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ABSTRACT

This paper provides exact (finite-sample) test critical values for carrying out tests of no cointegration versus some forms of nonlinear (threshold autoregressive) cointegration. The nonlinear models, which include threshold autoregressive and momentum threshold autoregressive behavior of deviations from long-run equilibrium, are easier to evaluate with the aid of the reported critical values. The results cover a variety of practical situations, with varying sample sizes, lag lengths, and number of time series.

JEL Classification Codes: C12, C3, C32

Keywords: Cointegration, nonlinear, hypothesis test, critical value.

1. Introduction

Asymmetric behavior in economic variables has attracted considerable attention in the last decades. Neftci (1984) showed that several measures of U.S. unemployment display asymmetric adjustment over the course of the business cycle. Unlike Neftci, Falk (1986) found little evidence in favor of asymmetry when he applied Neftci's method to real U.S. GNP, investment, and productivity and to industrial production in Canada, France, Italy, Germany, and the UK. Nevertheless, the most recent consensus seems to be in favor of asymmetric adjustment. Focusing on the asymmetric behavior of unemployment rates over the business cycle, Rothman (1992) showed that the primary source of asymmetry is the cyclical behavior of the unemployment rate in the manufacturing sector. Acemoglu and Scott (1994) have also shown asymmetries in the cyclical behavior of UK labor markets. Harris and Silverstone (1999) tested asymmetric adjustment in specifications of Okun's law. Enders and Dibooglu (2001) studied the long run Purchasing Power Parity with asymmetric adjustment using data from the post -Bretton Woods period. They showed that cointegration with threshold adjustment holds for a number of European countries on a bilateral basis. Furthermore, comparing the estimates of linear and asymmetric adjustment error-correction models, the authors showed that prices and exchange rates have markedly different adjustment patterns for positive deviations from the Purchasing Power Parity than for negative deviations. Ball and Mankiw (1994) provide a theoretical explanation for asymmetric adjustment of nominal prices. The authors present a menu-cost model in which positive trend inflation causes firms' relative prices to decline automatically between price adjustments. In this environment, the authors showed that shocks that raise firms' desired prices trigger larger responses than shocks that lower desired prices. Thus there is a large body of literature concerning asymmetric adjustment in economics[1].

This paper provides critical values for carrying out tests of asymmetric adjustment within a cointegration framework. Previous work such as Dibooglu and Enders (2001), and Enders (2003) provide critical values of for up to three variables. The objective of this paper is to extend the critical values of the empirical *F*-distribution of the null hypothesis of cointegration with asymmetric adjustment for up to five variables, various sample sizes, and 8 lagged changes. This should be useful for macroeconomic models with a larger set of variables. The critical values are derived for threshold autoregressive and momentum threshold autoregressive deviations from long-run equilibrium. Section 2 of the paper develops the econometric framework, while Section 3 presents the methodology. Section 4 concludes.

2. Threshold and Momentum Models of Cointegration

Suppose variables in the vector $\{y_{1t}, \ldots, y_{kt}\}$ are integrated of order 1. The Engle and Granger (1987) methodology entails estimating the regression,

$$y_{1t} = \beta_1 + \beta_2 y_{2t} + \ldots + \beta_k y_{kt} + \mu_t \tag{1}$$

and applying a unit root test to μ_t:

$$\Delta \mu_t = \rho \mu_{t-1} + \varepsilon_t \tag{2}$$

Cointegration implies that μ_t is stationary with mean zero and that $\rho = 0$. As such, equation (1) is an attractor such that its pull is strictly proportional to the absolute value of μ_{t-1}. The change in μ_t equals ρ multiplied by μ_{t-1} regardless of whether μ_{t-1} is positive or negative.

1 The second edition of W. Enders' book, *Applied Econometric Time Series* has a chapter on asymmetric adjustment; see, Enders (2003).

Similarly, the Johansen (1995) methodology begins with a specification of the form:

$$\Delta y_t = \pi y_{t-1} + \varepsilon_t \tag{3}$$

where y_t is the (k x 1) vector, π is a (k x k) matrix, and ε_t is a (k x 1) vector of normally distributed disturbances that may be contemporaneously correlated.

