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Modeling dynamics of legal relations with dynamic logic

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Abstract

The fundamental relations in private law are claims and duties. These legal relations can be changed by agents with the appropriate legal powers. We use propositional dynamic logic and ideas about propositional control from the agency literature to formalize these changes in legal relations. Our models are sets of states with functions specifying atomic facts, agents' abilities to change atomic facts, legal relations between agents concerning changing atomic facts and agents' powers. We present a formal language that allows us to describe models and changes of models caused by two kinds of actions: actions that change atomic facts and actions that change legal relations. Next, we present a sound and complete calculus for this language. The paper demonstrates that the perspective on actions borrowed from computer science can be used to shed interesting light on the dynamics of legal relations.

Keywords: legal relations, changes, actions, dynamic logic

1 Introduction

Private law (such as civil law), as opposed to public law (such as criminal law), deals with relationships between persons, either natural persons (individual human beings) or organizations (foundations, companies and so on). We will call such persons agents. In private law, the most fundamental relations among agents are *claims* and *duties*. The typical usage patterns for claims and duties are as follows: A *has a claim against B that B do/refrain from doing something* and B *has a duty towards A that B do/refrain from doing something*. Claims and duties can be defined in terms of each other by swapping agents' perspectives.

Agents can change legal relations, e.g. by signing contracts. *Powers* are agents' abilities to change legal relations. The typical usage patterns of powers are as follows: A *has the power against B to create/withdraw a claim that B do/refrain from doing something*; A *has the power against B to create/withdraw a duty that A do/refrain from doing something*. Note that the two kinds of powers are different: they concern claims and duties, respectively. Powers are also called *capacities* or *competences*.

The American legal scholar Wesley N. Hohfeld had a deep influence on how these conceptions were recognized and named. In his well-known work [17], Hohfeld extensively discussed claims, duties, powers and other concepts including *privileges*, *liabilities* and *immunities* by putting them

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in squares of opposition. The logical relations of these squares are jural correlatives and jural opposites. These squares are both descriptive and prescriptive: on the one hand, they describe how these conceptions are used in reality, on the other hand, they prescribe how they be used. Hohfeld's theory has become a part of the standard framework for analyzing legal relations (see [3], [29] and [31]).

Pioneering work on formalizing legal relations was done by Kanger [19, 20]. Later Lindahl [22] considerably refined Kanger's theory. Makinson [23] extensively commented on the works of Kanger and Lindahl. Sergot [32] provided an extensive survey of the literature before 2013. Recently, Markovich [24] and Dong and Roy [6] provided novel formalizations of several notions, all in the tradition of STIT logic. Frijters and De Coninck [9] presented a new way to deal with conflicts between directed duties, using the framework of term-modal logic from [8].

Legal relations can be changed by many factors. They can be changed directly by agents executing their powers. They can also be changed directly by legal institutions' decisions. Agents' behaviors of fulfillment or violation usually change legal relations. Finally, changes of legal relations can also be caused by factual changes in the world.

Formal work on the dynamics of legal relations is rather scarce. When defining powers, Markovich [24] applied the following idea: an agent has the power to perform a legal act if the act's performance has the appropriate legal consequence. Dong and Roy [6] handled powers in the following way: an agent has the power to create/withdraw a duty if there is an action of the agent which makes a difference to the duty.

In this paper, we will present a Logic for Dynamics of Legal Relations (DyLeR), using propositional dynamic logic and ideas about propositional control from the agency literature to construct the base of our formalization. Propositional dynamic logic goes back to [7]. For propositional control we base ourselves on [35] and [34]. We will extend the current literature on the formal representation of legal relations by showing how propositional dynamic logic can deal with many kinds of actions on legal relations.

Our approach to legal relations and their dynamics is as follows. We view the world as a collection of atomic facts. A state of the world is a distribution of facts, and change in the world is modeled as toggling atomic propositions from true to false or vice versa. Agents have abilities to perform such fact-changing actions. Some claims among agents concern agents performing fact-changing actions, but agents may also have powers to perform claim-changing actions.

The rest of this paper is structured as follows. In Section 2, we briefly discuss legal relations and their dynamics. In Section 3, we outline our approach to model dynamics of legal relations. The logic DyLeR is presented in Section 4. In Section 5, we compare our work to some main references of formalizing legal relations. In Section 6, we suggest some future work. Soundness and completeness of DyLeR are shown in an appendix (Section A).

2 Legal relations and their dynamics

2.1 Legal relations

Fundamental relations in private law are *one person having a claim against another person* and *one person having a duty towards another person* (see [27]). In general, to say that there is a legal relation between two persons is to say that one of them has a claim that the other do or do not do something, while the other correlatively has a duty to act as prescribed by the claim (see [21]).

Ownership of property entails a general claim for the owner against others that they do not infringe. Meanwhile, the others have duties to carry out what is required in these claims, namely

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to refrain from infringement. Written contracts or oral agreements with the subject matters of sale, lease, labor and so on contain claims and duties of different kinds: claims of the buyer against the seller on delivering goods with quantity and quality conforming to the agreement, claims of the lessor against the lessee on keeping the leased property in good condition claims, of the employee against the employer on paying the salary on time and so on.

Claims and duties are the fundamental legal relations that define private law. Creation or withdrawal of claims and duties refer to *powers* of agents. An agent who receives a selling offer has the power to accept the offer. Once the offer is accepted, some claims and duties are established. The owner of a property has the power to sell the property. Once the sale is concluded, all the claims of the former owner on the property and the corresponding duties of others are extinguished. An agent may have the power to make a testament to distribute her legacy after death. Once a testament is made, the beneficiaries have claims on the legacy as specified in the testament.

The direction of a power can be outward (a power to change the duties of others) or inward (a power to change duties of one's own), or it can be both at the same time. For example, a creditor may use her power to remit a debt, thereby outwardly dissolve a relevant duty of the debtor; alternatively, a creditor may use her power to promise to lend a certain amount of money, thereby inwardly impose a duty on the creditor herself. A lessor may use her power to accept the offer of a lease on her house, and by doing so she will create a duty for the lessee to pay, and in the meantime she will create another duty on herself to maintain the property in good condition. This is a typical case where both directions concur.

Remarks The concepts of claims, duties and powers have a long tradition. The correlative of claim–duty can be traced back to the notion of *vinculum juris* in Roman Law. A vinculum juris is a 'bond of the law' that binds people together by assigning to them the roles of *creditor* and *debitor*: the debitor owes the creditor duties to carry out (see [10, p. 1705] and [26, p. 126]).

Both the philosopher Jeremy Bentham and the legal scholar John Austin identified *rights to service* as correlative of *obligations* [22, p. 15, 16, 21]. Bentham also identified the *power of imperation* and the *power of de-imperation* [22, p. 198].

In this work, we only consider the concepts of claims, duties and powers and ignore other concepts discussed by Hohfeld. There are three reasons for this. In the first place, claims, duties and powers are the most central to the dynamics of legal relations. Secondly, other Hohfeldian conceptions are definable in terms of claims, duties and powers. For example, liabilities can be obtained from powers by swapping agents' perspectives. B *has the liability to A's creating/withdrawing a claim that B do/refrain from doing something* if and only if A *has the power against B to create/withdraw a claim that B do/refrain from doing something*. B *has the liability to A's creating/withdrawing a duty that A do/refrain from doing something* if and only if A *has the power against B to create/withdraw a duty that A do/refrain from doing something*. In the third place, not all Hohfeldian concepts are commonly used in legal contexts in the way specified by Hohfeld. Take liabilities and privileges as examples. Liabilities in tort law typically indicate *responsibilities* rather than correlatives of powers (see [12, p. 257–258]). Besides absence of duty, privileges in practice may also mean 'protection rights' that impose duties on others not to impede[1] [14, p. 443–444].

[1]The literature of deontic logic makes a distinction between weak and strong permission, also known as negative and positive permission [15, 28, 37]. As discussed by Halpin in [14, p. 451], those privileges in practice that involve 'protection rights' are positive permissions. This suggests that there are close connections between the two senses of privileges and the two senses of permissions. Similarly, there may also be a distinction between negative and positive immunity. The former refers to absence of power and the latter refers to an immunity explicitly given by law. We want to thank an anonymous reviewer for pointing this out.

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The outward and inward effects of power, though not explicitly stated by Hohfeld, are actually implied by his description. Hohfeld apparently acknowledged that to accept the offer of a contract is a power that can create contractual duties for the counterparty as well as for oneself (see [18, p. 55]). This suggests that a legal power may be used outwardly or inwardly.

As a side note we remark that Hohfeldian conceptions are not just applicable to private law. According to [2], Hohfeld's theory is also capable of providing insightful analyses to public law, as well as the 'complex modern relationship between private and public "rights"; and between public and private law as a whole'.

2.2 Dynamics of legal relations

Legal relations are changing all the time in various ways. Here is a brief overview:

- Executions of legal powers directly change legal relations. Note that many legal acts can be viewed as executions of powers: making legal statements, signing contracts, transferring ownership of properties and so on.
- Authoritative decisions can change legal relations as well. For example, in a suit for divorce, if the court decides to grant the divorce, the claims and duties based on marriage will then be terminated. Note that not all judicial decisions are concerned with changing legal relations. Some decisions only intend to clarify a particular controversy, say, whether the plaintiff has a particular claim or whether the defendant violates a duty towards the plaintiff.
- Fulfillment of one's duty may simply terminate the duty, as exemplified in the notion of *solutio* in Roman Law: fulfillment of one's duty unties the legal bond [25, p. 272].
- Delicts, the acts of breaching duties, often result in duties to repair the caused damage or to compensate for it (see [16, p. 24–28]).
- Changes of facts may cause legal relations to change. For instance, if a house is destroyed, all the claims and duties about the ownership of the house will vanish.

Below we provide three examples for the dynamics of legal relations. Later we will come back to the first two.

EXAMPLE 2.1
Adam was living in his apartment in Berlin. He wanted to rent out the apartment and move to another city. Beck wanted to rent an apartment in Berlin. Adam and Beck signed a lease for an indefinite period.

Let us see what happened in the scenario. A law is involved here: The German Civil Code (GCC).

- Before signing the contract, by Section 861 of GCC, Adam had a claim against Beck that Beck do not live in the apartment.
- By signing the contract, Beck had a claim against Adam that Adam do not live in the apartment.

