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# Multiple Model Adaptive Control Using Dual Youla-Kucera Factorisation

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**Abstract:** We propose a multi-model adaptive control scheme for uncertain linear plants based on the concept of model unfalsification. The approach relies on examining the ability of a pre-computed set of plant-controller candidates and choosing the one that is best able to reproduce observed in- and output signal samples. The ability to reproduce observations is measured as an easily computable signal norm. Compared to other related approaches, our procedure is designed to be able to handle significant measurement noise and closed-loop correlations between output measurements and control signals.

Keywords: Robust Adaptive Control; Switched Systems; Identification for Robust Control

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## 1. INTRODUCTION

As evidenced by the significant research interest generated over the last two decades, *multi-model adaptive control* (see e.g., Narendra et al. (1995), Narendra and Xiang (2000) and Kuipers and Ioannou (2010), among many others) have significant appealing properties in practical control settings, where some nominal model knowledge is available, but uncertainties, nonlinearities and time variations potentially might ruin an otherwise nice linear time-invariant (LTI) design.

In its simplest form, multi-model adaptive control involves a supervisor that switches among one of a finite number of controllers as more is learnt about the plant, until one of the controllers is finally selected and remains unchanged. Several research results on multi-model adaptive control have been reported in the past. Anderson et al. (2001) investigates indirect adaptive control of linear plants and employs the Vinnicombe nu-gap metric (Vinnicombe (2001)) to determine whether or not it is safe to switch to a new controller; however, one difficulty with this approach is that it is not trivial to compute the desired metric online from data. Dehghani et al. (2004) extends this work to so-called *Windsurfer* control (Lee et al. (1993, 1995)); in this concept, the initial control is by design chosen to be simple, conservative and very robust to uncertainties and disturbances. Then, as more and more data becomes available, the fidelity of model can be increased and the controller can gradually be tuned more aggressively as a consequence. Windsurfer control is thus related to multi-model adaptive control, but differs in the sense that candidate controllers are typically not computed *a priori*.

The multi-model adaptive control method recently presented in Baldi et al. (2011) extends previous work on adaptive control using *model unfalsification*-based control (Safonov and Tsao (1997); Battistelli et al. (2008)) to multi-input-multi-output systems. Roughly speaking, model unfalsification-based control revolves around the

basic premise that if a model and controller is unable to reproduce the observed behaviour of an actual closed-loop system, then the controller and/or plant model must be an inappropriate representation of the actual system and should therefore be replaced by a better candidate. Baldi et al. (2011) uses coprime factorisations of the controllers and plant models to find so-called virtual reference signals, by which to test the ability of candidates to reproduce the observations. The method clearly looks promising for high signal-to-noise ratios, but it is not quite clear how it will fare if the input/output data is contaminated with significant noise and the time in which data sets can be collected is limited (e.g., due to shaky stability margins).

The contribution of the present paper is to propose a reformulation of the method in Baldi et al. (2011) that eliminates this potential noise correlation problem in closed loop by exploiting a key feature of the *dual Youla-Kucera parameterisation* Youla et al. (1976); Kucera (1976) and choose the controller that provides the plant-controller pair with “smallest” model deviation in the form of an easily computable signal norm. In this respect, we borrow from the so-called *Hansen scheme* (Hansen et al. (1989); Tay et al. (1989); Anderson (1998); Ansay et al. (1999)).

The outline of the rest of the paper is as follows. Section 2 first provides some background on the Youla-Kucera factorisation. Then, in Section 3 we first outline the proposed switching strategy, which is based on manipulation of so-called virtual references, and then describe in details how this approach can be combined with open-loop-like identification to provide a novel switching criterion in Section 4. Next, Section 5 provides an illustrative example, and finally we conclude with some remarks in Section 6.

## 2. PRELIMINARIES

We start out by describing the basic notation and the Youla-Kucera factorisation, which will be used extensively in the sequel.

## 2.1 Problem setup and notation

We consider control loops of the form

$$y_k = G(z)u_k \quad (1)$$

$$u_k = K(z)y_k \quad (2)$$

where  $G(z)$  is a linear, possibly time-varying, discrete-time plant mapping input signals  $u_k \in \mathbb{R}^m$  to output signals  $y_k \in \mathbb{R}^p$ ,  $k \in \mathbb{Z}_+$  is the sample number and  $z$  is the time shift operator.  $K(z)$  is a controller, which at each sample  $k$  maps output measurements to control signals, such that the closed-loop behaviour achieves some specified performance.

