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## On the Calculation of the Exponential Bound Parameter for Phase Quantized 8-PSK

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Abstract\_The exponential bound parameter is numerically evaluated for 16-zone quantized 8-PSK for signal vectors bisecting decision regions and for signal vectors lying on decision boundaries. For decision regions of equal span, signal vectors lying on decision boundaries are superior, confirming the result of Parsons and Wilson. Signal vectors bisecting decision regions are superior if span of decision regions not containing signal vectors is optimized. Using the partial derivative, it is shown that the optimal configuration depends on SNR. Furthermore, the optimal configuration is not necessarily one in which the decision boundaries are straight lines.

#### I. INTRODUCTION

During the past decades, forward error correcting codes have become widely accepted for digital communications over additive white Gaussian noise (AWGN) channels, particularly satellite channels, which are power and bandwidth limited. More recently, the need for spectral efficiency on these channels has motivated the development of spectrally efficient coding schemes, such as trellis coded modulation (TCM), which use non-binary signaling. Often there is the additional requirement of constant envelope signaling, leading to the use of 8, 16, or 32 PSK. Theoretical predictions of information rates on AWGN channels generally assume the use of coding, and maximum likelihood detection using soft decisions. This in turn assumes a continuous signal vector space for the receiver; however, the design of practical decoders requires that the receiver space be quantized in some way.

Phase-quantized receivers are practical to build, and are an intuitively logical choice for use with PSK signaling. The merits of designing quantizers to optimize  $R_{\theta}$  have been well established [1]. Parsons and Wilson [2] have investigated the design of phase only quantizers to optimize  $R_{\theta}$  for PSK signaling. The purpose of this paper is to expand on the work of Parsons and Wilson, and to show that additional options exist for increasing  $R_{\theta}$ .

#### II. THE EXPONENTIAL BOUND PARAMETER

It has been shown [1,3,4] that the attainable performance of

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codes using outputs from a quantized channel is bounded by:

$$P_e \le C_r 2^{-N[R_0 - R]} \tag{1}$$

provided that  $R_0 > R$ . Here, N is the block length of the code, R is the number of data bits per symbol,  $R_0$  is the exponential bound parameter, and  $C_R$  is an empirically determined constant.

For a discrete channel having M equiprobable inputs  $\{s_i\}$ , and a finite set of outputs  $\{z_j\}$ ,  $R_0$  is calculated from the transition probabilities  $P(z_i|s_i)$  as follows:

$$R_0 = -\log_2 \left\{ \frac{1}{M^2} \sum_{j} \left[ \sum_{i=0}^{M-1} \sqrt{P(z_j | s_i)} \right]^2 \right\}$$
 (2)

Quantization degrades the channel but is often a practical necessity. The degradation is minimized when the quantizer is designed to maximize the exponential bound parameter.

The exponential bound parameter may be optimized by applying Lee's criterion [5], which states that if  $\rho$  is a point on the boundary between two decision regions  $D_a$  and  $D_b$  of an optimal quantizer then it is necessary that:

$$\sum_{m=0}^{M-1} \left[ \frac{1}{\sqrt{P(a|m)}} \sum_{i=0}^{M-1} \sqrt{P(a|i)} \right] f(r|m)$$

$$= \sum_{m=0}^{M-1} \left[ \frac{1}{\sqrt{P(b|m)}} \sum_{i=0}^{M-1} \sqrt{P(b|i)} \right] f(r|m)$$
 (3)

where f(x|m) is the probability density function of the received vector given that signal m was transmitted, P(a|m) is the probability that the quantizer will select  $D_a$ , given that m was transmitted, and P(b|m) is the probability that the quantizer will select  $D_b$ , given that m was transmitted. Lee's criterion indicates a local maximimum of  $R_0$ , and is a necessary but not sufficient for a global maximimum.

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### III. EXPONENTIAL BOUND PARAMETER FOR PHASE ONLY QUANTIZERS

Parsons and Wilson [2] have investigated the design of phase only quantizers, for M-ary PSK with M=4, 8, and 16; using quantizers of M, 2M and 4M zones, and have shown that Lee's criterion is met by quantizers in which the signal vectors lie on boundaries of decision regions as shown in Fig. 1, as opposed to quantizers in which the signal vectors bisect decision regions as shown in Fig. 2. This result is derived for 16 sector 8-PSK and then extended to 32-sector 8-PSK.