EXAMPLE 2.2
Miss Li was having dinner in a restaurant in Beijing. When passing by Miss Li's table, Miss Ma accidentally spilled her coffee over Miss Li's dress. Miss Ma apologized and said that she would clean the dress. Miss Li told her that she did not have to do it.

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A law is involved here: Civil Code of the People's Republic of China (CCC). What happened in this scenario is as follows:

- Before the accident, by Article 236 of CCC, Miss Li had a claim against Miss Ma that she do not spoil the dress.
- After the accident, by Article 237 of CCC, Miss Li had a claim against Miss Ma that she clean the dress.
- After Miss Li told Miss Ma that she did not have to clean the dress, the previous claim was withdrawn.

EXAMPLE 2.3
A company B in one country was considering buying a machine from a company A in another country. A sent B an offer indicating specifics about quality, price, payment, delivery and so on. B accepted the offer. B paid by the indicated date. Because of problems with the supply chain, A requested B that the delivery be delayed for certain days. B agreed. A delivered a machine to B by the newly agreed date. B found that the machine did not meet the agreed quality standards and claimed compensation from A.

What happened in this scenario? The United Nations Convention on Contracts for the International Sale of Goods (CISG) is involved here.

- By Article 15(1) and 23 of CISG, upon receiving the offer from A, B acquired the power to do the following: (1) creating a duty that B pay for a machine by some date and (2) creating a conditional duty that A deliver a well-functioning machine by some date after B has paid.
- When B accepted the offer, the duties mentioned above were created.
- When B paid, B fulfilled its duty, and A had a duty to deliver a qualified machine by some date.
- When B approved A's request, an old duty was withdrawn and a new one was created.
- When A delivered a malfunctioning machine, B had the power to create a claim for compensation.
- When B claimed for compensation, A had a duty to compensate.

Remarks Changes of legal relations have been extensively discussed in the literature. Lindahl in [22, p. 65] stated that it is 'a familiar fact' that legal relations change from time to time, 'for example by promise, contract, the decree of an authority, etc.', and changes in legal relations constitute an 'important area of legal problems'. Hage in [13, p. 203, 208–211] stated that claims and duties, as states of affairs in law, are dynamic in that law itself is a dynamic system of states of affairs, subject to change brought about by occurrence of events, e.g. 'contracts are signed' and 'property rights are acquired'.

3 Our approach to model dynamics of legal relations

3.1 Ontic aspects

Possible worlds In this paper we propose to view a possible world as a set of atomic propositions. Any change in the world is effected by changing propositional facts. This assumes a kind of Wittgensteinian view of the world: 'Die Welt ist die Gesamtheit der Tatsachen, nicht der Dinge.' ('The world is the totality of facts, not of things.'). Different possible worlds satisfy different sets of atomic propositions.

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Time flow Time flow is crucial for the dynamics of legal relations, which can be seen from Example 2.3 in Section 2.2. However, for simplicity, we do not explicitly introduce time in this work.

Agents We assume a finite number of agents.

Fact-changing actions Agents can perform some actions to change facts of the world. We call these fact-changing actions. In this work, we just consider an extremely simple kind of fact-changing actions: *swapping the truth value of an atomic proposition*. This notion is borrowed from the agency literature of [35] and [34].

Another kind of fact-changing actions is also natural: *sustaining the truth value of an atomic proposition*. It is non-trivial work to handle actions of swapping and sustaining the truth value of an atomic proposition at the same time, as they may interfere with each other. However, in this paper we will only consider actions that change facts.

We allow one agent to perform more than one fact-changing action at the same time. The reason for this will be explained later. We also allow more than one agent to perform fact-changing actions at the same time. Specifically, we allow different agents to change the truth value of a single atomic proposition simultaneously[2].

Refraining The notion of *refraining* is important in legal contexts. In this work, we do not introduce it, for this notion is not easy to capture well in dynamic logic. Work has been done on this in the literature, but in a different setting (compare [4]).

Abilities to perform fact-changing actions We assume that for every possible world and every agent, the agent is able to perform a given set of fact-changing actions in the possible world. Abilities matter. After all, legal issues cannot arise if agents do not have the ability to deviate from what the law prescribes.

Abilities are dependent on possible worlds. This allows for situations where changes to the world cannot be undone: you may be in a situation where you have the ability to break an egg, but there are no situations where you can mend a broken egg.

3.2 Normative aspects

Claims and duties We assume that for every possible world w, agents a and b, a has a set of claims against b in w, that is, b has a set of duties towards a in w. Every such duty concerns b performing some fact-changing actions.

We assume that duties have to be fulfilled immediately, that is, in the next step. The reason for this artificiality is that we do not explicitly model the flow of time.

Note that one agent might have duties to perform more than one fact-changing action at the same time. This is why we allow one agent to perform several fact-changing actions at once.

Situations can occur where an agent is legally obligated to act, but *unable* to perform the required action: think of a legal obligation to pay off a debt for an agent who is broke.

Note that fact-changing actions may affect duties, because duties may be dependent on facts.

Claim-changing actions Agents can perform some actions to create or withdraw claims and duties. We call them claim-changing actions. An example of a claim-changing action is signing a contract.

[2] There are two ways to understand this: *they collectively do this* and *they individually do this*. In this work, we mean the latter.

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We assume that claim-changing actions just change claims and duties. This implies that they do not change facts of the world, agents' abilities, or agents' powers.

In reality, just like fact-changing actions, claim-changing actions are also physical actions, so in a sense they affect the ontic state of the world. For example, a person who is signing a contract makes her name appear in the contract. However, we will disregard this for simplicity.

In reality, claim-changing actions have a rich temporal dimension. As we do not introduce time flow, we assume that claim-changing actions just change the set of claims of the present moment.

We allow one agent to change several claims or duties at the same time. We also allow more than one agent to change claims and duties at the same time. This is coincident with real scenarios. As discussed before, signing a contract can change many claims and duties at the same time. This is why we allow one agent to have more than one claim against another agent.

We do not allow fact-changing actions and claim-changing actions to be performed at the same time. The reason is that these categories of actions are different and we wish to avoid the complications that may arise from their simultaneous performance. For example, suppose that A accidently makes a dent in your car. At the moment you ask A to repair your car, B actually repairs it. What is the result now?

Powers to perform claim-changing actions We assume that for every possible world w, agents a and b, a has the power to perform a set of claim-changing actions in w, which change claims of a against b that b perform fact-changing actions, and a has the power to perform a set of claim-changing actions in w, which change duties of a towards b that a perform fact-changing actions.

Note that fact-changing actions affect the state of the world and legal powers depend on the state of the world. This implies that fact-changing actions change powers.

As mentioned above, we assume that claim-changing actions do not change powers. It is not clear whether this realistic. In some cases, this seems clear enough. *Making a testament* and *inviting a friend* are two examples. In principle, the testament and invitation can be created or withdrawn repeatedly. This may make people unhappy, but that is another matter. In some other cases, it seems that claim-changing actions do change powers. If someone damages my car, do I have the power to make/cancel a claim for a repair repeatedly? It would seem that once the claim for repair is revoked, this cannot be undone. A similar case would be cancellation of debts. If the US government would decide to cancel student debts, the government cannot at a later stage recreate these debts.

3.3 Dynamics

Fix a possible world. Some atomic propositions are true and some are false. Agents have abilities to perform fact-changing actions. Agents have claims against each other. Agents have powers to perform claim-changing actions. When agents act, the world evolves. When fact-changing actions are performed, facts are changed. As claims and powers are dependent on facts, the factual changes may also change these. When claim-changing actions are performed, claims are changed, but we will assume the facts of the world remain unaffected.

In Section 2.2, we mentioned three examples about dynamics of legal relations. Now we look closely at how the first two are handled by our approach.

The first example is about Beck renting an apartment from Adam in Berlin.

Before Adam and Beck signed the lease, the situation can be specified as follows:

- Some atomic propositions are true and some are false. *Adam lives in the apartment*. *Beck does not live in the apartment*.

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- Agents have abilities to perform fact-changing actions. *Adam has the ability to change (refrain to change) the truth value of 'Adam lives in the apartment'. Beck has the ability to change (refrain to change) the truth value of 'Beck lives in the apartment'.*
- Agents have claims against each other. *Adam has a claim against Beck that Beck refrain to change the truth value of 'Beck lives in the apartment'.*
- Agents have powers to perform claim-changing actions. *Adam has the power against Beck to withdraw the claim that Beck refrain to change the truth value of 'Beck lives in the apartment'. Adam has the power against Beck to create the duty that Adam change the truth value of 'Adam lives in the apartment'.*

Adam signing the lease with Beck means Adam performing the following claim-changing actions: (1) withdrawal of the claim of Adam against Beck that Beck refrain to change the truth value of '*Beck lives in the apartment*'; (2) creation of the duty of Adam towards Beck that Adam change the truth value of '*Adam lives in the apartment*'. After this, the situation can be specified as follows:

- The truth values of the atomic propositions mentioned above are not changed.
- Adam and Beck's abilities are not changed.
- The claim mentioned above dissolves. *Beck has a claim against Adam that Adam change the truth value of 'Adam lives in the apartment'.*
- Adam and Beck's powers are not changed.

The second example is about Miss Ma soiling Miss Li's dress.

Before Miss Ma soiled the dress, the situation can be specified as follows:

- Some atomic propositions are true and some are false. *Miss Li's dress is clean*.
- Agents have abilities to perform fact-changing actions. *Miss Li has the ability to change (refrain to change) the truth value of 'Miss Li's dress is clean'. Miss Ma has the ability to change (refrain to change) the truth value of 'Miss Li's dress is clean'.*
- Agents have claims against each other. *Miss Li has a claim against Miss Ma that she refrain to change the truth value of 'Miss Li's dress is clean'.*
- Agents have powers to perform claim-changing actions. *Miss Li has the power against Miss Ma to withdraw the claim that Miss Ma refrain to change the truth value of 'Miss Li's dress is clean'.*

Miss Ma contaminating Miss Li's dress means Miss Ma performing the following fact-changing action: changing the truth value of '*Miss Li's dress is clean*'. After this, the situation can be specified as follows:

- *Miss Li's dress is not clean*.
- Miss Li and Miss Ma's abilities are not changed.
- The claim mentioned above dissolves. *Miss Li has a claim against Miss Ma that she change the truth value of 'Miss Li's dress is clean'.*
- The power mentioned above is still there. *Miss Li has the power against Miss Ma to withdraw the claim that she change the truth value of 'Miss Li's dress is clean'.*

Miss Li saying that Miss Ma does not have to clean the dress means Miss Li performing the following claim-changing action: withdrawing the claim against Miss Ma that she change the truth value of '*Miss Li's dress is clean*'. After this, the situation can be specified as follows:

- *Miss Li's dress is still not clean*.
- Miss Li and Miss Ma's abilities are not changed.