The Johansen procedure entails the estimation of π and testing the null hypothesis that the rank of π equals zero. Again, under the alternative hypothesis [i.e., rank(π) $\neq$ 0] the adjustment process is symmetric around $y_t = 0$ in that for any $y_t \neq 0$, Δy_{t+1} always equals πy_t in expectation. Thus, πy_t can be viewed as an attractor such that its pull is strictly proportional to $\| y_t \|$.

However, the implicit assumption of symmetric adjustment is problematic if the adjustment towards the long-run equilibrium relationship is not linear. Enders and Granger (1998), and Enders and Siklos (2001) introduce asymmetric adjustment by letting the deviations from the long-run equilibrium in equation (1) behave as a Threshold Autoregressive (TAR) process. Thus, they replace (2) with:

$$\Delta \mu_t = I_t \rho_1 \mu_{t-1} + (1 - I_t) \rho_2 \mu_{t-1} + \varepsilon_t \tag{3}$$

where I_t is the Heaviside indicator such that:

$$I_t = \begin{cases} 1 \text{ if } \mu_{t-1} \geq \tau \\ 0 \text{ if } \mu_{t-1} < \tau \end{cases}$$

Asymmetric adjustment is implied by different values of ρ_1 and ρ_2; when μ_{t-1} is positive, the adjustment is $\rho_1\mu_{t-1}$, and if μ_{t-1} is negative, the adjustment is $\rho_2\mu_{t-1}$. A sufficient condition for stationarity of $\{\mu_t\}$ is for: $-2 < (\rho_1 , \rho_2) < 0$. Moreover, if the $\{\mu_t\}$ sequence is stationary, the least squares estimates of ρ_1 and ρ_2 have an asymptotic multivariate normal distribution if the value of the threshold is known (or consistently estimated). Thus, if the null hypothesis $\rho_1 = \rho_2 = 0$ is rejected, it is possible to test for symmetric adjustment (i.e., $\rho_1 = \rho_2$) using a standard *F*-test. Since adjustment is symmetric if $\rho_1 = \rho_2$, the Engle-Granger test for cointegration is a special case of (3).

Since the exact nature of the non-linearity may not be known, it is also possible to allow the adjustment to depend on the change in μ_{t-1} (i.e., $\Delta\mu_{t-1}$) instead of the level of μ_{t-1}. In this case, the Heaviside indicator of (4) becomes:

$$I_t = \begin{cases} 1 \text{ if } \Delta\mu_{t-1} \geq \tau \\ 0 \text{ if } \Delta\mu_{t-1} < \tau \end{cases}$$

Even though Hansen (1997) shows that setting the Heaviside indicator using $\Delta\mu_{t-1}$ can perform better than the specification using pure TAR adjustment, Enders and Granger (1998) and Enders and Siklos (2001) show that this specification is especially relevant when the adjustment is such that the series exhibits more "momentum" in one direction than the other. They call this model momentum-threshold autoregressive (M-TAR) model. Respectively, the *F*-statistics for the null hypothesis $\rho_1 = \rho_2 = 0$ using the TAR specification of (4) and the M-TAR specification of (5) are called Φ_μ and Φ_μ*. As there is generally no presumption as to whether to use (4) or (5), the recommendation is to select the adjustment mechanism by a model selection criterion such as the *AIC*.

If the errors in equation (3) are serially correlated, it is possible to use an augmented threshold model for the residuals. In this circumstance, equation (3) is replaced by:

$$\Delta\mu_t = I_t\,\rho_1\mu_{t-1} + (1-I_t)\,\rho_2\mu_{t-1} + \sum_{i=1}^{p}\beta_i\,\Delta\mu_{t-i} + \varepsilon_t \qquad (6)$$

The distributions of Φ_μ and Φ_μ* depend on the number of observations, the number of lags in equation (6) and the number of variables in the cointegrating relationship. The empirical *F*-distribution for the null hypothesis $\rho_1 = \rho_2 = 0$ is tabulated by Dibooglu and Enders (2001), and Enders (2003) for up to three variables. The objective of this paper is to calculate critical values of the empirical *F*-distribution (distributions of Φ_μ and Φ_μ*) for the null hypothesis $\rho_1 = \rho_2 = 0$ for up to five variables, various sample sizes, and 8 lagged changes. This should be useful for macroeconomic models with a larger set of variables.

