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## Automatic Design of Loop-Sorting-Systems

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# Automatic Design of
# Loop-Sorting-Systems

**Ph.D. Thesis**

**by**

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# Preface

This dissertation is submitted to the Faculty of Engineering, Science and Medicine at Aalborg University, Denmark in accordance to the degree *Doctor of Philosophy* in Mechanical Engineering.

This project is performed in corporation with Department of Mechanical and Manufacturing Engineering at Aalborg University, Department of Megatronic at University of Agder and in corporation with Crisplant A/S in the period 1st of May 2008 to 30st of April 2011.

The industrial Ph.D. project is partially founded by the Danish Agency for Science, Technology and Innovation and by the company Crisplant A/S.

I would like to use this opportunity to thank several persons who have contributed or helped with the project.

Primarily, I would like to thank my family and my lovely girlfriend for patience and support. Lone, I first of all owe you a big thank for being patient in busy periods, supporting in tough periods and helping on the project in the writing process.

Also, my main academic supervisor professor Michael Rygaard Hansen who has initiated this project, has supported me and provided inspiration. I appreciate the time you have invested in the project and in me. I wish you and your family the best and hope we will keep in contact.

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I would like to thank my former industrial supervisor Erik Steen Pedersen for guiding me through the project and giving me a 100% support. I would thank my final industrial supervisor Uffe Lykkegaard for your patience support.

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Søren Emil Sørensen
*Ulstrup, 1st of May, 2011.*

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# Abstract

This industrial Ph.D. project deals with the development of advanced engineering tools to help the design engineer at Crisplant A/S designing superior loop-sorting-systems (LSS).

Today, the development of LSS is performed manually through a design process which involves several iterations before a satisfying solution is reached. The large number of design iterations evolves from the combinatorial problem of choosing from several components of the standard product portfolio and simultaneously fulfilling several demands defined by the customer and by Crisplant. Despite the dedicated and skilled design engineers an optimum solution of the material handling system is probably never obtained due to the complexity of the task. In fact, some systems may encompass faults or inappropriateness which may first be discovered in the phase of installation. New computer based methods to support the design process have therefore become a necessity in order to be competitive and avoid solutions with faults.

The primary objective of the Ph.D. project has been the development of one or several tools to help the design engineer in the decision-making process of designing loop-sorting-systems. The aim has been to provide superior designs and to avoid the occurrence of unwanted dynamic effects.

A dynamic chain model has been developed with the purpose of being able to identify poor chain dynamics of an arbitrary shaped track layout. The proposed model applies the theory for unconstrained rigid multi-body dynamics using a force element formulation to model the body interactions. The chain model has been verified against experimental data of several full scale loop-sorting-systems using a special developed sensor cart. An implicit parameter identification method has been proposed to estimate the damping within the system. The implicit formulation applies optimization methods by minimizing the residual between measured and simulated data. By using the damping parameters of the chain models as design variables the best estimation has been obtained.

A tool for price and footprint optimization of the track layout has been developed. The tool uses the track length as objective function in which non-contact constraints and user defined constraints can be applied. A three stage approach has been utilized with success to avoid obstacles and reach an optimum solution according to the discrete track elements.

Finally, research has been devoted to finding a method to reduce unwanted chain dynamics in the track layout. The dynamic performance is optimized by changing the shape of the track layout using optimization methods. A kinematic model of the polygon action has been developed for derivation of the objective function.

# Resumé

Dette erhvervs-PhD-projekt beskæftiger sig med udviklingen af avancerede værktøjer, der kan hjælpe udvindingsingeniører ved Crisplant A/S med at opnå et forbedret loopsorteringssystem.

Udviklingen af loopsorteringssystemer er i dag udført manuelt gennem en designproces, som gennemgår flere iterationer før en acceptabel løsning er fundet. Det store antal af designiterationer opstår på grund af det kombinatoriske problem, hvor der skal vælges imellem adskillige komponenter fra standardprodukt-portføljen og samtidig opfylde flere krav specificeret af kunden og af Crisplant. Et optimalt loopsorteringssystem er aldrig nået på trods af engagerede og dygtige udviklingsingeniører. Derimod kan nogle systemer indeholde fejl eller uhensigtsmæssigheder, der i nogle tilfælde først bliver opdaget under installeringsfasen. Nye computerbaserede metoder, der kan hjælpe udviklingsingeniøren, er derfor blevet en nødvendighed for at være konkurrencedygtig og for at undgå løsninger med fejl.

Målet med dette PhD-projekt er at udvikle et eller flere værktøjer, der kan hjælpe i udviklingsprocessen af nye loopsorteringssystemer. Målet er at opnå overlegne designs og undgå problemer med uhensigtsmæssig dynamik i vognkæden.

En dynamisk model er udviklet med det formål at kunne identificere dårlig kædedynamik i ethvert tilfældigudformet skinnelayout. Den fremførte model anvender teorien for uafhængig stivlegemedynamik ved at anvende en deformationsafhængig formulering til at modellere interaktionen mellem de enkelte legemer. Kædemodellen er verificeret ved hjælp af eksperimentel data fra flere fuldskalaanlæg. Til dette er der anvendt en specialeudviklet målevogn. For at estimere systemets dæmpning er der blevet udviklet en implicit parameteri-dentifikationsmetode. Den implicitte formulering anvender optimeringsmetoder til at minimere fejlen mellem målt og simuleret data. Det bedste estimat af kædemodellens dæmpningsparametre er opnået ved at anvende disse som designvariabler.

Der er udviklet et værktøj til optimering af skinnelayoutet med hensyn til pris og footprint. Værktøjet anvender skinnelængden som kostfunktion, hvor der kan tilføjes kontaktbindinger og burgerspecificerede bindinger. Der er med succes anvendt en metode af tre optimeringsfaser for at passere forhindringer og for at kunne opnå løsninger med hensyn til de diskrete standardskinneelementer.

Afslutningsvist er der udviklet en metode, der vil sikre forbedret kædedynamik i skinnelayoutet. Ved brug af optimeringsmetoder er den dynamiske performance af vognkæden optimeret ved at ændre på formen af skinnelayoutet. Som evaluaringskriterie er en kinematisk model af polygoneffekten anvendt.

# Publications

This thesis is based on an collection of papers reproduced as-published only adapted to fit this printed format.

## Publications in Refereed Journals

- *Paper A*: S. E. Sørensen, M. R. Hansen, M. K. Ebbesen and O. Ø. Mouritsen, *Time domain simulation of large-scale material handling chains using an unconstrained formulation*, Journal of Multi-body Dynamics, 225, 95-110, 2011.

- *Paper B*: S. E. Sørensen, M. R. Hansen, M. K. Ebbesen and O. Ø. Mouritsen, *Implicit Identification of Contact Parameters in a Continuous Chain Model*, Journal of Modeling, Identification and Control, 32, 1, 1-15, 2011.

- *Paper C*: S. E. Sørensen, M. R. Hansen, M. K. Ebbesen and O. Ø. Mouritsen, *Layout Optimization of Tracks in Loop-Sorting-Systems, Journal of Automation in Construction*, Submitted April 2011.

- *Paper D*: S. E. Sørensen, M. R. Hansen, M. K. Ebbesen and O. Ø. Mouritsen, *Enhanced Chain Dynamics in Loop-Sorting-Systems by means of Layout Optimization and a Kinematic Model of the Polygon Action*, Journal of Structural and Multidisciplinary Optimization, 1-10, 2011.

## Publications in Proceedings

- *Paper E*: S. E. Sørensen, M. R. Hansen and M. K. Ebbesen, *An Experimentally Verified Model for Time Domain Simulation of Large Scale Material Handling Chains*. The 1st Joint International Conference on Multibody System Dynamics, May 25-27, 2010, Lappeenranta, Finland.

- *Paper F*: S. E. Sørensen, O. Ø. Mouritsen and M. K. Ebbesen, *An Improved Multibody Model of Closed Chains in Loop-sorter-systems*, Proceedings of the Twenty Second Nordic Seminar on Computational Mechanics, October 22-23, 2009, Aalborg, Denmark.

## Division of Work between the Authors

For all Søren Emil Sørensen did the main work and the co-authors contributed with valuable discussion and helpful comments.

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# Nomenclature

| | |
|---|---|
| $A_{Chord}$ | Area of enclosed chord |
| $A_{Poly}$ | Area of intersecting polygon |
| $\mathbf{a}$ | Lower bound |
| $\mathbf{b}$ | Upper bound |
| $d$ | Left or right hand side of track boundary |
| $F$ | Scalar force |
| $f(\mathbf{x})$ | Objective function |
| $\mathbf{g}_i(\mathbf{x})$ | Inequality constraints |
| $\mathbf{h}_i(\mathbf{x})$ | Equality constraints |
| $I_{in}$ | Ingoing intersection point |
| $I_{out}$ | Outgoing intersection point |
| $K$ | Stiffness |
| $l_{cart}$ | Length of cart |
| $m$ | Number of design variables |
| $m_1$ | Number of equality constraints |
| $m_2$ | Number of inequality constraints |
| $n$ | Number of vertices |
| $P_i$ | Intersection point |
| $q$ | Obstacle number |
| $R_i$ | Curve radius |
| $\mathbf{x}$ | Vector of design variables |
| $\Delta l_{step}$ | Step length |
| $\Delta P$ | Kinematic size of the polygon action |
| $\delta$ | Contact indentation |
| $\dot{\delta}$ | Contact indentation rate |
| $\lambda$ | Hysteresis damping factor |
| $\chi_q$ | Obstacle polygon |

# Abbreviations

| | |
|---|---|
| CAD | Computer-Aided Design |
| CoR | Coefficient of Restitution |
| FFT | Fast Fourier Transform |
| HC | Hunt and Crossly hysteresis damping model |
| HW | Herbert and McWhannell hysteresis damping model |
| LN | Lankarani and Nikravesh hysteresis damping model |
| LSS | Loop-Sorting-Systems |
| LW | Lee and Wang hysteresis damping model |
| NRMSD | Normalized Root Mean Square Deviation |
| PID | Proportional Integral Derivative |
| RFID | Radio Frequency Identification |

# Contents

# 1

# Introduction

The scientific contribution and industrial applicability of the current industrial Ph.D. project is addressed through a collection of state-of-the-art scientific methods developed for the design of LSS. The work presented in this thesis yields academic contribution in key areas of optimization methods and multi-body dynamics by which the company Crisplant A/S has gained new advanced methods to reach superior designs of LSS.

## 1.1 Crisplant A/S

Crisplant A/S was founded in 1945 by civil engineer Svend Christensen who started out producing industrial applications like cement-mixers, hydraulic jacks, etc. Crisplant has expanded significantly ever since and is today a global leading provider in custom-made material handling systems. Crisplant is specialized as main contractor using a standard product portfolio in the design of material handling systems. The main business segments are airports, post companies and warehouses providing solutions for medium sized items like luggage, post packages, flyers, clothing, etc. Over a thousand sorting solutions have been installed world wide having Europe, North America and Asia as main market sectors. Resent projects encompass customers like: Helsinki Airport, Turkish Post, Melbourne Airport, etc.

