



**Aalborg Universitet**

**AALBORG UNIVERSITY**  
DENMARK

## **A Discussion of Low Reynolds Number Flow for the Two-Dimensional Benchmark Test Case**

Weng, Miaocheng; Nielsen, Peter V.; Liu, Li

*Publication date:*  
2012

*Document Version*  
Publisher's PDF, also known as Version of record

[Link to publication from Aalborg University](#)

*Citation for published version (APA):*  
Weng, M., Nielsen, P. V., & Liu, L. (2012). *A Discussion of Low Reynolds Number Flow for the Two-Dimensional Benchmark Test Case*. Department of Civil Engineering, Aalborg University. DCE Technical reports No. 116

### **General rights**

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- ? Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- ? You may not further distribute the material or use it for any profit-making activity or commercial gain
- ? You may freely distribute the URL identifying the publication in the public portal ?

### **Take down policy**

If you believe that this document breaches copyright please contact us at [vbn@aub.aau.dk](mailto:vbn@aub.aau.dk) providing details, and we will remove access to the work immediately and investigate your claim.

# **A Discussion of Low Reynolds Number Flow for the Two- Dimensional Benchmark Test Case**

**M. C. Weng  
P. V. Nielsen  
L. Liu**





Aalborg University  
Department of Civil Engineering  
Group Name

**DCE Technical Report No. 116**

# **A Discussion of Low Reynolds Number Flow for the Two-Dimensional Benchmark Test Case**

by

**M. C. Weng  
P. V. Nielsen  
L. Liu**

December 2012

© Aalborg University

## Scientific Publications at the Department of Civil Engineering

**Technical Reports** are published for timely dissemination of research results and scientific work carried out at the Department of Civil Engineering (DCE) at Aalborg University. This medium allows publication of more detailed explanations and results than typically allowed in scientific journals.

**Technical Memoranda** are produced to enable the preliminary dissemination of scientific work by the personnel of the DCE where such release is deemed to be appropriate. Documents of this kind may be incomplete or temporary versions of papers—or part of continuing work. This should be kept in mind when references are given to publications of this kind.

**Contract Reports** are produced to report scientific work carried out under contract. Publications of this kind contain confidential matter and are reserved for the sponsors and the DCE. Therefore, Contract Reports are generally not available for public circulation.

**Lecture Notes** contain material produced by the lecturers at the DCE for educational purposes. This may be scientific notes, lecture books, example problems or manuals for laboratory work, or computer programs developed at the DCE.

**Theses** are monographs or collections of papers published to report the scientific work carried out at the DCE to obtain a degree as either PhD or Doctor of Technology. The thesis is publicly available after the defence of the degree.

**Latest News** is published to enable rapid communication of information about scientific work carried out at the DCE. This includes the status of research projects, developments in the laboratories, information about collaborative work and recent research results.

Published 2012 by  
Aalborg University  
Department of Civil Engineering  
Sohngaardsholmsvej 57,  
DK-9000 Aalborg, Denmark

Printed in Aalborg at Aalborg University

ISSN 1901-726X  
DCE Technical Report No. 116

# A Discussion of Low Reynolds Number Flow for the Two-Dimensional Benchmark Test Case

M. Weng, P. V. Nielsen and L. Liu  
Aalborg University

## Introduction.

The use of CFD in ventilation research has arrived to a high level, but there are some conditions in the general CFD procedure which do not apply to all situations in the ventilation research. An example of this is the turbulence models in Reynolds-averaged Navier-Stokes equations, i.e. (RANS) equations.

The flow in a ventilated room is generally assumed to be a fully developed turbulent flow, and this flow can be handled by most turbulence models. But in some areas of the room, including the occupied zone, a low Reynolds number flow can exist at a low room air supply velocity. Figure 1 shows measurements of the maximum velocity in the occupied zone of a room with mixing ventilation from a wall-mounted diffuser versus the air change rate. The flow is isothermal (Nielsen, 1992). Similarity principles state that any velocity, e.g. the maximum velocity in the occupied zone, is a linear function of the air change rate (or the supply velocity) when the flow is a fully developed isothermal turbulent flow. In Figure 1 this is the case for velocities larger than 0.25 m/s, but the figure indicates that the flow in the occupied zone is a low Reynolds number flow for velocities below 0.25 m/s (dotted line). The conventional turbulence models cannot accurately capture the flow in this low Reynolds number regime. On the other hand, an air change rate up to 5 covers most of the practical cases in air conditioning.