3. Critical Values of the Cointegration Test

In order to develop critical values that can be used to test for cointegration, we generated 50,000 random-walk processes of the following form:

$$y_{kt} = y_{kt-1} + \nu_{kt}, \quad k = 1, \dots, 5, \; t = 1, \dots, T \qquad (7)$$

For T = 50, 100, and 250, up to five sets of T normally distributed and uncorrelated pseudo-random numbers with standard deviation equal to unity were drawn to represent the $\{\nu_{kt}\}$ sequences. Randomizing the initial values of $\{ y_{kt} \}$, the next T values of each were generated using (7). For each of the 50,000 series, the TAR model given by (1), (4) and (5) was estimated.

Since the value of the threshold τ is typically unknown, for each of the 50,000 replications, we used Chan's (1993) method for obtaining the consistent estimate of the threshold. To find the consistent estimate of the threshold, we ordered the $\{\mu_{t-1}\}$ sequence from smallest to largest. Although any value of $\{\mu_{t-1}\}$ is a potential threshold, we consider only values of μ_{t-1} between the lowest 15% and the highest 85% values of the series as a potential threshold.

Estimate regressions in the form of (1) using each potential value of μ_{t-1} as a threshold. The value resulting in the lowest residual sum of squares is the estimate of the threshold $\hat{\tau}$. Using $\hat{\tau}$ as the threshold, compare the *F*-statistic for the null hypothesis $\rho_1 = \rho_2 = 0$ with the appropriate critical value shown in Table 1. For each estimated equation, we estimated ρ_1 and ρ_2 and recorded the *F*-statistic for the joint hypothesis $\rho_1 = \rho_2 = 0$ for the TAR and M-TAR models. These F-statistics are reported in Tables 1-8 for various values of sample sizes (*T*) and lag lengths *p*. For example, for $T = 100$, Table 1 shows that the Φ-statistic for the null hypothesis $\rho_1 = \rho_2 = 0$ exceeded 8.09 in approximately 5% of the 50,000 trials using a model augmented with 2 lagged changes in $\{\mu_t\}$.

4. Conclusions

The present work reports an extensive set of exact (finite sample) test critical values for nonlinear cointegration, and in the future these results may be further extended to cover an even greater variety of sample sizes, lag lengths, and number of time series. Also, while we report results for Gaussian time series, some applications may call for non-Gaussian models (allowing heavy-tailed distributions, etc.), for which a parallel set of critical values may be computed. As the sample size becomes very large, asymptotic theory becomes relevant for approximating exact critical values, and more work in this area is needed.

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Table 1. Distribution for the F-Statistic for the Null Hypothesis, $\rho_1 = \rho_2 = 0$, in the 2-variable case

T	LAGGED CHANGES											
	1 LAG			2 LAGS			3 LAGS			4 LAGS		
	90%	95%	99%	90%	95%	99%	90%	95%	99%	90%	95%	99%
THE TAR MODEL: Φ_μ												
50	6.35	7.54	10.29	6.12	7.25	9.84	5.99	7.13	9.84	5.83	6.98	9.56
100	5.95	6.99	9.39	5.80	6.82	9.04	5.79	6.77	9.01	5.66	6.66	8.97
150	5.94	6.98	9.29	5.79	6.82	9.02	5.76	6.77	8.98	5.78	6.76	8.93
200	6.03	7.05	9.35	5.97	6.96	9.25	5.99	7.02	9.27	5.89	6.88	9.05
250	6.14	7.11	9.38	6.09	7.08	9.19	6.10	7.10	9.37	6.07	7.08	9.32
500	6.41	7.39	9.66	6.44	7.47	9.64	6.35	7.36	9.54	6.38	7.40	9.63
THE M-TAR MODEL: Φ^*_μ												
50	7.22	8.49	11.55	6.88	8.06	10.91	6.79	8.04	10.72	6.54	7.76	10.50
100	6.97	8.15	10.67	6.84	7.95	10.35	6.77	7.87	10.34	6.61	7.73	10.14
150	6.75	7.87	10.40	6.62	7.71	10.09	6.58	7.65	10.06	6.54	7.62	9.96
200	6.62	7.72	10.04	6.58	7.64	9.97	6.52	7.62	9.94	6.46	7.51	9.85
250	6.61	7.76	10.15	6.51	7.57	9.91	6.50	7.59	9.84	6.41	7.44	9.71
500	6.52	7.55	9.93	6.47	7.53	9.76	6.46	7.54	9.77	6.42	7.47	9.72

