View metadata, citation and similar papers at core.ac.uk

brought to you by CORE

provided by VBN

Aalborg Universitet

Diversity-Multiplexing Trade-off for Coordinated Relayed Uplink and Direct Downlink Transmissions

Thai, Chan; Popovski, Petar; Sun, Fan; De Carvalho, Elisabeth

Published in:
Proceedings of the 2013 International Conference on Computing, Management and Telecommunications (ComManTel)

DOI (link to publication from Publisher):
10.1109/ComManTel.2013.6482368

Publication date:
2013

Document Version
Early version, also known as pre-print

Link to publication from Aalborg University

Citation for published version (APA):
Thai, C., Popovski, P., Sun, F., & De Carvalho, E. (2013). Diversity-Multiplexing Trade-off for Coordinated Relayed Uplink and Direct Downlink Transmissions. In *Proceedings of the 2013 International Conference on Computing, Management and Telecommunications (ComManTel)* IEEE Press. https://doi.org/10.1109/ComManTel.2013.6482368

General rights
Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

? Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
? You may not further distribute the material or use it for any profit-making activity or commercial gain
? You may freely distribute the URL identifying the publication in the public portal ?

Take down policy
If you believe that this document breaches copyright please contact us at vbn@aub.aau.dk providing details, and we will remove access to the work immediately and investigate your claim.

Diversity-Multiplexing Trade-off for Coordinated Relayed Uplink and Direct Downlink Transmissions

Chan Dai Truyen Thai, Petar Popovski, Fan Sun and Elisabeth de Carvalho
Department of Electronic Systems, Aalborg University
Email:{ttc, petarp, fs, edc@es.aau.dk}

***Abstract*—There are two basic principles used in wireless network coding to design throughput-efficient schemes: (1) aggregation of communication flows and (2) interference is embraced and subsequently cancelled or mitigated. These principles inspire design of Coordinated Direct/Relay (CDR) schemes, where each basic transmission involves two flows to a direct and a relayed user. Considering a scenario with relayed uplink and direct downlink, we analyze the Diversity-Multiplexing Tradeoff (DMT) calculating either the exact value or both upper/lower bounds. The CDR scheme is shown to have a higher diversity gain than the reference scheme at any multiplexing gain.**

I. Introduction

Two-Way Relay (TWR) Network Coding (NC) has recently emerged as one of the key generic techniques that can boost the throughput performance of wireless networks [1]–[3]. There are two basic principles used in designing throughput–efficient schemes with wireless network coding (NC) (1) *aggregation of communication flows* - NC operates by having the flows sent/processed jointly; (2) *intentional cancelable interference*: flows are allowed to interfere over the wireless channel, knowing *a priori* that the interference can be cancelled by the destination.

TWR NC using the principles mentioned above have been extensively discussed, analyzed and evaluated in many different aspects. The TWR scenario in which two users exchange messages over a relay and for which TWR NC has been proposed is actually only one of many scenarios in which the *principles of flow aggregation and a priori information* can be used. Several schemes are possibly proposed in many different scenarios using the same principles. One of them is a network with a base station (BS), a relay user (RS), a relayed user (U) and a direct user (V). In this scenario, we have considered different traffic sub-scenarios in each of which one user has an uplink or downlink message. In the proposed *coordinated direct/relay (CDR)* schemes, the transmissions for the relayed user and the direct user can be combined using the *principles of flow aggregation and a priori infomration*.

For example, for the traffic scenario of relayed uplink and direct downlink, in the first step of the proposed scheme of the scenario, termed S, user U transmits an uplink message to RS and BS transmits a downlink message to user V simultaneously. It becomes a Multiple Access Channel (MAC) at each receiver of RS and user V. Moreover, in the second step, RS decodes and relays the uplink message to BS. If user V can decode this message, its contribution in the first received signal can be cancelled. Therefore, the transmitted rates and durations are selected and optimized according to several conditions in several options. This will benefit the performance. Enabling such simultaneous transmissions improves the spectral efficiency and communication reliability.

The proposed schemes for other traffic scenarios are described in details in [5] (AF) and [6] (DF). The conventional scheme for each traffic scenario are defined as orthogonal transmissions for the two users. Definitely, TWR NC cannot be used here because there is only one message, uplink or donwlink, requested for the relayed user. In the full traffic scenario, each user has both uplink and downlink messages, the state-of-the-art conventional scheme is defined a combination of orthogonal transmissions for the two users and TWR NC scheme. In all full and non-full traffic scenarios, we have analyzed and compared the proposed scheme and the conventional scheme. The proposed scheme was shown to be better in terms of sum-rate and rate region.