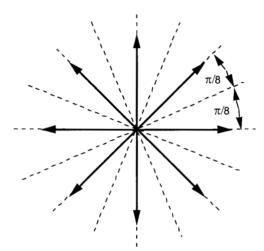


Fig. 1. First configuration for 16-sector 8-PSK.

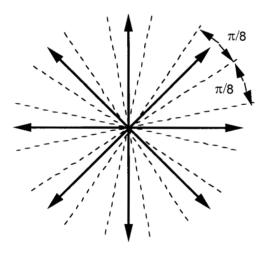


Fig. 2. Second configuration for 16-sector 8-PSK

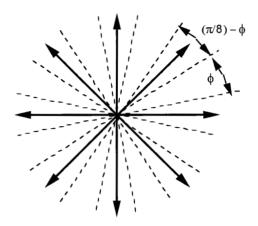


Fig 3. Third configuration for 16-sector 8-PSK.

As Lee's criterion does not guarantee a global opimimum, further optimization of  $R_{\theta}$  was sought and obtained using the configuration of Fig. 3. In this configuration the 8 sectors which encompass a signal vector have span of  $\phi$ , the 8 which do not, have span of  $\frac{\pi}{8}$  -  $\phi$ . The optimal value of  $\phi$  depends on the signal-to-noise ratio, and by selecting an appropriate value of  $\phi$ , it is possible to make  $R_{\theta}$  for configuration 3 exceed  $R_{\theta}$  for the configuration 1. Fig. 4 shows  $R_{\theta}$  for the configuration 3 as a function of  $\phi$  for  $\frac{E_{s}}{N_{\theta}} = 9$ , 10, and 11dB. Fig. 5 shows  $R_{\theta}$  as a function of  $\frac{E_{s}}{N_{\theta}}$  for the three configurations of 16-sector 8-PSK. Here,  $\phi$  is chosen to optimize  $R_{\theta}$  at  $\frac{E_{s}}{N_{\theta}} = 10dB$ . Note that if configuration 3 is optimized, it has the greatest value of  $R_{\theta}$  and the difference between configuration 3 and either 1 or 2 is

When  $R_0$  is optimized over  $\phi$ , which can be accomplished by calculating  $\frac{d}{d\phi}R_0$ , we find that Lee's criterion is not met at all points on the boundaries. Thus we suspect that further optimization would be obtained from a configuration in which the decision boundaries are not necessarily straight lines, although obtaining the exact configuration may be analytically intractable, and the gains minimal.

greater than that between 1 and 2.

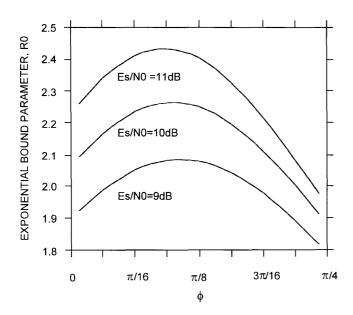


Fig. 4. Optimization of  $R_0$  for 16-sector 8-PSK.

#### 2.6 8 left to right: EXPONENTIAL BOUND PARAMETER, 2.5 configuration of figures 1,2 and 4 2.4 2.3 2.2 2.1 12 10 11 9 Es/N0 (dB)

Fig. 5.  $R_0$  for 3 configurations of 16-sector 8-PSK.

#### V. CONCLUSION

We have looked at the theoretical optimization of  $R_0$  for phase quantized PSK signaling and have also calculated numerical values of  $R_0$  for a few useful configurations. For 16sector 8-PSK, the optimal configuration is not necessarily one in which the decision boundaries are straight lines. With the constraint that the decision boundaries be straight lines, the optimal configuration is one in which half of the decision boundaries contain signal vectors, while the span of the null decision regions is optimized for SNR. For 32-sector 8-PSK, the value of  $R_0$  differs insignificantly among a number of configurations, and is in fact very close to the value of  $R_0$  for Therefore, we conclude that in the unquantized phase. construction of systems to implement phase-quantized 8-PSK, minute gains obtainable by fine adjustments in decision boundaries of the quantizer will be overshadowed by practical considerations in hardware development.

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