asymptotic analysis was restricted to the case where each subchannel had i.i.d. Rayleigh fading. In this paper, we extend this analysis to a larger class of fading distributions, which includes Rayleigh, Ricean and Nakagami fading.

II. Model Description

We consider a model of n users communicating to a single receiver. There are k available subchannels. Each subchannel may represent a single OFDM tone, or more likely a group of disjoint tones bundled together.[1] Each subchannel between each user and the receiver is modeled as a time-slotted, block-fading channel with frequency-flat fading and bandwidth W_c. This is reasonable when all the tones in a subchannel lie within a single coherence band; when this is not the case, then this can be viewed as an approximation in which the channel gain represents the "average" gain for the subchannel.

At each time t, the received signal on the jth subchannel is given by

$$y_j(t) = \sum_{i=1}^{n} \sqrt{H_{ij}(t)}x_{ij}(t) + z_j(t), \tag{1}$$

where $x_{ij}(t)$ and $H_{ij}(t)$ are the transmitted signal and channel gain for the ith user on subchannel j, and $z_j(t)$ is additive white Gaussian noise with power spectral density $\frac{N_0}{2}$. To simplify notation we assume that $N_0 W_c = 1$. The channel gains are assumed to be fixed during each time-slot and to randomly vary between time-slots, *i.e.* $H_{ij}(t) = H_{ij}$ for all $t \in [mT, (m+1)T]$, where T is the length of a time-slot. Here, $\{H_{ij}\}_{i=1,..,n,j=1,..,k}$ are assumed to be independent and identically distributed (i.i.d.) across both the users and subchannels

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[1]For example in 802.16, subchannels are formed by grouping a set of interleaved tones (the default mode) or by grouping adjacent tones (in the optional Band AMC mode).

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with a continuous probability density $f_H(h)$ on $[0, \infty)$.[2] We assume that $\mathbb{E}(H_{i,k}) < \infty$ and that $f_H(h) > 0$, for all $h > 0$ and is differentiable. It follows that the corresponding distribution function $F_H(h)$ is strictly increasing and twice differentiable. Let $\bar{F}_H(h) = 1 - F_H(h)$ denote the channel gain's complimentary distribution function. For example, if each subchannel experiences Rayleigh fading, then H will be exponentially distributed, and so $\bar{F}_H(h) = e^{-h/h_0}$, where $h_0 = \mathbb{E}(H_{i,k})$.

We focus on the case where at the start of each slot, each user i has perfect knowledge of $H_{i1}, ..., H_{ik}$, but no knowledge of the channel gains for any other users. For convenience, we drop the user subscript and let $\mathbf{H} = (H_1, ..., H_k)$ denote the vector of channel gains for an arbitrary user. Let $\mathbf{P}(\mathbf{h}) = (P_1(\mathbf{h}), P_2(\mathbf{h}), ..., P_k(\mathbf{h}))$ be a user's power allocation, where $P_j(\mathbf{h})$ indicates the power allocated to subchannel j given that $\mathbf{H} = \mathbf{h}$.[3] This power allocation must satisfy a total power constraint of $\check{P}$ across all subchannels in each time-slot, *i.e.*, $\sum_j P_j(\mathbf{h}) \leq \check{P}$, for all $\mathbf{h}$. No cooperation exists among users. In particular, all users are required to employ the same power allocation and transmission scheme; *i.e.*, they can not cooperate in selecting these allocations.

During each time-slot, we assume that at most one user can successfully transmit on each subchannel. If more than one user transmits on a given subchannel, a collision occurs and no packets are received. However, a packet sent over another subchannel without a collision will still be received, i.e., the information sent over each subchannel is independently encoded. Given that only one user transmits on subchannel j, let $R(\gamma_j)$ indicate the rate at which the user can reliably transmit as a function of the received power $\gamma_j = h_j P_j(\mathbf{h})$. We assume that $R(\gamma) := \log(1 + \gamma)$, which is proportional to the Shannon capacity of the subchannel during a given time-slot. We assume that there is no coding done over successive time-slots. Also, we do not consider any multiuser reception or power capture effects when multiple users transmit on a subchannel.

III. Optimal Distributed Power Allocation

Next we turn to the power allocation $\mathbf{P}(\mathbf{h})$ used by each user during each time-slot. To begin, consider the case where there is only $n = 1$ user who must allocate its power over the k available subchannels. In this case, for each channel realization $\mathbf{h}$, the power allocation that maximizes a user's throughput is the well-known "water-filling" allocation, $P_j(\mathbf{h}) = (\lambda - \frac{1}{h_j})^+$, where λ is chosen so that $\sum_{j=1}^k P_j(\mathbf{h}) = \check{P}$.

