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Department of Electrical and Computer Engineering

1-2007

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Abstract—**We compare the signal-to-noise ratio performances of an amplify-forward relay link and a direct link in a wireless sensor network. For a slow Rayleigh fading channel, an exact expression for the probability that the SNR of an amplify-forward relay link exceeds the SNR of a direct link is obtained. For a Rician fading channel, an upper bound for the corresponding probability is obtained. Numerical results indicate that, among the two fading channels considered, Rayleigh fading is more detrimental to the SNR performance of the amplify-forward relaying scheme.**

Keywords- cooperative relaying; amplify-forward; Rayleigh fading; Rician fading.

I. INTRODUCTION

In cooperative relaying, the availability of relay channels between a source (sensor) and a destination is exploited by appropriately combining signals arriving via various relay channels. A summary of cooperative diversity along with a chronological order of developments is provided by Laneman *et al.* in [1]. In a two-part paper, Sendonaris and Aazhang have provided the concept and the implementation aspects of cooperative diversity for wireless networks of mobile users [2]-[3].

Laneman *et al.* propose cooperative diversity for relaying in wireless sensor networks [1]. Their analysis primarily considered a single relay terminal (R) helping the communications between a source (S) and a destination (D), even though they point out possible extensions to situations involving multiple relays. They had formulated two schemes, namely, amplify and forward (AF) and decode and forward (DF) under fixed relaying procedures and two other schemes, namely, selection relaying and incremental relaying under adaptive relaying procedures. In selection relaying, depending on the quality of the link between the source and the relay, a decision as to whether the relay would retransmit the message it had received from the source would be made. When the quality is bad, the relay does not transmit but conserves it's transmit power and lets the destination to decode the direct reception from the source. In AF, the relay simply amplifies its received signal from the source and then transmits it to the destination. Since background noise (and any interference) is picked up along with the signal by the relay, its amplification process amplifies the noise in addition to the received signal. Despite this noise amplification, the overall gain in the performance (i.e., channel capacity or outage performance) of the AF relay scheme is documented in the literature [1], [4].

In this paper, we present an analysis that shows when a direct transmission from S to D may be preferable to the signal from the AF relay scheme. We are not combining the signals from the direct path and the relay path, but we are simply trying to determine, given the power constraints, when a relaying may or may not be useful.

II. ACHIEVABLE SNR IN S-TO-D AND AF SCHEMES IN FADING CHANNELS

Let P_T denote the transmit power from the source and define the channel gains (coefficients) in different links as follows: $\alpha_1, \alpha_2, \alpha_d$ for S to R, R to D, and S to D, respectively. If the power amplification gain in the AF scheme is g and if the noise power picked up by the relay receiver (destination receiver) is denoted as N_1 (N_2), then the effective received powers at the destination produced by AF and direct reception are given by

$$P_{R,AF} = \alpha_2 \{ g(S_1 + N_1) \}, \tag{1}$$

$$P_{R,d} = \alpha_d P_T , \tag{2}$$

where S_1 is the received power at the relay and is given by

$$S_1 = \alpha_1 P_T . \tag{3}$$

The effective SNR (i.e., the ratio of the signal power from the signal term in (1) and the noise power, which is the sum of the amplified noise power of the relay node, as received at the destination, and the noise power picked up by the destination node) of the AF relay path is given by

$$SNR_{AF} = \frac{g\,\alpha_1 \alpha_2 P_T}{g\,\alpha_2 N_1 + N_2}. \quad (4)$$

Similarly, for the direct path

$$SNR_d = \frac{\alpha_d P_T}{N_2}. \quad (5)$$

Then, from (4) and (5), for fixed channel coefficients, AF is better than the direct link (in terms of SNR), if $(g\,\alpha_1\alpha_2 - \alpha_d) > g\,\alpha_2\alpha_d\left(\frac{N_1}{N_2}\right)$. With the reasonable assumption of $N_1 = N_2$, AF has better SNR than the direct link when the following inequality is satisfied:

$$g\,\alpha_2(\alpha_1 - \alpha_d) > \alpha_d. \quad (6)$$

With a reasonable assumption of $\alpha_1 > \alpha_d$ (which corresponds to the signal power at R being larger than the signal power at D from a direct reception), it is clear that (6) can be satisfied by adequately increasing the amplifier gain g at R. That is, if there were no power limitation at R, increasing the power at R will eventually make the relay link better than the direct link. This happens irrespective of the fact that the noise picked up by the relay is also amplified by the relay (if $N_1 \neq N_2$, then α_1 has to be $> \alpha_d \frac{N_1}{N_2}$ for this to be true).