The schemes therefore have been so far considered through the prism of *spectrum efficiency* [4]–[6], in this paper, we will show that the schemes also enhance the *communication reliability* by calculating/bounding and analyzing diversity-multiplexing trade-off (DMT) functions for a conventional scheme and the CDR scheme using DF in traffic scenario of relayed uplink and direct downlink. From now on we simple say "the scenario" to refer to this scenario.

Because DF is used, the rate transmitted by the relay is not necessary equal to the rate it receives, the durations of the all hop transmissions are therefore different and subject to optimization of rates and outage probabilities. Enabling such simultaneous transmissions improves the spectral efficiency and communication reliability.

The rest of the paper is organized as follows. Section II presents the system model used. We describe and calculate rates of the reference and CDR schemes in Section III. Section IV calculates or bounds the DMT functions of the schemes. Section V presents and discusses the numerical results and Section VI concludes the paper.

II. System Model

We consider a scenario with one base station (BS), one relay (RS), and two users (U and V), see Fig. 1. All stations are half-duplex, single-antenna and all transmissions have a normalized bandwidth of 1 Hz. Each of the complex channels $h_i, i \in \{1, 2, 3, 4, 5\}$, is reciprocal. Only the transmitter and the receiver of a transmission know the channel of that transmission.

978-1-4673-2088-7/13/$31.00 ©2013 IEEE

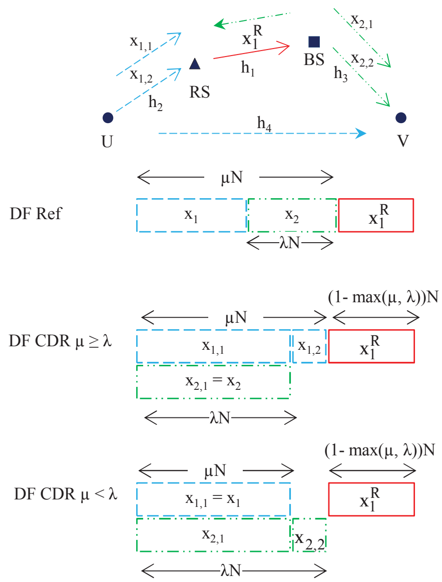

Fig. 1. Time slots in Reference E and CDR Scheme S. In each scheme, the transmissions represented by the rectangles in the same column are conducted simultaneously. The interference is represented by an arrow with thicker head.

BS has to receive message s_1 from user U via relay RS and send message s_2 to user V directly. Because we have a relayed uplink and a direct downlink, there are 3 hop transmissions which are U-RS, RS-BS, BS-V hop transmissions through channels h_2, h_1 and h_3. We assume that the transmit powers of the corresponding transmitters in those 3 hop transmissions are P_U, P_R and P_B. The noise at all stations is Additive White Gaussian Noise (AWGN) with $\mathcal{CN}(0,\sigma^2)$, $\sigma^2 = 1$, distribution. Denote $\gamma_i = \frac{|h_i|^2}{\sigma^2}$, $i \in \{1,2,3,4\}$.

In the reference scheme denoted as E, all transmissions are orthogonally multiplexed in time. In the CDR scheme denoted as S, transmissions of the two messages are combined in such a way that as much data as possible sent to the desiring stations in the same total time duration. D_U^i and D_V^i, $i \in \{E,S\}$ are maximal average rates over time for U and V respectively in the whole scheme i. The sum–rate is therefore estimated as $D_S^i = D_U^i + D_V^i$.

Each hop transmission period may consist of one transmission or two transmissions with different rates. λ and μ, $0 < \lambda, \mu < 1$, characterize different transmission durations and are defined as follows. In both schemes, there are λN symbols in the BS-V hop transmission where N is the total number of symbols in the whole scheme as in Fig. 1. In scheme E, there are totally μN symbols in U-RS and BS-V hop transmissions; if $\mu < \lambda$, there is no symbol in U-RS hop transmission and thus no data delivered from user U to BS. This case is therefore not considered. In scheme S, there are μN symbols in U-RS hop transmission. We have different definitions for μ in different schemes as above so that we can easily compare the rates of two schemes as shown later on. $R_U^i[j]$ and $R_V^i[j]$ are instantaneous rates in time slot j, $j \in \{1,2,3\}$ for user U and V respectively in scheme i, $i \in \{E,S\}$.