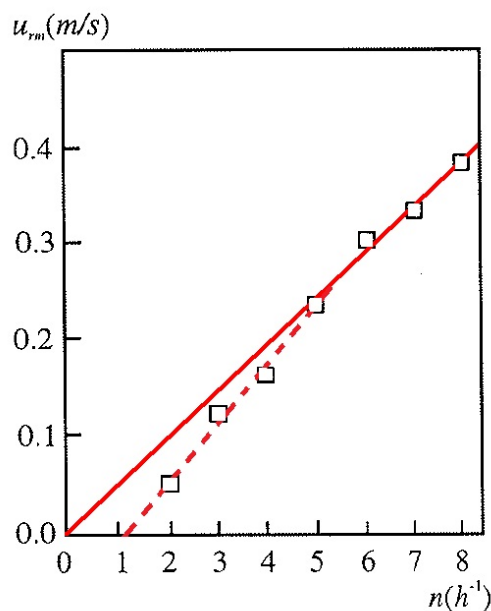


Figure 1. Measurements of maximum velocity in the occupied zone of a room versus air change rate (supply velocity) in the case of isothermal mixing ventilation. Proportionality between supply velocity and maximum velocity in the room indicates a fully developed turbulent flow in the occupied zone for a supply velocity larger than 0.25 m/s, and a low Reynolds number flow for lower velocities.

## Model and equation systems

The problems with a low Reynolds number flow will be addressed in the following IEA 2D test case.

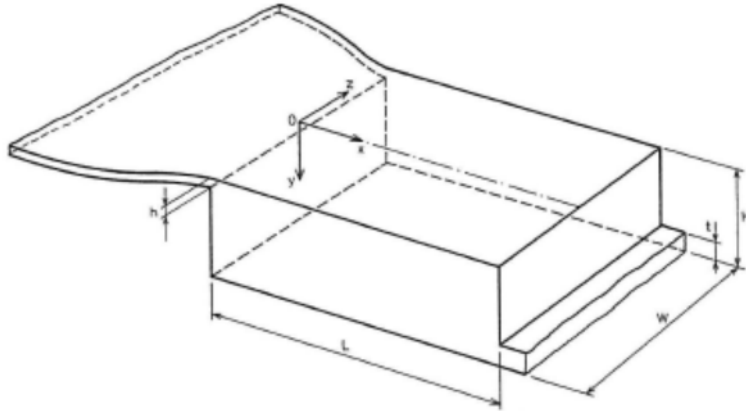


Figure 2. The two-dimensional benchmark test, also called the “IEA 2D test case”.

The geometry of this benchmark model is described as:

$$H = 3\text{m}, L = 9\text{m}, h = 0.168\text{m}, t = 0.48\text{m}$$

The air is supplied from the top slot in the model and the return air flow is through the bottom slot on the opposite position.

The boundary conditions for the following CFD predictions are given as:

Inlet velocity:  $u_0$

$$\text{Turbulent kinetic energy: } k_0 = 1.5(0.04 \cdot u_0)^2$$

$$\text{Dissipation: } \varepsilon_0 = \frac{k_0^{1.5}}{l_0} \quad \text{where } l_0 = h/10$$

$$\text{The Reynolds number is defined as } Re = \frac{h \cdot u_0}{\nu}$$

According to previous studies, the model has a two-dimensional steady state flow (Olmedo et al. 2010), and there are more than 50 papers related to this model including both experimental and numerical studies made during recent years.

## Mathematical description

The flow will be studied in the isothermal case and is fully described by the two-dimensional Reynolds-Averaged Navier-Stokes equations (RANS) without the energy equation.