If g is unbounded, could we still expect a similar result in a fading channel? Before we answer this question, we first observe that the probability, $P(g\,\alpha_2\alpha_1 > (1+g\,\alpha_2)\alpha_d)$, which equals $P\left(\frac{g\,\alpha_2}{1+g\,\alpha_2}\alpha_1 > \alpha_d\right)$, is a monotonic increasing function of g, since all the channel gains are positive. Hence, an upper bound to this probability is achieved at $g = \infty$. The corresponding upper bound is given by $P(\alpha_1 > \alpha_d)$.

A. *Rayleigh Fading Channels*

Consider a slow Rayleigh fading channel where the signal power received during the transmission of a symbol (or a packet) can be considered to be a sample of an exponential random variable (corresponding to a Rayleigh amplitude). Hence, let $\alpha_i, i = 1,2,d$ be independently distributed as exponential with mean values, $\lambda_i, i = 1,2,d$, respectively. For a given value of g, the probability that the SNR of AF is better than that of the direct link is given by (upon rearranging (6))

$$I = P(g\,\alpha_1\alpha_2 - \alpha_d(1+g\,\alpha_2) > 0). \quad (7)$$

Using the independence of various links and by conditioning on the variable, α_2, (7) can be evaluated as

$$I = \int_0^{\infty} P(g\,\alpha_1 x - \alpha_d(1+gx) > 0 \,|\, \alpha_2 = x) f_{\alpha_2}(x)dx \quad (8)$$

where $f_{\alpha_2}(x)$ is the probability density function of α_2. After some manipulations of the integral (8), we get (see Appendix)

$$I = \frac{1}{1+\left(\frac{\lambda_d}{\lambda_1}\right)}\left(1 - B\,e^B\,E_1(B)\right), \quad (9)$$

where

$$B = \frac{\lambda_d/\lambda_1}{g\,\lambda_2(1+\lambda_d/\lambda_1)}, \quad (10)$$

and $E_1(B)$ is the exponential integral defined by [5]

$$E_1(B) = \int_B^{\infty}\frac{e^{-z}}{z}dz. \quad (11)$$

As $g \to \infty, B \to 0,$ and $B\,E_1(B) \to 0$. Hence, the probability I is upper bounded by $I \leq \frac{1}{1+(\lambda_d/\lambda_1)}$.

That is, the probability that the SNR of AF exceeds the SNR of the direct link cannot approach 1 even if the amplification at the relay is infinite (unless λ_d/λ_1 is arbitrarily close to 0). This result has some similarity to the well known result in an adaptive power control scheme, which attempts to compensate for deep channel fades: in Rayleigh fading, the capacity of such an adaptive scheme is zero. Since AF relaying involves additional hardware and energy consumption, one can reasonable argue that the AF should be preferred only if the probability I exceeds some value, say 3/4? We will examine specific numerical solution of I, as a function of g, in the next section.

B. *Rician Fading*

Probability that the SNR of AF is better than the SNR of the direct link is given by the probability of the event specified by (6). Numerical calculation of this probability, when all the amplitudes of channel coefficients (amplitude coefficient is square root of the power coefficient) fade according to Rician distributions, is possible but complicated. Instead, we will compute the upper bound, which is attained for infinite amplification gain at a relay. Let $Z_1 = \sqrt{\alpha_1}, Z_d = \sqrt{\alpha_d}$, where Z_1, Z_d are distributed as Rice with parameters (A_1, σ_1^2), (A_d, σ_d^2), respectively [6]. Here A_i represents the amplitude of the line of sight (los) component of Rice distribution and σ_i^2 represents the average power of the fading component, i =1, d. Hence, the upper bound, $P_{u\,Rice} = P(\alpha_1 > \alpha_d)$ is given by

