



Aalborg Universitet

AALBORG UNIVERSITY  
DENMARK

## Three-dimensional Reciprocal Structures: Morphology, Concepts, Generative Rules

Parigi, Dario; Pugnale, Alberto

*Published in:*  
IASS-APCS 2012 Proceedings

*Publication date:*  
2012

*Document Version*  
Publisher's PDF, also known as Version of record

[Link to publication from Aalborg University](#)

*Citation for published version (APA):*  
Parigi, D., & Pugnale, A. (2012). Three-dimensional Reciprocal Structures: Morphology, Concepts, Generative Rules. In *IASS-APCS 2012 Proceedings: from spatial structures to space structures* The International Association for Shell and Spatial Structures.

### General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- ? Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- ? You may not further distribute the material or use it for any profit-making activity or commercial gain
- ? You may freely distribute the URL identifying the publication in the public portal ?

### Take down policy

If you believe that this document breaches copyright please contact us at [vbn@aub.aau.dk](mailto:vbn@aub.aau.dk) providing details, and we will remove access to the work immediately and investigate your claim.

# Three-dimensional reciprocal structures: morphology, concepts, generative rules

Dario PARIGI<sup>1</sup> and Alberto PUGNALE<sup>2</sup>

<sup>1</sup> Assistant Prof., Dep. of Civil Engineering, Aalborg University, Aalborg, DK, dp@civil.aau.dk

<sup>2</sup> Assistant Prof., Dep. of Civil Engineering, Aalborg University, Aalborg, DK, apu@civil.aau.dk

## Summary

This paper presents seven different three-dimensional structures based on the principle of structural reciprocity with superimposition joint and standardized un-notched elements. Such typology could be regarded as being intrinsically three-dimensional because elements sit one on top of the other, causing every configuration to develop naturally out-of the plane.

The structures presented here were developed and built by the students of the Master of Science in "Architectural Design" during a two-week long workshop organized at Aalborg University in the fall semester 2011.

**Keywords:** *structural reciprocity; complex spatial structures; morphology; conceptual design.*

## 1. Introduction

The principle of structural reciprocity, i.e. the use of load-bearing elements to compose a spatial configuration wherein they are mutually supported one another, has been known since antiquity. In the world of construction, the application of the principle of reciprocity requires:

- the presence of at least two elements allowing the generation of a certain forced interaction;
- that each element of the composition must support and be supported by another one;
- that every supported element must meet its support along the span and never in the vertices (in order to avoid the generation of a space grid with pin-joints) [1].

According to these generative rules, reciprocal configurations that use a superimposition joint, i.e. where un-notched bars sit on top and in the bottom of each other could be regarded as being intrinsically three-dimensional because they develop naturally out-of the plane, and their geometry is extremely difficult to predict and control.

This paper presents seven different three-dimensional configurations developed and built by the students of the Master of Science in "Architectural Design" during a two-week long workshop organized at Aalborg University in the fall semester 2011 [2]. Since the use of physical models is the most diffused tool to 'find' the form of reciprocal configuration due to the direct interaction with the designer, the students were called to explore the 3D peculiarity of reciprocal structures by building scale models and full-scale prototypes.

## 2. The concept of three dimensionality

In reciprocal configurations with superimposition joint elements sit on top and in the bottom of each other, and if they are un-notched their axes are not aligned and separated by a distance called eccentricity. This causes such configurations to develop naturally out-of plane, and their geometry becomes soon very difficult to predict and control because the position of each element at the same time determines, and is determined, by the position of adjacent ones.

An example of such configuration is the structures of Leonardo Da Vinci (Figure 1). Despite being not evident from the original plan representation, once elements are placed one on top of each other the structure develops out-of-plane as a dome-like structure.