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- The claim in the last situation dissolves.
- Miss Li and Miss Ma's powers are not changed.

4 Logic for Dynamics of Legal Relations

4.1 Language

DEFINITION 4.1 (Language Φ_{DyLeR}).
Let Agt be a nonempty finite set of agents and let a and b range over agents. Let Φ_0 be a countably infinite set of atomic propositions and let p range over it. Define the language Φ_{DyLeR} for Logic for Dynamics of Legal Relations (DyLeR) as follows:

Propositions:	ϕ	$::=$	$\mathsf{SP} \mid \neg\phi \mid (\phi \wedge \phi) \mid [\alpha]\phi$
Simple propositions:	SP	$::=$	$p \mid \top \mid \mathrm{AB}_a(a \uparrow p) \mid \mathrm{CL}_a^b(b \uparrow p) \mid$ $\mathrm{PO}_a^b(b \uparrow p) \mid \mathrm{PO}_a^b(a \uparrow p)$
Actions:	α	$::=$	$\mathsf{FA} \mid \mathsf{CA} \mid ?\phi \mid (\alpha;\alpha) \mid (\alpha \cup \alpha)$
Fact-changing actions:	FA	$::=$	$\mathsf{AFA} \mid (\mathsf{FA} \shortparallel \mathsf{AFA})$
Claim-changing actions:	CA	$::=$	$\mathsf{ACA} \mid (\mathsf{CA} \shortparallel \mathsf{ACA})$
Atomic fact-changing actions:	AFA	$::=$	$(a \uparrow p)$
Atomic claim-changing actions:	ACA	$::=$	$(a \curvearrowright \mathrm{CL}_a^b(b \uparrow p)) \mid (a \curvearrowright \mathrm{CL}_b^a(a \uparrow p))$

In the sequel, we also call propositions *formulas*.
Readings of the featured expressions of this language are as follows:

- $[\alpha]\phi$: For every way to do α, after α is done, ϕ is the case.
- $\mathrm{AB}_a(a \uparrow p)$: a has the ability to change the truth value of p.
- $\mathrm{CL}_a^b(b \uparrow p)$: a has a claim against b that b change the truth value of p.
- $\mathrm{PO}_a^b(b \uparrow p)$: a has the power against b to create or withdraw a claim that b change the truth value of p.
- $\mathrm{PO}_a^b(a \uparrow p)$: a has the power against b to create or withdraw a duty that a change the truth value of p.
- $\alpha \shortparallel \beta$: The action of doing α and β at the same time.
- $(a \uparrow p)$: The action of a changing the truth value of p.
- Note $(a \uparrow p)$ is not a proposition, although it contains a proposition p. Also note it means neither *making p true* nor *making p false*. If p is false, this action makes it true, and if p is true, this action makes it false.
- $(a \curvearrowright \mathrm{CL}_a^b(b \uparrow p))$: The action of a creating or withdrawing **a claim of** a **against** b that b change the truth value of p.
- Note $(a \curvearrowright \mathrm{CL}_a^b(b \uparrow p))$ is not a proposition, although it contains a proposition $\mathrm{CL}_a^b(b \uparrow p)$. Also note it means neither *creating the claim* nor *withdrawing it*. If a does not have the claim, this action creates it, and if a has the claim, this action withdraws it.
- $(a \curvearrowright \mathrm{CL}_b^a(a \uparrow p))$: The action of a creating or withdrawing **a duty of** a **towards** b that a change the truth value of p.

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Here are some derived expressions:

- We assume the usual definitions of $\bot, \vee, \rightarrow$ and $\leftrightarrow$.
- We define $\mathrm{DU}_a^b(a \uparrow p)$ as $\mathrm{CL}_b^a(a \uparrow p)$, indicating that a has a duty towards b that a change the truth value of p.
- We define $\langle\alpha\rangle\phi$ as $\neg[\alpha]\neg\phi$, indicating that there is a way to do α such that after α is done, ϕ is the case.

4.2 Models

DEFINITION 4.2 (Models for Φ_{DyLeR}).
A model for Φ_{DyLeR} is a tuple $\mathtt{M} = (\mathtt{W}, \mathtt{L}, \mathtt{AF}, \mathtt{CF}, \mathtt{PF}_O, \mathtt{PF}_I)$ where:

- $\mathtt{W}$ is a set of possible worlds with the same cardinality with $\mathcal{P}(\Phi_0)$ and $\mathtt{L} : \mathtt{W} \to \mathcal{P}(\Phi_0)$ is a bijective labeling function;
- $\mathtt{AF} : \mathtt{W} \times \mathsf{Agt} \to \mathcal{P}(\Phi_0)$ is an ability function;
- $\mathtt{CF} : \mathtt{W} \times \mathsf{Agt} \times \mathsf{Agt} \to \mathcal{P}(\Phi_0)$ is a claim function;
- $\mathtt{PF}_O : \mathtt{W} \times \mathsf{Agt} \times \mathsf{Agt} \to \mathcal{P}(\Phi_0)$ is an outward power function;
- $\mathtt{PF}_I : \mathtt{W} \times \mathsf{Agt} \times \mathsf{Agt} \to \mathcal{P}(\Phi_0)$ is an inward power function.

Models are understood as follows:

- Every possible world in $\mathtt{W}$ indicates a possible state of affairs of our world. The labeling function $\mathtt{L}$ specifies which atomic propositions are true at which possible worlds.
  The injectivity of $\mathtt{L}$ means that atomic facts determine possible worlds. That is to say, if two possible worlds satisfy the same atomic propositions, they are identical.
  The surjectivity of $\mathtt{L}$ denotes that every possibility of which atomic propositions are true is realized by a possible world. We require this to avoid the following: the agent is able to make p true/false at a possible world where p is false/true, but there are no possible worlds in the model where p is true/false.
  Note $\mathtt{W}$ is uncountably infinite, as Φ_0 is countably infinite.
- An atomic proposition p in $\mathtt{AF}(w, a)$ means that a has the ability to change the truth value of p at w.
- An atomic proposition p in $\mathtt{CF}(w, a, b)$ means that a has a claim against b that b change the truth value of p at w.
  Note that we do not rule out the possibility that an agent has a claim against herself. This might be redundant but not harmful.
- An atomic proposition p in $\mathtt{PF}_O(w, a, b)$ means that a has the power against b to create or withdraw a claim that b change the truth value of p at w.
- An atomic proposition p in $\mathtt{PF}_I(w, a, b)$ means that a has the power against b to create or withdraw a duty that a change the truth value of p at w.

For every model $\mathtt{M}$ and a possible world w of it, $(\mathtt{M}, w)$ is called a *pointed model*.

EXAMPLE 4.3 (Pointed models).
Figure 1 illustrates a pointed model $(\mathtt{M}, w_1)$. There are two agents, a and b, and three atomic propositions, p, q and r. There are eight states.

At $(\mathtt{M}, w_1)$: (1) p and q are true; (2) a has the ability to change the truth values of p and q; b has the ability to change the truth values of q and r; (3) a has a claim against b that b change the truth

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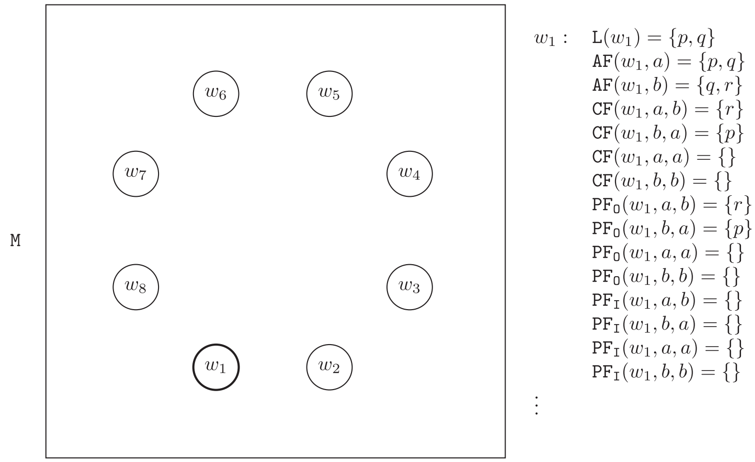

FIGURE 1. A pointed model $(\mathtt{M}, w_1)$

value of r; b has a claim against a that a change the truth value of p; (4) a has the power against b to withdraw the claim that b change the truth value of r; b has the power against a to withdraw the claim that a change the truth value of p.

4.3 Semantics

Let $(\mathtt{M}, w)$ be a pointed model where $\mathtt{M} = (\mathtt{W}, \mathtt{L}, \mathtt{AF}, \mathtt{CF}, \mathtt{PF}_O, \mathtt{PF}_I)$. We say that a fact-changing action FA is *available* at $(\mathtt{M}, w)$ if for every $(a \uparrow p)$ occurring in FA, $p \in \mathtt{AF}(w, a)$, and a claim-changing action CA is *available* at $(\mathtt{M}, w)$ if (1) for every $(a \curvearrowright \mathrm{CL}^b_a(b \uparrow p))$ occurring in CA, $p \in \mathtt{PF}_O(w, a, b)$, and (2) for every $(a \curvearrowright \mathrm{CL}^a_b(a \uparrow p))$ occurring in CA, $p \in \mathtt{PF}_I(w, a, b)$.

The semantics for Φ_{DyLeR} is by mutual recursion with Definitions 4.4 and 4.5.