When there are multiple users, if more than one user transmits on a subchannel, a collision results and no data is received. Following [7], we consider an Aloha-based approach, where each user transmits on each subchannel with a certain probability p. Since each subchannel is i.i.d., it is reasonable to require that each user transmits with the same probability p in each slot and on each subchannel. The probability of some user successfully transmitting on one subchannel is then $np(1-p)^{n-1}$. Given this probability, for each subchannel j, each user chooses a subset $\mathcal{H}_j$ of the possible realizations of $\mathbf{H}$ with $\Pr(\mathbf{H} \in \mathcal{H}_j) = p$. The user then only transmits on subchannel j when $\mathbf{H} \in \mathcal{H}_j$. To maximize the total throughput, each user will choose channel states in each set $\mathcal{H}_j$ that can achieve higher transmission rates. However, the transmission rate that can be achieved also relies on the specific power allocation, *e.g.* if a state $\mathbf{h}$ is in both $\mathcal{H}_j$ and $\mathcal{H}_l$, the user must allocate power across both subchannels, while if $\mathbf{h}$ is in only one set, the user can use all the available power on the corresponding subchannel. For a given power allocation, $P_j(\mathbf{h})$, the expected transmission rate on subchannel j, conditioned on a user successfully transmitting on that subchannel is given by

$$\begin{aligned}&\mathbb{E}_{\mathbf{H}}\left(R(H_j P_j(\mathbf{H}))\middle|\mathbf{H} \in \mathcal{H}_j\right)\\ &\quad = \mathbb{E}_{\mathbf{H}}\left(R(H_j P_j(\mathbf{H}))\middle|P_j(\mathbf{H}) > 0\right),\end{aligned}$$

where we have used that the channel gains are independent across users. We now specify the following *distributed optimal throughput problem*:

$$\begin{aligned}\max_{\mathbf{P}(\mathbf{H}),p}\ & np(1-p)^{n-1}\sum_{j=1}^k \mathbb{E}_{\mathbf{H}}\left(R(H_j P_j(\mathbf{H}))\middle|P_j(\mathbf{H}) > 0\right)\\ \text{s.t.}\quad & \sum_{j=1}^k P_j(\mathbf{h}) \leq \check{P},\ \forall \mathbf{h}\\ & \Pr\{P_j(\mathbf{H}) > 0\} = p,\ j = 1, ..., k.\end{aligned} \tag{2}$$

The objective in (2) is the average sum throughput for all n users over all k subchannels. This is optimized over the transmission probability p and the power allocation $(P_1(\mathbf{H}), P_2(\mathbf{H}), ..., P_k(\mathbf{H}))$, which is used by each user. The second constraint ensures that the sets $\mathcal{H}_j$ all have probability p. When this constraint is met, it follows that

$$p\mathbb{E}_{\mathbf{H}}\left(R(H_j P_j(\mathbf{H}))\middle|P_j(\mathbf{H}) > 0\right) = \mathbb{E}_{\mathbf{H}}\left(R(H_j P_j(\mathbf{H}))\right).$$

Hence, the objective in (2) can also be written as

$$n(1-p)^{n-1}\sum_{j=1}^k \mathbb{E}_{\mathbf{H}}\left(R(H_j P_j(\mathbf{H}))\right). \tag{3}$$

For a given channel realization $\mathbf{h}$, let $(h_{(1)}, h_{(2)}, ..., h_{(k)})$ denote the ordered channel gains from the largest to the smallest, with any ties broken arbitrarily. It can be shown that the solution to (2) is symmetric and so it will just depend on this ordered sequence in each time-slot. Given this ordered sequence, for $j \leq l \leq k$, let $R^l_{(j)}(\mathbf{h})$ denote the rate achievable over the jth best channel when the transmitter uses the optimal (water-filling) power allocation over only the l best channels. In other words, $R^l_{(j)}(\mathbf{h}) = \log(1 + P_{(j)}(\mathbf{h})h_{(j)})$, where $P_{(j)}(\mathbf{h}) = (\lambda - \frac{1}{h_{(j)}})^+$ and λ is chosen such that $\sum_{j=1}^l P_{(j)}(\mathbf{h}) = \check{P}$.