In order to avoid the out-of plane deviation notched elements can be introduced to re-align elements axis and eliminate the eccentricity. This solution was adopted in the past to get configurations that develop in plane as in the slab by Johannis Wallis, as shown in Figure 2. Another solution to reduce geometric complexity is to align elements axis by not superimposing one on top and in the bottom of each other, as in the slab designed by Sebastiano Serlio (Figure 3).

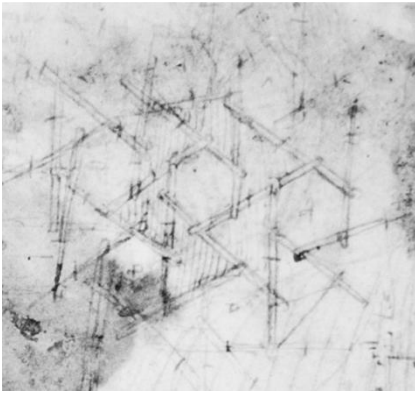


Figure 1: Da Vinci L, un-notched elements

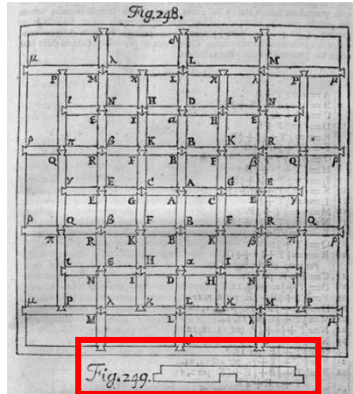


Figure 2: Wallis J, notched elements

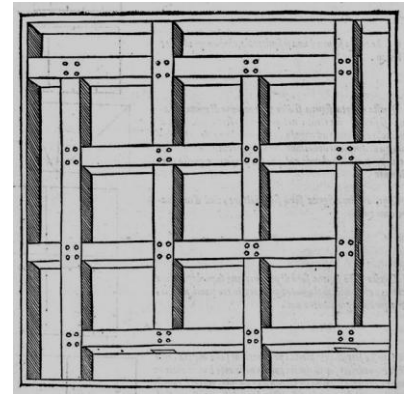


Figure 3; Serlio S, aligned axis

However such intrinsic three dimensionality could be regarded as an opportunity if the overall geometry of the reciprocal configuration can be predicted. Hybrid optimization strategies proved to be an effective computational tool in controlling their geometry, such that complex three dimensional configurations with potentially any shape can be obtained with the use of standardized un-notched round elements [3].

In this framework, the use of physical models is a tool that triggers the exploration of new typologies because of the direct interaction of the designer with the complex geometry of the model itself. The structures presented here were developed using physical scale models and built in a full scale prototype with the aim to explore and test how three dimensionality of superimposition joint could lead to reciprocal configuration:

1. that expand in more than one direction or it has no direct reference to a surface;
2. where every joint is a reciprocal superimposition joint.

The aim is to stimulate design thinking in a more spatial way, with a typology that is intrinsically three dimensional. This allow to understand the potential of such configurations to create architectural spaces by excluding those typologies whose final result is predictable.

As an example of structures that can expand in one direction is Da Vinci's bridge represented in Figure 4: such configuration can be expanded indefinitely along one direction by repeating the basic fan. Placing two or more bridges side by side we can cover a larger span in the transversal direction, however the direction of expandability remains the longitudinal one.

Surface structures could be easily obtained by creating regular or non regular patterns repeating along two directions. Such typology was excluded because the results that could be achieved by using physical models are predictable and lead to dome-like structures.

Another type of structure that was explicitly excluded by rule no. 2 is the one obtained by superimposition of different fans - with this generative rule the joint between two different fans is not reciprocal, being constituted by two bars that does not interlock with each other, instead they merely touch and need to be fixed with other connecting methods.