DEFINITION 4.4 (Action interpretation).
Let $(\mathtt{M}, w)$ and $(\mathtt{M}', w')$ be two pointed models where $\mathtt{M} = (\mathtt{W}, \mathtt{L}, \mathtt{AF}, \mathtt{CF}, \mathtt{PF}_O, \mathtt{PF}_I)$ and $\mathtt{M}' = (\mathtt{W}', \mathtt{L}', \mathtt{AF}', \mathtt{CF}', \mathtt{PF}'_O, \mathtt{PF}'_I)$. Let α be an action. Define $(\mathtt{M}, w) \xrightarrow{\alpha} (\mathtt{M}', w')$, indicating $(\mathtt{M}, w)$ is transformed to $(\mathtt{M}', w')$ by α, as follows:

$$(\mathtt{M}, w) \xrightarrow{\mathsf{FA}} (\mathtt{M}', w') \quad \Leftrightarrow \quad \textit{(1) } \mathsf{FA} \textit{ is available at } (\mathtt{M}, w), \textit{(2) } \mathtt{M}' = \mathtt{M}, \textit{and (3) for every } p \in \Phi_0, \; p \in \mathtt{L}(w') \textit{ iff } \Big(p \notin \mathtt{L}(w) \Leftrightarrow (a \uparrow p) \textit{ occurs in } \mathsf{FA} \textit{ for some } a\Big);$$

$$(\mathtt{M}, w) \xrightarrow{\mathsf{CA}} (\mathtt{M}', w') \quad \Leftrightarrow \quad \textit{(1) } \mathsf{CA} \textit{ is available at } (\mathtt{M}, w), \textit{(2) } w' = w, \textit{(3) } \mathtt{M}' \textit{ differs from } \mathtt{M} \textit{ at most at } \mathtt{CF}', \textit{and (4) for every } p \in \Phi_0, x \in \mathtt{W} \textit{ and } a, b \in \mathsf{Agt}, p \in \mathtt{CF}'(x, a, b) \textit{ iff } \Big(p \notin \mathtt{CF}(x, a, b) \Leftrightarrow \big(x = w \textit{ and } (a \curvearrowright \mathrm{CL}^b_a(b \uparrow p)) \textit{ or } (b \curvearrowright \mathrm{CL}^b_a(b \uparrow p)) \textit{ occurs in } \mathsf{CA}\big)\Big);$$

$$(\mathtt{M}, w) \xrightarrow{?\phi} (\mathtt{M}', w') \quad \Leftrightarrow \quad (\mathtt{M}, w) = (\mathtt{M}', w') \textit{ and } \mathtt{M}, w \Vdash \phi;$$

$$(\mathtt{M}, w) \xrightarrow{\alpha_1;\alpha_2} (\mathtt{M}', w') \quad \Leftrightarrow \quad \textit{there is } (\mathtt{M}'', w'') \textit{ such that } (\mathtt{M}, w) \xrightarrow{\alpha_1} (\mathtt{M}'', w'') \textit{ and } (\mathtt{M}'', w'') \xrightarrow{\alpha_2} (\mathtt{M}', w');$$

$$(\mathtt{M}, w) \xrightarrow{\alpha_1 \cup \alpha_2} (\mathtt{M}', w') \quad \Leftrightarrow \quad (\mathtt{M}, w) \xrightarrow{\alpha_1} (\mathtt{M}', w') \textit{ or } (\mathtt{M}, w) \xrightarrow{\alpha_2} (\mathtt{M}', w').$$

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DEFINITION 4.5 (Truth).
Let $(\mathtt{M}, w)$ be a pointed model.

$$\begin{array}{rcl}
\mathtt{M}, w \Vdash p & \Leftrightarrow & p \in \mathtt{L}(w) \\
\mathtt{M}, w \Vdash \top & & \\
\mathtt{M}, w \Vdash \mathrm{AB}_a(a \uparrow p) & \Leftrightarrow & p \in \mathtt{AF}(w, a) \\
\mathtt{M}, w \Vdash \mathrm{CL}_a^b(b \uparrow p) & \Leftrightarrow & p \in \mathtt{CF}(w, a, b) \\
\mathtt{M}, w \Vdash \mathrm{PO}_a^b(b \uparrow p) & \Leftrightarrow & p \in \mathtt{PF}_O(w, a, b) \\
\mathtt{M}, w \Vdash \mathrm{PO}_a^b(a \uparrow p) & \Leftrightarrow & p \in \mathtt{PF}_I(w, a, b) \\
\mathtt{M}, w \Vdash \neg\phi & \Leftrightarrow & \textit{not}\ \mathtt{M}, w \Vdash \phi \\
\mathtt{M}, w \Vdash (\phi \wedge \psi) & \Leftrightarrow & \mathtt{M}, w \Vdash \phi\ \textit{and}\ \mathtt{M}, w \Vdash \psi \\
\mathtt{M}, w \Vdash [\alpha]\phi & \Leftrightarrow & \textit{for all}\ (\mathtt{M}', w')\ \textit{with}\ (\mathtt{M}, w) \xrightarrow{\alpha} (\mathtt{M}', w') : \mathtt{M}', w' \Vdash \phi
\end{array}$$

We define *validity* as usual. For every set Γ of formulas and formula ϕ of Φ_{DyLeR}, we use $\Gamma \models \phi$ to express that the inference from Γ to ϕ is valid. We use DyLeR to denote the set of all valid formulas of Φ_{DyLeR}.

The first two clauses in the definition of action interpretation need some explanations.

The following facts can be easily checked:

FACT 4.6
$(\mathtt{M}, w) \xrightarrow{\mathsf{FA}} (\mathtt{M}', w')$ if and only if (1) FA is available at $(\mathtt{M}, w)$, (2) $\mathtt{M}' = \mathtt{M}$, and (3) for every $p \in \Phi_0$, if $(a \uparrow p)$ occurs in FA for some a, then $p \notin \mathtt{L}(w') \Leftrightarrow p \in \mathtt{L}(w)$, and if not, then $p \in \mathtt{L}(w') \Leftrightarrow p \in \mathtt{L}(w)$.

FACT 4.7
$(\mathtt{M}, w) \xrightarrow{\mathsf{CA}} (\mathtt{M}', w')$ if and only if (1) CA is available at $(\mathtt{M}, w)$, (2) $w' = w$, (3) $\mathtt{M}'$ differs from $\mathtt{M}$ at most at $\mathtt{CF}'$, and (4) for every $p \in \Phi_0$, $x \in \mathtt{W}$ and $a, b \in \mathsf{Agt}$, if $x = w$ and $(a \curvearrowright \mathrm{CL}_a^b(b \uparrow p))$ or $(b \curvearrowright \mathrm{CL}_a^b(b \uparrow p))$ occurs in CA, then $p \in \mathtt{CF}'(x, a, b) \Leftrightarrow p \notin \mathtt{CF}(x, a, b)$, and if not, then $p \in \mathtt{CF}'(x, a, b) \Leftrightarrow p \in \mathtt{CF}(x, a, b)$.

The relation $\xrightarrow{\mathsf{FA}}$ is defined between possible worlds in the same model. $(\mathtt{M}, w) \xrightarrow{\mathsf{FA}} (\mathtt{M}', w')$ indicates that w' is the result of changing w by the fact-changing action FA. FA just changes the truth values of the atomic propositions occurring in FA.

The relation $\xrightarrow{\mathsf{FA}}$ is a partial function. How? Suppose $(\mathtt{M}, w) \xrightarrow{\mathsf{FA}} (\mathtt{M}', w')$ and $(\mathtt{M}, w) \xrightarrow{\mathsf{FA}} (\mathtt{M}'', w'')$. By Fact 4.6, $\mathtt{M}' = \mathtt{M}'' = \mathtt{M}$, and w' and w'' satisfy the same atomic propositions. By injectivity of the labeling function of $\mathtt{M}$, $w' = w''$. Then $(\mathtt{M}', w') = (\mathtt{M}'', w'')$. The reason for that $\xrightarrow{\mathsf{FA}}$ might not be full lies in that FA might be unavailable at $(\mathtt{M}, w)$. In this case, $(\mathtt{M}, w) \xrightarrow{\mathsf{FA}} (\mathtt{M}', w')$ for no $(\mathtt{M}', w')$.

The relation $\xrightarrow{\mathsf{CA}}$ is defined between the same state in different models. $(\mathtt{M}, w) \xrightarrow{\mathsf{CA}} (\mathtt{M}', w')$ indicates that $\mathtt{M}'$ is the result of changing $\mathtt{M}$ by the claim-changing action CA at w. CA just changes the claim of a against b that b change the truth value of p, where $(a \curvearrowright \mathrm{CL}_a^b(b \uparrow p))$ or $(b \curvearrowright \mathrm{CL}_a^b(b \uparrow p))$ occurs in CA.

The relation $\xrightarrow{\mathsf{CA}}$ is also a partial function. Suppose $(\mathtt{M}, w) \xrightarrow{\mathsf{CA}} (\mathtt{M}', w')$ and $(\mathtt{M}, w) \xrightarrow{\mathsf{CA}} (\mathtt{M}'', w'')$. By Fact 4.7, the claim functions of $\mathtt{M}'$ and $\mathtt{M}''$ are the same and $w' = w''$. Note $\mathtt{M}'$ and $\mathtt{M}''$ are different at most at claim functions. Then $(\mathtt{M}', w') = (\mathtt{M}'', w'')$. The reason for that $\xrightarrow{\mathsf{CA}}$ might not be full is that CA might not be available at $(\mathtt{M}, w)$.

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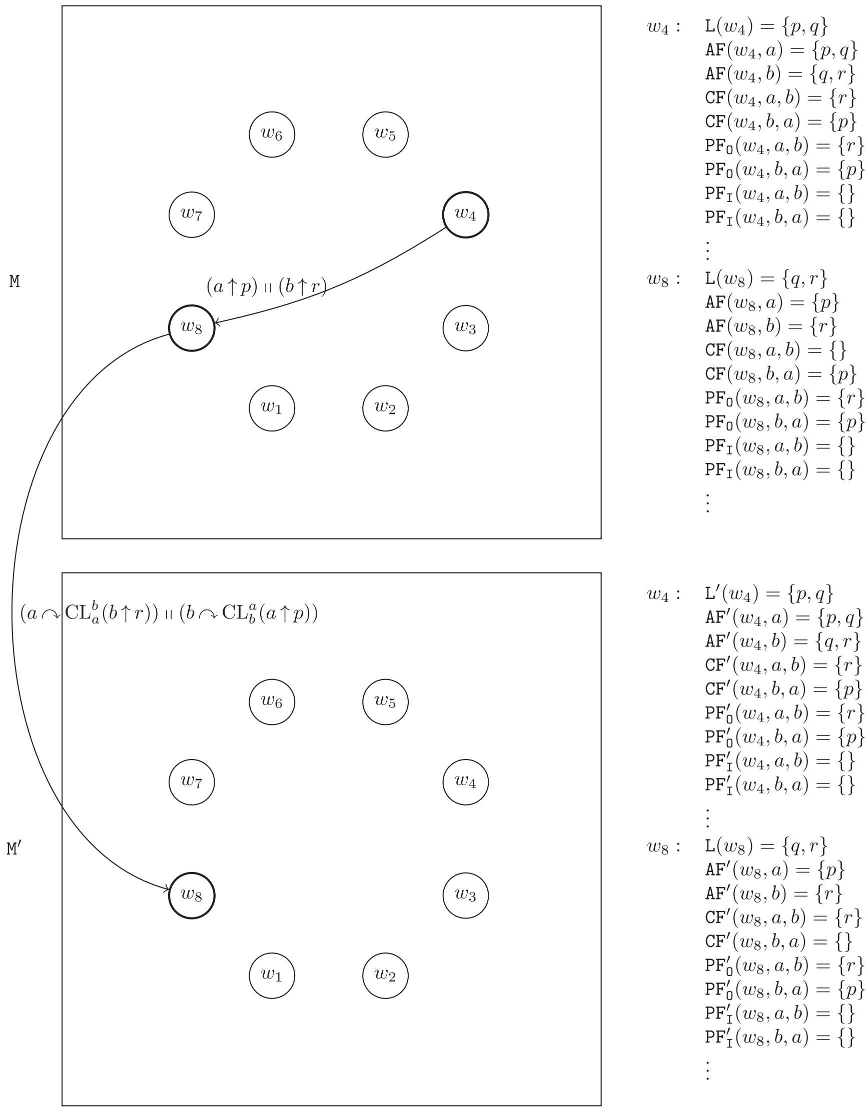