Let  $\mathbb{S}^n$  denote the linear subspace of  $\ell_2$  consisting of all real-valued  $n$ -dimensional sequences of finite 2-norm. For any element  $x = \{x_k\}, k = 0, 1, 2, \dots$  in  $\mathbb{S}$ , we will define its *truncation at sample  $\kappa$*  as  $\bar{x}^\kappa = \{x_0, x_1, \dots, x_\kappa, 0, 0, \dots\}$ . The *truncated 2-norm* of  $x$  is then defined as

$$\|\bar{x}^\kappa\|_2 = \sqrt{\sum_{k=0}^{\kappa} x_k^T x_k}$$

For notational convenience, we will in general not write the  $z$ - and  $k$ -dependencies in the following, as long as the meaning can be understood from the context. Capital letters denote systems, while small letters denote signals. Furthermore, the plant-controller interconnection (1)–(2) will be written as  $[G, K]$ .

## 2.2 Model-controller parameterisation

First off, it is well known that the plant and controller (1)–(2) can be factorised as

$$G = NM^{-1} = \tilde{M}^{-1}\tilde{N} \quad (3)$$

and

$$K = UV^{-1} = \tilde{V}^{-1}\tilde{U} \quad (4)$$

Each pair of factors  $(N, M)$ ,  $(\tilde{N}, \tilde{M})$ ,  $(U, V)$  and  $(\tilde{U}, \tilde{V})$  are chosen such that they are stable and mutually *coprime*, i.e., have no common unstable zeros. The invertible factors  $M, \tilde{M}, V$  and  $\tilde{V}$  must be square, which determines the input-output dimensions of each factor.

Given a plant and one stabilising controller, it is now possible to parameterise all stabilising controllers as follows, see e.g. Anderson (1998):

*Lemma 1.* Let a plant  $G = NM^{-1}$ , with  $N$  and  $M$  coprime and stable, be stabilised by a controller (in positive feedback loop)  $K = UV^{-1}$ , with  $U$  and  $V$  coprime and stable. Then the set of all stabilising controllers for  $G$  is given as

$$\begin{aligned} \mathbf{K} &= \{K(Q) = (U + MQ)(V + NQ)^{-1}\} \\ &= \left\{K(Q) = (\tilde{V} + Q\tilde{N})^{-1}(\tilde{U} + Q\tilde{M})\right\} \end{aligned} \quad (5)$$

where  $Q$  is any stable system of appropriate dimensions.

$Q$  in Lemma 1 is known as a *Youla-Kucera parameter*. Figure 1 is a block diagram representation of Eqn. (5).

Not surprisingly, there also exists a dual parameterisation of all plants stabilised by a given controller:

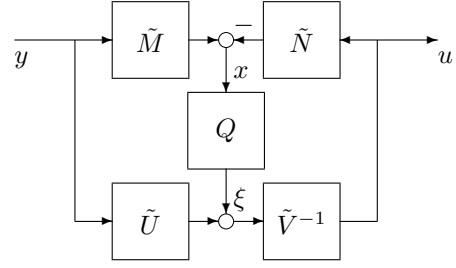


Fig. 1. Youla-Kucera parameterisation of all controllers stabilising the plant  $G = \tilde{M}^{-1}\tilde{N}$ .

*Lemma 2.* Let a plant  $G = NM^{-1}$ , with  $N$  and  $M$  coprime and stable, be stabilised by a controller (in positive feedback loop)  $K = UV^{-1}$ , with  $U$  and  $V$  coprime and stable. Then the set of all plants stabilised by  $K$  is given as

$$\begin{aligned} \mathbf{G} &= \{G(S) = (N + SV)(M + SU)^{-1}\} \\ &= \left\{G(S) = (\tilde{M} + \tilde{U}S)^{-1}(\tilde{N} + \tilde{V}S)\right\} \end{aligned} \quad (6)$$

where  $S$  is any stable system of appropriate dimensions.

$S$  in Lemma 1 is known as a *dual Youla-Kucera parameter*. This situation is depicted in Figure 2.