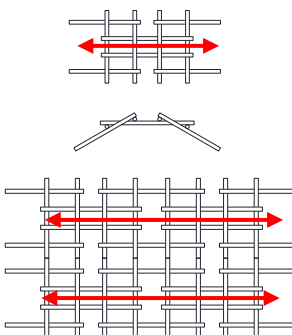


Figure 4: Da Vinci's bridge

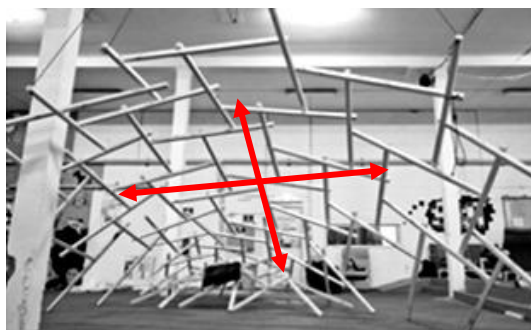


Figure 5: reciprocal surface

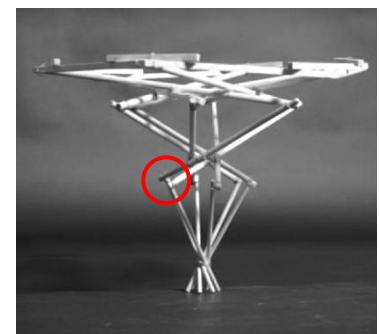


Figure 6: fans superimposition by Popovic Larsen [4]

### 3. The structures

The students at Aalborg University developed seven reciprocal structures, which interpret in different declination the assigned task. Different strategies to generate three dimensional structures that have no reference to a surface are analyzed below, highlighting:

1. the generative rule;
2. the three dimensional potential;
3. relation between the physical model and the created space.

The identification of those elements aims to generalize in a systematic way the results, also suggesting the extension and application of the generative rules to this class of structures to obtain potentially infinite number of variations and re-combinations.

#### 3.1 Structure 1 - "Bugs"

##### 3.1.1 The generative rule

This structure is generated starting from the classic typology of the Leonardo Da Vinci Bridge (Figure 1) consisting of 6 longitudinal and 4 transversal interlocked bars, which can expand along one direction only. The generative rule is the elimination of two transversal bars and the rotation of four longitudinal bars in order to meet and to be supported directly by the two longitudinal bars (Figure 2).

##### 3.1.2 The three dimensional potential

With the rotation of the four longitudinal bars (the 'legs' of the 'bug') the elemental fan can expand into four different directions. Each leg can be extended both upward and downward, enabling the possibility to create three dimensional patterns of elements that develop in three dimensions at different heights.

##### 3.1.3 Relation between the physical model and the created space

The realized model highlighted the difficulty in controlling the geometry of such configuration and the need of an absolute precision in order for the bars of different bugs to meet at the desired position.

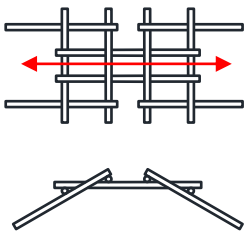


Figure 1: Leonardo bridge fan

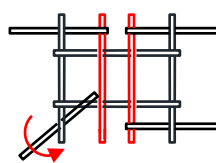


Figure 2: the generative rule

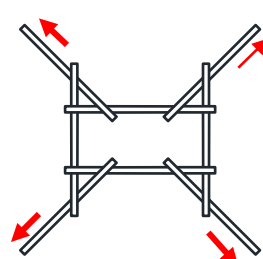


Figure 3: the elemental fan (the 'bug')