FIGURE 2. The pointed models $(\mathtt{M}, w_4)$ and $(\mathtt{M}, w_8)$ have the relation $\xrightarrow{(a\uparrow p)\shortparallel(b\uparrow r)}$, and the pointed models $(\mathtt{M}, w_8)$ and $(\mathtt{M}', w_8)$ have the relation $\xrightarrow{(a\curvearrowright \mathrm{CL}^b_a(b\uparrow r))\shortparallel(b\curvearrowright \mathrm{CL}^a_b(a\uparrow p))}$.

EXAMPLE 4.8 (Action interpretation).
Figure 2 illustrates action interpretation. There are two agents, a and b, and three atomic propositions, p, q and r. The situations at the pointed models $(\mathtt{M}, w_4), (\mathtt{M}, w_8), (\mathtt{M}', w_4)$ and $(\mathtt{M}', w_8)$ are partly specified.

4.4 *Axiomatization*

We first define some auxiliary notations and notions.

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Let $\alpha_1; \ldots; \alpha_n$ be a sequence of fact-changing or claim-changing actions. Recursively define $(\alpha_1; \ldots; \alpha_n)^{\mathbf{avai}}$, the *availability condition* of $\alpha_1; \ldots \alpha_n$, in the following way:

- $\mathsf{FA}^{\mathbf{avai}} = \mathrm{AB}_{a_1}(a_1 \uparrow p_1) \wedge \cdots \wedge \mathrm{AB}_{a_i}(a_i \uparrow p_i)$, where $\mathsf{FA} = (a_1 \uparrow p_1) \shortparallel \cdots \shortparallel (a_i \uparrow p_i)$;
- $\mathsf{CA}^{\mathbf{avai}} = \mathrm{PO}^{b_1}_{a_1}(b_1 \uparrow p_1) \wedge \cdots \wedge \mathrm{PO}^{b_i}_{a_i}(b_i \uparrow p_i) \wedge \mathrm{PO}^{d_1}_{c_1}(c_1 \uparrow q_1) \wedge \cdots \wedge \mathrm{PO}^{d_j}_{c_j}(c_j \uparrow q_j)$, where $\mathsf{CA} = (a_1 \curvearrowright \mathrm{CL}^{b_1}_{a_1}(b_1 \uparrow p_1)) \shortparallel \cdots \shortparallel (a_i \curvearrowright \mathrm{CL}^{b_i}_{a_i}(b_i \uparrow p_i)) \shortparallel (c_1 \curvearrowright \mathrm{CL}^{c_1}_{d_1}(c_1 \uparrow q_1)) \shortparallel \cdots \shortparallel (c_j \curvearrowright \mathrm{CL}^{c_j}_{d_j}(c_j \uparrow q_j))$;
- $(\alpha_k; \ldots; \alpha_n)^{\mathbf{avai}} = \alpha_k^{\mathbf{avai}} \wedge [\alpha_k](\alpha_{k+1}; \ldots; \alpha_n)^{\mathbf{avai}}$.

For example, $((a \uparrow p); (b \uparrow q))^{\mathbf{avai}} = \mathrm{AB}_a(a \uparrow p) \wedge [(a \uparrow p)]\mathrm{AB}_b(b \uparrow q)$.

We say that a sequence of fact-changing actions $\mathsf{FA}_1; \ldots; \mathsf{FA}_n$ is a *skip action* if for every p occurring in it, p occurs in an even number of FA_is. For example, $(a \uparrow p); (b \uparrow p)$ is a skip action. Intuitively, skip actions do not change anything. Note that a skip action may be unavailable at a possible world.

Let Δ be a finite set of formulas. We use $\bigwedge \Delta$ to express the conjunction of all the formulas in Δ. If Δ is empty, let $\bigwedge \Delta = \top$. We use $\neg\Delta$ to denote $\{\neg\phi \mid \phi \in \Delta\}$. Note if Δ is empty, $\bigwedge \neg\Delta = \top$.

By *literals*, we mean atomic propositions p or their negations $\neg p$.

We say that two sequences of fact-changing actions $\mathsf{FA}_1; \ldots; \mathsf{FA}_n$ and $\mathsf{FA}'_1; \ldots; \mathsf{FA}'_m$ are *alternatives* to each other if for every atomic proposition p, p occurs in an odd number of FA_is if and only if p occurs in an odd number of FA'_js. For example, $(a \uparrow p)$ and $(b \uparrow p); (c \uparrow p); (d \uparrow p)$ are alternatives to each other, and $(a \uparrow p); (b \uparrow p)$ and $(c \uparrow q); (d \uparrow q)$ are alternatives to each other. Intuitively, if two fact-changing actions are alternatives to each other, they change atomic facts in the same way. Note that it may happen that two fact-changing actions are alternatives to each other but one of them is available and the other is not.

DEFINITION 4.9 (An axiomatic system for DyLeR).

Axioms:

1. Axioms for the Propositional Logic
2. Normality axioms for $[\alpha]$:
   (a) $[\alpha](\phi \to \psi) \to ([\alpha]\phi \to [\alpha]\psi)$
3. Axioms for test, sequence and choice:
   (a) $[?\phi]\psi \leftrightarrow (\phi \to \psi)$
   (b) $[\alpha_1; \alpha_2]\phi \leftrightarrow [\alpha_1][\alpha_2]\phi$
   (c) $[\alpha_1 \cup \alpha_2]\phi \leftrightarrow ([\alpha_1]\phi \wedge [\alpha_2]\phi)$
4. Axioms for $[\mathsf{FA}]$ and $[\mathsf{CA}]$ meeting boolean operators:
   (a) $[\mathsf{FA}]\neg\phi \leftrightarrow (\mathsf{FA}^{\mathbf{avai}} \to \neg[\mathsf{FA}]\phi)$
   (b) $[\mathsf{FA}](\phi \wedge \psi) \leftrightarrow ([\mathsf{FA}]\phi \wedge [\mathsf{FA}]\psi)$
   (c) $[\mathsf{CA}]\neg\phi \leftrightarrow (\mathsf{CA}^{\mathbf{avai}} \to \neg[\mathsf{CA}]\phi)$
   (d) $[\mathsf{CA}](\phi \wedge \psi) \leftrightarrow ([\mathsf{CA}]\phi \wedge [\mathsf{CA}]\psi)$
5. Axioms for $[\mathsf{CA}]$ meeting $[\mathsf{FA}_1] \ldots [\mathsf{FA}_n]$:
   (a) $[\mathsf{CA}][\mathsf{FA}_1] \ldots [\mathsf{FA}_n]\mathsf{SP} \leftrightarrow ((\mathsf{FA}_1; \ldots; \mathsf{FA}_n)^{\mathbf{avai}} \to [\mathsf{CA}]\mathsf{SP})$, where $\mathsf{FA}_1; \ldots; \mathsf{FA}_n$ is a skip action
   (b) $[\mathsf{CA}][\mathsf{FA}_1] \ldots [\mathsf{FA}_n]\mathsf{SP} \leftrightarrow (\mathsf{CA}^{\mathbf{avai}} \to [\mathsf{FA}_1] \ldots [\mathsf{FA}_n]\mathsf{SP})$, where $\mathsf{FA}_1; \ldots; \mathsf{FA}_n$ is not a skip action

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6. Axioms for $[\mathsf{CA}]$ meeting simple propositions:
   (a) $[\mathsf{CA}]p \leftrightarrow (\mathsf{CA}^{\mathbf{avai}} \rightarrow p)$
   (b) $[\mathsf{CA}]\top \leftrightarrow \top$
   (c) $[\mathsf{CA}]\mathrm{AB}_a(a \uparrow p) \leftrightarrow (\mathsf{CA}^{\mathbf{avai}} \rightarrow \mathrm{AB}_a(a \uparrow p))$
   (d) $[\mathsf{CA}]\mathrm{CL}_a^b(b \uparrow p) \leftrightarrow (\mathsf{CA}^{\mathbf{avai}} \rightarrow \neg\mathrm{CL}_a^b(b \uparrow p))$, where $(a \curvearrowright \mathrm{CL}_a^b(b \uparrow p))$ or $(b \curvearrowright \mathrm{CL}_a^b(b \uparrow p))$ occurs in CA
   (e) $[\mathsf{CA}]\mathrm{CL}_a^b(b \uparrow p) \leftrightarrow (\mathsf{CA}^{\mathbf{avai}} \rightarrow \mathrm{CL}_a^b(b \uparrow p))$, where neither $(a \curvearrowright \mathrm{CL}_a^b(b \uparrow p))$ nor $(b \curvearrowright \mathrm{CL}_a^b(b \uparrow p))$ occurs in CA
   (f) $[\mathsf{CA}]\mathrm{PO}_a^b(b \uparrow p) \leftrightarrow (\mathsf{CA}^{\mathbf{avai}} \rightarrow \mathrm{PO}_a^b(b \uparrow p))$
   (g) $[\mathsf{CA}]\mathrm{PO}_a^b(a \uparrow p) \leftrightarrow (\mathsf{CA}^{\mathbf{avai}} \rightarrow \mathrm{PO}_a^b(a \uparrow p))$
7. Special axioms for $[\mathsf{FA}]$:
   (a) $\mathsf{FA}^{\mathbf{avai}} \leftrightarrow \langle \mathsf{FA} \rangle \top$
   (b) $\langle \mathsf{FA} \rangle \phi \rightarrow [\mathsf{FA}]\phi$
   (c) $(\bigwedge \Delta \wedge \bigwedge \Gamma) \rightarrow [\mathsf{FA}](\bigwedge \Delta \wedge \bigwedge \neg\Gamma)$, where Δ is a finite set of literals such that none of their atomic propositions occur in FA, and Γ is a finite set of literals such that all of their atomic propositions occur in FA
   (d) $((\mathsf{FA}_1;\ldots;\mathsf{FA}_n)^{\mathbf{avai}} \wedge (\mathsf{FA}'_1;\ldots;\mathsf{FA}'_m)^{\mathbf{avai}}) \rightarrow ([\mathsf{FA}_1]\ldots[\mathsf{FA}_n]\phi \leftrightarrow [\mathsf{FA}'_1]\ldots[\mathsf{FA}'_m]\phi)$, where $\mathsf{FA}_1;\ldots;\mathsf{FA}_n$ and $\mathsf{FA}'_1;\ldots;\mathsf{FA}'_m$ are alternatives to each other
   (e) $\phi \rightarrow [\mathsf{FA}_1]\ldots[\mathsf{FA}_n]\phi$, where $\mathsf{FA}_1;\ldots;\mathsf{FA}_n$ is a skip action