Mass conservation equation

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0$$

Momentum conservation equation (Navier-Stokes equations)

$$\frac{\partial(\rho u)}{\partial t} + u \frac{\partial(\rho u)}{\partial x} + v \frac{\partial(\rho u)}{\partial y} = \frac{\partial}{\partial x} \left( \mu_{eff} \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial y} \left( \mu_{eff} \frac{\partial u}{\partial y} \right) - \frac{\partial p}{\partial x}$$

$$\frac{\partial(\rho v)}{\partial t} + u \frac{\partial(\rho v)}{\partial x} + v \frac{\partial(\rho v)}{\partial y} = \frac{\partial}{\partial x} \left( \mu_{eff} \frac{\partial v}{\partial x} \right) + \frac{\partial}{\partial y} \left( \mu_{eff} \frac{\partial v}{\partial y} \right) - \frac{\partial p}{\partial y}$$

The equation system describes laminar flow when  $\mu_{eff}$  is equal to the physical viscosity  $\mu$ , and it describes the averaged values in fully developed turbulent flow when  $\mu_{eff}$  is equal to  $\mu + \mu_t$ , where the distribution of the turbulent viscosity  $\mu_t$  is given from the  $k$ - $\varepsilon$  equations.

A laminar flow prediction can thus be made by the above equations as well as a prediction with fully developed turbulent flow.

The  $k$ - $\varepsilon$  turbulence model is only valid for fully turbulent flow and it is therefore not possible to make predictions in the regime between the laminar and the fully developed turbulent flow. A Low-Reynolds  $k$ - $\varepsilon$  model (LRN) sheds some light on this area. The LRN model is developed for wall boundary layer and is basically a  $k$ - $\varepsilon$  model with variable “constants” ( $c_\mu$  and  $c_2$ ) adjusted by a local Reynolds number  $R_t$ . Launder and Sharma (1978) introduce a version of an LRN model with the variable “constants”  $f_\mu$  and  $f_2$  given as functions of the local turbulent Reynolds number  $R_t$

$$f_\mu = \exp(-3.4/(1 + R_t/50)^2)$$

$$f_2 = 1 - 0.3 \exp(-R_t^2)$$

A high turbulence  $k$ - $\varepsilon$  model, which is used in this paper, may be considered a special version of an LRN model with  $f_\mu$  and  $f_2$  equal to 1.0. Those conditions are fulfilled in practice when  $R_t$  exceeds 400 everywhere in the flow domain, as seen from the above equations. Nielsen (1974 and 1995). The local turbulent Reynolds number can be expressed by

$$R_t = \mu_t / (c_\mu f_\mu \mu)$$

Fully developed turbulent flow ( $R_t > 400$  and  $c_\mu = 0.09$ ) corresponds to the ratio

$$\mu_t / \mu > 40$$

The value of this ratio made it possible to express the quality of a CFD prediction with a  $k$ - $\varepsilon$  turbulence model. An LRN model is identical to a  $k$ - $\varepsilon$  model when the ratio is larger than 40, and the  $k$ - $\varepsilon$  model is therefore a valid model in this case. When the ratio is smaller than 40, the local turbulent Reynolds has such a level that the  $k$ - $\varepsilon$  model is insufficient to describe the flow with the level of low Reynolds number turbulence which exist in the area. The following chapter addresses the possibilities of using the ratio  $\mu_t / \mu$  for quality control of a prediction.



### Predictions at low Reynolds numbers based on a $k-\varepsilon$ model.

Figure 3 shows the predictions at Reynolds numbers from 5000 down to 500.