Approximately 500 employees work at headquarter in Aarhus while a corresponding number of employees work in subsidiary companies all over the world. The organization of Crisplant encompasses a matrix structure divided into two major divisions: A project division where customer specific material handling solutions are developed and installed and a product division developing, manufacturing and maintaining the standard product portfolio. The size of a project normally varies in turnover between 0.01 and 1.0 billion DKK and is concluded throughout a period of a few months up to several years. A project is performed by a team who design the material handling system in close corporation with the customers. A system is uniquely adapted to the customers' demands using as many in-house standard products as possible. The main products in the standard product portfolio are the CrisBag solution and the loop-sorting-system. The CrisBag solution is a product specialized for the airport segment and is used for sorting, long distance transportation and storage of luggage. LSS are specialized for high capacity sorting of medium sized items with large variation is size, weight and surface texture.

Competition in the business segment of material handling system is tough and Crisplant is in constant fights to get new market shares. Some of the main competitors are

VanDerLande [56] BEUMER [4] and SIMENS [48] who either are specialized as main contractors or manufacture similar products.

In the appointed period of the industrial Ph.D. project Crisplant has changed ownership three times. In the summer of 2009 the family owned company BEUMER bought Crisplant from the investment company Melrose PLC. In addition to changing the name of the company from FKI Logistex to Crisplant the new owners have carried out significant changes to the organizational structure as well as changes to the business strategy. An integration process between Crisplant and BEUMER has been commenced with the aim of taking the best from both worlds into a joint cooperation.

## 1.2   The Topology of LSS

The LSS consists of three main components: the loop-sorter, inductions and chutes, see Figure 1.1.

The loop-sorter consists of an arbitrarily shaped track in which a closed chain of carts drives with a steady speed between $1.5m/s$ and $3.0m/s$. The track layout is defined by the design engineer by choosing between several different discrete types of standard track elements like: straight tracks, horizontal curved tracks, vertical curved tracks and spiral tracks. The track layout is shaped according to the sorting task and forms a loop. The layout is normally located $3m$ to $5m$ above the ground floor secured either to the ground or to the roof sealing. The chain of carts is propelled by several linear motors equally distributed along the track. The chain of carts is maintained at steady speed by a PID controller which changes the output motor force according to the reference speed.

A cart is manufactured in aluminum and shaped like a T, see Figure 1.2. It consist of a main profile and an end profile. Two steering wheels and two driving wheels are mounted on the end profile interfacing to the track sides. Each cart is connected to another cart at both ends of the main profile through a spherical plain bearings. The length of the carts may be between $500mm$ and $1250mm$ with a multiple of $50mm$ depending on the sorting task. A carrier is mounted on top of each cart with the purpose to transport items from the induction area to the sorting area. The carrier may be either a tilting tray or a cross belt system depending on the mix of the items.

The main purpose of an induction is to induct items onto the carriers in the most efficient and precise way. Inductions are normally allocated in groups where the combined task is to occupy as many empty carriers passing through as possible. Induction of items onto the carrier may be performed by top loading, cross loading or loading manually. Due to its high capacity cross loading is the solution frequently used.

The main purpose of chutes is the ability to handle items discharged from the sorter and guide them for further treatment. The number of chutes is normally very high in order to have as many destinations as possible along the track. The chute design is not very standardized caused by large variation in the demands to functionalities. Chutes may encounter functionalities as storage for accumulation of items, multiple storage or transportation of items from several meters above ground.

Sorting is performed by inducting items onto the carriers by which the chain of carts

A)



B)



C)
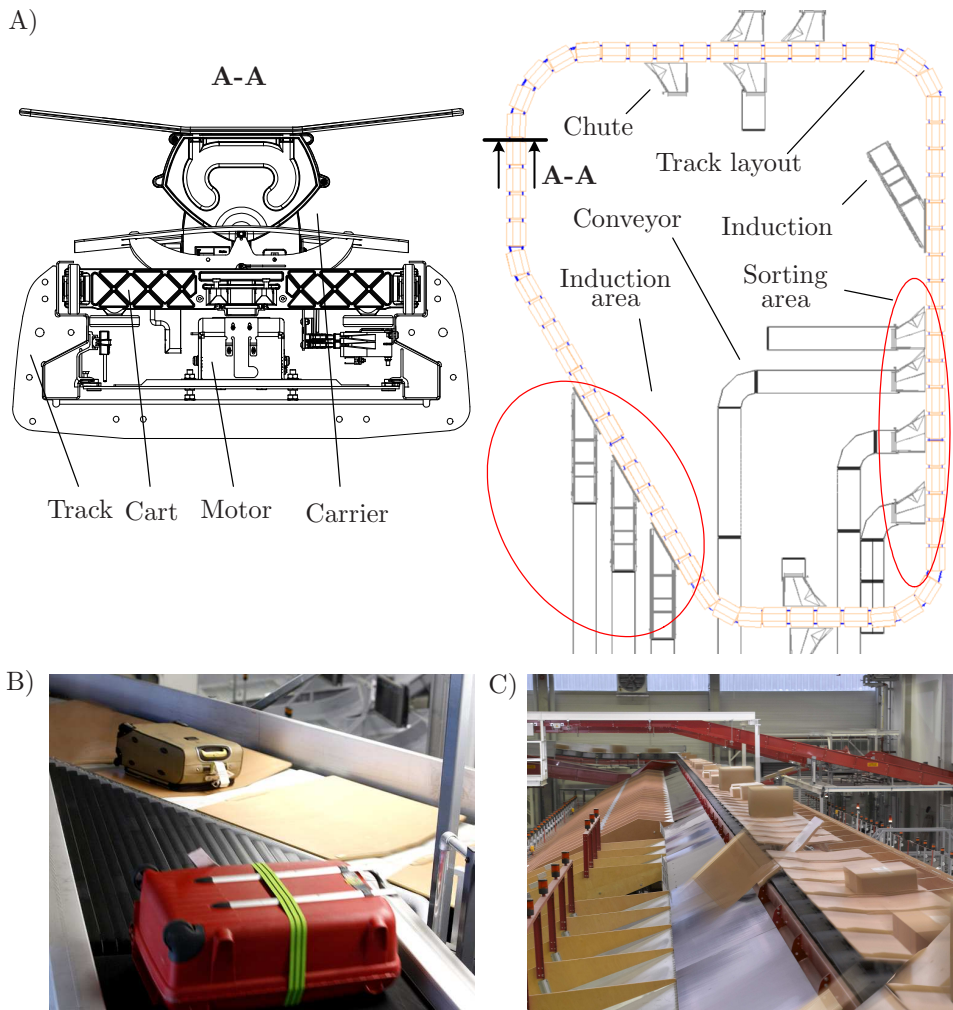


**Figure 1.1**: The topology of LSS. A) Track layout with chutes and inductions. B) Cross-loading of items onto chain of carts. C) Sorting area with several chutes located along the track side.

transports each item to a defined destination along the track. The destination of each item is identified through a bar-code or RFID tags on each item which is scanned at the inductions or along the track. The discharge mechanism is activated on the carrier when the item reaches the defined destination where it is discharged into a chute for further treatment.
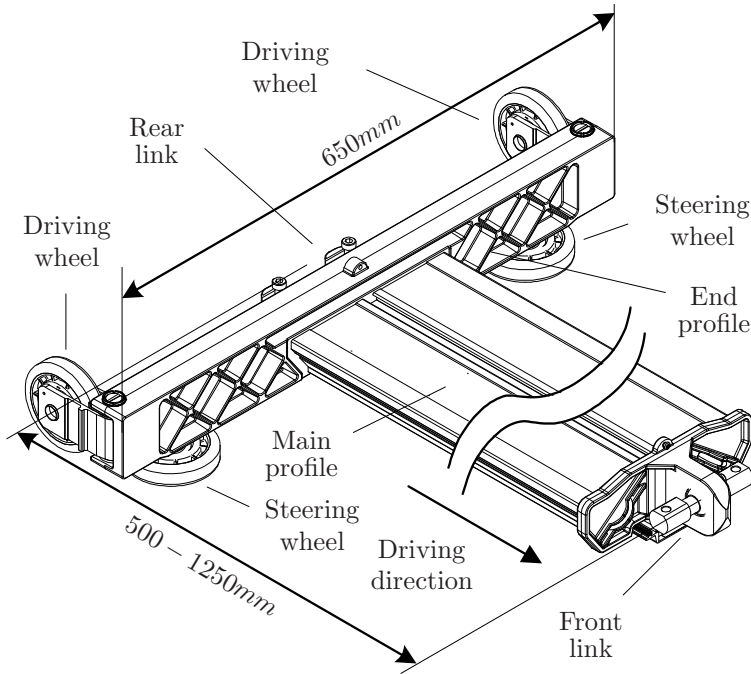
**Figure 1.2**: The design of a cart in the loop-sorter.

## 1.3  Background and Motivation

There is a certain amount of competition in the business segment of LSS and it is important to outdistance the competitors and maintain the leading position on the market. This is obtained by constantly improving the ability to compete, not only on price, but also on criteria such as:

- Increased capacity, i.e. number of sorted items over time.
- Standardization of subsystems and components. This entail reduced price and delivery time and ensure maintenance proven technology.
- Handling larger variation in the item mix.
- Reduced noise.
- More gentle treatment of items.
- Reduced operation and maintenance cost.
- Reduce tests with physical models and increase tests with virtual models i.e. reduced cost.
- Reduce research and design time, i.e. reduced delivery time.
- Increasing operation time, i.e. reduce down-time.
- Minimize footprint. (Footprint is the amount of space each machinery use on the floor)

One way of meeting these criteria is by enhancing the ability to design customized LSS.

### 1.3.1   Development of LSS

The process of developing sorting solutions encompasses numerous decisions by the design engineer to fulfill the demands defined by the customer and by Crisplant. These demands includes:

- Price
- Capacity
- Footprint
- Item mix
- System performance
- Environmental conditions
- Running reliability
- Etc.

Presently, these demands are evaluated through CAD drawings, spreadsheets, simulation tools and through plain judgments by the individual design engineer.

In conjunction with the demands the amount of design variables in a system is considerable. They encounter integer, discrete and continuous variables in which several are interdependent. The large amount of design variables and their interdependency make it impossible to asses the whole system at once by which the design process is divided into several sub iterations.

Today LSS are developed through a manual process that encompasses several design iterations with numerous expert parties involved. Despite the design engineers' dedicated work an optimum sorting solution is never obtained but may on the contrary include undetected faults and inappropriateness.

### 1.3.2   System Faults and Inappropriateness

The majority of these faults or inappropriateness are discovered and ratified in the development process through simple means. However, some are not discovered until the phase of installation. These faults and inappropriateness may include:

- Noise from the chain of carts, inductions and chutes.
- Increased wear on mechanical components.
- Unnecessary complex layout of the sorting system.

Faults and inappropriateness which are discovered in the phase of installation are critical as they often induce huge expenses to ratify.

A known inappropriateness in LSS is poor dynamics in the chain of carts due to the polygon action. Poor chain dynamics introduces large vibration in the track structure

and unwanted noise. This lead to increased wear and lower durability while the unwanted jabbing noise often encompasses unsatisfied customers. Experience show that sorting systems infected with poor dynamic are very complex and expensive to address. Therefore, being able to predict and remove poor chain dynamics in the development period induces large savings, higher durability and more satisfied customers.