Inference rules:

1. Modus ponens: from $\phi \rightarrow \psi$ and ϕ, we can get ψ;
2. Action generalization: from ϕ, we can get $[\alpha]\phi$.

Derivability is defined as before. For every set Γ of formulas and formula ϕ of Φ_{DyLeR}, we use $\Gamma \vdash_{\mathsf{DyLeR}} \phi$ to indicate that ϕ is derivable from Γ by this system.

THEOREM 4.10 (Soundness of DyLeR).
The axiomatic system for DyLeR is sound.

THEOREM 4.11 (Strong completeness of DyLeR).
The axiomatic system for DyLeR is strongly complete.

The proofs for the two results can be found in Section A in the appendix.

4.5 The way this formalization works

In Section 2.2, we mentioned three examples about the dynamics of legal relations, and in Section 3.3, we analyzed the first two by our approach. Now let us make some parts of the analysis formal. The main reason we cannot carry out the formalization in full is that we do not introduce refrainment in our formalization.

The first example is about Beck renting an apartment from Adam in Berlin.

We use a and b to represent Adam and Beck respectively, and use l_a to represent *Adam lives in the apartment*. Figure 3 indicates that two pointed models $(\mathtt{M}, w_1)$ and $(\mathtt{M}', w_1)$ are in a relation $\xrightarrow{(a \curvearrowright \mathrm{CL}_b^a(a \uparrow l_a))}$. The relation $\xrightarrow{(a \curvearrowright \mathrm{CL}_b^a(a \uparrow l_a))}$ represents the action of *Adam creating a duty towards Beck that Adam change the truth value of* l_a. The pointed models $(\mathtt{M}, w_1)$ and $(\mathtt{M}', w_1)$ respectively indicate the situations before and after the action.

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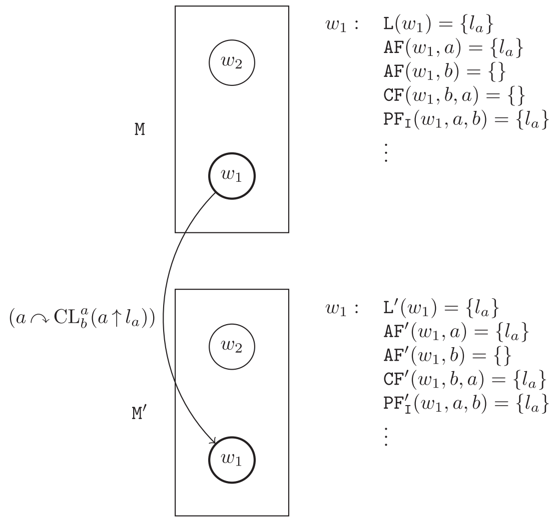

FIGURE 3. Two pointed models for Example 2.1

We can see that at $(\mathtt{M}, w_1)$: (1) *Adam lives in the apartment*; (2) *Adam has the ability to change the truth value of 'Adam lives in the apartment'*; (3) Adam does not have a duty towards Beck; (4) *Adam has the power against Beck to create a duty that Adam change the truth value of 'Adam lives in the apartment'*.

We can see that at $(\mathtt{M}', w_1)$: (1) *Adam lives in the apartment*; (2) *Adam has the ability to change the truth value of 'Adam lives in the apartment'*; (3) *Adam has a duty towards Beck that Adam change the truth value of 'Adam lives in the apartment'*; (4) *Adam has the power against Beck to withdraw the duty that Adam change the truth value of 'Adam lives in the apartment'*.

The second example is about Miss Ma soiling Miss Li's dress.

We use l and m to represent Miss Li and Miss Ma respectively, and use c to represent *Miss Li's dress is clean*. Figure 4 indicates that two pointed models $(\mathtt{M}, w_1)$ and $(\mathtt{M}, w_2)$ are in a relation $\xrightarrow{(m \uparrow c)}$, and $(\mathtt{M}, w_2)$ and a pointed model $(\mathtt{M}', w_2)$ are in a relation $\xrightarrow{(l \curvearrowright \mathrm{CL}^m_l(m \uparrow c))}$. The relation $\xrightarrow{(m \uparrow c)}$ indicates the action of *Miss Ma soiling Miss Li's dress* and the relation $\xrightarrow{(l \curvearrowright \mathrm{CL}^m_l(m \uparrow c))}$ indicates the action of *Miss Li withdrawing the claim against Miss Ma that Miss Ma change the truth value of 'Miss Li's dress is clean'*. The pointed models $(\mathtt{M}, w_1)$, $(\mathtt{M}, w_2)$ and $(\mathtt{M}', w_2)$ respectively indicate the situations before Miss Ma soiling the dress, after Miss Ma soiling the dress and after Miss Li withdrawing the claim against Miss Ma.

We can see that at $(\mathtt{M}, w_1)$: (1) *Miss Li's dress is clean*; (2) *Both Miss Li and Miss Ma have the ability to make the dress dirty*; (3) There are no claims between them; (4) They have no powers.

We can see that at $(\mathtt{M}, w_2)$: (1) *Miss Li's dress is not clean*; (2) *Both Miss Li and Miss Ma have the ability to make the dress clean*; (3) *Miss Li has a claim against Miss Ma that Miss Ma clean the dress*; (4) *Miss Li has the power against Miss Ma to withdraw the claim that Miss Ma clean the dress*.

We can see that at $(\mathtt{M}', w_2)$: (1) *Miss Li's dress is not clean*; (2) *Both Miss Li and Miss Ma have the ability to make the dress clean*; (3) There are no claims between them; (4) *Miss Li has the power against Miss Ma to create the claim that Miss Ma clean the dress*.

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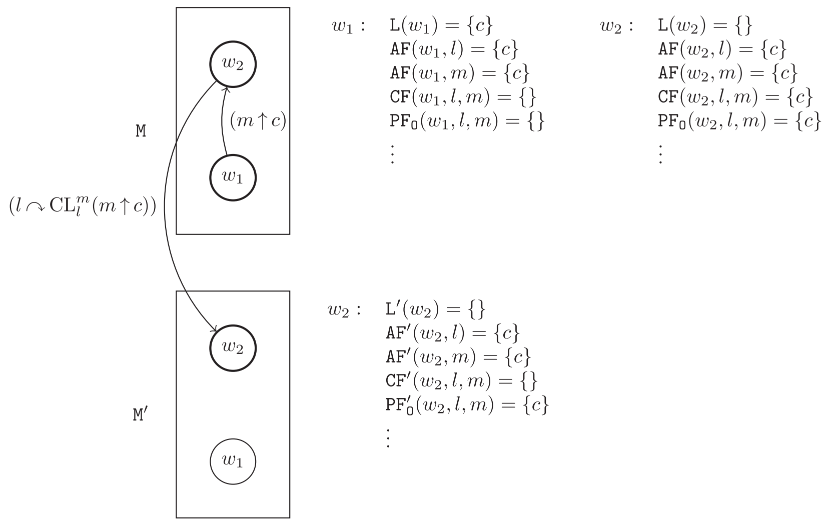

FIGURE 4. Two pointed models for Example 2.2

5 Comparison to related work

Here we briefly compare our work to some main references on formalizing legal relations.

Kanger was the first to formally represent legal relations and his theory can be found in [19] and [20]. Kanger uses Shall Do(b,F) to indicate that a, which is implicit, has a claim against b that b bring it about that F, where F is a formula. The treatment of Do(b,F) is from a modal logic presented in [5]. The operator Shall is the box operator in Standard Deontic Logic, proposed in [36].

Kanger focuses on the following question: Which legal relations are possible between two agents? For instance, is it possible that both Shall Do(b,F) and Shall Do(a,F) are true? Kanger provides a detailed analysis of this question.

As pointed out by Makinson [23] and Sergot [32], there are two limitations to this theory. Firstly, legal relations are not explicitly between agents in this work. As a result, scenarios with more than two agents cannot be represented. Also, this theory does not deal with powers.

Lindahl provides a further development of Kanger's theory in [22]. In this work, Lindahl discusses Kanger's question in much more detail. In addition, Lindahl discusses changes in legal relations. However, Lindahl does not essentially formalize the notion of counterparts either, nor does he provide a formal representation of legal powers (although powers are mentioned).

In the literature, Kanger's and Lindahl's theories are treated as a whole: the Kanger–Lindahl theory [32].

Two recent works on formalization of legal relations deserve mention: [24] and [6].

In [24], Markovich uses $\mathbf{O}_{a\to b}E_a\phi$ to express that a has a duty towards b that a see to it that ϕ. Here ϕ is a formula. E_a and $\mathbf{O}_{a\to b}$ are handled in the same way as the Kanger–Lindahl theory. Note that for every pair of agents a and b, there is an operator $\mathbf{O}_{a\to b}$. In this way, Markovich makes counterparts explicit.