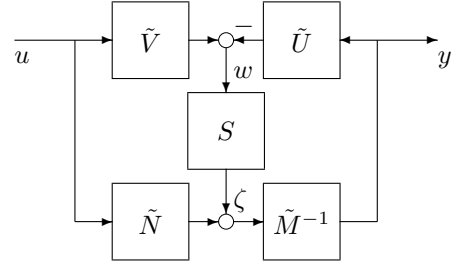


Fig. 2. Dual Youla-Kucera parameterisation of all plants stabilised by the controller  $K = \tilde{V}^{-1}\tilde{U}$ .

Now consider the closed-loop interconnection of the plant and controller in Figures 1 and 2, i.e., setting  $u$  and  $y$  in Fig. 1 equal to  $u$  and  $y$  in Fig. 2. Then we observe that the following relations must hold:

$$w = \tilde{V}u - \tilde{U}y = \tilde{V}\tilde{V}^{-1}(\xi + \tilde{U}y) - \tilde{U}y = \xi$$

and

$$x = \tilde{M}y - \tilde{N}u = \tilde{M}\tilde{M}^{-1}(\zeta + \tilde{N}u) - \tilde{N}u = \zeta$$

The closed loop  $[G(0), K(0)]$  is stable by construction. Furthermore, we see that since the matrix  $\begin{bmatrix} \tilde{M} & \tilde{U} \\ \tilde{N} & \tilde{V} \end{bmatrix}$  is invertible over  $\mathcal{RH}_\infty$ , the stability of the interconnection  $[G(S), K(Q)]$  is, in fact, *equivalent* to the stability of the simpler loop  $[S, Q]$ .

## 3. MULTI-MODEL ADAPTIVE CONTROL SCHEME

The goal of multi-model adaptive control is to handle uncertainties and disturbances by switching between different possible controllers that match a given observed input-output behaviour of the plant in some sense.

### 3.1 Controller switching

To make this more precise, we now extend the class of control loops under consideration slightly as follows:

$$y_k = G_\theta u_k \quad (7)$$

$$u_k = K_{\sigma_k} y_k \quad (8)$$

where  $G_\theta$  is a linear plant similar to (1), but affected by some disturbance, uncertainty or time variation represented by some unknown, possibly time varying parameter  $\theta$ .

Let  $\mathcal{K} = \{K_0, K_1, \dots, K_N\}$  denote a set of *a priori*-designed linear controllers. Each candidate controller  $K_i \in \mathcal{K}$  is designed for a particular value of  $\theta$ , say,  $\theta_i$ , providing specified stability and performance for  $[G_{\theta_i}, K_i]$ .  $\sigma_k \in \mathcal{N} = \{0, 1, \dots, N\}$  is a *switching sequence* that determines which of the controllers is active at sample  $k$ . The switching sequence is generated by some sort of *supervisor*, whose task it is to pick the “best suited” controller in some appropriate sense. This essentially means finding a set of switching rules that achieve good performance in the presence of uncertainties and noise.

To this end, at each sample step  $k$ , the supervisor evaluates a family of test functionals  $J_i : \mathbb{S}^m \times \mathbb{S}^p \rightarrow \mathbb{R}_+, i \in \mathcal{N}$ , where  $J_i = J_i(\bar{u}^k, \bar{y}^k)$  loosely speaking quantifies how well the  $i$ 'th potential plant-controller pair  $[G_{\theta_i}, K_i]$  suits the data up to the current sample.

In particular, one first chooses  $\sigma_0 \in \mathcal{N}$ . At every  $k$  the supervisor then computes the least index  $i \in \mathcal{N}$  such that  $J_{i_*} \leq J_i, i \in \mathcal{N}$ , whereupon the subsequent switching index  $\sigma_{k+1}$  is given by

$$\sigma_{k+1} = \begin{cases} \sigma_k & \text{if } J_{\sigma_k} < J_{i_*} + h \\ i_* & \text{otherwise} \end{cases} \quad (9)$$

where  $h > 0$  is a hysteresis constant.

Assuming the above switching strategy is employed to switch controllers in the plant-controller interconnection (7)–(8), the so-called Hysteresis Switching Logic lemma Morse et al. (1992) then states that as far as  $G_\theta$  is constant,  $\{\sigma_k\}$  admits a limit and there exist  $i$  such that  $J_i$  is bounded, then, for any initial condition and reference sequence, switching will cease in finite time. That is, the supervisor will eventually settle upon a stabilising controller in the set  $\mathcal{K}$ , if such a controller exists.