Lemma 1: As l increases, $\sum_{i=1}^l R^l_{(i)}(\mathbf{h}) - \sum_{i=1}^{l-1} R^{l-1}_{(i)}(\mathbf{h})$ is non-increasing.

[2] In an OFDM system different sub-carriers will typically experience correlated fading. However, if each subchannel is a large enough group of sub-carriers, then this independence assumption is reasonable.

[3] If a user does not transmit on channel j, then $P_j(\mathbf{h}) = 0$.

Given a "threshold rate" $R_{th} > 0$ for each channel realization $\mathbf{h}$, we introduce the following problem:

$$\max_{l=1,\ldots,k} l$$
$$\text{s.t.} \quad \sum_{i=1}^{l} R^{l}_{(i)}(\mathbf{h}) - \sum_{i=1}^{l-1} R^{l-1}_{(i)}(\mathbf{h}) \geq R_{th} \tag{4}$$

If this problem has no feasible solution, we define the solution to be $l = 0$. When $k = 1$, the constraint in (4) is $R^{1}_{(1)}(\mathbf{h}) \geq R_{th}$, i.e., the rate when only transmitting on the best channel should be greater than R_{th}. For $k = 2$, the constraint in (4) becomes $R^{2}_{(1)}(\mathbf{h}) + R^{2}_{(2)}(\mathbf{h}) - R^{1}_{(1)}(\mathbf{h}) \geq R_{th}$, which means that the increase in the total rate from using the best two channels versus only using the best channel should be greater than R_{th}. In general, the objective of (4) is to find the maximal number of channels l, such that the gain in the sum rate from transmitting on the l best channels instead of only the $l-1$ best channels is at least R_{th}. From Lemma 1 it follows that if l^* solves (4), then any $l < l^*$ will also be feasible.

For a given R_{th}, let $\mathbf{P}^{R_{th}}(\mathbf{h})$ be the power allocation that corresponds to solving (4) for each channel realization $\mathbf{h}$; i.e. this will be a water-filling allocation over the l best channels, where l is the solution to (4) for each given realization (note l may change with each realization). The following proposition relates this to the solution of (2).

Proposition 1: There exists a constant $R_{th} > 0$ such that $\mathbf{P}^{R_{th}}(\mathbf{h})$ is also the optimal solution to (2).

This proposition specifies the form of the optimal power allocation; the corresponding transmission probability is given by $p = \Pr(P^{R_{th}}_{i}(\mathbf{H}) > 0)$. It follows from this proposition that the optimal solution to (2) can be found by solving (4) for a given R_{th}, and then iteratively searching for the optimal R_{th}. An algorithm for solving (4) for a given R_{th} and channel realization $\mathbf{h}$ is given in [7]. This algorithm uses the property in Lemma 1, to converge to the optimal solution to (4) in at most k iterations. The optimal value of R_{th} must still be found via a numeric search; however, we note that this search is now only a one-dimensional search, instead of a k-dimensional search over the possible power allocations. For a given n and k, the optimal power allocation could be determined offline using this procedure. For a large number of channels k this will result in a large computational cost. Next, we introduce a simpler sub-optimal algorithm and analyze its performance.

IV. Sub-optimal Power Allocation and Asymptotic Analysis

We consider a simplified distributed scheme, where instead of finding a threshold rate R_{th} and solving (4), we set a threshold h_{th} on the channel gain. Each user then transmits on the kth subchannel when its gain is greater than h_{th}, resulting in the transmission probability $p = \bar{F}_H(h_{th})$. If a user has more than one subchannel whose gain is higher than the threshold, then the total power $\check{P}$ will be allocated equally to each of these subchannels.[4] Given that a user transmits on i subchannels, we assume it transmits at a constant rate of $R_i(p) := R(\bar{F}_H^{-1}(p)\frac{\check{P}}{i})$ on each subchannel. This is a lower bound on the achievable rate and simplifies our analysis.