Markovich deals with powers by the following idea: an agent has the power to perform a legal act if and only if the act's performance causes the corresponding legal consequence. She defines $\mathbf{P}_{a\to b}E_aF^c$, meaning that a has the power to do E_aF^c, as $[E_aF^c]C_b$. Here E_aF^c intuitively represents

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the action of a making a legal statement. $[E_aF^c]$ is the public announcement operator of Public Announcement Logic, proposed independently in [30] and [11]. C_b intuitively expresses that b *has/does not have* some *duty/privilege* related with the content of F^c. $[E_aF^c]C_b$ means the following: after a making a legal statement, the corresponding legal consequence obtains.

In [6], Dong and Roy use $O_{j\to i}Do_j\phi$ to indicate that j has a duty towards i to see to it that ϕ. Here Do_j is the S5 operator and $O_{j\to i}$ is the obligation operator in the preference-based deontic logic proposed in [33].

They deal with powers in the following way: an agent has the power to change a legal relation if and only if there is a legal act for the agent to do such that after the act is done, the change comes. They use $\neg T(j,k,\psi) \wedge \bigvee_{a\in A} < \mathcal{A}_i, a > T(j,k,\psi)$ to indicate that i has the power to introduce a legal relation $T(j,k,\psi)$ between j and k, and use $T(j,k,\psi) \wedge \bigvee_{a\in A} < \mathcal{A}_i, a > \neg T(j,k,\psi)$ to indicate that i has the power to cancel a legal relation $T(j,k,\psi)$ between j and k. Here $T(j,k,\psi)$ can be any expressible legal relation. For example, it can be $O_{j\to k}Do_j\phi$. A is intuitively a set of deontic actions such as issuing a parking ticket. The tuple $(\mathcal{A}_i, a)$ represents a deontic action of i. $< \mathcal{A}_i, a > \phi$ means that the action $(\mathcal{A}_i, a)$ is executable, and after it is done, ϕ is the case. $< \mathcal{A}_i, a >$ is handled as the action operator in dynamic epistemic logic, proposed in [1]. The purpose of defining powers with respect to three agents is to reflect that legal institutions can change legal relations.

Our way of representing legal relations is different from these works in the following respects.

In all these works, duties concern actions of seeing to a formula, which are handled by the results of their performance. In our work, duties concern actions of changing the truth value of an atomic proposition, which are handled by transitions of states. The two ways of dealing with actions are in different traditions: the former is in the line of STIT logic and the latter is in the line of propositional dynamic logic.

Dong and Roy deal with duties by the obligation operator in preference-based deontic logic and others deal with duties by the obligation operator in Standard Deontic Logic. Both of the two operators involve relational semantics and its philosophical underpinnings. We deal with duties in a direct fashion: the claim function specifies which atomic actions agents are obligated to do.

We also deal with powers directly: the power function specifies which claim-changing actions agents are able to perform. Both Markovich's work and Dong and Roy's work define powers by using consequences of acts, which is a more indirect way.

The indirect way has two problems. Conceptually, whether one agent has the power to perform a legal act determines whether the legal act's performance changes related legal relations, and it is not the other way around. There are cases where an agent does not have the power to perform a legal act, but her (unlawful) performance of the legal act still causes the related consequence. An example can be seen in Article 172 in the Civil Code of the People's Republic of China: 'Where an actor still performs an act of agency without a power of agency, beyond his or her power of attorney, or after his or her power of attorney terminates, the act shall be valid if the opposite party has reason to believe that the actor has the power of attorney.'[3]

6 Further work

Legal relations are notoriously complex. Since in this paper we have ignored much of this complexity, there is ample scope for further work.

[3] The translation is cited from http://www.pkulaw.com.

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Enrichment of actions We have introduced two types of atomic actions in this work, namely atomic fact-changing actions and atomic claim-changing actions, and four action constructors, namely concurrency, test, composition and choice. This leaves out a number of important actions.

As mentioned before, actions may have the effect of *sustaining the truth values of atomic propositions*. It makes good sense to introduce such actions. However, they cannot be added without further ado, as they might interfere with our fact-changing actions. One would have to introduce the notion of constraints. For instance, if an agent is able to change the truth value of p, then other agents should not be able to sustain the truth value of p, and vice versa.

As seen above, the notion of *refraining* is important in legal contexts, but this is not yet part of our machinery.

We disallow simultaneous fact-changing and claim-changing actions. Further study of real scenarios would be needed to investigate how these interact.

Test, sequence, choice, and *iteration* form a natural package in propositional dynamic logic, but in the interest of simplicity we have not considered iteration in this paper.

Powers and their dynamics In the above, we have treated the powers of an agent as her abilities to change claims and duties between her and other agents. This is a simplification. As pointed out by Hohfeld, there are many scenarios where a power can be used to generate or withdraw other powers. For instance, Hohfeld considered the creation of an agency relation as one party using its power 'to create agency powers in another party' [18, p. 52]. Furthermore, an agent may use her power to change claims and duties between other agents. For example, if an attorney by power of agency signs a contract with a counterparty, then the contractual claims and duties actually exist between the principal (the one represented by the attorney) and the counterparty (see also [18, p. 52]).

In this work, we assume that claim actions do not change powers. As mentioned previously, this needs more study. As just discussed, powers can change powers. A general question arises: 'What is the dynamics of powers?'

Introduction of time flow Finally, we have ignored the flow of time, while time plays an important role in legal contexts. Claims, duties and powers usually have a temporal dimension. Indeed, duties are often framed in terms of what may happen in time, as in: *Whenever the tenant causes damage to the apartment, she has a duty to compensate*. Also, fulfillment and violation of duties essentially involve time.

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A Proofs for soundness and completeness of DyLeR

A.1 Soundness of DyLeR

The following fact, which has an easy proof, says that skip actions never change possible worlds and other fact-changing actions always change possible worlds.

LEMMA A.1

1. If $\mathsf{FA}_1;\ldots;\mathsf{FA}_n$ is a skip action, then for every $(\mathtt{M},w)$ and $(\mathtt{M},w')$, if $(\mathtt{M},w) \xrightarrow{\mathsf{FA}_1;\ldots;\mathsf{FA}_n} (\mathtt{M},w')$, then $w=w'$;
2. If $\mathsf{FA}_1;\ldots;\mathsf{FA}_n$ is not a skip action, then for every $(\mathtt{M},w)$ and $(\mathtt{M},w')$, if $(\mathtt{M},w) \xrightarrow{\mathsf{FA}_1;\ldots;\mathsf{FA}_n} (\mathtt{M},w')$, then $w \neq w'$.

The following fact, which has an easy proof, says that if two fact actions are alternatives to each other, they cause the exact change.

LEMMA A.2
Let $\mathsf{FA}_1;\ldots;\mathsf{FA}_n$ and $\mathsf{FA}'_1;\ldots;\mathsf{FA}'_m$ be two sequences of fact-changing actions available at $(\mathtt{M},w)$ which are alternatives to each other. Then for every w' of $\mathtt{M}$, $(\mathtt{M},w) \xrightarrow{\mathsf{FA}_1;\ldots;\mathsf{FA}_n} (\mathtt{M},w')$ if and only if $(\mathtt{M},w) \xrightarrow{\mathsf{FA}'_1;\ldots;\mathsf{FA}'_m} (\mathtt{M},w')$.

THEOREM A. 1 (Soundness of DyLeR).
The axiomatic system for DyLeR is sound.

PROOF. It suffices to show that all the axioms are valid and that the two inference rules preserve validity.

It is easy to see that the two inference rules preserve validity.

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There are seven groups of axioms. Validity of the first four groups of axioms can be easily verified and we skip the details.

(5a) $[\mathsf{CA}][\mathsf{FA}_1]\dots[\mathsf{FA}_n]\mathsf{SP} \leftrightarrow ((\mathsf{FA}_1;\dots;\mathsf{FA}_n)^{\mathbf{avai}} \to [\mathsf{CA}]\mathsf{SP})$, where $\mathsf{FA}_1;\dots;\mathsf{FA}_n$ is a skip action.

Fix a pointed model $(\mathtt{M}, w)$.

Assume $\mathtt{M}, w \not\Vdash [\mathsf{CA}][\mathsf{FA}_1]\dots[\mathsf{FA}_n]\mathsf{SP}$. Then $\mathtt{M}, w \Vdash \mathsf{CA}^{\mathbf{avai}}$ and $\mathtt{M}', w \not\Vdash [\mathsf{FA}_1]\dots[\mathsf{FA}_n]\mathsf{SP}$, where $(\mathtt{M}, w) \xrightarrow{\mathsf{CA}} (\mathtt{M}', w)$. Then $\mathtt{M}', w \Vdash (\mathsf{FA}_1;\dots;\mathsf{FA}_n)^{\mathbf{avai}}$ and $\mathtt{M}', w' \not\Vdash \mathsf{SP}$, where $(\mathtt{M}', w) \xrightarrow{\mathsf{FA}_1;\dots;\mathsf{FA}_n} (\mathtt{M}', w')$. By Fact 4.7, $\mathtt{M}, w \Vdash (\mathsf{FA}_1;\dots;\mathsf{FA}_n)^{\mathbf{avai}}$. By Lemma A.1, $w = w'$. Then $\mathtt{M}', w \not\Vdash \mathsf{SP}$. Then $\mathtt{M}, w \not\Vdash [\mathsf{CA}]\mathsf{SP}$. Then $\mathtt{M}, w \not\Vdash (\mathsf{FA}_1;\dots;\mathsf{FA}_n)^{\mathbf{avai}} \to [\mathsf{CA}]\mathsf{SP}$.

Assume $\mathtt{M}, w \not\Vdash (\mathsf{FA}_1;\dots;\mathsf{FA}_n)^{\mathbf{avai}} \to [\mathsf{CA}]\mathsf{SP}$. Then $\mathtt{M}, w \Vdash (\mathsf{FA}_1;\dots;\mathsf{FA}_n)^{\mathbf{avai}}$ and $\mathtt{M}, w \not\Vdash [\mathsf{CA}]\mathsf{SP}$. Then $\mathtt{M}, w \Vdash \mathsf{CA}^{\mathbf{avai}}$ and $\mathtt{M}', w \not\Vdash \mathsf{SP}$, where $(\mathtt{M}, w) \xrightarrow{\mathsf{CA}} (\mathtt{M}', w)$. By Fact 4.7, $\mathtt{M}', w \Vdash (\mathsf{FA}_1;\dots;\mathsf{FA}_n)^{\mathbf{avai}}$. By Lemma A.1, $(\mathtt{M}', w) \xrightarrow{\mathsf{FA}_1;\dots;\mathsf{FA}_n} (\mathtt{M}', w)$. Then $\mathtt{M}', w \not\Vdash [\mathsf{FA}_1]\dots[\mathsf{FA}_n]\mathsf{SP}$. Then $\mathtt{M}, w \not\Vdash [\mathsf{CA}][\mathsf{FA}_1]\dots[\mathsf{FA}_n]\mathsf{SP}$.