We denote $C_1 = \log_2(1 + P_R\gamma_1)$, $C_2 = \log_2(1 + P_U\gamma_2)$, $C_3 = \log_2(1 + P_B\gamma_3)$ and $C_4 = \log_2(1 + P_U\gamma_4)$ which are the maximal rates in RS-BS, U-RS, BS-V and U-V hop transmission if there is no interference. Denote $C_{2-1} = \log_2\left(1 + \frac{P_U\gamma_2}{P_B\gamma_1+1}\right)$ which is the maximal rate for decoding a desired signal from user U over h_2 treating the interference from BS over channel h_1 as noise at RS. Similarly, we denote $C_{1-2} = \log_2\left(1 + \frac{P_B\gamma_1}{P_U\gamma_2+1}\right)$, $C_{3-4} = \log_2\left(1 + \frac{P_B\gamma_3}{P_U\gamma_4+1}\right)$ and $C_{4-3} = \log_2\left(1 + \frac{P_U\gamma_4}{P_B\gamma_3+1}\right)$. Denote $C_{1,2} = \log_2(1 + P_B\gamma_1 + P_U\gamma_2)$ which is the maximal total rate for multiplexing in time between two modes: the first mode decodes the signal from user U treating the signal from BS as noise, cancels its contribution and decode the other signal while the second mode decode in the opposite order because $C_{1,2} = C_{2,1} = C_{1-2} + C_2 = C_{2-1} + C_1$ [7]. $|x|$ is the number of symbols in symbol stream x and $\log$ means base 2 logarithm.

III. Scheme Description

In scheme E, the sum–rate will be calculated and optimized based on parameter μ. In scheme S we have two more parameters, $R_U^S[1]$ and $R_V^S[1]$, to optimize.

A. Reference Scheme

First, user U encodes s_1 to x_1 with rate $R_U^E[1]$ and transmits it to RS as seen in Fig. 1, RS receives $y_R[1] = h_2x_1 + z_R[1]$. Second, BS encodes s_2 to x_2 with rate $R_V^E[3]$ and transmits it to user V, user V receives $y_V[3] = h_3x_2 + z_V[3]$. Third, RS decodes x_1 to s_1, re-encodes it to x_1^R with rate $R_U^E[2]$ and transmits it to BS, BS receives $y_B[2] = h_1x_1^R + z_BU[2]$. Since x_1, x_2 and x_1^R are transmitted with power P_U, P_R and P_B, the rates $R_U^E[1]$, $R_V^E[3]$ and $R_U^E[2]$ are selected as the maximal rates over the corresponding channels $R_U^E[1] = C_2$, $R_V^E[3] = C_3$ and $R_U^E[2] = C_1$.

Since all transmissions are performed separately, the duration for RS-BS transmission is $(1-\mu)N$. The U-RS, RS-BS and BS-V transmissions therefore have durations of $(\mu-\lambda)N$, $(1-\mu)N$ and λN symbols respectively. On the other hand, the corresponding maximal rates are C_1, C_2 and C_3 respectively thus the maximal rate transmitted through them are respectively

$$D_{U_1}^E = (\mu - \lambda)C_2, \quad D_{U_2}^E = (1-\mu)C_1, \quad D_V^E = \lambda C_3. \tag{1}$$

Because the rate transmitted from user U to BS end-to-end is the minimum of the rate transmitted in BS-RS and RS-U transmissions, the sum–rate transmitted for two users is $D_S^E = \min\left(D_{U_1}^E, D_{U_2}^E\right) + D_V^E$. Because the information of all channels is not available, μ is optimized to have the highest sum–rate based on the distributions of the channels $D_{So}^E = \max_{\lambda,\mu} \bar{D}_S^E(\lambda,\mu)$, where $\bar{D}_S^E$ is the average of D_S^E over all channel realizations, is the maximum sum-rate.

B. CDR Scheme

First, user U transmits $x_{1,1}$ with rate $R_U[1]$ to RS and BS transmits $x_{2,1}$ with rate $R_V[1]$ in $\min(\mu,\lambda)N$ symbols simultaneously. RS and V therefore respectively receive $y_R[1] =$

$h_2x_{1,1} + h_1x_{2,1} + z_R[1]$, $y_V[1] = h_4x_{1,1} + h_3x_{2,1} + z_V[1]$. Second, in $|\mu - \lambda|N$ symbols, U transmits $x_{1,2}$ to RS $y_R[2] = h_2x_{1,2} + z_R[2]$ if $\mu \geq \lambda$ or BS transmits $x_{2,2}$ to V $y_V[2] = h_3x_{2,2} + z_V[2]$ $\mu\,\lambda$ interference-free with maximal rates of the corresponding channels C_2 and C_3 respectively (see Fig. 1). Third, RS decodes $x_{1,1}$ and $x_{1,2}$, re-encodes and forwards them to BS $y_B[3] = h_1x_3^R + z_B[3]$. Since BS and RS cannot transmit and receive at the same time, the RS-BS transmission cannot be performed simultaneously with any other transmission, it starts only after both U-RS and BS-V transmissions are finished (the first $\max(\mu, \lambda)N$ symbols). Thus $|x_1^R| = |x_{1,1}^R| + |x_{1,2}^R| = (1 - \max(\mu, \lambda))N$. We consider two cases as follows.