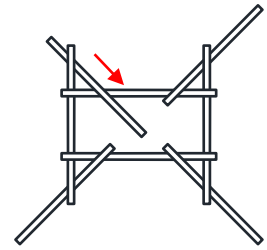


Figure 4: three dimensionality

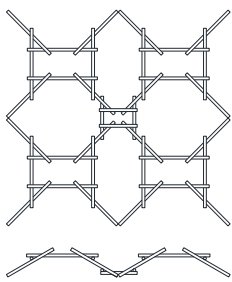


Figure 5: array of bugs, plan(top) and elevation

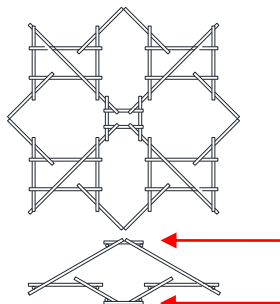


Figure 6: A three dimensionaonl array, plan(top) and elevation

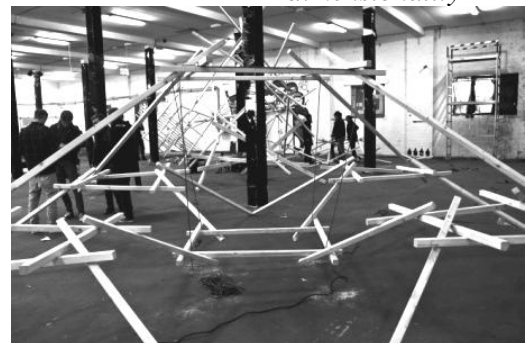


Figure 7: the full scale prototype

### 3.2 Structure 2 - "Matrix"

#### 3.2.1 The generative rule

The structure is based on the repetition of the reciprocal pattern represented in Figure 11 along two orthogonal directions (Figure 8 and 9) at every node intersection. The structure is based on the node of Figure 10 that allows to deal with three bars coming from three different orthogonal directions.

#### 3.2.2 The three dimensional potential

The structure can be expanded indefinitely in the space along every direction.

#### 3.2.3 Relation between the physical model and the created space

The prototype is a cube with extending legs elements that suggest the indefinite expandability of such configuration along three directions (Figure 12). The final model appears as an orthogonal 'architectural frame' (Figure 13). It suggest also the possibility to generate different space frames with different starting two dimensional patterns.

