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Abstract- **In this paper we introduce a new switched order statistics CFAR test (SW-OS) for detecting a radar target in the presence of nonhomogeneous clutter and/or multiple interfering targets situation. Whereas a switching CFAR test (S-CFAR) was recently proposed in the literature for addressing a similar background scenario, unlike the S-CFAR test, the test proposed here does not utilize the test cell statistic in classifying the cells surrounding the test cell as homogeneous or not. The SW-OS test has some similarity to the selection and estimation (SE) test, which was co-authored by the second author of this paper, but is simpler to design. Probability of detection performance results obtained for Rayleigh clutter and Rayleigh target indicate that the SW-OS performs nearly as good as an order statistic test (OS) in the homogeneous background condition and performs much better than OS under the presence of many interfering targets. When compared to S-CFAR, SW-OS performs slightly worse in the homogeneous background, but performs better under the condition of many interfering targets.**

Index terms: **CFAR detection, Order statistic, switched CFAR test.**

I. INTRODUCTION

Procedures for constant false alarm rate (CFAR) detection of radar targets in clutter have been investigated extensively since late1960's (see [1], [2] for a review). Variations of cell averaging CFAR tests do not perform well in multiple interfering targets or in step-clutter transitions [1]-[3]. Almost all the tests that perform reasonably well in a variety of background clutters are based on order statistics of clutter cells surrounding the test cell [2]. Some of them have appeared in the literature as censored mean level detector, variably trimmed mean, Ll CFAR, MAX family of OS-CFAR, selection and estimation (SE) test, an intelligent CFAR processor not based on order statistic, distributed CFAR test, and OS-CFAR detector for Weibull clutter [4]-[12]. Even though ordering of samples with currently available economical processors should not be a computational burden, the switching CFAR test (S-CFAR) was proposed recently as a new and competitive test that does not require ordering of samples [13]. However, the S-CFAR test utilizes the test cell sample along with the samples from the reference (surrounding) cells in order to classify the background, i.e, the reference cells, as from an homogeneous clutter or not. In the second stage of the decision process, the test cell sample needs to be used again in order to make a determination of the presence or the absence of a target in the test cell [13]. While the use of test cells in both stages introduces statistical dependency, the author of this paper was able to derive analytical expression for detection probability of Rayleigh-target in Rayleigh-clutter without too much difficulty. However, philosophically speaking, it must be possible to classify the background clutter as homogeneous or not, without having to take into account the test cell sample. Hence, in this paper, we revisit the approach taken in [8], wherein the SE test was proposed. Whereas both SE and the switched order statistic (SW-OS) method proposed here depend on the ordered values of the samples of the reference cells, the SW-OS is simpler to design.

In section II we propose the SW-OS test and derive expression for the probability of detection of a Rayleigh target in homogeneous Rayleigh clutter. In section III, the performances of a simple OS test, S-CFAR and SW-OS test are compared for homogeneous and interfering targets situations. These results indicate superior performance of SW-OS test in interferers involving many targets. Conclusions from this study are presented in section IV.

II SWITCHED ORDER STATISTICS TEST

Let $X_1, X_2,.., X_N$ denote the samples of the reference cells coming out of a radar signal processor, where N indicates the total number of reference cells. Let Y_i denote the i^{th} order statistic of the samples, $X_1, X_2,.., X_N$, viz., Y_i is the i^{th} largest value when $X_1, X_2,.., X_N$ are ordered in an ascending order, $i = 1,2,.., N$. By appropriately choosing two order statistics, Y_l and Y_k, $l > k$, the proposed switched order statistic (SW-OS) test classifies the reference cell as homogeneous (hypothesis H_2) or nonhomogeneous (hypothesis H_1) according to the following rule:

$$Y_l \underset{H_2}{\overset{H_1}{\gtrless}} \beta Y_k, \tag{1}$$

where $\beta \geq 1$ is a constant. In designing the test, β needs to be determined along with l and k. When all the reference samples are from homogeneous background, with high probability Y_l will be less than βY_k, whereas the converse will be true when there are significant number of interferers in the reference cells. Since a simple OS test based on Y_l, with $l \approx \frac{3}{4}N$ will provide a reasonable probability of detection in homogeneous background, while tolerating up to $N/4$ interfering targets, this choice of l seems reasonable for the SW-OS test also. The choice of k depends on whether the situation of a step clutter transition in the middle of the reference window is important. Assuming that this is the case, a choice of $k \approx \frac{N}{2}$ is reasonable. Having fixed l and k through qualitative requirements, an approximate value of β can be obtained by requiring that the probability of falsely declaring a nonhomogeneous background, given that the reference samples had actually come from the homogeneous clutter, be below a number. This number could be chosen to be in the range, say (0.0001, 0.001). Of course, the final choice for β is arrived by looking at the detection performance in various environments. Finally, if we denote the test cell sample as Z, the decision rule for declaring the presence of a target in the test cell is given by

Decide target present iff

$$\begin{cases} Z \geq c_1 Y_l \text{ when } H_2 \text{ is decided in (1)} \\ Z \geq c_2 Y_k \text{ when } H_1 \text{ is decided in (1)} \end{cases}, \qquad (2).$$

where c_1 and c_2 are two positive constants. Using the customary definition of probability of false alarm (P_F) as the probability of the test declaring the presence of a target in the test cell, when it is in fact absent, an expression for P_F for the test (2), as a function of l, k, β, and c_1 and c_2 can be obtained. Since other parameters, except c_1 and c_2, could be fixed as stated earlier, the variations of P_F, as c_1 and c_2 are varied, could be studied. However, to make the design of the test simple, in the sequel we assume that $c_1 = c_2$. In the remainder of the paper, we assume that the clutter cells as well as the target returns have Rayleigh amplitudes. Moreover, the reference samples are statistically independent and are all independent of the test sample.

2.1 Probability of Detection in Homogeneous Rayleigh clutter.

First, we determine the probability, P_b, that the test (1) decides hypothesis H_1, given that all the reference samples are from homogeneous Rayleigh clutter, viz., $X_1, X_2, .., X_N$ are all i.i.d exponential with identical mean, which can be assumed to be unity, without any loss of generality. Using the general theory of order statistic, the joint density of order statistics Y_k, Y_l can be written as [14]:

$$f_{k,l}(y_k, y_l) = \binom{N}{l}\binom{l}{k-1}(l-k+1)(l-k) \cdot F^{k-1}(y_k)\left(F(y_l) - F(y_k)\right)^{l-k-1} \cdot \left(1 - F(y_l)\right)^{N-l} f(y_k) f(y_l), \quad 0 < y_k < y_l \qquad (3).$$

where $f(x) = e^{-x}$ and $F(x) = 1 - e^{-x}$, $x \geq 0$. Use of binomial expansion and simplification of (3) yields, with $0 < y_k < y_l$,

$$f_{k,l}(y_k, y_l) = \binom{N}{l}\binom{l}{k-1}(l-k+1)(l-k) \cdot \sum_{p=0}^{k-1}\sum_{m=0}^{l-k-1}\binom{k-1}{p}\binom{l-k-1}{m}(-1)^{p+m} \cdot \left\{e^{-(\gamma_1 y_k + \gamma_2 y_l)}\right\}, \quad 0 < y_k < y_l \qquad (4).$$

where $\gamma_1 = p + l - k - m$, $\gamma_2 = (N + m - l + 1)$. Hence,

$$P_b = P(Y_l > \beta Y_k \mid H_2) = \int_0^{\infty}\int_{\beta x}^{\infty} f_{k,l}(x, y)\, dy\, dx \qquad (5).$$

Using (4) in (5), and doing the integration, P_b can be obtained as the right hand side of (4), with the expression within the curly brackets replaced by $\left\{\frac{1}{\gamma_2(\gamma_1 + \gamma_2 \beta)}\right\}$.

Numerical evaluation yields P_b = 2.908E-4, when l = 19, k = 10, β = 11,.