### 1.3.3   Motivation to this Ph.D. Project

The development of customer-made LSS forces Crisplant to have skilled employees with high knowledge about the standard product portfolio, know-how on designer rules among subsystems and know-how on detecting possible faults. The high demands and the large degree of manual operations make the development process both time-consuming and costly. The time and cost associated with the development of LSS are highly competitive parameters that can do the difference whether or not a new market share is acquired. The motivation to this Ph.D. project is to develop new methods for Crisplant which will enhance the process of designing LSS and avoid faults and inappropriateness. These methods may help ensure Crisplant remains competitive.

## 1.4   Research Objectives

The objective of this project is *to develop one or several automatic or semiautomatic computer based design tools to help the design engineer reach customized LSS design efficiently and robustly.* The developed methods should result in a more streamlined design processes that reduce the amount of manual design iterations, reduce the time of development, minimize faults within the final design and reduce the overall communication network within the organization of Crisplant. Main emphasis is put on poor chain dynamics in order to find an efficient method to identify and rectify this problem. The research objective can be specifically outlined as:

- To study the possibilities of simulating the chain dynamics.
- To investigate a tool for optimization of track layouts.
- To investigate a tool for ratifying poor chain dynamics.

## 1.5   Research Approach

The objectives are divided into three parts that are treated separately:

### 1.5.1   Simulation of Chain Dynamics

The objective of this research is to develop a model capable of providing accurate results of the chain dynamics and capable of analyzing any track layout and cart configuration in a fast and robust way. The large movement in relation to the small deformation within the chain structure makes the use of rigid multi-body dynamics ideal to model the chain dynamics. Thus, research will be carried out on state-of-the-art methods of rigid multi-body chain models. This study will be used in the development of a dynamic chain simulation model.

The numerous shifting body interactions introduced by the topology of the LSS makes the use of accurate contact models critical in predicting the chain dynamics. Study of contact models will therefore be conducted and tested to find the superior model. The dynamic chain model is subjected to extensive verification to ensure the reliability and efficiency. Verification is conducted by comparing experimental data with simulated data. The experimental date is calculated on true sorting systems with several different track layouts, different track lengths and different cart configurations to have sufficient statistic material.

A high reliability of the chain model is important by means of which identification of model parameters is treated. Due to the complex model parameters an implicit method is used to obtain the best estimate.

### 1.5.2   Optimization of Track Layout

An optimization tool for minimization of price and footprint is developed. A study is performed within the scientific topics of trajectory, layout, shape and path optimization methods. This survey will apart from dealing with geometric optimization methods also treat the subject of geometric contact formulations. The conducted survey gives inspiration to an efficient and robust method for optimization of the track layout.

Obstacles and the building layout are included as they often introduce several limitations in the shape of the track layout. A constraint formulation for avoiding building and obstacle collision is therefore developed. The non-contact constraints must match into the optimization problem to maintain a continuous formulation. Also, the optimization problem must be able to handle the discrete angles and radii of the track curves.

High flexibility of the parametric track model is an important issue which must be solved in order to have an efficient optimization method. Consequently, research on parameterization of the track layout is carried out. The parametric formulation must introduce the least amount of design variables, with the least extent of interdependency. The parametric formulation must simultaneously be applicable for every shape of track layouts.

### 1.5.3   Optimization of Chain Dynamics

Optimization of chain dynamics focuses on the development of a tool to help the design engineer in reaching a track layout without poor chain dynamics. It is known from investigations of existing LSS that the dynamic performance of chain is highly dependent on the shape of the track layout due to the interference of the polygon action between curves. This tool must therefore automatically identify the extent of poor dynamics and hereby change the shape of the track layout until an improved shape is obtained.

The main concern of this research focuses on how to evaluate the dynamic performance of the chain. An obvious choice is to use the dynamic chain model. However, despite the simplicity of multi-body models the computational efficiency may not necessarily be sufficient in relation to the number of design evaluations applied by the optimization method. As the dynamic chain simulation model may be unsuited as evaluation criteria other means to evaluate the chain performance become necessary. Consequently, main part of this research concentrates on finding a simple and robust objective function

representing the chain performance.

The developed evaluation criterion is used in an optimization formulation which minimizes the chain dynamics by changing the shape of the track layout. The parametric model, developed for optimization of the track layout, is also used in this optimization problem. The dynamic chain simulation model is utilized to verify the applicability of the proposed optimization method.

## 1.6  Outline of Thesis

This introduction is further treated in the following technical chapters as the state-of-the-art, methodology and results of the conducted research are presented. Simulations of chain dynamics are described in Chapter 2 whereas Chapter 3 treats optimization of the track layout. A conclusion describing the scientific contribution of the project is presented in Chapter 4 along with a discussion of further work. Appendices in the final part of the thesis outline the collection of papers published in peer reviewed scientific journals.

# 2

# Simulation of Chain Dynamics

Part of the Ph.D. project is subject to the development of a chain model capable of simulating the dynamics in any arbitrary shaped track layout. The conducted research focuses on applying state-of-the-art methods to obtain an efficient and robust model as a consequence of being able to optimize the track layout in terms of dynamic performance. The conducted research also concerns identification of model parameters and verification of the model accuracy.

## 2.1 The Chain Model

Chain dynamics has undergone extensive research in literature. Some researchers perform experimental measurements on chains [9], others predict the noise from chains [60] and yet others work on modeling of the chain dynamic [40]. Additionally, dynamic performance in chain drives is discussed in more populist literature, see [1]. This research concentrates on developing a dynamic chain model which is capable of approximating the force vibrations in the chain of carts of an LSS.

### 2.1.1 State-of-the-art Chain Models

The majority of conducted research on simulation of chain dynamics is in applications of tracked vehicles and in power transmitting systems as chain-drives.

Nakanishi et. al. propose a spatial model in [35] of a heavy load earthmoving tracked vehicle. In the model Nakanishi et. al. use a simple planar model of the track utilizing an open chain approach to avoid the use of a Newton-Raphson solver and the occurrence of numerical instability. The planar model of the track is connected to a spatial flexible model of the chassis utilizing force elements in the contact between sprocket, idler rollers, track and the ground.

A full spatial dynamic model of a tracked vehicle is presented by Choi et al. in [8, 29]. The tracked vehicle is divided into rigid bodies as chassis, sprocket, roller, idler and track links. Each link of the track is joined together by rigid revolute joints. To avoid a system of differential and algebraic equations a joint coordinate formulation is utilized by cutting the track open at a selected secondary joint and replacing the revolute joint with a force element. With the body chassis and the two tracks as three kinematically decoupled subsystems, contact is defined through force element formulations.

Simulation results in [8, 29] appear promising; thus, comparisons with experimental data on real applications are missing. However, in [45, 44] two similar simulation models of armored tracked vehicles show fine accordance between measured and simulated data.

The stiff design of the track and the short distance between each joint in a tracked vehicle support the assumption of a perfect rigid revolute joint between each link. The assumptions of perfect rigid revolute joints are improper in applications like chain-drives. The discrete structure of the chain causes the well-known polygon action with the associated variation in chain speed, see e.g. [32]. The polygon action is due to the transition of the links from a smooth line into a series of chords of a circle arc and vice versa. The purpose of chain-drive simulation models is to investigate vibrations or noise caused by effects like the polygon action. Therefore, flexibility in the local region of the contact points has to be included to capture the actual vibrations.

A planar model of a continuously changing chain-drive is presented in [52] by Srnik et. al. To take the longitudinal flexibility of the chain into account Srnik et. al. treat each link as a rigid body connected to the neighboring links through a force element formulation of linear springs and dampers. Contact between the chain pins and two pulleys is defined using a quasi spatial formulation of the normal and frictional forces as well as the deformation of the pulley sheaves. Convincing results from simulation of a continuously changing chain-drive system with 197 degree of freedom are presented. In order to investigating the effect of wear in the continuously changing chain-drive system [51] has further extended the model from [52] by including both clearance in the joints and frictional effects like static friction and the Stribeck effect.

A simulation model of a chain-drive in contact with sprockets, guide-bars and rollers is presented by Pedersen in [38, 37]. The planar model utilizes unconstrained rigid forward dynamics with effort on the modeling of contact forces between the bodies. Simulations of a chain-drive on a marine diesel engine are presented. However, results are only compared with an analytical chain-drive model. A corresponding two dimensional chain-drive model is presented by Hippmann et. al. in [19]. This model uses the same multi-body formulation as [38, 37] combined with various techniques to reduce the computational time of the simulations. A comparison is performed between simulation and measured data of the cam shaft motion and tensional force. According to Hippmann et. al. the agreement with the measured data is both satisfying and sufficient, enabling the engine designer to further optimize the system dynamics.

A kinematically decoupled formulation has also been used for simulation of continuous timing belts. The model presented in [6, 31] utilizes a two-dimensional model of a discretized timing belt with the purpose of investigating the acoustic radiation from the impact contact between the belt and the pulleys. Verification with different experimental setups shows reasonable results, although the absence of torsional vibration modes in the planar model introduces an offset between the simulated and experimental data.

The chain design of the LSS differs in two ways from the design of the previous described chain types. A chain-drive has the single purpose of transferring mechanical power from one shaft to another. However, the purpose of the chain in the LSS is to continuously transport items to several different destinations along the chain. Therefore, a system of distributed power stations is used in order to even out forces along the chain. Another difference between the two chain types is the guidance of the chain. Rather than being

guided by sprockets, rollers and idlers, the chain of an LSS is guided by a single smooth continuous track that encloses the entire chain.

A dynamic chain model of LSS is presented by Ebbesen et al. in [10, 11, 12]. As in [52, 37, 38, 19] Ebbesen et al. apply a linear force element approach in the contact between chain links and in the contact between the chain and the track. The accuracy of the multibody model is validated by Ebbesen et. al. via experiments on different layouts utilizing accumulated fatigue (Rainflow Counting) and FFT (Fast Fourier Transformation) as means for comparison. In [39] Petersen et. al. further develop the multi-body model presented in [10, 11, 12] by introducing a beam finite element model of the track, taking flexibility of the track into account. Results from Petersen et. al. demonstrate that the flexible track approach only introduces minor influence on the results which does not justify the extended complexity and computational effort.

### 2.1.2 Multi-body Model of Chain Dynamics

The developed model uses the theory of unconstrained rigid multi-body dynamics in [36] assuming both carts and track to be rigid bodies. The interactions between carts and track are modeled using a force element approach by which deformation is assumed to occur in a small region of the contact points between the interacting bodies.