### 3.2 Introduction of virtual reference

This idea is now combined with the concept of model falsification, in which a candidate model is tested against plant measurements and rejected in case it fails to reproduce the observations. Here, instead of attempting to reproduce output values as a function of inputs, we look at the input-output data produced by the plant and controller currently in the loop, and select the plant-controller pair  $[G_{\theta_i}, K_i]$  that best matches the observed behaviour.

Assume now that an external reference sequence  $r \in \mathbb{S}^p$  is introduced as shown in Figure 3. Let

$$\bar{\psi}_i^k = \begin{bmatrix} K_i(\bar{y}^k, \bar{r}^k) \\ [G_{\theta_i}, K_i](\bar{y}^k, \bar{r}^k) \end{bmatrix} \quad (10)$$

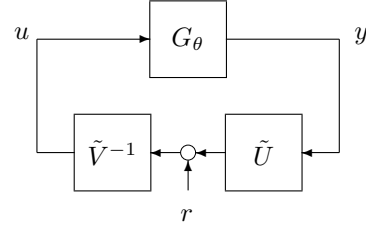


Fig. 3. Introduction of reference in control loop with  $K = \tilde{V}^{-1}\tilde{U}$

denote the collection of in- and output samples (behavioural data) that would have been generated up to sample  $k$  if the  $i$ 'th controller had been in closed loop with the plant  $G_{\theta_i}$ , while

$$\bar{\varphi}^k = \begin{bmatrix} \bar{u}^k \\ \bar{y}^k \end{bmatrix} \quad (11)$$

denotes the collection of actual observations of in- and output samples up to sample  $k$ . An obvious choice of plant-controller pair would then be the one that minimizes the difference between  $\bar{\psi}_i$  and  $\bar{\varphi}$ , i.e., letting  $J_i$  in (9) be given by

$$J_i(\bar{u}^k, \bar{y}^k) = \max_{r \neq 0} \|\bar{\psi}_i^k - \bar{\varphi}^k\|_2 \quad (12)$$

Unfortunately, even if we were able to apply some specific  $r$ , simply applying the same reference sequence to all plant-controller pairs would not allow us to determine whether or not different  $r$ 's would have been able to generate the observed behaviours for other plant-controller pairs. To avoid this problem, we introduce a *virtual reference* for each candidate plant-controller pair as known from model unfalsification Battistelli et al. (2008), and use that for evaluating  $J_i$ . Let  $\tilde{V}^{-1}$ ,  $\tilde{U}$  and  $r$  in Figure 3 be replaced by  $\tilde{V}_i^{-1}$ ,  $\tilde{U}_i$  and the virtual reference  $\tilde{r}_i$ , respectively. Then we compute

$$\tilde{r}_i = \tilde{V}_i u - \tilde{U}_i y \quad (13)$$

for all  $i \in \mathcal{N}$ , replace  $\bar{r}^k$  by  $\tilde{r}^k$  in (10), use the resulting  $\bar{\psi}_i^k$  to evaluate (12) for each plant-controller pair, and pick the controller corresponding to the pair that best matches the observed behaviour.

### 3.3 Noisy data

As shown in Baldi et al. (2011), the above approach works well as long as the measurements are largely uncorrupted by noise. However, it is a commonly known problem in closed-loop identification that noise may be fed back through a controller operating in closed loop with the plant and make the input correlated with the output samples.

To illustrate this, consider the situation in Figure 4. Here, the output measurements are contaminated by the additive noise signal  $\nu_y$ . From the block diagram,

$$u = \tilde{V}_i^{-1} \left( r + \tilde{U}_i(\nu_y + G_{\theta_i} u) \right) = (\tilde{V}_i - \tilde{U}_i G_{\theta_i})^{-1} \left( r + \tilde{U}_i \nu_y \right)$$

which results in (13) becoming unbiased by  $\nu_y$  in the best case where the “correct”  $i$  has been selected. For the other plant-controller pairs, however,  $G_{\theta_i} \neq \tilde{M}_j^{-1} \tilde{N}_j$ , which means that the noise is not canceled out, causing a mismatch between the true and virtual references.