In homogeneous clutter, the probability of detection for the test (2) can be obtained from

$$\begin{aligned} P_D = {} & P\left(Z \geq c_1 Y_l \,\middle|\, \text{decide } H_2, \text{true } H_2, \text{target present}\right) \\ & \cdot P\left(\text{decide} H_2 \,\middle|\, H_2\right) \\ & + P\left(Z \geq c_1 Y_k \,\middle|\, \text{decide } H_1, \text{true } H_2, \text{target present}\right) \\ & \cdot P\left(\text{decide} H_1 \,\middle|\, H_2\right) \end{aligned} \qquad (6).$$

Upon denoting the two summands on the right hand side of (6) as T_1 and T_2,

$$T_1 = \int_0^{\infty} P\left(Y_l < \min\left(\frac{z}{c_1}, \beta Y_k\right) \middle| Z = z, \text{true } H_2 \right) \cdot f_Z\left(z \mid \text{target present}\right) dz \quad (7).$$

$$T_2 = \int_0^{\infty} P\left(Y_k < \min\left(\frac{z}{c_2}, \frac{Y_l}{\beta}\right) \middle| Z = z, \text{true } H_2 \right) \cdot f_Z\left(z \mid \text{target present}\right) dz \quad (8),$$

where $f_Z\left(z \mid \text{target present}\right) = \frac{1}{\theta} e^{-\frac{z}{\theta}}, \quad z > 0$, and θ equals one plus target signal power to noise clutter power ratio (SNR). Upon denoting the probabilities inside the integral in (7) and (8), respectively, as P_1 and P_2,

$$P_1 = \int_0^{z/(\beta c_1)} \int_x^{\beta x} f_{k,l}(x,y)\, dy\, dx + \int_{z/(\beta c_1)}^{z/c_1} \int_x^{z/c_1} f_{k,l}(x,y)\, dy\, dx \quad (9).$$

$$P_2 = \int_0^{\beta z/c_2} \int_0^{y/\beta} f_{k,l}(x,y)\, dx\, dy + \int_{\beta z/c_2}^{\infty} \int_0^{z/c_2} f_{k,l}(x,y)\, dx\, dy \quad (10).$$

Using (4) and (6) through (10) and some simplification, a final expression for the probability of detection is obtained as the right hand side of (4) with the expression within the curly brackets replaced by

$$\left\{ \begin{array}{l} \dfrac{\beta - 1}{(\gamma_1 + \gamma_2 \beta)(\gamma_1 + \gamma_2)} + \dfrac{c_1}{\gamma_1 (\gamma_1 + \gamma_2)(\theta(\gamma_1 + \gamma_2) + c_1)} \\ - \dfrac{\beta^2 c_1}{\gamma_1 (\gamma_1 + \gamma_2 \beta)(\theta(\gamma_1 + \gamma_2 \beta) + \beta c_1)} + \dfrac{\theta}{\gamma_2 (\theta(\gamma_1 + \gamma_2 \beta) + c_2)} \end{array} \right\}.$$

The expression for the probability of false alarm, P_F, is simply the expression for P_D, with θ replaced by 1 (corresponding to SNR of zero).

III PROBABILITY OF DETECTION PERFORMANCE OF SW-OS, OS, AND S-CFAR TESTS

In this section we analyze the detection performances of SW-OS, OS, and S-CFAR tests. The design parameters of all tests that guarantee a specified false alarm probability in homogeneous clutter can be obtained through appropriate analytical expressions, viz., (41) in [1] for OS test, (20) in [12] for S-CFAR test, and by the procedure stated below equation (10), for SW-OS test. Once appropriate design parameters are obtained, the detection performances of various tests under nonhomogeneous background are obtained by generating independent exponential samples of $X_1, X_2, .., X_N$, with N_I of them having a mean value of one plus interfering target power to noise (low clutter) power ratio (INR) and the rest having a mean value of 1. The target signal Z is generated to be an independent exponential sample with mean equal to SNR. Then, appropriate tests are implemented using the generated samples to determine if these tests detect the presence of target correctly or not. A count of the number of times a test detects the target, divided by the total number of times the samples are generated, provides an estimate of the probability of detection for that test. The number of reference cells, N, equals 24. We take k to be 10 so that SW-OS would tolerate a little more than $N/2$ interfering targets. For l, two choices, 18 and 19 were considered. l is fixed at 19, because this choice provides a slightly larger detection probability when seven interfering targets are present in the reference cells (see Fig. 6).