A flowchart of the main operation within the simulation of the chain dynamics is illustrated in Figure 2.1.
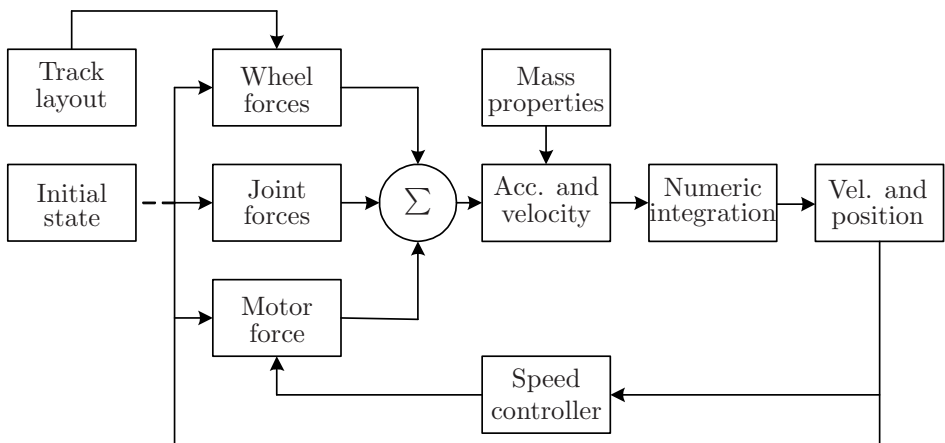


**Figure 2.1**: Numeric operations of the dynamic chain simulation model.

An algorithm is developed to define the initial state of the chain on the basis of any track layout and cart configuration. The track layout is imported from CAD software by which a model of the track centre line is generated. The initial state of the carts is obtained kinematically using a Newton-Raphson solver to find the number of carts and position of each cart along the track centre line. Seven points of contact is defined on each cart which is divided into three types of force evaluations, see Figure 2.1. The wheel contact is identified by finding the shortest distance between the track centre line and the

wheel centre. If the wheel is in contact with the track the contact force is evaluated by a penalty method using the indentation and indentation rate between the wheel and the track sides. The rear and front joint contact is modeled using the same penalty method to compute the contact force. The joint contact force is evaluated in two directions as the spherical plain bearing connecting the two carts differs in flexibility in axial and radial direction.

The algorithm searching the wheel contact is also utilized to identify those carts located above a linear motor. A cart located above a linear motor is applied a motor force at the bottom of the cart. A speed controller defines the motor force using the chain speed as control parameter.

Summing up all contact forces the kinematically decoupled carts yield a set of six independent equations of motions. The differential equation of motion is integrated numerically to obtain the velocity and position of each cart at the next time iteration, see Figure 2.1. A fifth order explicit Cash-Karp-Runge-Kutta solver with an adaptive step size controller described in [41] is utilized.

## 2.2   Implementation of a Force Element Formulation

Implementation of a superior force element model proves vital to reach a robust and accurate simulation of the chain dynamics. The force element formulation is important in order to capture the local flexibility of the cart and the energy dissipated in the contact period. Implementation of state-of-the-art force elements formulation is therefore commenced.

Contact models play a vital role in a variety of scientific fields. The importance of accurate, reliable and fast contact models is shown by the wide range of literature dating back to 1882 where [18] proposed one of the first contact models. The complex mechanics of interacting bodies make contact difficult to model as kinematics, geometry, material and surface properties have to be taken into account to get an accurate interpretation. Models in literature are therefore simplified through assumptions that quantify specific types of body interactions. Assumptions like, pure static contact, low impact velocity, pure impact, point contact, no plastic deformation and pure linear elastic deformation are often used as means to simplify the model and speed up the computational time.

A common used force element formulation is the linear spring and damper denoted the Kelvin-Voigt model, see [21, 53]. This model applies the penalty method by which the magnitude of the contact force is dependent on the indentation $\delta$ and indentation rate $\dot{\delta}$ between the contacting bodies. The Kelvin-Voigt model is a frequently used contact model and applies well in applications with both compression and tension between the contacting bodies. In impact contact the Kelvin-Voigt model does on the other hand perform poorly. An infinite force gradient may occur at the instant of contact as the indention rate is high, see Figure 2.2. Likewise tension between the interacting bodes may occur right before separation as the indentation rate is negative and the contact indentation approaches zero.

Another contact formulation is the Hunt-Crossly model, see [53]. The Hunt-Crossly contact model is a continuous force model derived from the local indentation and
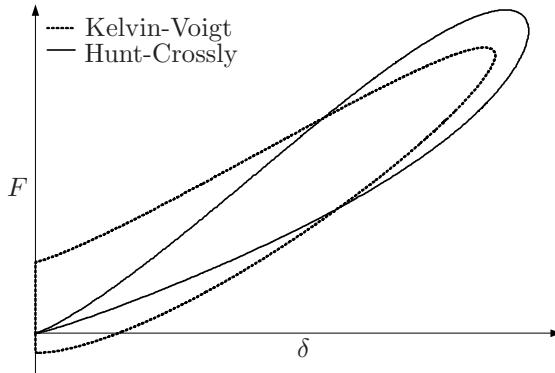
**Figure 2.2**: Difference between the Kelvin-Voigt and Hunt-Crossly contact force models.

indentation rate at the point of contact and evolves around the pure elastic contact model formulated by Hertz, as

$$F = K \cdot \delta^n + \lambda \cdot \dot{\delta} \cdot \delta^n \qquad (2.1)$$

in which $K$ and $n$ are the stiffness coefficient and the power exponent computed from material and geometric properties in the local contact region of the contacting bodies. A hysteresis shaped contact force is obtained replacing the common linear viscous damper with a hysteresis damping factor $\lambda$ and $\delta^n$ by which the enclosed area of the contact force versus indentation correspond to the energy dissipated in the period of contact, see Figure 2.2.

The Hunt-Crossly contact formulation is implemented in the dynamic chain model to avoid the unwanted effects of the Kelvin-Voigt model. However, one concern of the Hunt-Crossly model is the formulation of a proper hysteresis damping factor, $\lambda$ in relation to the coefficient of restitution (CoR), $e$. An overview of five different proposed formulations of the hysteresis damping factor is presented in [59]. In general two different approaches are used to define the hysteresis damping factor. The hysteresis damping factor in [21, 26, 27] derive from an energy approach by comparing the work dissipated by the Hunt-Crossly model to the energy dissipated by the CoR. A relationship between $\lambda$ and $e$ is obtained in [17, 30, 14] by combining the equation of motion of the colliding bodies with the Hunt-Crossly contact model. Through a range of assumptions [17, 30] obtain a closed form solution whereas [14] presents the exact open form solution.

As stated by [59], the proposed models of the hysteresis damping factor have never been compared to find the superior model despite their extensive use in literature. Consequently, four of the models in [59] have been tested on the dynamic chain model to identify a superior model. The tested models are: Herbert and McWhannell (HW) in [17], Hunt and Crossly (HC) in [21], Lankarani and Nikravesh (LN) in [26, 27] and Lee and Wang (LW) in [30]. These models are further described in paper B.

## 2.3   Sensor Cart for Verification of the Chain Model

Experimental data from actual LSS are used to verify the dynamic chain model. Part of the project is therefore dedicated the development of an experimental setup for measurements of chain dynamics.

The method adopted measures the chain dynamics by a special sensor cart inserted in the chain of carts. Hereby, the sensor cart captures the chain dynamics as it drives within the chain of carts.

The special sensor cart is designed and manufactured with the aim of applying as many sensors as possible to have sufficient data when verifying the chain model. A total of 14 sensors are build into the sensor cart which encompasses:

- Steering wheel forces in $\eta$-direction
- Running wheel forces in $\zeta$-direction
- Joint forces in $\xi$- $\eta$- and $\zeta$- direction
- Angle of driving wheel suspension
- Rotational velocity of wheels
- Temperature of the iron core of the cart

All force sensors use strain gauges to measure shear strain in a thin web design in which variations in temperature are compensated utilizing a full Wheatstone bridge, see [20]. The angle of the driving wheel suspension is captured through a small gear connected to a potentiometer while the wheel speed is captured from two inductive sensors on each wheel, see Figure 2.3. Measurements are conducted in steady speed condition using a photocell arrangement to trigger start and stop of one lap in the track.

Experimental data is collected 22 times on seven different track layouts. The seven different track layouts encompass a broad variation in track length, number of curves, densities of curves, cart configurations and number of level changes. The 22 different measurements are obtained by variations in different chain speeds and chain tensions.

Experience show the fact that the only applicable data for the verification of the chain model are the measured forces. The angel of the driving wheel suspension and the temperature of the iron core turn out to be insignificant while the rotational velocity of the wheels lack the degree of precision.

Apart form providing data for the verification of the dynamics chain model the conducted measurements have also provided vital new information for the structural design of supports for the track layout.

### 2.3.1   Verification of the Chain Model

Verification of the developed chain model is conducted by comparing the experimental data with the simulated date. In general the normalized root mean square deviation (NRMSD) between the measured and simulated forces are around or less than 10% for all layouts and all measured forces. This proves the accuracy and applicability of the developed chain model.
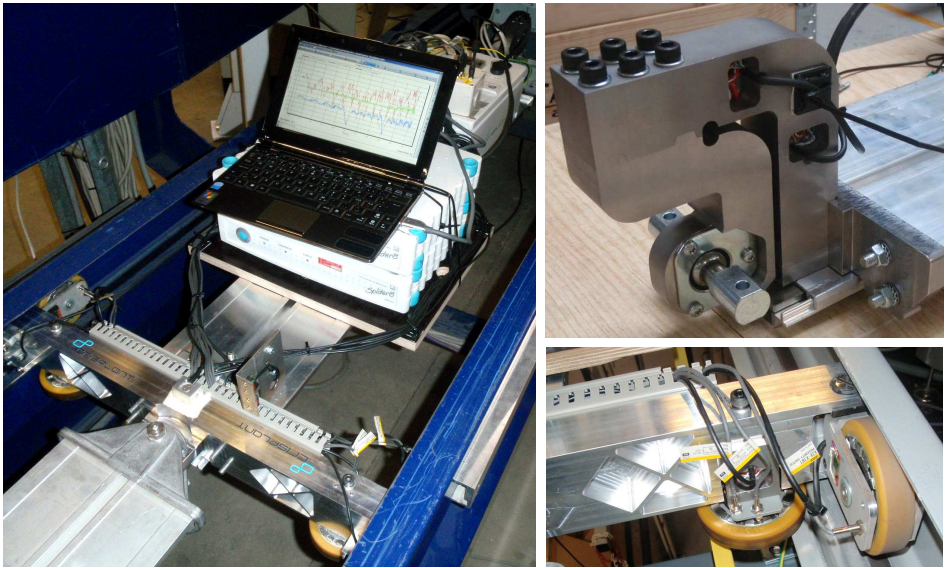
**Figure 2.3**: Design of the sensor cart. Left: The sensor cart mounted into the track. Upper right: Transducer for measurements of joint forces. Lower right: Force transducers and inductive sensors on the steering wheel and the driving wheel.