The total throughput using this scheme is a function of k, n and p. For $i = 1, \ldots, k$, let $q_{k,p}(i)$ be the probability one user has i subchannels above the threshold $h_{th} = \bar{F}_H^{-1}(p)$, *i.e.*,

$$q_{k,p}(i) = \binom{k}{i}(p)^i(1-p)^{k-i}.$$

Among these i subchannels, for $j = 1, \ldots, i$, let $\omega_{p,i}(j)$ be the probability a user transmits successfully on exactly j subchannels, *i.e.* the probability there is no collision on exactly j subchannels, given that i are above the threshold. This is given by

$$\omega^n_{p,i}(j) = \binom{i}{j}\left[(1-p)^{n-1}\right]^j\left[1-(1-p)^{n-1}\right]^{i-j}.$$

The average sum throughput of this system is then given by

$$s(k,n,p) = n\sum_{i=1}^{k} q_{k,p}(i)\sum_{j=1}^{i}\omega^n_{p,i}(j)jR_i(p).$$

Note that $\omega^n_{p,i}(j)$ is a binomial probability mass function (p.m.f.) and so $\sum_{j=1}^{i}\omega^n_{p,i}(j)j = (1-p)^{n-1}i$. Therefore,

$$s(k,n,p) = n(1-p)^{n-1}\sum_{i=1}^{k}\binom{k}{i}(p)^i(1-p)^{k-i}iR_i(p). \tag{5}$$

We consider how the sum throughput of this scheme and the optimal distributed scheme scales in three asymptotic regimes. We define two sequences $f(m)$ and $g(m)$ to be *asymptotically equivalent*, denoted by $f(m) \asymp g(m)$, if $\lim_{m\to\infty}\frac{f(m)}{g(m)} = c$. In the special case where $c = 1$, we say that they are *strongly asymptotically equivalent* and denote this by $f(m) \overset{\sim}{\asymp} g(m)$. This implies that both sequences asymptotically grow at the same rate and have the same first order constant. For our analysis, we make an additional assumption on the tail of the fading distribution. Specifically, we assume that as $h \to \infty$,

$$f_H(h) \asymp f'_H(h), \tag{6}$$

where $f'_H(h) = \frac{d}{dh}f_H(h)$. This is satisfied by any fading distribution that has an exponential tail, which is the case for most common fading models such as Rayleigh, Ricean and Nakagami fading.

Lemma 2: For any continuous, differentiable fading density f_H that satisfies (6), then the following conditions hold: (*a.*) $\bar{F}_H(h) \asymp f_H(h)$, (*b.*) $\lim_{h\to\infty}\frac{\bar{F}_H(h)}{hf_H(h)} = 0$, and (*c.*) $\lim_{h\to\infty}\frac{d}{dh}\left[\frac{\bar{F}_H(h)}{f_H(h)}\right] = 0$.
These conditions follow directly from evaluating the limits using L'Hospital's rule.

We also compare the distributed approaches to an optimal centralized system that maximizes the throughput in every slot.

[4] Other similar equal power allocation approaches for multi-carrier systems have been studied, see e.g. [8].

This is given by:[5]

$$\begin{aligned}&\max_{\{P_{ij},c_{ij}\}} \sum_{i=1}^{n}\sum_{j=1}^{k} R(P_{ij}c_{ij}h_{ij})\\ &\text{s.t.} \quad \sum_{j=1}^{k} P_{ij}c_{ij} = \check{P},\ \forall i,\\ &\qquad \sum_{i=1}^{n} c_{ij} \le 1,\ \forall j,\ c_{nk}\in\{0,1\},\ \forall i,j.\end{aligned} \tag{7}$$

Here, the integer variables, c_{ij}, indicate when user i is assigned to subchannel j; the second constraint ensures that at most one user is assigned to each subchannel.

Let $s_{ct}(k,n)$ be the average sum throughput obtained by the optimal centralized scheduling policy. Denote the throughput of the optimal distributed policy by $s^*(k,n)$ and the optimal throughput of the threshold-based algorithm by $s(k,n,p^*)$, where p^* is the transmission probability that optimizes $s(k,n,p)$. For all n and k, we have,

$$s\left(k,n,\tfrac{1}{n}\right) \le s(k,n,p^*) \le s^*(k,n) \le s_{ct}(k,n), \tag{8}$$

where the first term is the throughput with a transmission probability of $1/n$.

First, we consider the case where k is fixed and n increases.

Proposition 2: Given any finite k, as $n\to\infty$, $s(k,n,\frac{1}{n})$, $s(k,n,p^*)$, $s^*(k,n)$ and $\frac{1}{e}s_{ct}(k,n)$ are all strongly asymptotically equivalent to $\frac{k}{e}\log\left(1+\check{P}\bar{F}_H^{-1}(\frac{1}{n})\right)$.