If $C_1 > C_5$, RS can decode x_1 when one of the two conditions follows occurs:

- RS decodes x_1 treating x_2 as noise: $R_U[1] \leq C_{2-1}$.
- RS decodes both x_1 and x_2 according to MAC [7]: $R_U[1] \leq C_2$, $R_V[1] \leq C_1$, $R_U[1] + R_V[1] \leq C_{1,2}$.

Similarly, user V can decode x_2 when one of the two conditions follows occurs:

- V decodes x_2 treating x_1 as noise: $R_V[1] \leq C_{3-4}$.
- V decodes both x_1 and x_2 according to MAC: $R_U[1] \leq C_4$, $R_V[1] \leq C_3$, $R_U[1] + R_V[1] \leq C_{3,4}$.

If $C_1 \leq C_5$, because RS transmit x_1^R with rate C_1, therefore both BS and V can decode x_1^R. Using the information about s_1, the interference in the first slot at V can be completely cancelled thus the condition so that U can decode x_2 in the first slot in the case $C_1 > C_5$ is replaced by $R_V[1] \leq C_3$.

The rates transmitted in U-RS, RS-BS, BS-V transmissions and the sum-rate transmitted to two users are respectively $D_{U_1}^S = \min(\mu, \lambda)R_U[1] + (\mu - \min(\mu, \lambda))C_2$, $D_{U_2}^S = (2 - \max(\mu, \lambda))C_1$, $D_V^S = \min(\mu, \lambda)R_V[1] + (\lambda - \min(\mu, \lambda))C_3$ with $R_U[1]$ and $R_V[1]$ satisfying all the conditions mentioned above. The sum-rate is $D_S^S = \min(D_{U_1}^S, D_{U_2}^S) + D_V^S$. $D_{So}^S = \max_{\lambda,\mu} \bar{D}_S^S(\lambda, \mu)$, where $\bar{D}_S^S$ is the average of D_S^S over all channel realizations, is the maximum sum-rate.

Above we consider the cases when V has to decode at least s_1 or s_2 in slots 1 and 2 or in slot 3. The case when V does not need to decode any of them can be achieved by using combining two replicas of the information sent originally by U, each encoded with a different codebook (one used by U and the other by RS). However, such a scheme is outside the scope of this paper.

IV. Diversity-Multiplexing Trade-off Analysis

We first introduce the DMT definition and notations and after that estimate the DMT functions for E and S.

A. DMT Definition and Notations

A scheme is said to achieve spatial multiplexing gain r and diversity gain d if the data rate $\lim_{\rho\to\infty} \frac{R(\rho)}{\log \rho} = r$ where ρ is the corresponding SNR when fading is not considered, and the average outage probability

$$\lim_{\rho\to\infty} -\frac{\log P_e(\rho)}{\log \rho} = d. \tag{2}$$

Throughout the rest of the paper, we use the symbol $\doteq$ to denote exponential equality, i.e. we write $f(\rho) \doteq \rho^b$ to denote $\lim_{\rho\to\infty} \frac{\log f(\rho)}{\log \rho} = b$ and $\dot{\geq}$, $\dot{\leq}$ are similarly defined. Therefore, equation (2) can be written as in [8]

$$P_e(\rho) \doteq \rho^{-d}. \tag{3}$$

We also denote $(x)^+ = \max(0, x)$ and $\log$ refers to $\log_2$ if not stated otherwise. If $\lim_{\rho\to\infty} \frac{m}{n} = 1$, we write $m \sim n$ when $\rho \to \infty$. Because $\rho^a + \rho^b \sim \rho^{\max(a,b)}$ when $\rho \to \infty$, we often use $\rho^a + \rho^b \doteq \rho^{\max(a,b)}$ or $\rho^a + 1 = \rho^a + \rho^0 \doteq \rho^{\max(a,0)} = \rho^{a^+}$. Moreover,

$$\Pr[\gamma_i < x] = \int_0^x e^{-t}\mathrm{d}t = 1 - e^{-x} \sim x \quad \text{when} \quad x \to 0. \tag{4}$$

Therefore,

$$\Pr\left[\gamma_i < \rho^{-a}\right] \doteq \rho^{-a^+} \quad \text{when} \quad \rho \to \infty. \tag{5}$$

We investigate on the outage probability when the system tries to achieve a certain target rate pair (D_U^t, D_V^t). Here to consider the diversity-multiplexing trade-off we examine how fast the outage probability decays when the rate increases as a multiple of $\log \rho_i$, where $i \in \{B, R, U\}$, $\rho_i = \frac{P_i}{\sigma^2}$ when station i transmits with power P_i and $\sigma^2 = 1$ is noise power at a receiver. We assume $P_B = \rho$ and $P_R = P_U = \rho^\beta$ as in [9]. We assume that β is always known at all stations.