One of the test layouts which are used for verification of the chain model is shown in Figure 2.4. The two parallel closed lines illustrate the shape of the track while arrows within the track show level changes. Arrows outside the track layout show the driving direction of the chain while figures along the track show the distance from the trigger point of the conducted measurements. This track layout is $212m$ long, it has three levels and consists of seven horizontal curves turning left and three horizontal curves turning right. The chain consists of 171 carts each $1250mm$ long which in total weigh $7ton$ and are propelled by 13 motors equally distributed along the track. The chain simulation model is capable of simulating one lap on the track in $\sim 21sec$.
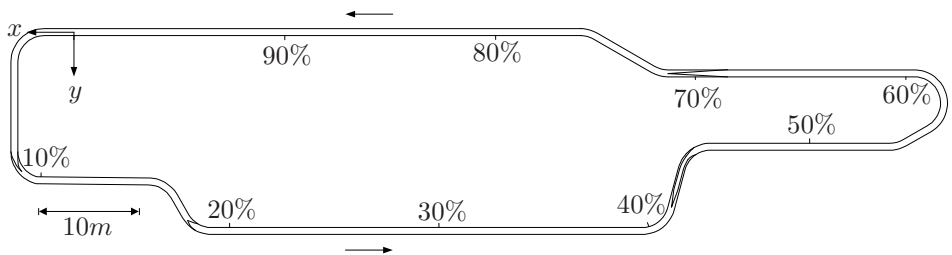


**Figure 2.4**: One of seven test layouts used for verification of the chain model. Paper A

The measured and simulated longitudinal joint forces are shown in Figure 2.5 in which the x-axis is the position along the track and the y-axis is the force. A fine accordance

between the measured and simulated forces is reached. The number of fluctuations is the same while the amplitude of the simulated force is smaller than the measured force along some part of the track.
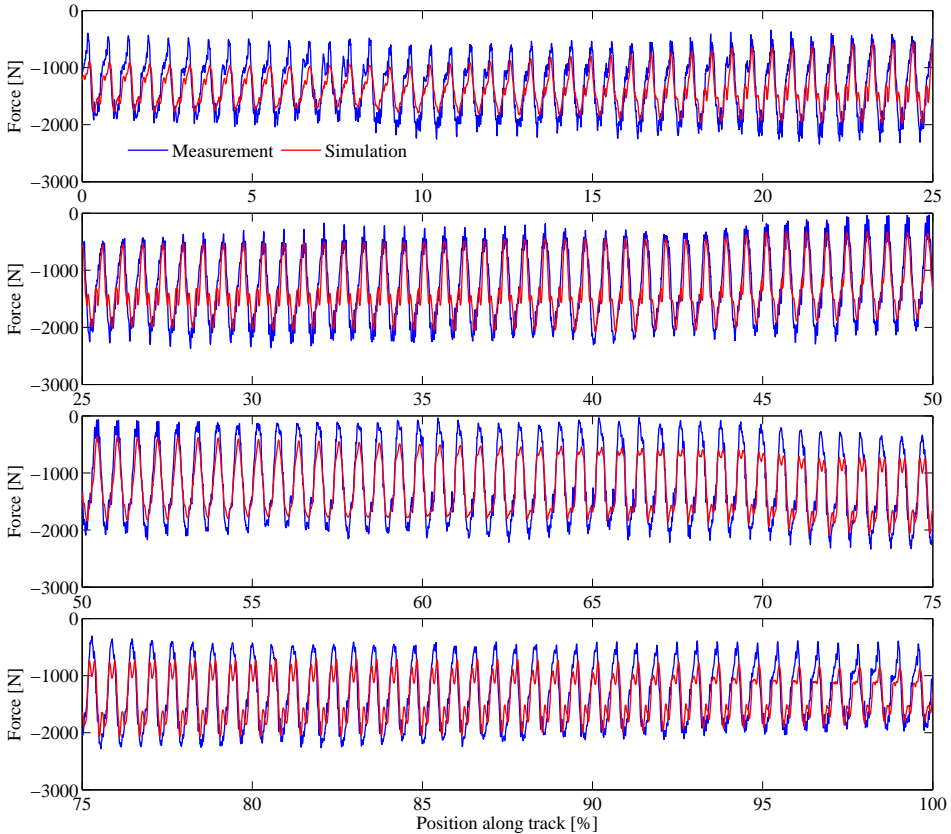


**Figure 2.5**: Measured and simulated longitudinal joint forces. Paper A

The correspondence between measured and simulated steering wheel forces is illustrated in Figure 2.6. A positive force encompasses contact on the left steering wheel while a negative force show contact on the right steering wheel. It is shown that a very fine accordance with the true chain dynamics is reached.

The performed verification of the dynamic chain model show several other encouraging results which confirm the applicability of the developed model. Several other results from the conducted verification are found in paper A, E and F.
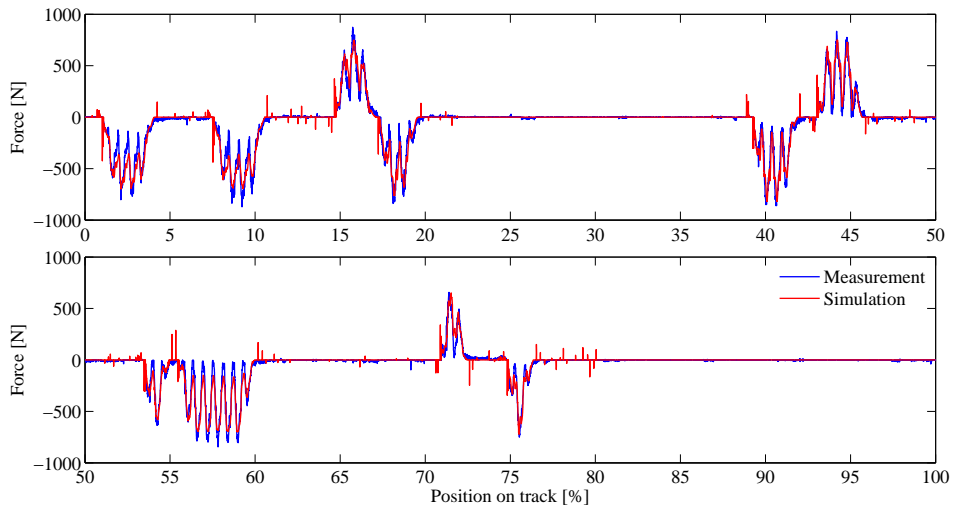
**Figure 2.6**: Left and right steering wheel forces for one lap on the track layout at steady speed. Paper A

## 2.4 Identification of Model Parameters

The majority of model parameter in the chain model is obtained by simple means. The geometric data and mass properties of the carts are obtained through CAD software while the stiffness and rolling resistance of the wheels are provided by the wheel manufacture. The joint flexibility is experimentally obtained thought the used of a static test machine, see Figure 2.7. The experimental setup captures the joint force in relation to the applied deformation through the use of a Spider8 and a laptop. A non-linear regression scheme is hereafter utilized to find the stiffness parameters of the joint.

The chain model encompasses several contact elements which dissipate energy. The complex structure of LSS makes it difficult to construct an experimental setup representing the average damping in the system design. Other means are therefore utilized to identify the damping parameters within the chain model.

### 2.4.1 Identification of Damping Parameters

The identification of damping parameters in large mechanical structures are complicated tasks as the dissipated energy within the system depends on the configurations of materials, structural design and internal contact. Damping is often a joint term which covers several effects of energy losses like internal heating, plastic deformation, viscoelastic effects and the impact deformation wave.

Several researchers have paid their attention to identification of damping parameters. In [59, 50] extensive experimental work on solid spheres bouncing off a flat plate is conducted by the use of high speed cameras to identify the pre and post impact velocity. Also, in [25] a more complex experimental setup of two colliding solid spheres is performed with
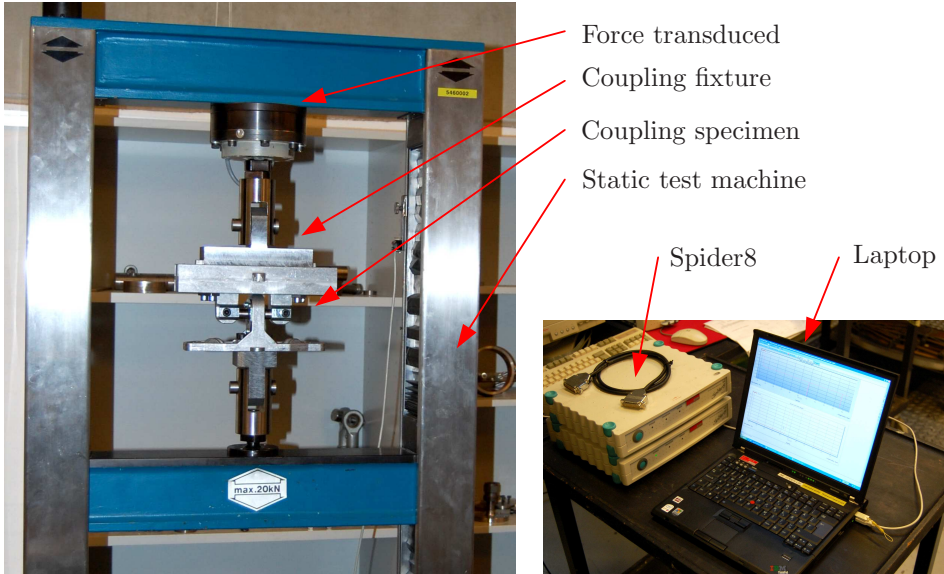
Force transduced

Coupling fixture

Coupling specimen

Static test machine

Spider8        Laptop

**Figure 2.7**: Experimental setup for identification of joint flexibility.

emphasis on the frictional effects during impact.

The resonance method is another popular method used to determine the system damping in more complicated structural designs, see [54]. An experimental setup capable of isolating both stiffness and damping parameters from the system response in a micro scale structure using a sophisticated experimental setup as presented in [47].

Direct experimental methods for parameter identification are infeasible means in large and complex structures as equipment normally has a limited range of use. Implicit parameter identification methods are therefore substitutionally used. An implicit method for model parameter identification of an induction motor is presented in [55]. The exact parameters of the motor model are identified using a differential evolution algorithm by minimizing the residual between measured and simulated current. The use of parameter optimization also proves to be efficient in other well-defined models like for example magnetorheological fluid dampers and magnetic hysteresis characteristics of construction steel, see [24, 28].

An implicit parameter identification method is also used on contiguous multi-body models. A gradient based optimization method for estimation of model parameters in a multi-body vehicle model is presented in [46]. A reasonable estimation of both stiffness and damping parameters for the suspension system of a full scale multi-body vehicle model is obtained by minimizing the difference between measured and simulated accelerations.

### 2.4.2 Implicit Parameter Identification

An implicit parameter identification method is proposed to estimate the damping parameters in the chain model. The damping parameters are estimated by minimizing the difference between simulated and measured data using optimization methods as means. The objective function is formulated as the root mean square of the residual between the seven measured forces and there matching simulated forces.

Ten design variables are utilized to perform the parameter identification. Four of the design variables are dedicated the damping parameters in terms of the CoR while the remaining six design variables are dedicated to layout dependent parameters. The layout dependent parameters encounters the sorter speed, the chain tension, the wheels rolling resistance, location of trigger point and the proportional and integral gain of the speed controller.