$$P_{u\,Rice} = 1 - \frac{\gamma^2}{1+\gamma^2}\left(1 - Q\left(\sqrt{b}, \sqrt{a}\right)\right) - \frac{1}{1+\gamma^2} Q\left(\sqrt{a}, \sqrt{b}\right), \quad (12)$$

where $a = \frac{\psi}{\gamma^2 + 1}$, $\psi = \frac{A_d^2}{\sigma_d^2}$,

$b = a r_d^2$, $r_d = \frac{A_1}{A_d}$ $\gamma = \frac{\sigma_1}{\sigma_d}$. When the los components vanish, Rice distributions become Rayleigh distributions and the ratio $\frac{\lambda_d}{\lambda_1}$ of section II (i) is equivalent to $\frac{1}{\gamma^2}$. Hence, the upper bound is a function of the three parameters, ψ, r_d and γ. A numerical study of the behaviors of this upper bound and the exact probability for the Rayleigh case is presented in the following section.

III. NUMERICAL RESULTS

Using MATLAB® we numerically evaluated the exact probability that the SNR of AF is greater than the SNR of direct link for the Rayleigh fading case and also the upper bound (Eq. 12) for the Rician fading case. Fig.1 shows the probability as a function of $g\ \lambda_2$, for Rayleigh fading, when the ratio of the average SNR of S-to-R and that of the direct link from S-to-D is 10 dB. As to be expected, this probability increases monotonically as the relay amplification gain g (more precisely, $g\ \lambda_2$) increases and reaches the maximum value of 1/(1+1/10) at $g = \infty$. In order to justify the additional hardware and energy expenses associated with relaying, a practical criterion may be to employ relaying only if this probability exceeds certain value, say 0.75. For $\frac{\lambda_1}{\lambda_d}$ of 10 dB, this probability is 0.75 or more if $g\ \lambda_2$ is at least 0.58 dB. When $\frac{\lambda_1}{\lambda_d}$ is only 5 dB, in order that this probability is 0.75 or higher, $g\ \lambda_2$ has to exceed 20.1 dB. We can observe a nonlinear dependence of g on $\frac{\lambda_1}{\lambda_d}$, with lower values of the latter dictating much larger values of the power gain at the relay, in order to achieve a specific relay link quality.

For Rician fading, the upper bound on probability, equation (12), is plotted against r_d for different values of ψ in Fig. 2 ($\gamma = 10$ dB) and in Fig. 3 ($\gamma = 0$ dB). When $r_d = 0$ dB and ψ approaches zero (i.e., ψ in dB approaches $-\infty$), the fading channel becomes Rayleigh and the upper bound in Fig. 2 matches with the asymptotic value in Fig. 1. With sufficient strength in the direct component of Rician amplitude in the S-to-R link, it is possible to achieve the upper bound of one to the probability. Certainly, the presence of direct component is beneficial to the relaying procedure. Fig. 3 corresponds to the case where the ratio of the average SNR of the fading component of S-to-R and that of S-to-D is one. When the ratio of the direct component amplitudes of S-to-R and S-to-D is one, irrespective of the relative strengths of the direct and the fading components, the upper bound to probability, which is achieved with infinite gain at the relay, becomes 0.5.

IV. CONCLUSION

In this paper, for an amplify-forward relaying scheme, we evaluated and studied the variation of the probability that the SNR of amplify-forward link exceeds the SNR of the direct link. While an exact expression for the probability was derived for the Rayleigh fading case, only an upper bound, corresponding to an infinite power gain at the relay, was calculated for the Rician fading case. Of the two fading channels, it is observed that the Rayleigh fading is more detrimental to the relaying scheme. Specific sensor network configuration and specific modulation / coding scheme may ultimately determine if amplify-forward relaying in certain paths in such a network.

APPENDIX

Here we show the steps that lead to the derivation of (9). Since $\alpha_1, \alpha_2, \alpha_d$ are all independent, the conditional probability function in the integrand of (8) becomes the unconditional probability, $P_1 = P\left(g\,\alpha_1 x - \alpha_d (1+gx) > 0\right)$.