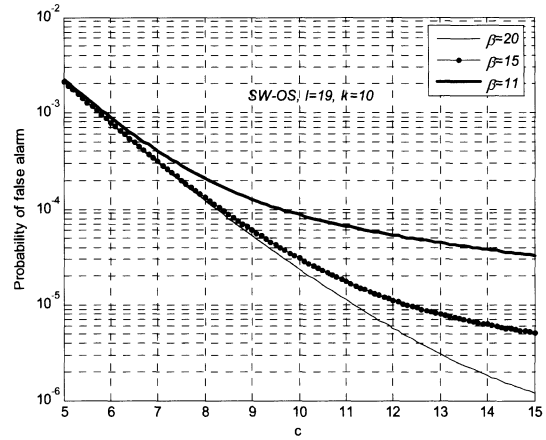

Fig 1. SW-OS probability of false alarm.

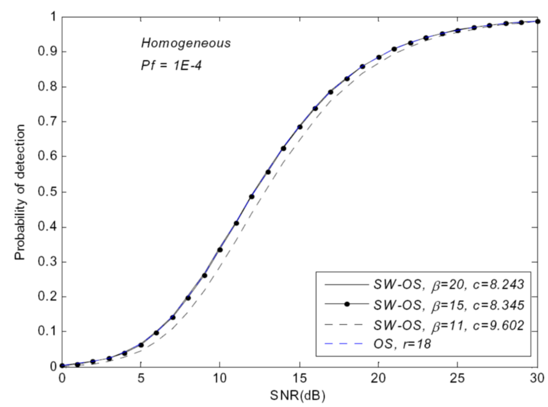

Fig 2. Probability of detection in homogeneous background, SW-OS, OS.

Fig. 1 shows the variation of false alarm probability as a function of c, for $l = 19$, $k = 10$, and three values of β. As explained earlier, β is chosen by targeting a reasonable value of P_b. In Fig. 2 we show the probability of detection of SW-OS at $P_F = 10^{-4}$, for homogeneous clutter and for three values of β. As mentioned earlier, for the sake of simplicity of design, we consider only the case, $c_1 = c_2 = c$. It is observed that $\beta = 11, c = 9.602$ provides a slightly lower detection probability when compared to $\beta = 20, c = 8.243$. The detection performance of SW-OS, for $\beta = 11, c = 9.602$, although below that of OS, for r=18, is reasonably close to the performance of the OS test.

Fig. 3 shows the detection performance for the same parametric values in Fig. 2, but with five interfering targets, with interfering target power to noise ratio (INR) equal to SNR of the target. As before, $\beta = 11, c = 9.602$ provides a slightly smaller detection probability when compared to the other combinations and the OS test. Fig. 4 shows the probabilities of detection of SW-OS and OS tests under seven interferers situation. The OS test was designed with a rank order of r = 18, N = 24, which implies that this test could tolerate only up to six interfering targets. We can observe that the OS test provides a detection probability of only 0.43, as the SNR asymptotically approaches infinity. Superiority of the SW-OS test with $\beta = 11, c = 9.602$ over other two combinations of (β, c) values and the OS test can be observed. For 11 interfering targets situation (Fig. 5), SW-OS completely outperforms OS. Since the SW-OS was designed to reasonably differentiate between the homogeneous and nonhomogeneous situation, corresponding to a large number of interfering targets approximating half the reference window size, the switched order statistic test (SW-OS) is able to handle the case of a large number of interfering targets much more efficiently than a simple OS test. Fig. 6 shows that l = 19 is a better choice than l = 18, when seven interfering targets are present (of course, each case is designed to provide false alarm probability of 10^{-4}). Figs. 7, 8, 9 and 10 show detection performance comparison of SW-OS and S-CFAR. S-CFAR was designed with $(\alpha = 0.5, \beta = 12.293)$ in order to achieve the same false alarm probability of 10^{-4} [13]. Although S-CFAR provides a larger detection probability than SW-OS in five interfering targets situation (and only a slightly larger detection probability in homogeneous background), SW-OS performs better than S-CFAR under 9 and 11 interfering targets.