Since the stations have different transmit powers, the target rates have different expressions. We calculate the DMT function for the scheme when the target rate for user i is given by $D_i^t = r_i \log P_j$ where r_i is the corresponding multiplexing gain and P_j is the transmit power of the transmitter i.e. for direct downlink $P_j = P_B = \rho$ and for relayed uplink $P_j = P_U = P_R = \rho^\beta$. The target rate for user U is $D_U^t = r_U \log \rho^\beta$ and the target rate for user V is $D_V^t = r_V \log \rho$ where r_U and r_V are multiplexing gains for user U and V respectively. The scheme is in outage when the maximal achievable rate for user U or user V is smaller than the respective target rate.

An outage of a certain pair (λ, μ) is defined as when that pair cannot support a target rate pair (R_U^t, R_V^t). Therefore λ and μ can be selected such that the average outage probabilities over all channel realizations are the smallest ones $(\lambda_o, \mu_o) = \min_{\lambda,\mu} \Pr[\mathcal{O}]$ where $\mathcal{O}$ is the event of outage of the scheme.

B. Reference Scheme

As described in section III, the rates for U-RS, BS-V and RS-BS hop transmissions are respectively

$$\begin{cases} D_{U_1}^E = (\mu - \lambda)\log(1 + \gamma_2\rho^\beta) \\ D_V^E = \lambda \log(1 + \gamma_3\rho) \\ D_{U_2}^E = (1 - \mu)\log(1 + \gamma_1\rho^\beta). \end{cases} \tag{6}$$

The delivery of s_1 from user U to BS is in outage when one of these two conditions occurs $D_{U_1}^E < D_U^t$ and $D_{U_2}^E < D_U^t$. We call these two events $\mathcal{O}_{U_1}^E$ and $\mathcal{O}_{U_2}^E$ respectively. The probability of the first event is

$$\begin{aligned} &\Pr\left[\mathcal{O}_{U_1}^E\right] = \Pr[(1 + \gamma_2\rho^\beta)^{\mu-\lambda} < \rho^{r_U\beta}] \\ &\doteq \Pr[(\gamma_2\rho^\beta)^{\mu-\lambda} < \rho^{r_U\beta}] = \Pr[\gamma_2 < \rho^{\frac{r_U}{\mu-\lambda}-1}]. \end{aligned} \tag{7}$$

Because $\rho \to \infty$, this probability decays only when $\frac{r_U}{\mu-\lambda}-1<0$. It means that we have a positive diversity gain corresponding to this transmission only when $1-\frac{r_U}{\mu-\lambda}>0$. In the other case, the diversity gain is 0 therefore we can write the diversity gain as a function of $\left(1-\frac{r_U}{\mu-\lambda}\right)^+$. On the other hand, because of (5), we can write $\Pr\left[\mathcal{O}_{U_1}^E\right] \doteq \rho^{-\left(1-\frac{r_U}{\mu-\lambda}\right)^+}$. Similarly, the probability of the second event is $\Pr\left[\mathcal{O}_{U_2}^E\right] \doteq \rho^{-\left(1-\frac{r_U}{1-\mu}\right)^+}$.

According to the scheme description, the rate of user V is $D_V^E = \lambda \log(1+\rho\gamma_3)$. It is in outage when $D_V^E < D_V^t$. We call this event $\mathcal{O}_V^E$. With some derivations, we have $\Pr\left[\mathcal{O}_V^E\right] \doteq \rho^{\left(\frac{r_V}{\lambda}-1\right)^+}$.

The scheme is in outage when one of the conditions $\mathcal{O}_{U_1}^E$, $\mathcal{O}_{U_2}^E$ and $\mathcal{O}_V^E$ occurs i.e. $\mathcal{O}^E = \mathcal{O}_{U_1}^E \cup \mathcal{O}_{U_2}^E \cup \mathcal{O}_V^E$. Denoting $d_1 = \beta\left(1-\frac{r_U}{\mu-\lambda}\right)^+$, $d_2 = \left(1-\frac{r_V}{\lambda}\right)^+$, $d_3 = \beta\left(1-\frac{r_U}{1-\mu}\right)^+$,

$$\Pr[\mathcal{O}^E] \doteq \Pr[\mathcal{O}_{U_1}^E] + \Pr[\mathcal{O}_{U_2}^E] + \Pr[\mathcal{O}_V^E] \doteq \rho^{-\min\{d_1,d_2,d_3\}} \tag{8}$$

The first equation comes from the fact that the events $\mathcal{O}_{U_1}^E$, $\mathcal{O}_{U_2}^E$ and $\mathcal{O}_V^E$ are independent due to the independence of γ_1, γ_2 and γ_3 and that the product of any two or three of the probabilities $\Pr[\mathcal{O}_{U_1}^E]$, $\Pr[\mathcal{O}_{U_2}^E]$ and $\Pr[\mathcal{O}_V^E]$ decays faster than each of them. The second equation is due to that the smaller terms are negligible and the probability is determined by the largest term in the sum when $\rho \to \infty$. According to the definition in (3), the diversity gain is therefore $d^E = \min(d_1, d_2, d_3)$.