where

$$L_1 = \begin{bmatrix} -0.782 \\ -0.551 \\ -0.195 \end{bmatrix}, L_2 = \begin{bmatrix} -0.816 \\ -0.596 \\ -0.264 \end{bmatrix}, L_3 = \begin{bmatrix} -0.853 \\ -0.648 \\ -0.343 \end{bmatrix},$$

$$L_4 = \begin{bmatrix} -0.893 \\ -0.703 \\ -0.443 \end{bmatrix}, L_5 = \begin{bmatrix} -0.94 \\ -0.735 \\ -0.625 \end{bmatrix}$$

and

$$F_1 = [-0.506 \quad -0.954 \quad -1.1]$$

$$F_2 = [-0.717 \quad -1.3 \quad -1.17]$$

$$F_3 = [-0.988 \quad -1.79 \quad -1.25]$$

$$F_4 = [-1.32 \quad -2.44 \quad -1.34]$$

$$F_5 = [-1.71 \quad -3.28 \quad -1.44].$$

The factorisation of a controller-model pair is found as

$$\begin{bmatrix} \tilde{V}_i & -\tilde{U}_i \\ -\tilde{N}_i & \tilde{M}_i \end{bmatrix} = \begin{bmatrix} A_{\theta_i} + L_i C & -B & L_i \\ F_i & I & 0 \\ C & 0 & I \end{bmatrix} \quad (20)$$

Figs. 6 and 7 illustrate the performance of the switching scheme. The true reference is chosen as a series of random steps in the interval  $[-1; 1]$ , and the noise and unknown parameter are arbitrarily chosen as  $\rho^2 = 10^{-2}$  and  $\theta = 0.75$ .  $h$  is chosen by hand to  $h = 0.25$ .

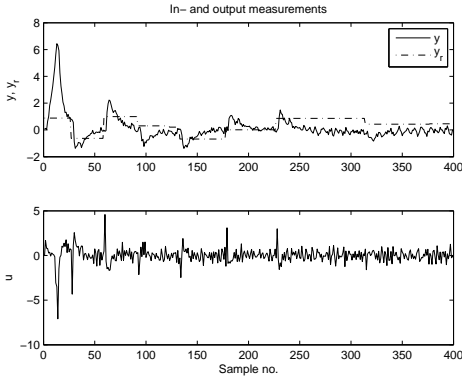


Fig. 6. Reference  $r_k$ , output  $y_k$  and input  $u_k$

The top trace of Figure 7 shows the switching sequence. The first controller is arbitrarily selected as  $\sigma_0 = 1$ . Data is collected for a few samples, before the switching scheme is activated; about 5 samples later, the test functional  $J_1$  has increased enough to satisfy  $J_1 < J_4 + h$ , and the controller switches to the gains  $F_4$  and  $L_4$ . As seen in Figure 6, this controller quickly drives the output back to the reference value. The bottom trace of Figure 7 shows the development of the five test functionals.

Next, a number of simulation tests are now performed. For each series, for 40 values of the real  $\theta$ , 50 simulations are performed, 10 for each controller. Every simulation consists of 400 samples where a reference signal,  $r$ , formed by filtering white noise with variance 0.01 through  $\tilde{U}_i$  is added between the factors, i.e.  $u = \tilde{V}_i(r + \tilde{U}_i y)$ . For every simulation, the method in Baldi et al. (2011) and the method in Section 4 are applied to determine the best of the five controllers.

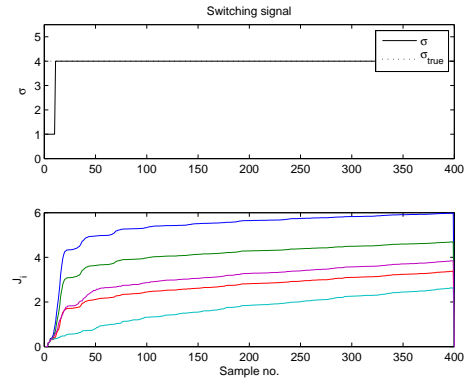


Fig. 7. Switching sequence  $\sigma_k$  and test functionals  $J_i(\bar{u}^k, \bar{y}^k)$

In the first series,  $\rho^2 = 0$ , i.e. there is no noise, and the two schemes pick the same controller all the time for any particular value of  $\theta$ . Figure 8 shows the resulting choices. We will consider this as the optimal choice.