Table 2. Distribution for the F-Statistic for the Null Hypothesis, $\rho_1 = \rho_2 = 0$, in the 2-variable case

T	LAGGED CHANGES											
	5 LAGS			6 LAGS			7 LAGS			8 LAGS		
	90%	95%	99%	90%	95%	99%	90%	95%	99%	90%	95%	99%
THE TAR MODEL: Φ_μ												
50	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
100	5.62	6.64	8.87	5.53	6.52	8.76	5.49	6.50	8.72	5.37	6.33	8.48
150	5.71	6.70	8.88	5.61	6.61	8.76	5.61	6.60	8.80	5.55	6.53	8.76
200	5.87	6.92	9.10	5.87	6.83	9.11	5.80	6.77	9.00	5.78	6.76	8.95
250	6.01	7.02	9.14	6.03	7.05	9.31	5.99	7.00	9.18	5.90	6.88	9.01
500	6.38	7.42	9.69	6.32	7.34	9.55	6.35	7.32	9.53	6.31	7.30	9.45
THE M-TAR MODEL: Φ^*_μ												
50	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
100	6.55	7.64	10.02	6.42	7.49	9.95	6.36	7.42	9.87	6.27	7.33	9.65
150	6.42	7.49	9.82	6.33	7.41	9.71	6.30	7.38	9.69	6.20	7.26	9.49
200	6.41	7.47	9.74	6.37	7.40	9.74	6.32	7.36	9.60	6.27	7.32	9.60
250	6.40	7.44	9.83	6.36	7.41	9.73	6.31	7.31	9.60	6.27	7.30	9.45
500	6.45	7.46	9.70	6.36	7.41	9.68	6.38	7.38	9.70	6.38	7.39	9.63

Table 3. Distribution for the F-Statistic for the Null Hypothesis, $\rho_1 = \rho_2 = 0$, in the 3-variable case

T	LAGGED CHANGES											
	1 LAG			2 LAGS			3 LAGS			4 LAGS		
	90%	95%	99%	90%	95%	99%	90%	95%	99%	90%	95%	99%
THE TAR MODEL: Φ_μ												
50	7.70	9.09	12.29	7.26	8.56	11.41	6.97	8.22	11.06	6.61	7.78	10.53
100	7.17	8.34	10.94	6.97	8.09	10.61	6.86	7.98	10.39	6.65	7.73	10.21
150	7.10	8.23	10.71	7.00	8.11	10.53	6.90	8.04	10.44	6.77	7.87	10.18
200	7.24	8.40	10.77	7.18	8.30	10.69	7.11	8.22	10.61	7.00	8.11	10.50
250	7.37	8.50	10.79	7.27	8.43	10.99	7.22	8.30	10.66	7.16	8.27	10.62
500	7.60	8.73	11.16	7.54	8.66	11.06	7.53	8.63	10.97	7.49	8.58	11.03
THE M-TAR MODEL: Φ^*_μ												
50	8.49	9.92	13.23	8.04	9.42	12.42	7.82	9.10	12.00	7.41	8.65	11.47
100	8.25	9.55	12.13	7.98	9.20	11.88	7.88	9.09	11.77	7.62	8.79	11.34
150	7.98	9.20	11.82	7.83	9.04	11.55	7.72	8.92	11.47	7.55	8.71	11.22
200	7.90	9.10	11.71	7.77	8.95	11.42	7.70	8.86	11.40	7.57	8.71	11.20
250	7.85	9.05	11.58	7.75	8.93	11.42	7.75	8.91	11.29	7.61	8.77	11.26
500	7.78	8.98	11.51	7.71	8.85	11.24	7.69	8.84	11.31	7.68	8.83	11.19

Table 4. Distribution for the F-Statistic for the Null Hypothesis, $\rho_1 = \rho_2 = 0$, in the 3-variable case