The parameter identification is performed for all 22 measurements to take into account variations within the layouts. Also, four formulation of the hysteresis damping factor is tested to find the superior model. A comparison of the optimum results among the four hysteresis damping models is performed. A box-plot of the optimum results of the 22 measurements for the four contact models is shown in Figure 2.8.
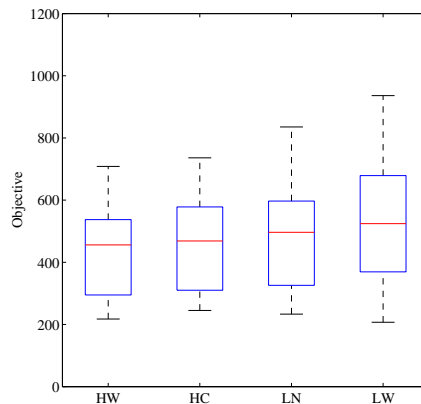


**Figure 2.8**: Box-plot of the final objectives for the four contact models. Paper B

None of the four models proves significant in simulating the contact forces. Nonetheless, the HW model is some magnitudes better than the other three models.

The optimum damping are analyzed by comparing the optimum CoR for the 22 measurements. Three wheel types and two joint configurations are used on the seven test layouts - by which only one of the wheel types and only one of the joint configurations are utilized as they posses sufficient statistical material. A box-plot of the CoR for the joint in axial direction and the CoR of the steering wheel for the four models of the hysteresis damping factor are shown in Figure 2.9 in which the dotted lines indicate the upper and lower bound applied in the optimization problem.
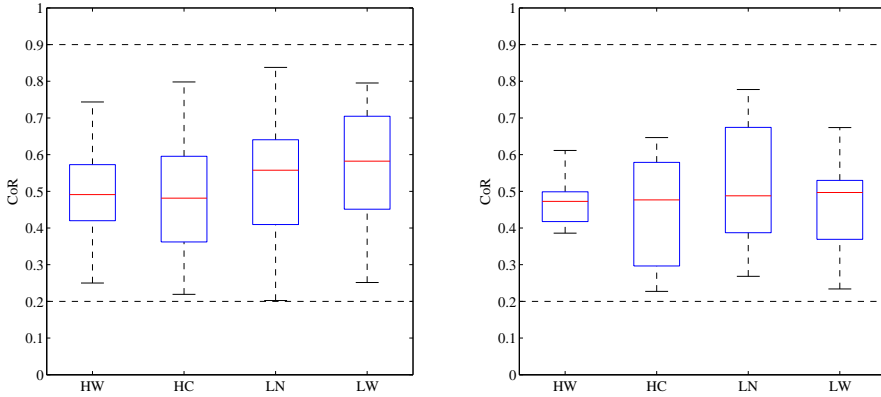
**Figure 2.9**: Box-plot of the CoR for the four models of the hysteresis damping model. Left: The joint contact in axial direction. Right: The steering wheel contact. Paper B

Corresponding data of the cart joint in radial direction and the driving wheel is found in paper B.

Overall, the HW model has the best accordance with the actual dynamics and induces the smallest variation within the damping parameters. For that reason this model is preferred in the future use of the dynamic chain model.

## 2.5 Summary

Extensive research on the development of a chain simulation model is commenced. The main conclusive remarks drawn from this work are:

- The theory of unconstrained rigid multi-body dynamics is efficient to model the complex dynamics in the chain of cart in LSS.

- An efficient contact formulation pays a vital role in predicting the chain dynamics by which the Hunt-Crossly model is adopted.

- An experimental setup for actual LSS is developed which makes it possible to measure several data of the chain dynamics in a steady speed condition.

- An efficient implicit method is proposed using optimization techniques to identify damping parameters of the chain model.

# 3

# Optimization of Track Layouts

This passage addresses the development of new tools for optimization of LSS. Two optimization problems are proposed in which both are concentrated on the track layout. The first problem optimizes the price and footprint of the track layout taking various constraint functions into account. The aim of this tool is to give the design engineer a fast and reliable method in reaching the optimal shape taking obstacles and discrete standard track element into account. The second optimization problem deals with the dynamic performance in the chain of carts. The aim of the second problem is to give the design engineer a tool which efficiently minimizes the chain dynamics in the track layout.

## 3.1 Geometric Optimization Problems

A variety of engineering disciplines addresses geometric problems of path, trajectory, layout or shape optimization.

In [7] attention is paid to path optimization of a five-axis milling machine with emphasis on obtaining an enhanced surface quality. The presented method utilizes a gradient based approach minimizing the movement generated by each rotation axis of the machine. Collisions are addressed utilizing a spatial collision model presented in [22]. By representing the tool profiles using a ray tracking technique contacts are identified by searching through a set of polygons defining the surface of each obstacle. In robotic motion optimization techniques are employed to obtain a collision-free trajectory, [42]. The optimum trajectory is reached by minimizing smoothness and accelerations between two pre-specified points. Collision is included using a penalty formulation. Convincing results of the optimal spatial trajectory for a robotic arm and a quadruped robot are presented using a gradient based solver.

Aircraft trajectory planning is addressed as a planer optimization problem in [43]. Dynamics are modeled using a point mass formulation subject to constraints of maximum speed and force acting upon the aircraft. Collision avoidance constraints are imposed by an exclusion region around the vehicle which forces the aircraft to turn when approaching an obstacle. The optimal trajectory is obtained by minimizing the time of reaching a predefined waypoint.

A layout optimization method is presented in [2] utilizing a simulation model to address both cost and time of the material handling process. The location of each machining unit is obtained by a slicing technique using the genetic algorithm to find the optimal

solution. A corresponding layout problem is addressed in [3] by optimizing the location of components in a shelter cabinet taking the mass distribution, the energetic network and the components' accessibility into consideration. The multi-objective problem is solved using the genetic algorithm by which the overlapping areas of the intersecting bodies are used as contact constraints. Also, a two-stage strategy for layout optimization of satellite modules is presented in [58]. The objective is to minimize the residual between desired and current moment of inertia of the satellite modules. Collisions are defined as volumetric constraints of the overlapping volumes between box and cylindrical shapes. Convincing results from two layout problems are presented while testing five different non-gradient optimization algorithms.

A method for optimization of chain performance in LSS is presented in [13, 11]. Assuming dynamic vibrations in LSS to originate from the polygon action within the chain of carts, Ebbesen et al. minimize the global polygon action using a kinematic simulation model. To parameterize the track layout Ebbesen et al. utilize an inherent formulation consisting of track lengths and radii of the curves as design variables.

## 3.2   Parameterization of Track Layouts

Parameterization of the track layout has previously been proposed by Ebbesen et al. in [13, 11] using the length of the straight track sections and the radii of the curves as design variables. This parametric formulation proves efficient for the simple track layout although the inherent design variables introduce a number of equality constraints in order to maintain the closed loop of the track.

For that reason a new approach is developed to get a formulation applicable for any track layouts. This parametric formulation is reduced a planer case as level changes within the track layout only have very few configurations in the standard product portfolio.

The proposed parametric formulation applies the Euclidian coordinates of the intersection point of the neighboring straight tracks section along with a fillet radius of each curve, see Figure 3.1. This induces three design variables for each intersection point defined as $P_i(x_i, y_i, R_i)$. An arbitrary shaped track layout is herby defined by a set of independent points $P_1, P_2, ..., P_n$ in which $n$ is the number of vertices.

A feasible track layout is maintained by applying geometric inequality constraints to the optimization problem. This is obtained by constraining angles between the straight track sections and constraining the distance between each vertex of the intersecting straight track sections. A more detailed description of the parametric formulation of track layouts and the geometric constraints is provided in Paper C.

Apart from providing a set of independent design variables the parametric formulation gives a new a more intuitive way of designing a track layout. Today a track layout is designed by outlining each track elements until a closed loop is reached. This induces several rearrangements of each track section as several design iterations are necessary before reaching a satisfying solution. The new parametric formulation offers an intuitive design process where the track layout is easily rearranged by changing the Euclidian coordinates of the intersection points or the radii of the curves. Expansions of the track layout are also easily made by introducing one or several new intersection points to the
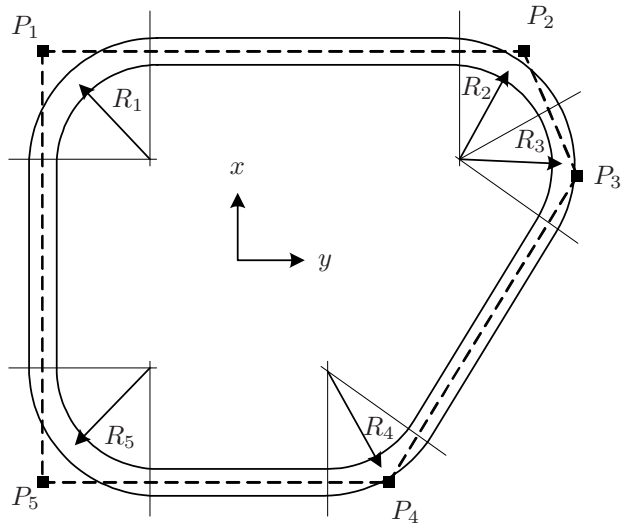
**Figure 3.1**: The intersection point between neighboring straight track sections are used as design variables. Paper C

vector of the design variables.

## 3.3   Optimization of Track Layouts

Price and footprint of the track layout are optimized by changing the length of the track layout. This is possible as price and footprint are linear dependent with the track length. Hence, the optimization problem is formulated as

$$
\begin{aligned}
\underset{\mathbf{x}}{\text{minimize}} \quad & f(\mathbf{x}), \qquad \mathbf{x} \in \mathbb{R}^m \\
\text{subject to} \quad & \mathbf{h}_i(\mathbf{x}) = \mathbf{0}, \ i = 1, \ldots, m_1 \\
& \mathbf{g}_i(\mathbf{x}) \leq \mathbf{0}, \ i = 1, \ldots, m_2 \\
& \mathbf{a} \leq \mathbf{x} \leq \mathbf{b}
\end{aligned}
\tag{3.1}
$$

In which $f(\mathbf{x})$ is the objective function and $\mathbf{h}_i(\mathbf{x})$ and $\mathbf{g}_i(\mathbf{x})$ are equality and inequality constraint functions and $\mathbf{x}$ is the design variables.

Apart from the geometric constraints introduced by the parametric formulation two types of implicit constraint functions are formulated. They include:

- Constraints of track sections defined by the user.
- Non-contact constraints to avoid building and obstacle intersection.

The user defined constraints are introduced as some straight track sections may be well defined in advance. Five types of user defined constrained functions are proposed by which either the length, the rotation, the parallel movement, the perpendicular movement or all degree of freedom can be restrained on each straight track section. The non-contact constraints are developed to avoid collision with the building layout or obstacles. Non-contact constraints are further described in next section.

A three-stage approach is adopted to pass obstacles and obtain a discrete solution of the track layout. Each stage encompasses a new optimization problem in which various constraint formulations are applied. The three stages include:

1. To minimize the track length subject to user defined constraints and building collision constraints.
2. To include obstacle collision constraints.
3. To apply penalty function in order to reach a discrete solution.

Angles and radii of the horizontal curves are designed with the discrete multiple of $2.5°$ and $1.0m$. These discrete geometries are relaxed in order to get a continuous optimization problem. A penalty method proposed in [16] is used to regain the discrete solution at the third stage of the optimization problem. This penalty method utilizes a saw tooth function by which curve angles and radii are forced towards the discrete multiple.