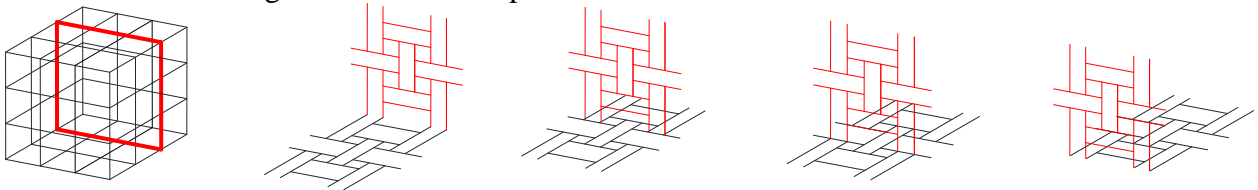


Figure 8

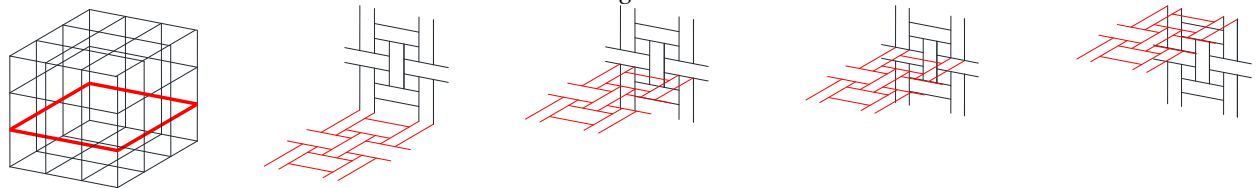


Figure 9

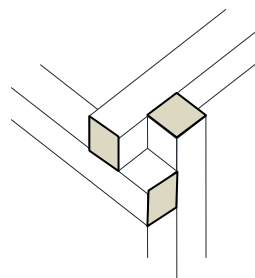


Figure 10

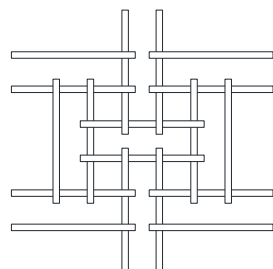


Figure 11

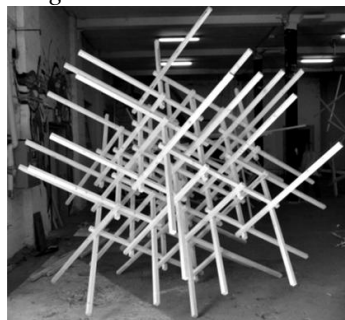


Figure 12



Figure 13

### 3.3 Structure 3 - "Neural network"

#### 3.3.1 The generative rule

The generative rule is the combination of a positive (Figure 13) and a negative (Figure 14) reciprocal fan by interlocking (Figure 15).

#### 3.3.2 The three dimensional potential

The potential of such generative rule relies on the possibility to conceive it as a three dimensional node that can expand along six different direction, three in the bottom (correspondent to the three bars of the positive fan), and three in the top (corresponding to the three bars of the negative fan). The two interlocked fans can also rotate relatively to the other, thus they can vary the direction they are pointing at.



### 3.3.3 Relation between the physical model and the created space

The realized structure demonstrates that the node enable the possibility to freely design spatial structures by combining the nodes at different heights and position, with almost unlimited possibilities for expandability. This concept can be seen as an open-ended structure in which the issue is related to the design of its ending points.