IV CONCLUSIONS

In this paper we proposed a switched order statistic CFAR detector for detecting a radar target in the presence of a significant number of interfering targets or varying clutter background. For detecting a Rayleigh target in Rayleigh clutter, the analysis performed shows that SW-OS performs much better than a simple OS test and the S-CFAR for multiple interfering targets.

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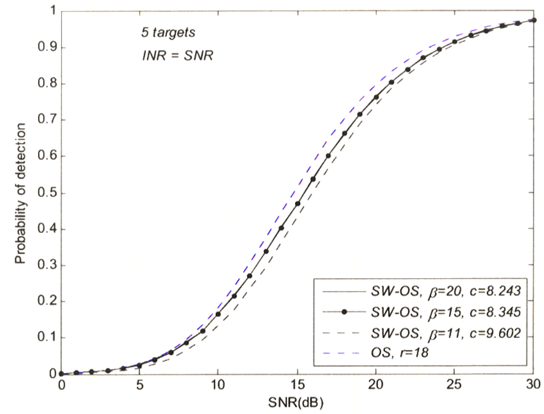

Fig 3. Probability of detection for SW-OS and OS, five interfering targets.

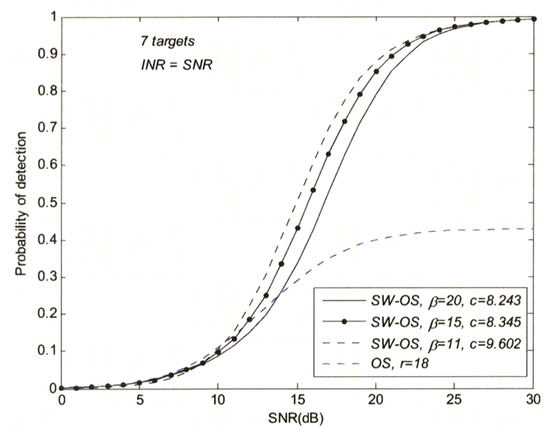

Fig 4. Probability of detection for SW-OS and OS, twelve interfering targets.

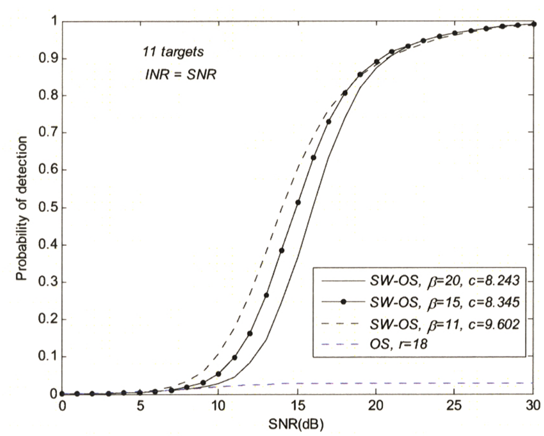

Fig 5. Probability of detection for SW-OS and S-CFAR, five interfering targets.

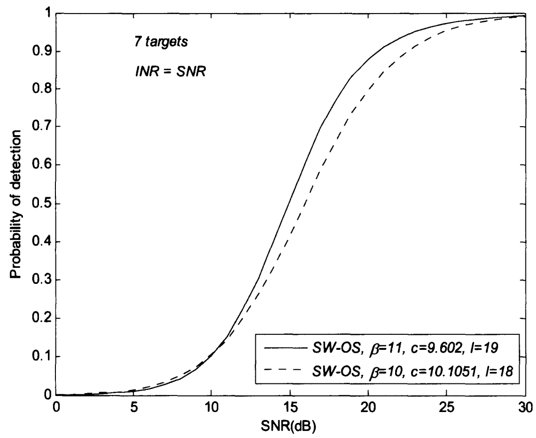

Fig 6. Probability of detection for SW-OS, seven interfering targets.