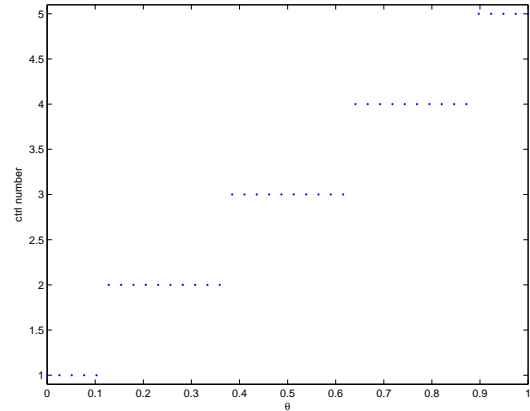


Fig. 8. Controller selection with no noise.

Now the noise is increased to  $\rho^2 = 10^{-6}$ . For each value of  $\theta$ , by treating the controller selection as values between 1 and 5, we can take mean and deviation of the 40 tests. Ideally the mean should be the same as in the previous series and the deviation should be zero.

Figure 9 shows the means plus/minus one standard deviation. The method in Baldi et al. (2011) is shown by the dashed lines, the method in Section 4 the solid lines. The dots show the optimal selection. Both methods still perform very well.

However, when the noise level is increased to  $\rho^2 = 10^{-4}$  as shown in Figure 10, then the method in Baldi et al. (2011) not only gets a significant variance of the selection, it also shows a strong bias towards the controllers with lower numbers. The method presented here also shows some deviations, but on average the correct controller is selected.

## 6. DISCUSSION

In this paper, we proposed a novel scheme for adaptive control of linear systems subject to significant uncertainties and/or time variations. The approach relies on examining

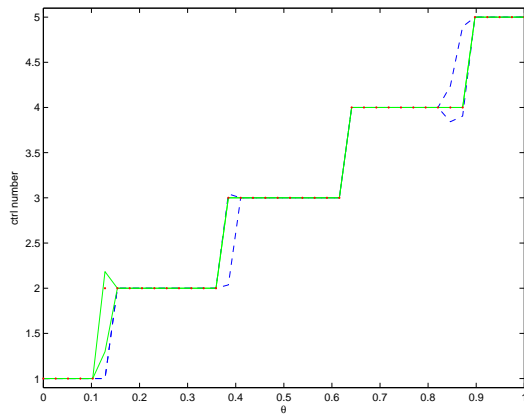


Fig. 9. Controller selection with weak noise.

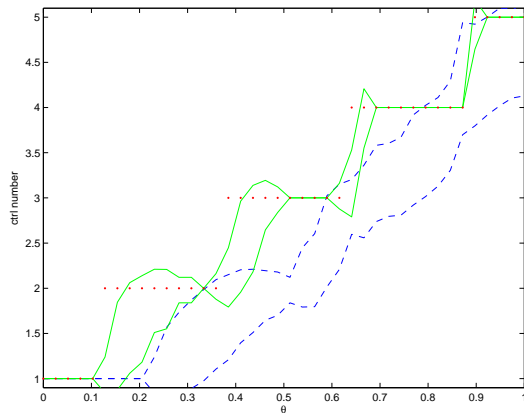


Fig. 10. Controller selection with strong noise.

the ability of a pre-computed set of plant-controller candidates and choosing the one that is best able to reproduce observed in- and output signal samples. The ability to reproduce observations is measured as an easily computable signal norm. Compared to other related approaches, our procedure is designed to be able to handle significant measurement noise and closed-loop correlations between output measurements and control signals. However, it must be pointed out that simulation studies indicate that as far as the signal-noise is high or the closed-loop operation permits long data sequences to be obtained, our approach does not appear to provide significantly better results than the method presented in Baldi et al. (2011).

Like most adaptive control algorithms, the method suggested here is initially formulated under the assumption that the plant is time-invariant. However, there is an underlying requirement that the adaptive controller has the capability to track time variations in the plant, which are generally relatively slow compared to the input-output dynamics. To do so, it may be advantageous to introduce exponential forgetting in the evaluation of  $J_i$ .

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