T	LAGGED CHANGES											
	5 LAGS			6 LAGS			7 LAGS			8 LAGS		
	90%	95%	99%	90%	95%	99%	90%	95%	99%	90%	95%	99%
THE TAR MODEL: Φ_μ												
50	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
100	6.55	7.61	10.04	6.36	7.44	9.86	6.30	7.35	9.61	6.12	7.15	9.42
150	6.73	7.82	10.14	6.60	7.67	9.94	6.49	7.54	9.10	6.43	7.49	9.72
200	6.94	8.03	10.37	6.82	7.91	10.29	6.76	7.83	10.34	6.69	7.76	10.03
250	7.10	8.18	10.52	7.04	8.12	10.51	6.93	8.03	10.27	6.89	7.95	10.21
500	7.46	8.60	10.97	7.40	8.49	10.89	7.44	8.51	10.76	7.41	8.50	10.94
THE M-TAR MODEL: Φ^*_μ												
50	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
100	7.55	8.73	11.39	7.34	8.49	11.08	7.20	8.33	10.87	7.06	8.15	10.61
150	7.47	8.62	11.11	7.35	8.43	10.85	7.28	8.39	10.87	7.17	8.29	10.70
200	7.56	8.68	11.13	7.42	8.55	11.08	7.39	8.50	10.87	7.22	8.34	10.79
250	7.57	8.76	11.21	7.45	8.56	11.02	7.40	8.49	10.95	7.31	8.40	10.86
500	7.63	8.76	11.17	7.58	8.72	11.09	7.53	8.66	11.06	7.52	8.65	10.95

Table 5. Distribution for the F-Statistic for the Null Hypothesis, $\rho_1 = \rho_2 = 0$, in the 4-variable case

T	LAGGED CHANGES											
	1 LAG			2 LAGS			3 LAGS			4 LAGS		
	90%	95%	99%	90%	95%	99%	90%	95%	99%	90%	95%	99%
THE TAR MODEL: Φ_μ												
50	8.98	10.53	13.91	8.28	9.63	12.85	7.92	9.31	12.39	7.36	8.64	11.56
100	8.45	9.77	12.63	8.09	9.36	12.05	7.92	9.13	11.82	7.61	8.84	11.39
150	8.40	9.65	12.42	8.17	9.41	11.97	8.04	9.22	11.88	7.84	9.00	11.61
200	8.53	9.71	12.44	8.33	9.57	12.17	8.27	9.47	12.13	8.11	9.24	11.74
250	8.54	9.79	12.58	8.46	9.66	12.28	8.41	9.62	12.05	8.31	9.45	12.01
500	8.81	10.03	12.73	8.74	9.92	12.60	8.69	9.86	12.40	8.70	9.92	12.44
THE M-TAR MODEL: Φ^*_μ												
50	9.85	11.47	14.91	9.05	10.53	13.81	8.75	10.18	13.58	8.14	9.53	12.58
100	9.55	10.89	13.81	9.12	10.46	13.33	8.91	10.21	13.03	8.62	9.84	12.66
150	9.28	10.63	13.44	9.01	10.28	12.99	8.86	10.08	12.76	8.62	9.88	12.46
200	9.19	10.46	13.17	8.98	10.28	13.04	8.83	10.07	12.68	8.72	9.95	12.50
250	9.10	10.36	13.09	8.96	10.17	12.90	8.87	10.16	12.82	8.72	9.90	12.54
500	9.05	10.30	12.99	8.99	10.24	12.81	8.92	10.18	12.76	8.80	10.03	12.54

Table 6. Distribution for the F-Statistic for the Null Hypothesis, $\rho_1 = \rho_2 = 0$, in the 4-variable case

T	LAGGED CHANGES											
	5 LAGS			6 LAGS			7 LAGS			8 LAGS		
	90%	95%	99%	90%	95%	99%	90%	95%	99%	90%	95%	99%
THE TAR MODEL: Φ_μ												
50	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
100	7.45	8.60	11.15	7.19	8.29	10.71	7.02	8.15	10.57	6.80	7.89	10.22
150	7.65	8.80	11.26	7.48	8.60	10.91	7.40	8.56	10.95	7.19	8.29	10.66
200	7.95	9.13	11.65	7.78	8.93	11.38	7.76	8.91	11.37	7.61	8.71	11.14
250	8.22	9.40	12.01	8.03	9.19	11.71	8.00	9.13	11.68	7.87	8.96	11.34
500	8.63	9.83	12.36	8.53	9.73	12.20	8.49	9.67	11.98	8.48	9.66	12.12
THE M-TAR MODEL: Φ^*_μ												
50	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
100	8.40	9.66	12.25	8.17	9.36	12.08	8.05	9.26	11.85	7.75	8.91	11.52
150	8.47	9.67	12.34	8.29	9.46	12.07	8.16	9.33	11.93	7.97	9.10	11.66
200	8.60	9.82	12.39	8.42	9.60	12.23	8.32	9.51	12.05	8.19	9.35	11.81
250	8.63	9.84	12.40	8.53	9.76	12.20	8.45	9.62	12.12	8.33	9.49	12.02
500	8.77	10.04	12.65	8.72	9.87	12.43	8.72	9.88	12.41	8.58	9.78	12.19