The continuous non-linear optimization problem is applied on two different solvers to test the robustness of the proposed formulation. One is the sequential quadratic algorithm *fmincon* provided by MatLab [34] and the other is the stochastic complex algorithm described in [5, 33]. The *fmincon* algorithm is a sequential quadratic constraint formulation in which the vector gradient is derived by the forward difference method. The complex method is an unconstrained evolutionary algorithm where constraints are treated through the use of a penalty formulation.

### 3.3.1 Non-contact Constraints

The space limitation defined by the building layout and obstacles are taken into account in the optimization problem. This is done by applying the equality constraints formulation

$$0 = A_q(\mathbf{G}_d, \chi_q) \tag{3.2}$$

in which $A_q$ is the contact area, $d$ is the left or right hand side of the track boundary, $\mathbf{G}$ and $\chi$ is the $q$'th obstacle. The non-linear equality constraint of Equation 3.2 is formulated for each obstacle and track side.

One applicable method to compute the contact area can be the use of polygon chipping algorithms. Polygon chipping algorithms are efficient means to compute the intersecting areas between polygons, see [57, 15, 49]. However, discritization of the track curves causes discontinuities in the contact constraint formulation. Consequently, an analytical expression of the curves' geometry is utilized. Though, the track layout is represented by a left and right hand side by a set of straight and curved lines. Obstacles are defined by a polygon which may be arbitrary shaped using a set of vertices.

An algorithm to search the contact area of Equation 3.2 is developed. It computes the contact area between the track side and the obstacle in four steps. The initial step searches for all intersection points between the track side and the boundary of the obstacle and classifies them as being either ingoing or outgoing, see Figure 3.2 A. A loop is hereafter commenced for each ingoing intersection point by which the second step finds the matching outgoing point of the enclosed contact area. The third step computes the polygon between the overlapping lines and all areas of the enclosed chords of the intersecting track sections, see Figure 3.2 B. At the fourth step all contact areas are summed up. This procedure is repeated between each track side and the obstacles.
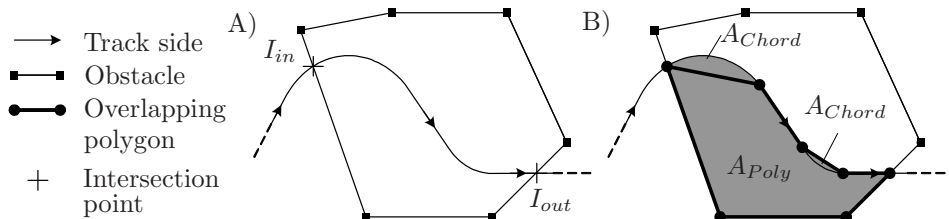


**Figure 3.2**: The contact area is searched and computed through four steps. A) All intersection points between the track side and the obstacle are searched and classified. B) The contact area is obtained by summing up the areas of the enclosed chords and the area of the overlapping polygon.

A profound description of the non-contact constraint formulation is provided in paper C.

### 3.3.2 Results

The proposed optimization method is tested on three layouts by which results from only one is presented in this section.

The initial track layout is shown in Figure 3.3. The shape and width of the track layout is shown by the two parallel blue lines. The track layout is $214.0m$ and consists of ten curves. The blue crosses illustrate intersection vertices of the neighboring straight track section by which 30 design variables are utilized to represent the track layout. The black dotted lines show the applied user defined constraints. The straight track sections between $P_2$ to $P_3$ and $P_8$ to $P_9$ are constrained in all degrees of freedom while the straight section between $P_4$ and $P_5$ is constrained in length and rotation. The red polygon shows the building layout while the green polygons illustrate obstacles. The presented results are obtained by the use of the *fmincon* solver.

The optimum track layout of the first optimization stage is shown in Figure 3.4. This particular optimization problem may be divided into a left and right hand side due to the constrained track section. The right hand side is the most complex part to solve due to the constrained track section between $P_4$ and $P_5$. However, the *fmincon* algorithm is robustly capable of finding the shortest track layout. Part of the track intersects one of the obstacles in the lower left corner as the non-contact constraints are disabled at stage one.
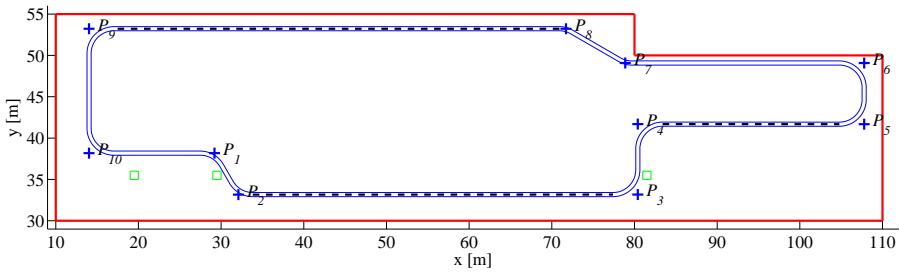
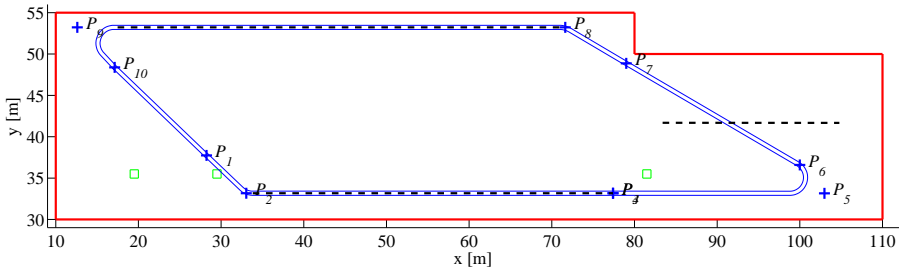**Figure 3.3**: Initial track layout with building and obstacles.



**Figure 3.4**: Optimum track layout after optimization stage one.

The optimum result reached in stage two is shown in Figure 3.5. Only the position of $P_1$ is changed to avoid collision with the obstacle.
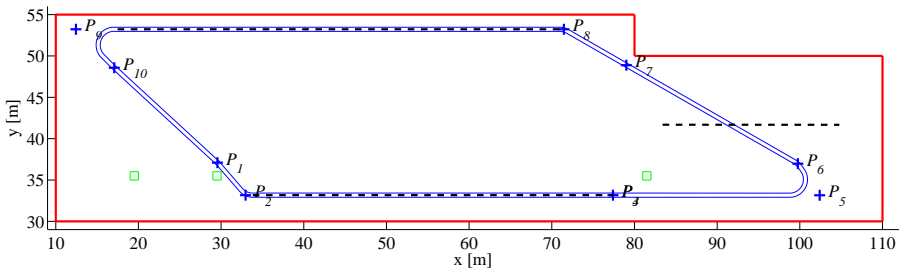


**Figure 3.5**: Optimum track layout after optimization stage two.

The optimum track layout of the third and final stage in the optimization process is shown in Figure 3.6. Several small changes to the track layout are performed in order to fulfill the multiple of the discrete curve angel and curve radius.
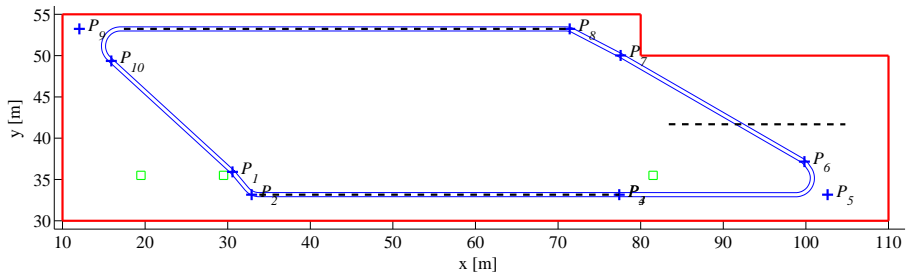
**Figure 3.6**: Optimum track layout after optimization stage three.

The proposed optimization method for minimizing prize and footprint of the track layout proves efficient with the *fmincon* solver. The *fmincon* solver always reaches the exact optimum solution through few design iterations. The complex algorithm also finds improved solutions but is most often not as accurate as the *fmincon* solver. The complex algorithm also involves some problems in reaching the discrete solution at stage three, due to the complex interdependence between the sweep angles and curve radius. More results are presented in paper C along with discussions on the proposed optimization method.

## 3.4  Optimization of Chain Dynamics

Poor dynamics in the chain of carts evolve from the discrete links which introduce the geometric polygon action. Polygon action is a known issue in chains and is subject to thorough research in literature, see [32]. The polygon action is a forced velocity variation evolving from the discrete chain when it transits from going in a straight direction to becoming a chord of a circle arc as it changes direction in a curve. The size of the forced velocity variation is dependent on the length of the chain links, the curve radius and the sweep angle. As each curve in the track layout introduces polygon action the interference between curves may induce constructive interference which amplifies the polygon action in some areas along the track layout.

The polygon action in combination with the low flexibility of the cart design cause some track layout to perform poorly as large vibrations are observed. The large vibrations cause increased wear on mechanical components like wheels, joints and the track sections. Occasionally, it also introduces a jabbing noise form the steering wheels which most of all result in unsatisfied customer. One option to improve the chain dynamics might be to reduce the length of the cart or use larger radii of the curves in the track layout. However, reducing the length of the carts is undesirable in some LSS due to the item mix. The use of bigger curve radius is likewise undesirable as it is pure overhead in relation to the sorting capacity, prize and footprint.

Another way of reducing vibrations in the track would be to redesign the cart paying grate attention in finding the optimal flexibility and damping of the cart structure. Redesign of the existing cart is not considered since this project concentrates on the existing standard

product portfolio.

Consequently, the objective is to reduce the chain dynamics by changing the shape of the track layout in order to attempt destructive interference between curves and decrease the polygon action through changes to the angles of the curves.

Optimization of the chain dynamics has previously been proposed in [11] by changing the shape of the track layout. Ebbesen et al. utilize a kinematic model of the polygon action as objective function changing the radii of the curve and the length of the straight section to find an optimal solution.

The proposed kinematic model utilizes an open chain approach in which the chain of carts is moved along the centre line of the track layout. The approach utilized by Ebbesen et al. appears promising though they only use the polygon action of the last cart in the chain as objective function.

### 3.4.1 Optimization Approach

Evaluation of the chain dynamics is most accurately obtained by the use of the proposed dynamic chain model presented in Chapter 2. However, the chain model proves inappropriate because of the computational time spend on each evaluation. Instead, a kinematic model similar to the one proposed in [11] is adopted. Three new formulations of the objective function are proposed as the formulations presented in [11] only take the polygon action of the last cart in the chain into account.

The dynamic chain model is utilized to verify the applicability of the three objective formulations. This is commenced by comparing the chain dynamics of the initial track layout with the optimum track layout.

User defined constraints and non-contact constraints are not included in the proposed optimization problem as the main purpose is to verify the applicability of the kinematic objective formulation. The optimization problem is formulated as

$$
\begin{aligned}
\underset{\mathbf{x}}{\text{minimize}} \quad & f(\mathbf{x}), \qquad \mathbf{x} \in \mathbb{R}^m \\
\text{subject to} \quad & \mathbf{g}_i(\mathbf{x}) \leq \mathbf{0},\ i = 1, \ldots, m_2
\end{aligned}
\tag{3.3}
$$

in which $f(\mathbf{x})$ is the objective function evaluating the dynamic performance in the chain of carts and $\mathbf{x}$ is the design variables of the track layout.