In other words, asymptotically there is no difference in the first-order performance compared to the optimal distributed approach when using the simplified scheme or from choosing $p=\frac{1}{n}$ instead of the optimal p^*. The throughput for each distributed approach asymptotically increases like $\frac{k}{e}\log(1+\check{P}\bar{F}_H^{-1}(\frac{1}{n}))$, as does $\frac{1}{e}$ times the throughput with the optimal centralized scheduler. In other words, the distributed approaches all grow at the same rate as the centralized approach and asymptotically the ratio of their throughputs approach $\frac{1}{e}$, the contention loss in a standard slotted Aloha system. As an example, for the case of i.i.d. Rayleigh fading on each channel the throughput in each case will increase at rate $O(\log(\log(n)))$.

The second regime we consider is when n is fixed and k increases.

Proposition 3: Given any finite n, as $k\to\infty$, $s(k,n,p^*)$, $s^*(k,n)$, $s_{ct}(k,n)$ are all strongly asymptotically equivalent to $n\check{P}\bar{F}_H^{-1}(\frac{1}{k})$.

Again the threshold based approach is strongly asymptotically equivalent to the optimal distributed approach. In this case, it is also asymptotically equivalent to the optimal centralized system; i.e. there is no loss of $\frac{1}{e}$. Intuitively, this is because as the number of channels increases, the probability of collision becomes negligible. In this case, for a Rayleigh fading channel each of these terms grows like $O(\log(k))$ as $k\to\infty$, with a first order constant that is linear in n.

[5] This is similar to a problem studied for centralized OFDM systems in [3].

The last regime we consider is where both k and n increase with fixed ratio $\frac{k}{n}=\beta$.

Proposition 4: If $\frac{k}{n}=\beta$, as $n\to\infty$, $s(\beta n,n,\frac{1}{n})$, $s(\beta n,n,p^*)$, $s^*(\beta n,n)$ and $\frac{1}{e}s_{ct}(\beta n,n)$ are all strongly asymptotically equivalent to $\beta n e^{-1}\log\left(1+\check{P}\bar{F}_H^{-1}(\frac{1}{n})\right)$.

As in Proposition 2, once again compared to the centralized scheme there is an asymptotic penalty of $1/e$ due to the contention, and a transmission probability of $p=\frac{1}{n}$ is asymptotically optimal for the distributed system. For Rayleigh fading channels the throughput now grows like $O(n\log(\log(n)))$, as $n\to\infty$, with a first order constant that is linear in β.

V. Numerical Examples

We next give some numerical examples to illustrate the performance of the optimal and simplified distributed algorithms with a finite number of channels and users. All the results in this section are for an i.i.d. Rayleigh fading model, with $E(H_{ij})=1$, and a total power constraint of $\check{P}=1$. The performance is averaged over multiple channel realizations. Figure 1 shows the average throughput achieved by the optimal distributed power allocation scheme from Section III compared to the simplified power allocation scheme in Section IV. The throughput of both approaches is shown as a function of the number of users for a system with $k=10$ channels. As the number of users increases, both throughputs increase and the difference between the two curves decreases.

Figure 2 shows upper and lower bounds on the ratio of the average throughput of the optimal distributed scheme $s^*(k,n)$ to the centralized scheme $s_{ct}(k,n)$ defined in (7) as a function of the number of users, for $k=5$ and 10 channels. Calculating $s_{ct}(k,n)$ requires solving the optimization problem in (7) for every channel realization, which is complicated due to the integer constraints. Instead we compare $s^*(k,n)$ to upper and lower bounds on $s_{ct}(k,n)$. We upper bound $s_{ct}(k,n)$ by relaxing the total power constraint on the channels, $\sum_k P_{nk}c_{nk}=\check{P}$. Instead, we allow each user to transmit with $P_{nk}=\check{P}$ over each channel. The maximum throughput is achieved for this relaxed system by letting the best user on each channel transmit at each time. To lower bound $s_{ct}(k,n)$, we still choose the best user to transmit on each subchannel, but if one user is chosen to transmit on more than one subchannel, its power is divided equally across these channels. The resulting throughput is then a lower bound on $s_{ct}(k,n)$. Figure 2 shows that as the number of users increases, the two bounds approach each other. It can be seen that the ratio of the throughputs of the distributed to the optimal scheme is decreasing as the number of users increases and is larger than the limiting value of $1/e$ (see Proposition 2) for all finite n. As the number of the channels, k, increases, the throughput ratio also increases for a fixed n. This is due to the increased frequency diversity with more channels.