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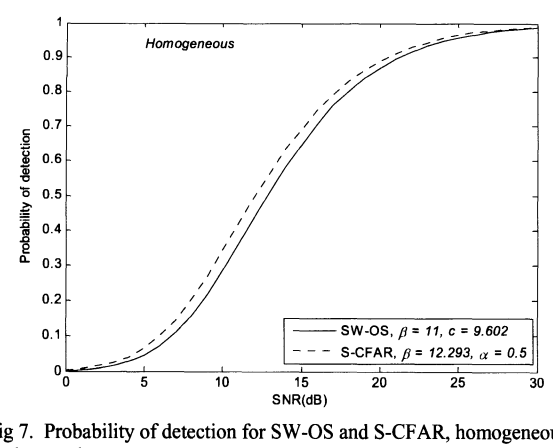

Fig 7. Probability of detection for SW-OS and S-CFAR, homogeneous background.

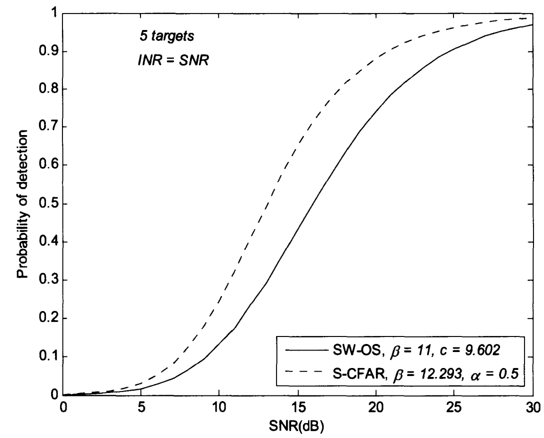

Fig 8. Probability of detection SW-OS and S-CFAR, five interfering targets.

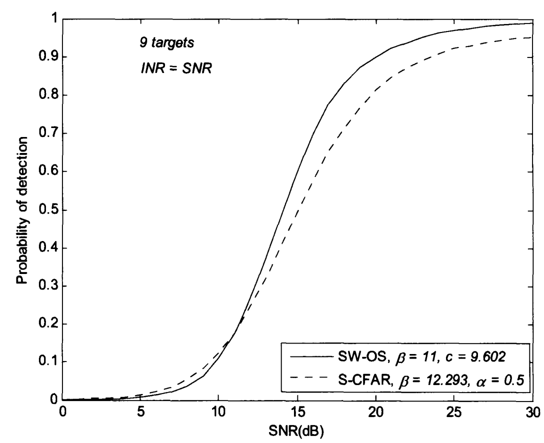

Fig 9. Probability of detection SW-OS and S-CFAR, nine interfering targets

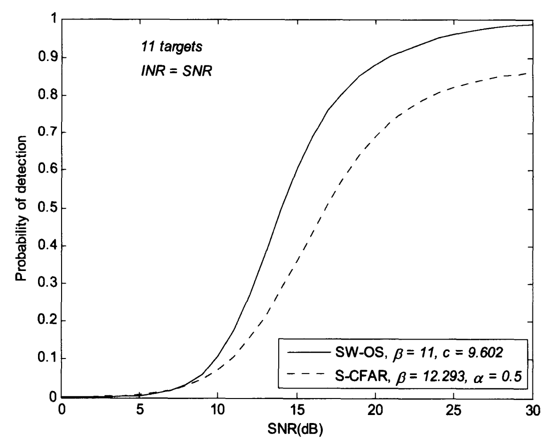

Fig 10. Probability of detection SW-OS and S-CFAR, eleven interfering targets.