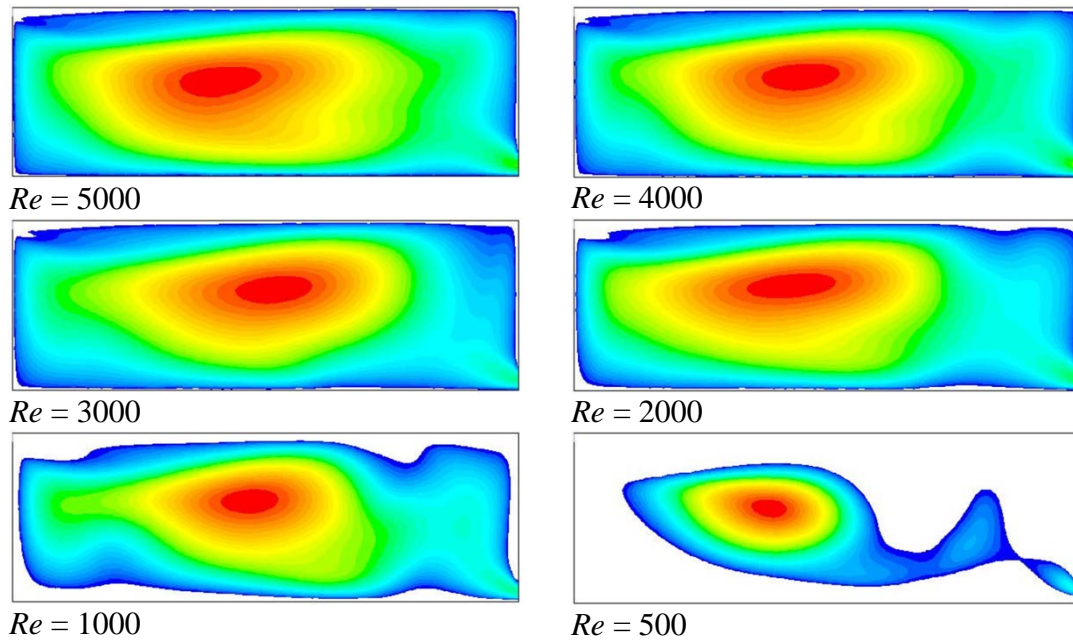


Figure 3. Prediction of the flow in the IEA 2D test case. Areas with a value of the turbulent viscosity lower than  $40 \mu$  are indicated as a white area.

The predictions show that, even for the Reynolds number of 5000, there will be areas with a turbulent viscosity lower than  $40 \mu$ . This is particularly the case for the inlet opening, because the given inlet conditions correspond to a turbulent intensity of 4 %, which is far from the turbulent flow. Le Dréau et al. (2012) have shown that  $\mu_t/\mu$  is about 5 to 10 in the inlet area. This low level of turbulence is maintained in the constant velocity core in front of the opening because there will not be any production of turbulent kinetic energy in this area. The wall jet below the ceiling will immediately obtain a high turbulent level due the production in the velocity gradients. This turbulent level will be transported around in the room in addition to further production.

The predictions for  $Re = 4000$  show that  $\mu_t/\mu$  is larger than 40 in most of the domain, and it is also similar to the predictions for  $Re = 5000$ . Therefore it can be considered to be a fully developed flow.

The predictions for  $Re = 500$  and 1000 shows large areas where  $\mu_t/\mu$  is smaller than 40 and it is only in the center of the recirculating flow that a high turbulence is predicted. Those predictions will be false because they are connected to an area with an insufficient turbulence model. The  $k-\varepsilon$  cannot be used at those low Reynolds numbers.

### Predictions with the $k-\varepsilon$ turbulent model in situations close to laminar flow.

It is often expressed that the  $k-\varepsilon$  model can give a sufficient solution at very low velocities (laminar flow regime). This situation will be addressed in this section.