If we let Y_1, Y_d to denote $g\,x\,\alpha_1, (1+g\,x)\,\alpha_d$, respectively, then Y_1, Y_d are independent and distributed as exponentials with means $m_1 = gx\lambda_1$, $m_d = (1+gx)\lambda_d$, respectively. Hence, $P_1 = P(Y_1 > Y_d) = \frac{1}{1 + \left(\frac{1+gx}{gx}\right)\left(\frac{\lambda_d}{\lambda_1}\right)}$. After substituting this expression for P_1 in (8) and executing simple algebraic manipulations, we get an equivalent expression for (8):

$$I = \int_0^\infty \frac{1 - \frac{\lambda_2 B}{x + \lambda_2 B}}{\left(1 + \frac{\lambda_d}{\lambda_1}\right)\lambda_2} e^{-x/\lambda_2} dx,$$

where B is defined in (10). Using the definition (11) in the evaluation of the above integral leads to (9).

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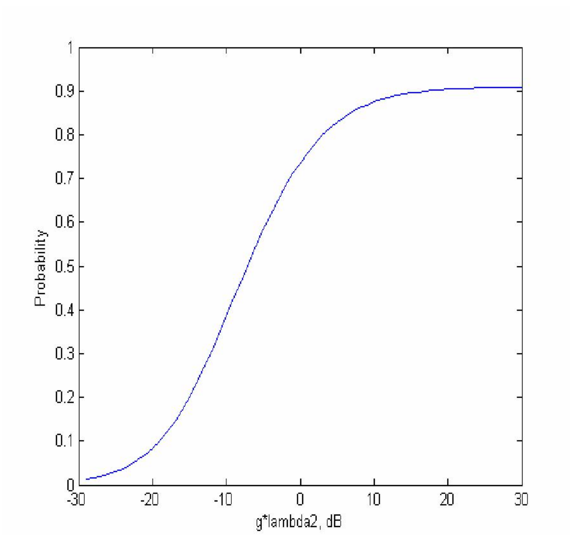

Fig. 1 Probability that SNR of AF is greater than the SNR of direct link, Rayleigh Fading

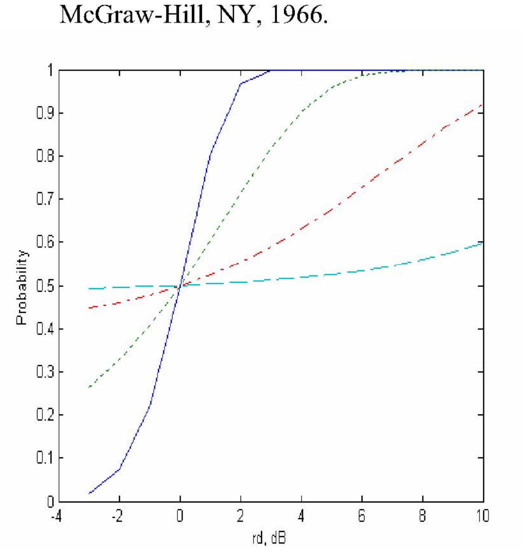

Fig. 3 Upper bound on probability, Eq. (12) Vs r_d dB, Rician fading, $\gamma = 0$ dB

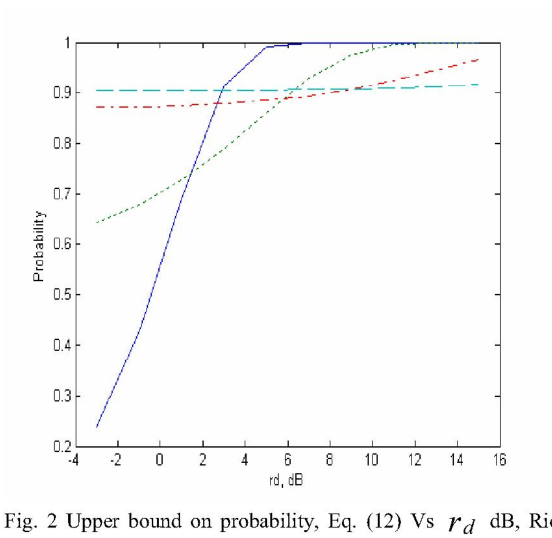

Fig. 2 Upper bound on probability, Eq. (12) Vs r_d dB, Rician fading, $\gamma = 10$ dB