Table 7. Distribution for the F-Statistic for the Null Hypothesis, $\rho_1 = \rho_2 = 0$, in the 5-variable case

T	LAGGED CHANGES											
	1 LAG			2 LAGS			3 LAGS			4 LAGS		
	90%	95%	99%	90%	95%	99%	90%	95%	99%	90%	95%	99%
THE TAR MODEL: Φ_μ												
50	10.29	11.88	15.54	9.25	10.75	14.10	8.81	10.22	13.51	8.10	9.46	12.54
100	9.77	11.14	14.17	9.16	10.45	13.24	8.97	10.25	13.02	8.50	9.73	12.43
150	9.62	10.96	13.80	9.33	10.65	13.54	9.16	10.45	13.16	8.77	10.02	12.73
200	9.76	11.08	13.90	9.55	10.87	13.70	9.35	10.65	13.45	9.17	10.40	13.06
250	9.83	11.18	13.87	9.69	10.99	13.80	9.57	10.83	13.52	9.34	10.57	13.20
500	10.07	11.34	14.08	9.90	11.17	14.03	9.86	11.15	13.81	9.78	11.05	13.67
THE M-TAR MODEL: Φ^*_μ												
50	11.07	12.69	16.47	10.03	11.54	15.11	9.64	11.10	14.60	8.90	10.26	13.44
100	10.74	12.16	15.18	10.20	11.60	14.64	9.98	11.34	14.25	9.52	10.81	13.72
150	10.55	12.00	14.87	10.15	11.51	14.23	9.93	11.29	14.19	9.66	10.99	13.74
200	10.43	11.78	14.56	10.14	11.49	14.40	9.97	11.26	14.10	9.75	11.05	13.83
250	10.39	11.70	14.43	10.10	11.43	14.32	10.00	11.33	14.11	9.84	11.12	13.82
500	10.27	11.60	14.45	10.15	11.47	14.22	10.09	11.37	14.03	10.05	11.31	14.07

Table 8. Distribution for the F-Statistic for the Null Hypothesis, $\rho_1 = \rho_2 = 0$, in the 5-variable case

T	LAGGED CHANGES											
	5 LAGS			6 LAGS			7 LAGS			8 LAGS		
	90%	95%	99%	90%	95%	99%	90%	95%	99%	90%	95%	99%
THE TAR MODEL: Φ_μ												
50	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
100	8.29	9.50	12.19	7.94	9.11	11.66	7.66	8.83	11.38	7.38	8.51	11.05
150	8.67	9.88	12.48	8.40	9.61	12.13	8.26	9.41	11.90	8.01	9.12	11.64
200	8.99	10.29	12.92	8.77	9.96	12.66	8.66	9.84	12.36	8.47	9.64	12.19
250	9.19	10.44	13.01	9.07	10.30	12.96	8.95	10.21	12.76	8.79	9.98	12.39
500	9.77	11.03	13.80	9.69	10.96	13.56	9.57	10.83	13.51	9.49	10.78	13.44
THE M-TAR MODEL: Φ^*_μ												
50	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
100	9.26	10.53	13.35	8.94	10.23	12.83	8.72	9.96	12.63	8.40	9.65	12.30
150	9.43	10.68	13.41	9.17	10.43	13.15	9.05	10.25	12.92	8.81	10.01	12.62
200	9.60	10.91	13.55	9.37	10.64	13.42	9.25	10.48	13.11	9.03	10.23	12.79
250	9.69	11.02	13.72	9.48	10.72	13.30	9.44	10.68	13.32	9.26	10.49	13.06
500	9.94	11.21	13.94	9.83	11.14	13.80	9.74	11.06	13.73	9.67	10.92	13.59

Note: NA indicates not available. We do not provide the critical values for the model with more than 5 lags using only 50 observations.