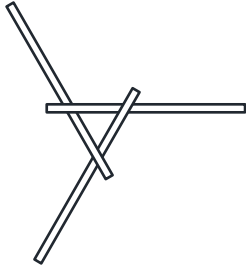


Figure 13: Positive fan

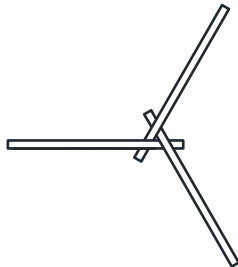


Figure 14: Negative fan

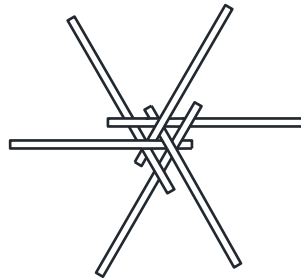


Figure 15: Combined positive and negative fan

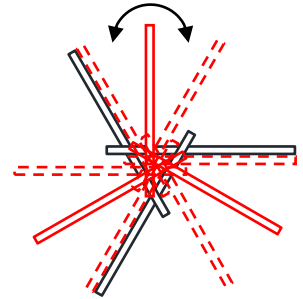


Figure 16: relative rotation of the two interlocked fans

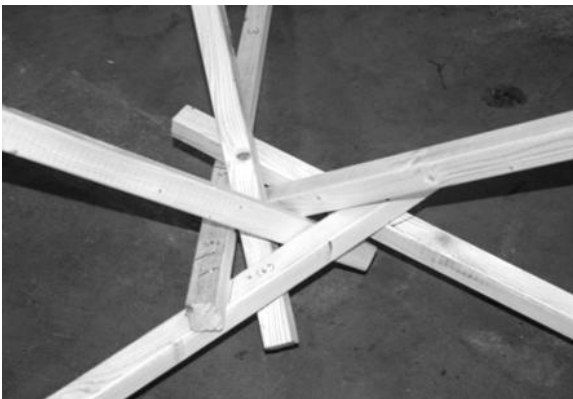


Figure 17

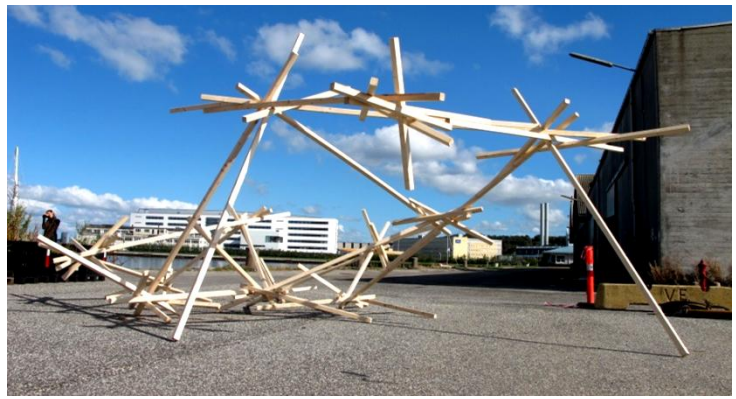


Figure 18

## 3.4 Structure 4 - "Hypar"

### 3.4.1 The generative rule

Starting from a three bars fan, an outer layer of bars is added, each one interlocking with two bars of the inner layer. Many layers could be added until physical limitations are reached (Figure 19).

### 3.4.2 The three dimensional potential

The structure has three high points and three low points, and it develops in the space along these directions. The geometry of such configuration turned out to be extremely difficult to control, because of the interdependence of each element with the others.

### 3.4.3 Relation between the physical model and the created space

The scale model (Figure 20) and the final prototype define a space with sculptural qualities (Figure 22). Bars touches towards their ends and not along their span (Figure 21).

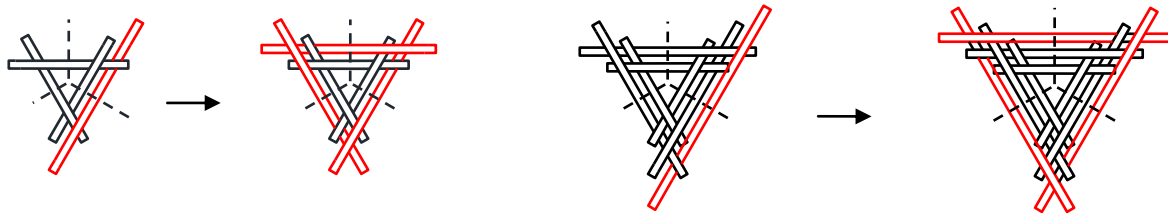


Figure 19

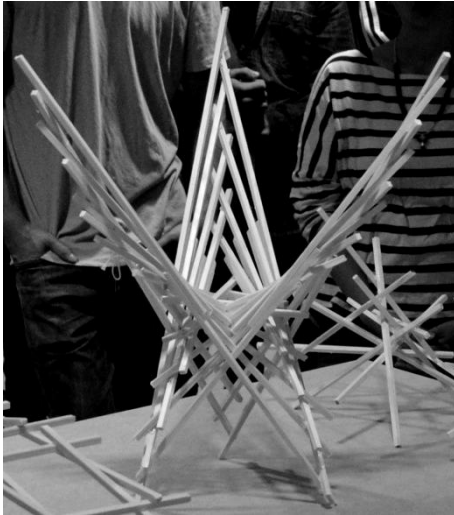


Figure 20



Figure 21



Figure 22

### 3.5 Structure 5 - "Star frame"

#### 3.5.1 The generative rule

Starting from a six bars array where bars extend in both direction before and after the contact point (Figure 23b/c), an outer array of bars is added (Figure 23e) where each bar supported by two bars facing opposite in the first array (Figure 23d). A final array of six bars is added (Figure 23 g), where each bar is supported by one bar belonging to the first array and one to the second array (Figure 23f).

#### 3.5.2 The three dimensional potential

The configuration, despite the rigorous methods required to generate it, creates a seemingly random "cloud" of bars (Figure 25). Each bar point into different direction, and it has no reference to surfaces. The configuration seems to be a closed configuration, as it does not allow for further expandability

#### 3.5.3 Relation between the physical model and the created space

The 1:1 prototype was easily scaled up from the model. The integrity of the space it creates descends from the fact that the top elements are interlocked and constitutes a whole with the elements that goes to the ground.