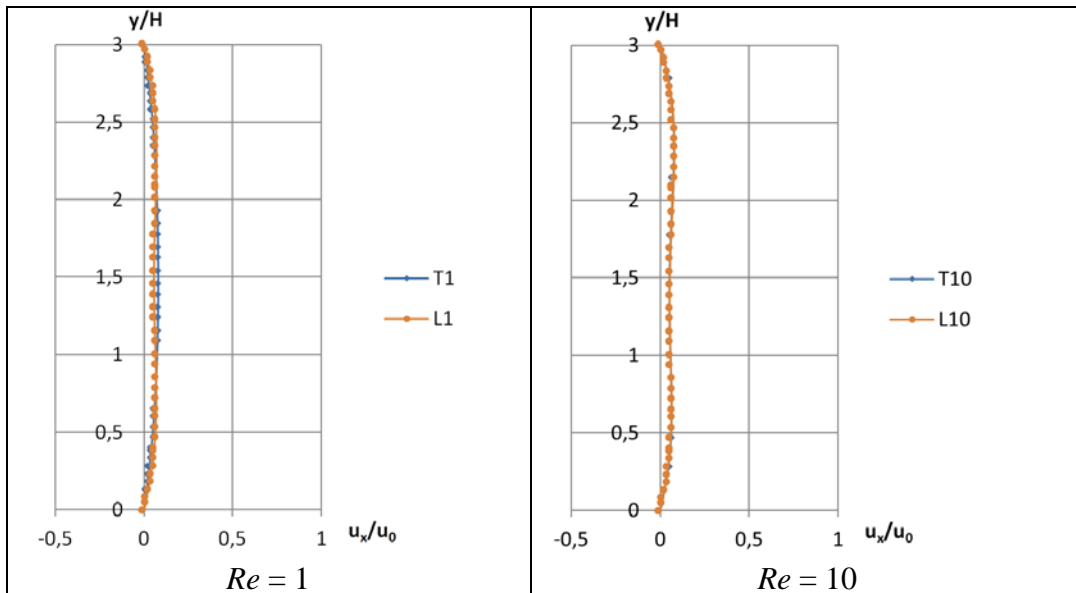


Figure 4. Velocity distribution at  $x/H = 2.0$  predicted by RANS equations and a  $k-\epsilon$  model as well as by the laminar equations.

Figure 4 shows that the velocity distribution found by the RANS equations and a  $k-\epsilon$  model are identical to the solution found by the laminar equations at the extreme low velocities ( $Re = 1, 10$ ). This effect can be explained if we look into the distribution of  $\mu_t$  and  $\mu + \mu_t$ . Figure 5 shows that the predicted turbulent viscosity  $\mu_t$  from the  $k-\epsilon$  model is close to 0.0 for  $Re = 1$  and 10. The molecular viscosity  $\mu$  will be large compared to  $\mu_t$  and the RANS equations will be similar to the equations for laminar flow and will therefore predict laminar flow.

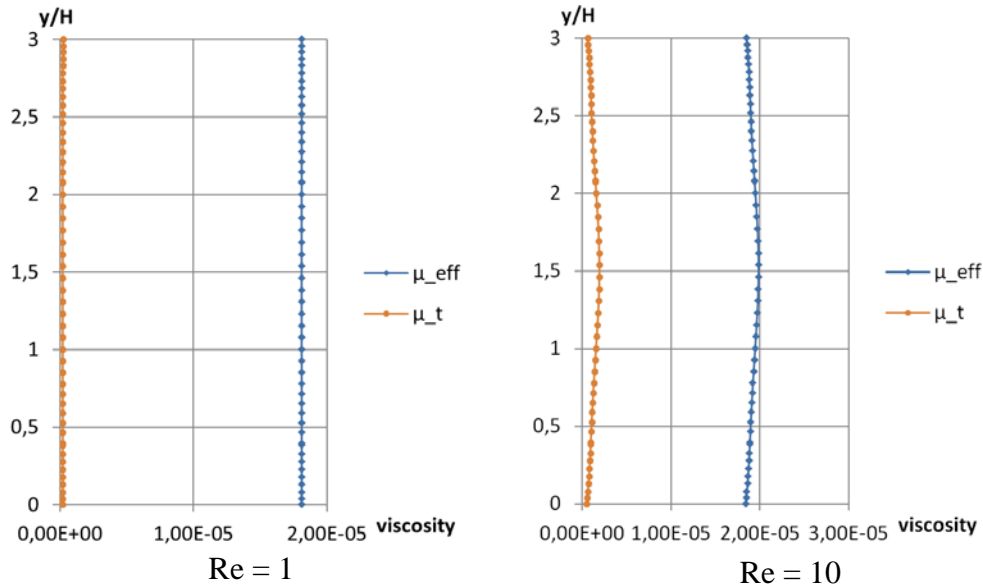


Figure 5. Distribution of  $\mu_t$  and  $\mu_{eff} = (\mu + \mu_t)$  for  $Re = 1$  and 10, at the position  $x/H = 2.0$ .