Each element in the min function above is actually the diversity gain of each hop transmission (U-RS, BS-V and RS-BS) in the reference scheme. It increases when the corresponding time assigned to it increases ($\mu-\lambda$, λ and $1-\mu$). However, the diversity gain of the scheme is the minimum of them, therefore the time durations assigned to 3 hop transmissions should be balanced such that the minimum diversity gain is maximal. Obviously, the diversity gain can be improved by selecting the right values of λ and μ. This is how we improve the diversity gain when the transmitter and the receiver of a transmission do not know the channels not related to them. $d_o^E = \max_{\lambda,\mu} d^E(\lambda,\mu)$ is the maximum DMT function scheme E can achieve. It is obvious that d^E is maximized when $d_1 = d_2 = d_3$ roots of which are optimal μ and λ

C. CDR Scheme

In the first time slot, there are two decoding options at RS: (Option 1) decode $x_{1,1}$ or x_1 treating the interference x_2 or $x_{2,1}$ from BS as noise, (Option 2) decode the interference from BS first, cancel its contribution and decode the desired signal. At user V, there are also two similar decoding options: (Option a) decode the signal from user U first and (Option b) decode the signal from BS first. In total, we have four options (1a, 1b, 2a, 2b) however multiplexing of more than one of those options in time with any time ratio also give an achievable rate pair (D_U^S, D_V^S).

On the other hand when $\rho \to \infty$, C_1, C_2, C_3 and $C_4 \to \infty$. If $\beta_V < 1$, $\lim_{\rho\to\infty} C_{1-2} = \lim_{\rho\to\infty} C_{3-4} = \infty$ and $\lim_{\rho\to\infty} C_{2-1} = \lim_{\rho\to\infty} C_{4-3} = 0$. Therefore in case of $\rho \to \infty$ only two decoding options are relevant: 1a (both RS and user V decode the signal from user U first, for larger β) and 2b (both RS and user V decode the signal from BS first, for larger β). In two cases below ($\mu \geq \lambda$ and $\mu < \lambda$), we consider these two options when calculating the DMT function for scheme S. The subscript $i/j-k$, $i,j,k \in \{B,R,U,V\}$ means decoding the signal for user j treating the signal for user k as noise at station i and subscript i/j means similarly but decoding without interference. Similarly to the derivations for Reference Scheme we have the following results.

- $\mu \geq \lambda$
  - Option 1a: the rates of three hop transmissions U-RS, BS-V and RS-BS

$$\begin{cases} D_{R/V-U}^S &= \lambda \log\left(1+\frac{\gamma_1\rho}{\gamma_2\rho^\beta+1}\right) \\ D_{U_1}^S = D_{R/U}^S &= \mu \log\left(1+\gamma_2\rho^\beta\right) \\ D_{V/V-U}^S &= \lambda \log\left(1+\frac{\gamma_3\rho}{\gamma_4\rho^\beta+1}\right) \\ D_{U_2}^S = D_{B/U}^S &= (1-\mu)\log\left(1+\gamma_1\rho^\beta\right). \end{cases} \tag{9}$$

    There is an outage when one of the following conditions occurs $D_{R/V-U}^S < D_V^t$, $D_{U_1}^S < D_U^t$, $D_{V/V-U}^S < D_V^t$ and $D_{U_2}^S < D_U^t$. We have the DMT function $d^S > d^{S-\mathrm{LB}-1} = \min(d_4, d_5, d_3)$ with $d_4 = \left(1-\frac{r_V}{\lambda}-\beta\right)^+$ and $d_5 = \beta\left(1-\frac{r_U}{\mu}\right)^+$.
  - Option 2b: the rates of three hop transmissions U-RS, BS-V and RS-BS

$$\begin{cases} D_{U_1}^S = D_{R/U-V}^S &= \lambda \log\left(1+\frac{\gamma_2\rho^\beta}{\gamma_1\rho+1}\right) \\ &+ (\mu-\lambda)\log\left(1+\gamma_2\rho^\beta\right) \\ D_{V/U-V}^S &= \lambda \log\left(1+\frac{\gamma_4\rho^\beta}{\gamma_3\rho+1}\right) \\ &+ (\mu-\lambda)\log\left(1+\gamma_4\rho^\beta\right) \\ D_V^S = D_{V/V}^S &= \lambda \log(1+\gamma_3\rho) \\ D_{U_2}^S = D_{B/U}^S &= (1-\mu)\log\left(1+\gamma_1\rho^\beta\right). \end{cases} \tag{10}$$