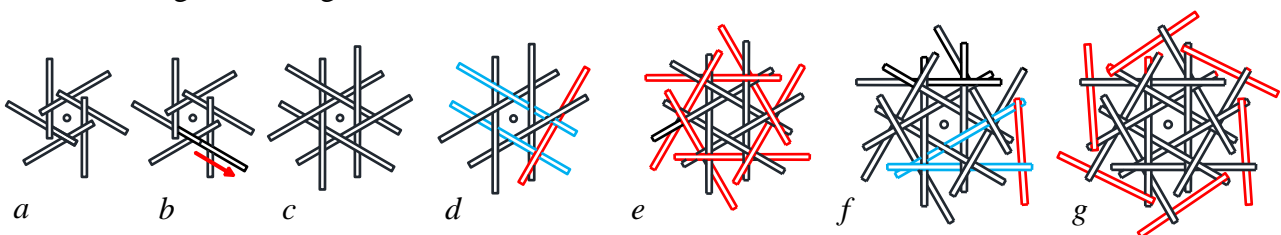


Figure 23



Figure 24



Figure 25



Figure 26

### 3.6 Structure 6 - "Flame"

#### 3.6.1 The generative rule

Starting from a three bars fan, a first array of three bars is added (Figure 27): each bar touch the ground at one extreme. A second array is added where each bar being supported by the first array and the three bars fan. A the third array is added, each bar being supported by the first and the second array. The generative rule suggest that this operation can be repeated again starting now from the third array, a three bars fan similar to the starting one.

#### 3.6.2 The three dimensional potential

Depending on the angles created between bars, the model opens up the possibility to create a fully reciprocal tower, indefinitely expandable. However, the realized prototype closes up representing a sculptural flame.

#### 3.6.3 Relation between the physical model and the created space

The configuration encloses a three dimensional space taking advantage of the three dimensionality of the reciprocal superimposition joint. The step from the scale model to the prototype highlighted that the geometry is extremely difficult to control, because of the interdependence of one element with the other. In the realized prototype the tower closes up in the fourth array and does not offer the possibility to continue indefinitely, however further experimentation could lead to a fully expandable structure.

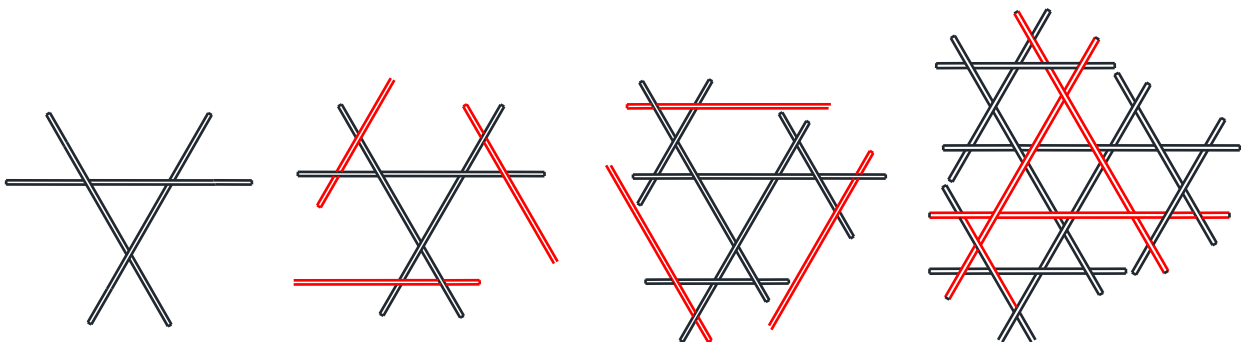


Figure27

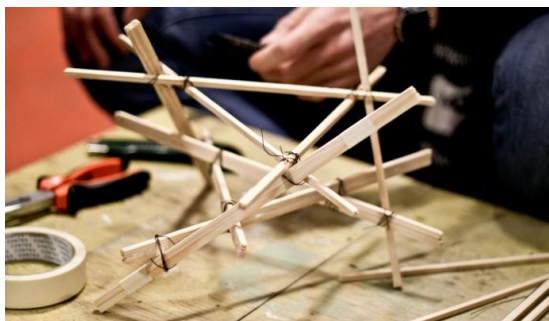


Figure 28



Figure 29



### 3.7 Structure 7 – "Wave"

#### 3.7.1 The generative rule

The wave combines four bars fan and three bars fan. The two typologies are assembled in an orthogonal direction with respect to each other as shown in Figure 30.