The proposed parametric formulation in Section 3.2 is utilized in the optimization problem. However, the radii of the curves are not applied because the standard product portfolio only encompasses three discrete options and the most obvious choice would be to use the largest curve radius to reduce the polygon action. The inequality constraints in Equation 3.3 encompasses the geometric constraints introduced by the parametric formulation.

### 3.4.2 A Kinematic Model of the Polygon Action

The kinematic model approximates the polygon action through the geometric length variations introduced by the discrete chain of carts. The model assumes an open chain by which the initial cart and final cart do not affect each other. The kinematic model only require the length of the cart and a geometric formulation of the track centre line to compute the polygon action in which a very efficient algorithm is developed.

From an arbitrarily chosen starting point on a straight track section the position of each cart is computed by finding the location of the next cart's front joint along the track centre line, see Figure 3.7. The location of each front joint is numerically obtained using a Newton-Raphson solver to find the point on the centre line which holds the length of the cart form the point of the previous cart front joint, see [23].

The initial cart is moved in small steps, $\Delta l_{step}$ by which the positions of the remaining carts are computed. This is repeated until the first cart has moved a distance of its own length. An approximate magnitude of the polygon action, $\Delta P$ is hereby extracted by evaluating the relative variation in the joint position according to the step length of the initial cart.
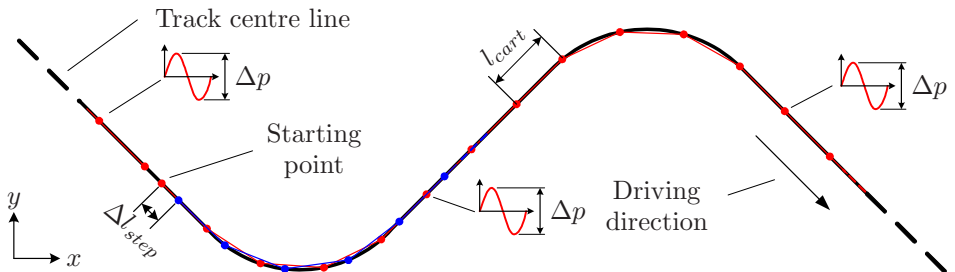


**Figure 3.7**: The kinematic model computes the polygon action by moving the chain of carts along the center line of the track layout.

The polygon action of one cart along each straight track section is utilized in the objective formulation as the polygon action is the same for every cart after each curve. The starting point is moved to each straight track section in the layout to capture all variations of the kinematic polygon action. As a result of this a layout with four curves encompasses polygon action in each of the four straight track sections four times for each starting point. This gives rise to 16 values of the polygon action which are composed differently in the three tested objective functions. They include:

1. The sum of square of all computed polygon actions.

2. The sum of square of the polygon action after the first three curves.

3. The sum of squares of the polygon action for the last cart in the chain.

### 3.4.3 Results

The proposed objective functions are tested on a track layout where poor dynamics in the chain of carts is acknowledged. The average sized track layout is $214.0m$ and consists of ten curves, see Figure 3.8. Ten vertices corresponding to 20 design variables are utilized

to parameterize the layout. The complex algorithm is utilized to solve the optimization problem in which the inequality constraints are applied through a penalty formulation. The non-linear design space introduced by the polygon action causes several local minima by which the optimization problem is repeated ten times for each objective function.

The best optimum track layout obtained with objective function 1 is shown in Figure 3.8. The optimum track layouts differ insignificantly from the initial track layout. This is known from the fact that the phases of the polygon action between each curve vary within the length of the cart in which small changes in the design variables lead to destructive interference.
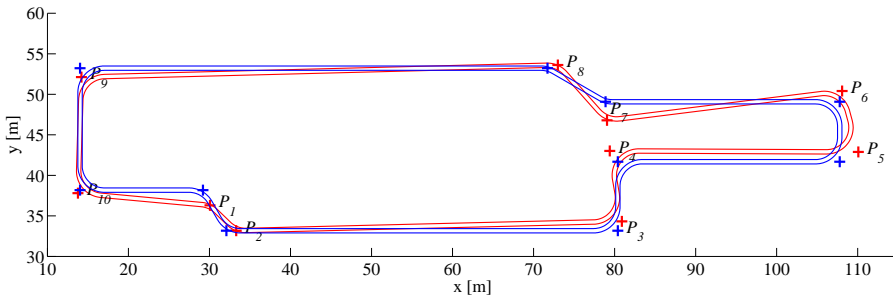


**Figure 3.8**: Different between initial (blue line) and optimum (red line) track layout.

All there tested objective functions reduce the geometric polygon action significantly. This reduction is verified through dynamic simulations of the initial and optimum track layout. A part of the simulated longitudinal joint forces of the initial and optimum track layout is shown in Figure 3.9. The verification result shows that the amplitude of the chain vibrations is reduced considerably and that the main frequency of vibration has changed.



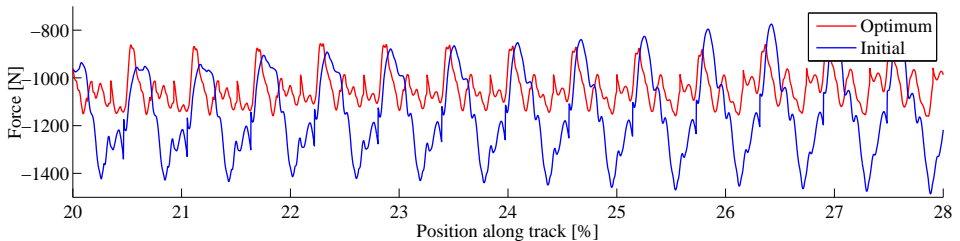**Figure 3.9**: Longitudinal joint forces of the initial and optimum track layout. Paper D

The reduced vibration level is also shown by the forces on the steering wheels, see Figure 3.10. The fluctuations of the steering wheel forces are reduced considerably.

A comparison of the three objective functions shows that the reduced dynamics are most significant with objective function 1. This formulation is therefore recommended for
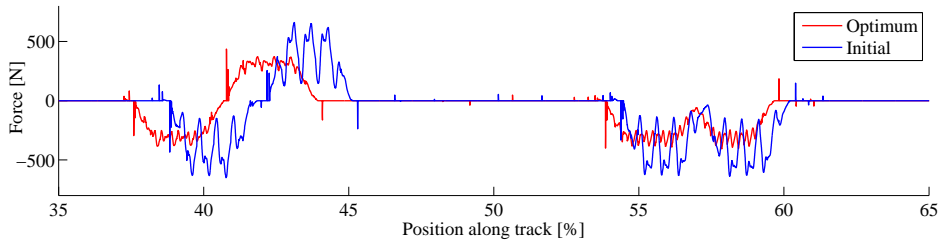
**Figure 3.10**: Steering wheel forces of initial and optimum track layout. Paper D

future use in optimization of the chain dynamics.

## 3.5 Summary

This chapter has dealt with the development of design tools which help the design engineer in reaching optimized track layouts. The main outcome of the research carried out can be summarized into:

- A robust parametric formulation is obtained by applying the intersection points of the neighboring straight track sections.
- A three stage approach is an efficient method to optimize prize and footprint of a track layout taking user defined constraints and collision constraints into account.
- Applying a kinematic model of the polygon action is an efficient method to optimize the dynamic performance within the chain of carts.

# 4

# Conclusions

The main research of this industrial Ph.D. has been the development of tools to help the design engineer in reaching superior designs of LSS. Three different engineering tools have been developed. They are:

- A chain model capable of predicting the dynamics in a robust and efficient way for any arbitrary shaped track layout.
- A tool for optimization of the track layout evaluating the price and footprint including non-contact constraints of obstacles and the building layout.
- A tool for optimization of the chain dynamics using a kinematic model of the polygon action as evaluation criteria.

All three models have proved efficient in assisting the design engineer reaching superior layouts of LSS.

The dynamic chain model has been developed with the aim of being able to predict the chain dynamics. The developed model proves accurate and robust by which a simulation is easily performed by simple means. The accuracy has been confirmed through extensive verification tests with a high conformance. The robustness of the model has been tested along with a verification where several variations in cart configurations and track layouts has been applied.

Optimization of the track layout proves to be a non-linear optimization problem with several implicit constraint formulations involved. The utilized three-stage approach is an efficient mean to reach an optimum track layout where obstacles can be passed and the discrete optimum solution achieved. Optimization results on three track layouts with different constraints formulations are convincing.

The proposed formulation for optimization of the chain dynamics appears efficient. As the dynamic chain model is inefficient as evaluation criteria a kinematic model of the polygon action has been utilized. Three formulations of the objective formulations have been tested in which all have proved to decrease the chain dynamics. However, verification through the use of the dynamic chain model shows that objective function one is superior.

## 4.1 Contributions

The main scientific contribution within the design of the dynamic chain model has been:

- A scalable multibody model of the chain of carts in LSS.
- Extensive test of state-of-the-art force element formulations with emphasis on a superior model of the hysteresis damping factor.
- The development of an advanced sensor cart capable of measuring chain forces, wheel speed and the fork angle of the driving wheels.
- An implicit optimization formulation for identification of damping parameters.

The main scientific contributions derived from the developed engineering tools have been:

- A new parametric formulation maintaining the closed loop track which yields a set of independent design variables.
- A new method for optimization of track layouts in terms of price and footprint with the use of a three stage strategy.
- A non-contact constraint method using a continuous formulation of the overlapping contact areas.
- A method for improving the chain dynamics using a kinematic model of the polygon action.

## 4.2 Further Work

The work in this industrial Ph.D. project has been divided in analysis and design optimization. This choice was made at an early stage of the Ph.D. project due to the extensive interest in solving problems with poor chain dynamics. Though, further work on enhanced automation of the design process can focus on three main topics:

1) The price and footprint of equipment which treat items to and from the LSS can be included in the optimization problem. The additional equipment can be added as it is part of the combinatorial design problem and include large cost of the overall system. Consequently, the position of chutes and inductions along the track section should be considered along with the shape of the adjacent conveyor layout. By including the additional equipment it becomes possible to find the best trade-off between the length of the conveyor and the track layout along with an optimal position of components along the track layout. However, the main concern is the formulation of constraints that efficiently handles collision between subsystems.

2) The capacity of LSS is one of the most critical evaluation criteria. The development of an efficient capacity model is able to enhance the applicability of automatic design tools. The huge possibilities in the design of LSS, the advanced induction algorithm, the possibility of item recirculation and the statistical variation of items make the development of an accurate and efficient capacity model very complex.

3) Finally, a tool capable of picking the right set of standard components from the standard product portfolio in regard to the demands of the system can be developed.

This combinatorial problem can help the design engineer in choosing the least expensive set of components. The problem may not address position and shape of components but only consider the combinatorial problem. However this tool is complex as it requires an efficient integer optimization algorithm and an efficient capacity model.

Research to improve the accuracy and the computational speed of the dynamic chain model can still be conducted. Various extensions to improve the accuracy of the model can be investigated as for example joint clearance, sliding of wheels and the rotational degree of freedom of the driving wheel fork. Also, the computational efficiency might be improved by applying a new superior numeric solver.

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