    There is an outage when one of the following conditions occurs $D_{U_1}^S < D_U^t$, $D_{V/U-V}^S < D_U^t$, $D_V^S < D_V^t$, and $D_{U_2}^S < D_U^t$. We have the DMT function $d^S > d^{S-\mathrm{LB}-2} = \min(\max(d_1, d_6), d_2, d_3))$ with $d_6 = \left(\beta\left(1-\frac{r_U}{\mu}\right)-\frac{\lambda}{\mu}\right)^+$.
- $\mu < \lambda$
  - Option 1a: the rates of three hop transmissions U-RS, BS-V and RS-BS

$$\begin{cases} D_{R/V-U}^S &= \mu \log\left(1+\frac{\gamma_1\rho}{\gamma_2\rho^\beta+1}\right) \\ &+ (\lambda-\mu)\log(1+\gamma_1\rho) \\ D_{U_1}^S = D_{R/U}^S &= \mu \log\left(1+\gamma_2\rho^\beta\right) \\ D_V^S = D_{V/V-U}^S &= \mu \log\left(1+\frac{\gamma_3\rho}{\gamma_4\rho^\beta+1}\right) \\ &+ (\lambda-\mu)\log(1+\gamma_3\rho) \\ D_{U_2}^S = D_{B/U}^S &= (1-\lambda)\log\left(1+\gamma_1\rho^\beta\right). \end{cases} \tag{11}$$

    Similar to Option 1a in the first case, we have the DMT function $d^S > d^{S-\mathrm{LB}-3} = \min(\max(d_7, d_8), d_5, d_9)$ with $d_7 = \left(1-\frac{r_V}{\lambda}-\beta\frac{\mu}{\lambda}\right)^+$, $d_8 = \left(1-\frac{r_V}{\lambda-\mu}\right)^+$ and $d_9 = \beta\left(1-\frac{r_U}{1-\lambda}\right)^+$.

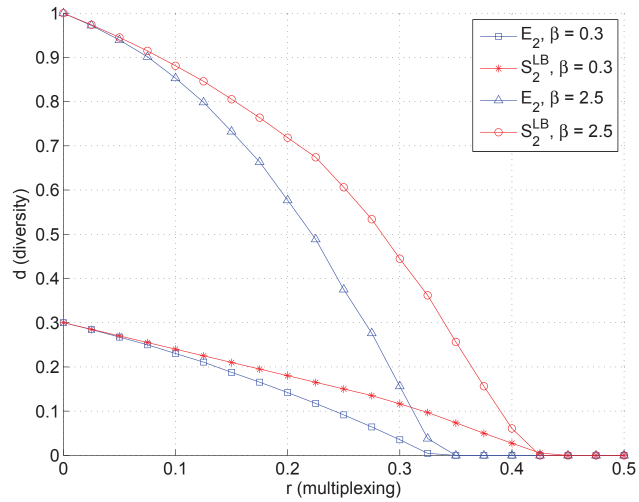

Fig. 2. Maximum DMT function d_o^E and lower bound of the maximum DMT funciton $d_o^{S-\mathrm{LB}}$ at $\beta = 0.3$ and $\beta = 2.5$ with $r_U = r_V = r$.

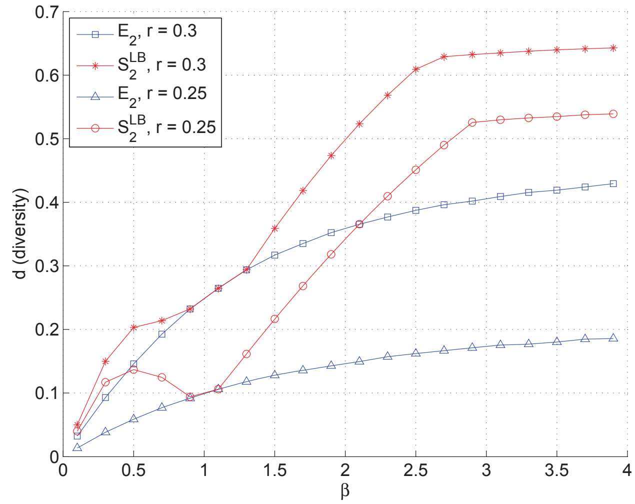

Fig. 3. d_o^E and $d_o^{S-\mathrm{LB}}$ $r = 0.3$ and $r = 0.25$.