#### 3.7.2 The three dimensional potential

Despite the structure derives directly from a surface it is three dimensional because their quadrilateral and triangular fans develops into two orthogonal directions.

#### 3.7.3 Relation between the physical model and the created space

The structure encloses a space where the interaction between the two typologies of fans are responsible of the overall stability and manifest itself in the different aesthetic qualities in the exterior and the interior (Figure 31).

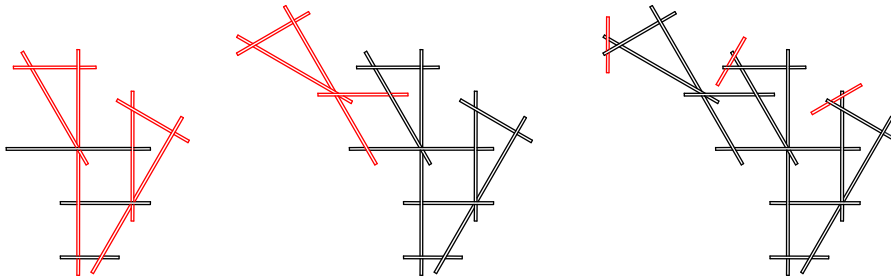


Figure 30

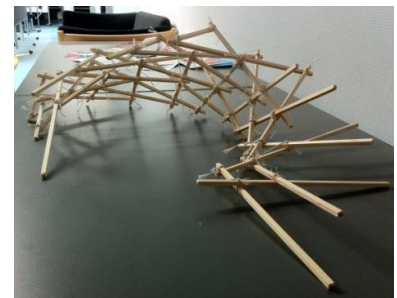


Figure 31

## 4. Conclusions

The structures presented here are based on the principle of reciprocity and are built using un-notched standardized elements. Each structure interprets differently the intrinsic three dimensionality of reciprocal structures with superimposition joint, and their generative rules are reported in the paper as schemes whose aim is to generalize the concepts behind them.

It should be underlined that small variations on the position of elements and the variation of the value of their thickness could influence dramatically the geometry of the configuration. However, those elements were voluntarily omitted from our considerations in order to focus on the topology of the connections, and not on the infinite variations that can be achieved by changing such parameters.

## 5. Acknowledgements

The authors would like to thank the Department of Architecture, Design & Media Technology, Aalborg University, for providing us working spaces and funds, the students of the 1<sup>st</sup> semester of the Master of Science in "Architectural Design" at Aalborg University for their intense participation to the workshop, Prof. Poul Henning Kirkegaard for encouraging us in organizing this activity and Nathalie Balfroid and Marie Frier for their active collaboration during the whole process.

## References

- [1] Pugnale A, Parigi D, Kirkegaard PH, and Sassone M, "The principle of structural reciprocity: history, properties and design issues", *Proceeding of the IABSE-IASS symposium 2011 "Taller, Longer, Lighter"*, London, United Kingdom, 20-23 September 2011.
- [2] Pugnale A, Parigi D, "Approaching technical issues in architectural education", *Proceedings of the IASS-APCS 2012*, Seoul, Korea, 21-24 May 2012.
- [3] Parigi D, Kirkegaard PH, Sassone M, "Hybrid optimization in the design of reciprocal structures", *Proceedings of the IASS-APCS 2012*, Seoul, Korea, 21-24 May 2012.
- [4] Popovic Larsen O, *Reciprocal Frame Architecture*, Elsevier, Burlington, 2008.