Figure 3 shows upper and lower bounds on the ratio of the throughput of the optimal distributed scheme to that of the optimal centralized approach as the number of channels increases, for a system with $n=5$ and 10 users. In this case, we upper bound $s_{ct}(k,n)$ by the information theoretic capacity

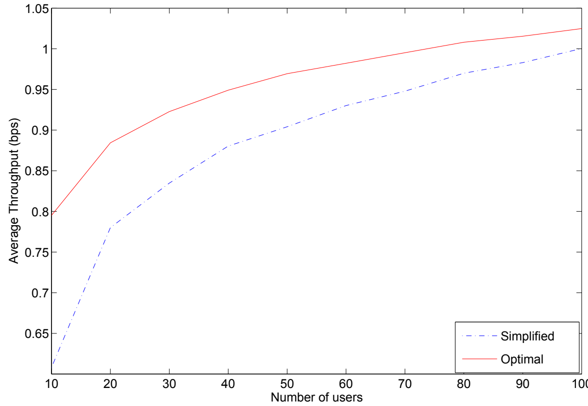

Fig. 1. Average throughput (bps) per channel of the optimal distributed scheme and the simplified distributed scheme as a function of the number of users for $k = 10$ channels.

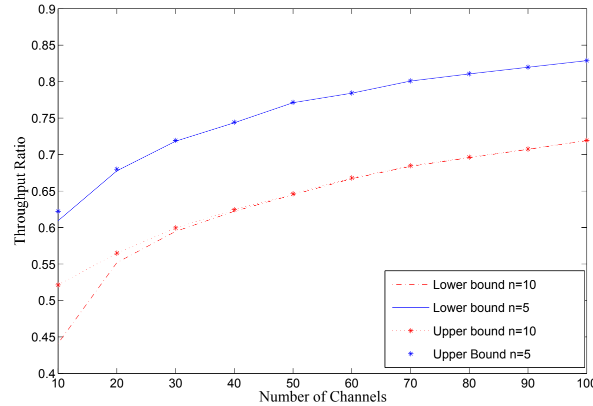

Fig. 3. Lower and upper bounds on the ratio of the average throughputs of the optimal distributed scheme to the optimal centralized scheme versus the number of channels, for $n = 5$ and 10 users.

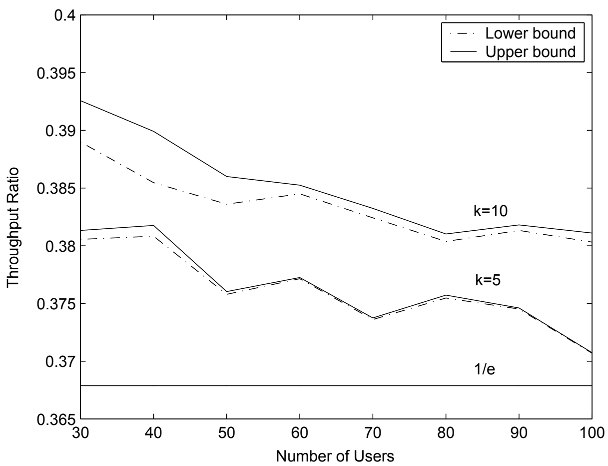

Fig. 2. Lower and upper bounds on the ratio of average throughputs of the optimal distributed scheme to the optimal centralized scheme versus the number of users, for $k = 5$ and 10 channels.

of this multi-access system. In other words, joint decoding is used when multiple users transmit on the same channel. We lower bound $s_{ct}(k, n)$ by only allowing the user who has the best channel to transmit on a channel. Figure 3 shows that as the number of channels increases, the two bounds quickly converge. The throughput ratio increases as the number of channels increases. From Proposition 3, as k increases, these bounds should approach 1. In this asymptotic regime, the convergence appears to be much slower than in Figure 2.

VI. Summary

We have presented distributed algorithms for resource allocation in an OFDMA wireless network, where each user only has knowledge of its own channel gains. Using a contention model, an optimal distributed algorithm is characterized. A simplified distributed approach is also given. In three different asymptotic regimes, the simplified algorithm is shown to be asymptotically equivalent to the optimal distributed algorithm. Both algorithms are also shown to scale at the same rate as the optimal centralized scheduler. These results suggest that it is possible to develop near optimal approaches for scheduling and power allocation without requiring a centralized controller with complete channel knowledge. There are several important issues that we have not addressed here. For example, we have not considered asymmetric models, where the fading is not identically distributed across the channels or the users, or models where the fading is correlated across the channels. We also assumed that each user knows the fading distribution; in practice, an adaptive approach would be required to estimate this distribution.

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