– Option 2b: the rates of three hop transmissions U-RS, BS-V and RS-BS

$$\begin{cases} D_{U_1}^S = D_{R/U-V}^S & = \mu \log\left(1 + \frac{\gamma_2 \rho^\beta}{\gamma_1 \rho + 1}\right) \\ D_{V/U-V}^S & = \mu \log\left(1 + \frac{\gamma_4 \rho^\beta}{\gamma_3 \rho + 1}\right) \\ D_V^S = D_{V/V}^S & = \lambda \log\left(1 + \gamma_3 \rho\right) \\ D_{U_2}^S = D_{B/U}^S & = (1-\lambda) \log\left(1 + \gamma_1 \rho^\beta\right). \end{cases} \quad (12)$$

Similar to Option 2 in the first case, we have the DMT function $d^S > d^{S-\mathrm{LB}-4} = \min(d_2, d_9, d_{10})$.

In summary, $d^{S-\mathrm{LB}} =$

$$\max\left(d^{S-\mathrm{LB}-1}, d^{S-\mathrm{LB}-2}, d^{S-\mathrm{LB}-3}, d^{S-\mathrm{LB}-4}\right). \quad (13)$$

$d_o^{S-\mathrm{LB}} = \max_{\lambda,\mu} d^{S-\mathrm{LB}}(\lambda, \mu)$ is the lower bound of the maximum DMT function that scheme S can achieve. Similarly to scheme E, optimal μ and λ can also be written in close forms however not presented here due to its cumbersomeness.

V. Numerical Results

We search d_o^E and $d_o^{S-\mathrm{LB}}$ with μ and λ getting values from 0 to 1 with resolution $\delta\mu = \delta\lambda = 0.001$. As shown in Fig. 2, although having the same maximum diversity gain of 1 ($\beta = 2.5$) and of 0.3 ($\beta = 0.3$), scheme S has a higher diversity gain at any multiplexing gain and also the higher maximum multiplexing gain than scheme E. In scheme E, because the three hop transmissions are conducted orthogonally in time and $r_U = r_V$, the maximum DMT function or the minimum outage probability is achieved when the durations of the three transmissions are equally balanced. The maximum multiplexing gain is thus $\frac{1}{3}$.

Fig. 3 shows the DMT functions versus β, when β is near 1, the signal from user U and the signal from BS equally strong when $\rho \to \infty$, the successive interference cancellation does not help therefore scheme S has the same DMT function with scheme E at these values.

VI. Conclusion

We describe and calculate the user rates and sum-rate of the reference and CDR schemes for relayed uplink and direct downlink transmission. We calculate the Diversity-Multiplexing Trade-off functions either the exact value or both upper/lower bounds of the schemes. The CDR scheme is shown to have a higher DMT function at any multiplexing gain as well as the higher maximum multiplexing gain than the reference scheme.

Acknowledgment

This work is supported by the Danish Research Council for Technology and Production, grant nr. $09-065035$.

References

[1] P. Popovski and H. Yomo, "Bi-directional Amplification of Throughput in a Wireless Multi–Hop Network," IEEE VTC, Spring 2006.
[2] S. Katti, S. Gollakota and D. Katabi "Embracing Wireless Interference: Analog Network Coding," ACM SIGCOMM, 2007.
[3] H. Ning, C. Ling and K. Leung, "Wireless Network Coding with Imperfect Overhearing," arXiv:1003.4270v1 [cs.IT] 22 Mar 2010.
[4] C. Thai and P. Popovski, "Coordinated Direct and Relay Transmission with Interference Cancelation in Wireless Systems," IEEE Comm. Letters, vol. 15, no. 4, April 2011, pp. 416-418
[5] C. Thai, P. Popovski, M. Kaneko and E. Carvalho, "Coordinated Transmissions to Direct and Relayed Users in Wireless Cellular Systems," in Proc. IEEE ICC, Kyoto, Japan, Jun 2011.
[6] C. Thai and P. Popovski, "Coordination of Regenerative Relays and Direct Users in Wireless Cellular Networks," in Proc. IEEE ISWCS'11, Aachen, Germany, Nov 2011.
[7] D. Tse and R. Viswanath, "Fundamentals of Wireless Communications," Chap. 8, Cambridge Press, 2005.
[8] L. Zheng and D. Tse, "Diversity and multiplexing: A fundamental tradeoff in multiple antenna channels," IEEE Trans. Inform. Theory, vol. 49, pp. 1073-1096, May 2003.
[9] M. Ansari, A. Bayesteh and A. Khandani, "Diversity-multiplexing tradeoff in Z-channel," in Proc. in IEEE CWIT, pp. 21-24, June 2007.