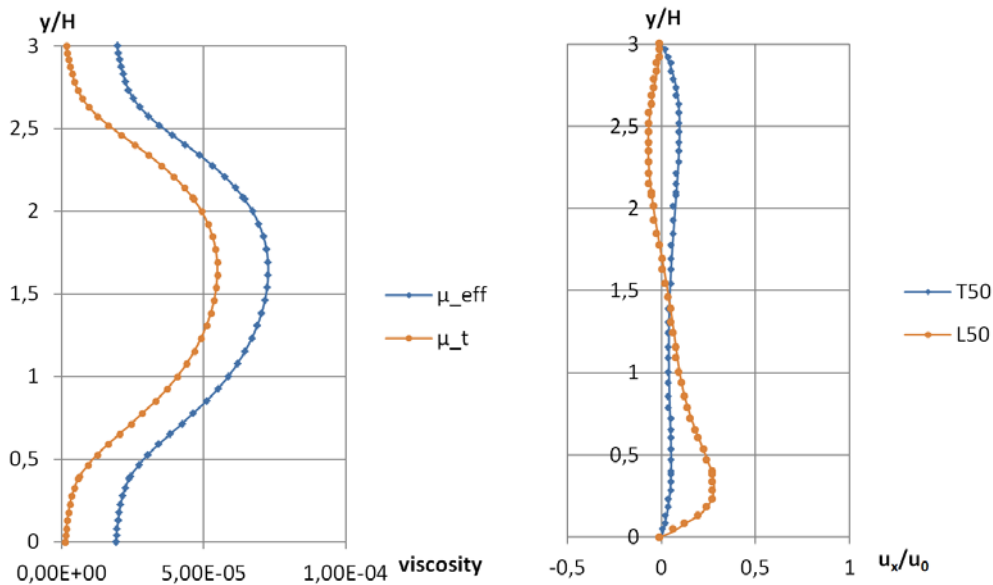


Figure 6. Distribution of  $\mu_t$  and  $\mu_{eff} = (\mu + \mu_t)$  for  $Re = 50$ , as well as velocity distribution at the position  $x/H = 2.0$ .

The prediction of  $\mu_t$  in the RANS equations will amplify with an increasing Reynolds number and will be about 2 to 4 times larger than the molecular viscosity  $\mu$  (but of course far from 40), see figure 6. The velocity distribution shows two very different results. The RANS solution is surely wrong and only measurements can determine the real solution.

## Conclusion

In the past couple of years, CFD prediction has played an important role in ventilation research and more and more people use this tool. The turbulence model selected in the CFD simulation becomes particular important, and in this paper it is shown that the standard  $k-\varepsilon$  model is not valid for all cases although the  $k-\varepsilon$  model is widely used in ventilation research in CFD prediction.

The inability of the  $k-\varepsilon$  model in the low velocity regime, which exists in ventilation research, has been demonstrated in the IEA Annex 20 2D test case. When the inlet Reynolds number is larger than 4000, the prediction in CFD code with the standard  $k-\varepsilon$  model reflects the turbulent flow in the real flow. When the inlet Reynolds number is less than 10, the prediction in CFD code with the standard  $k-\varepsilon$  model is similar to the laminar model, but when the inlet Reynolds number is from 10 to 4000, it is most likely impossible to make predictions with the standard  $k-\varepsilon$  model.

## Literature

Le Dreau, J, Heiselberg, P and Nielsen, PV, Simulation with Different Turbulence Models in an Annex 20 Benchmark Test using Star-CCM+. Aalborg University. Department of Civil Engineering, Aalborg. DCE Technical Reports, nr. 147, 2012

B. E. Launder and B. I. Sharma, Letters in Heat and Mass Transfer, 1: 129, 1978.

Peter V. Nielsen. Flow in Air Conditioned Rooms: Model experiments and numerical solution of the flow equations. PhD thesis, Technical University of Denmark, 1974.

Peter V. Nielsen. Specification of a Two-dimensional Test Case. Department of Building Technology and Structure Engineering. Aalborg University, 1990. ISBN 0902-7531 R9040.

Peter V. Nielsen. Air distribution systems: room air movement and ventilation effectiveness, International Symposium on Room Air Convection and Ventilation Effectiveness, ISRACVE, Tokyo, Japan, 1992.

Peter V. Nielsen. Airflow in a world exposition pavilion studied by scale-model experiments and computational fluid dynamics. ASHRAE Transactions, 101(2), 1118-1126, 1995.

Olmedo, I. and P.V. Nielsen, Analysis of the IEA 2D test. 2D, 3D, steady or unsteady airflow? DCE Technical Report No. 106, Aalborg University, Department of Civil Engineering, 2010.