(5b) $[\mathsf{CA}][\mathsf{FA}_1]\dots[\mathsf{FA}_n]\mathsf{SP} \leftrightarrow (\mathsf{CA}^{\mathbf{avai}} \to [\mathsf{FA}_1]\dots[\mathsf{FA}_n]\mathsf{SP})$, where $\mathsf{FA}_1;\dots;\mathsf{FA}_n$ is not a skip action.

Fix a pointed model $(\mathtt{M}, w)$.

Assume $\mathtt{M}, w \not\Vdash [\mathsf{CA}][\mathsf{FA}_1]\dots[\mathsf{FA}_n]\mathsf{SP}$. Then $\mathtt{M}, w \Vdash \mathsf{CA}^{\mathbf{avai}}$ and $\mathtt{M}', w \not\Vdash [\mathsf{FA}_1]\dots[\mathsf{FA}_n]\mathsf{SP}$, where $(\mathtt{M}, w) \xrightarrow{\mathsf{CA}} (\mathtt{M}', w)$. Then $\mathtt{M}', w \Vdash (\mathsf{FA}_1;\dots;\mathsf{FA}_n)^{\mathbf{avai}}$ and $\mathtt{M}', w' \not\Vdash \mathsf{SP}$, where $(\mathtt{M}', w) \xrightarrow{\mathsf{FA}_1;\dots;\mathsf{FA}_n} (\mathtt{M}', w')$. By Fact 4.7, $\mathtt{M}, w \Vdash (\mathsf{FA}_1;\dots;\mathsf{FA}_n)^{\mathbf{avai}}$ and $(\mathtt{M}, w) \xrightarrow{\mathsf{FA}_1;\dots;\mathsf{FA}_n} (\mathtt{M}, w')$. By Lemma A.1, $w \neq w'$. By Fact 4.7, $\mathtt{M}, w' \not\Vdash \mathsf{SP}$. Then $\mathtt{M}, w \not\Vdash [\mathsf{FA}_1]\dots[\mathsf{FA}_n]\mathsf{SP}$. Then $\mathtt{M}, w \not\Vdash \mathsf{CA}^{\mathbf{avai}} \to [\mathsf{FA}_1]\dots[\mathsf{FA}_n]\mathsf{SP}$.

Assume $\mathtt{M}, w \not\Vdash \mathsf{CA}^{\mathbf{avai}} \to [\mathsf{FA}_1]\dots[\mathsf{FA}_n]\mathsf{SP}$. Then $\mathtt{M}, w \Vdash \mathsf{CA}^{\mathbf{avai}}$ and $\mathtt{M}, w \not\Vdash [\mathsf{FA}_1]\dots[\mathsf{FA}_n]\mathsf{SP}$. Then $\mathtt{M}, w \Vdash (\mathsf{FA}_1;\dots;\mathsf{FA}_n)^{\mathbf{avai}}$ and $\mathtt{M}, w' \not\Vdash \mathsf{SP}$, where $(\mathtt{M}, w) \xrightarrow{\mathsf{FA}_1;\dots;\mathsf{FA}_n} (\mathtt{M}, w')$. Assume $(\mathtt{M}, w) \xrightarrow{\mathsf{CA}} (\mathtt{M}', w)$. By Fact 4.7, $\mathtt{M}', w \Vdash (\mathsf{FA}_1;\dots;\mathsf{FA}_n)^{\mathbf{avai}}$. By Fact 4.7, $(\mathtt{M}', w) \xrightarrow{\mathsf{FA}_1;\dots;\mathsf{FA}_n} (\mathtt{M}', w')$. By Lemma A.1, $w \neq w'$. By Fact 4.7, $\mathtt{M}', w' \not\Vdash \mathsf{SP}$. Then $\mathtt{M}', w \not\Vdash [\mathsf{FA}_1]\dots[\mathsf{FA}_n]\mathsf{SP}$. Then $\mathtt{M}, w \not\Vdash [\mathsf{CA}][\mathsf{FA}_1]\dots[\mathsf{FA}_n]\mathsf{SP}$.

Validity of the axioms (6a), (6b), (6c), (6f) and (6g) are easy to check.

(6d) $[\mathsf{CA}]\mathrm{CL}_a^b(b \uparrow p) \leftrightarrow (\mathsf{CA}^{\mathbf{avai}} \to \neg\mathrm{CL}_a^b(b \uparrow p))$, where $(a \curvearrowright \mathrm{CL}_a^b(b \uparrow p))$ or $(b \curvearrowright \mathrm{CL}_a^b(b \uparrow p))$ occurs in CA.

Fix a pointed model $(\mathtt{M}, w)$, where $\mathtt{M} = (\mathtt{W}, \mathtt{L}, \mathtt{AF}, \mathtt{CF}, \mathtt{PF}_O, \mathtt{PF}_I)$.

Assume $\mathtt{M}, w \not\Vdash [\mathsf{CA}]\mathrm{CL}_a^b(b \uparrow p)$. Then $\mathtt{M}, w \Vdash \mathsf{CA}^{\mathbf{avai}}$ and $\mathtt{M}', w \not\Vdash \mathrm{CL}_a^b(b \uparrow p)$, where $(\mathtt{M}, w) \xrightarrow{\mathsf{CA}} (\mathtt{M}', w)$, where $\mathtt{M}' = (\mathtt{W}, \mathtt{L}, \mathtt{AF}, \mathtt{CF}', \mathtt{PF}_O, \mathtt{PF}_I)$. Assume $\mathtt{M}, w \Vdash \neg\mathrm{CL}_a^b(b \uparrow p)$. Then $p \notin \mathtt{CF}(w, a, b)$. By Fact 4.7, $p \in \mathtt{CF}'(w, a, b)$. Then $\mathtt{M}', w \Vdash \mathrm{CL}_a^b(b \uparrow p)$. We have a contradiction. Then $\mathtt{M}, w \not\Vdash \neg\mathrm{CL}_a^b(b \uparrow p)$. Then $\mathtt{M}, w \not\Vdash \mathsf{CA}^{\mathbf{avai}} \to \neg\mathrm{CL}_a^b(b \uparrow p)$.

Assume $\mathtt{M}, w \not\Vdash \mathsf{CA}^{\mathbf{avai}} \to \neg\mathrm{CL}_a^b(b \uparrow p)$. Then $\mathtt{M}, w \Vdash \mathsf{CA}^{\mathbf{avai}}$ and $\mathtt{M}, w \Vdash \mathrm{CL}_a^b(b \uparrow p)$. Assume $(\mathtt{M}, w) \xrightarrow{\mathsf{CA}} (\mathtt{M}', w)$, where $\mathtt{M}' = (\mathtt{W}, \mathtt{L}, \mathtt{AF}, \mathtt{CF}', \mathtt{PF}_O, \mathtt{PF}_I)$. Assume $\mathtt{M}', w \Vdash \mathrm{CL}_a^b(b \uparrow p)$. Then $p \in \mathtt{CF}'(w, a, b)$. By Fact 4.7, $p \notin \mathtt{CF}(w, a, b)$. Then $\mathtt{M}, w \not\Vdash \mathrm{CL}_a^b(b \uparrow p)$. We have a contradiction. Then $\mathtt{M}', w \not\Vdash \mathrm{CL}_a^b(b \uparrow p)$. Then $\mathtt{M}, w \not\Vdash [\mathsf{CA}]\mathrm{CL}_a^b(b \uparrow p)$.

(6e) $[\mathsf{CA}]\mathrm{CL}_a^b(b \uparrow p) \leftrightarrow (\mathsf{CA}^{\mathbf{avai}} \to \mathrm{CL}_a^b(b \uparrow p))$, where neither $(a \curvearrowright \mathrm{CL}_a^b(b \uparrow p))$ nor $(b \curvearrowright \mathrm{CL}_a^b(b \uparrow p))$ occurs in CA.

Fix a pointed model $(\mathtt{M}, w)$, where $\mathtt{M} = (\mathtt{W}, \mathtt{L}, \mathtt{AF}, \mathtt{CF}, \mathtt{PF}_O, \mathtt{PF}_I)$.

Assume $\mathtt{M}, w \not\Vdash [\mathsf{CA}]\mathrm{CL}_a^b(b \uparrow p)$. Then $\mathtt{M}, w \Vdash \mathsf{CA}^{\mathbf{avai}}$ and $\mathtt{M}', w \not\Vdash \mathrm{CL}_a^b(b \uparrow p)$, where $(\mathtt{M}, w) \xrightarrow{\mathsf{CA}} (\mathtt{M}', w)$, where $\mathtt{M}' = (\mathtt{W}, \mathtt{L}, \mathtt{AF}, \mathtt{CF}', \mathtt{PF}_O, \mathtt{PF}_I)$. Then $p \notin \mathtt{CF}'(w, a, b)$. By Fact 4.7, $p \notin \mathtt{CF}(w, a, b)$. Then $\mathtt{M}, w \not\Vdash \mathrm{CL}_a^b(b \uparrow p)$. Then $\mathtt{M}, w \not\Vdash \mathsf{CA}^{\mathbf{avai}} \to \mathrm{CL}_a^b(b \uparrow p)$.

Assume $\mathtt{M}, w \not\Vdash \mathsf{CA}^{\mathbf{avai}} \rightarrow \mathrm{CL}_a^b(b \uparrow p)$. Then $\mathtt{M}, w \Vdash \mathsf{CA}^{\mathbf{avai}}$ and $\mathtt{M}, w \not\Vdash \mathrm{CL}_a^b(b \uparrow p)$. Then $p \notin \mathtt{CF}(w, a, b)$. Assume $(\mathtt{M}, w) \xrightarrow{\mathsf{CA}} (\mathtt{M}', w)$, where $\mathtt{M}' = (\mathtt{W}, \mathtt{L}, \mathtt{AF}, \mathtt{CF}', \mathtt{PF}_O, \mathtt{PF}_I)$. By Fact 4.7, $p \notin \mathtt{CF}'(w, a, b)$. Then $\mathtt{M}', w \not\Vdash \mathrm{CL}_a^b(b \uparrow p)$. Then $\mathtt{M}, w \not\Vdash [\mathsf{CA}]\mathrm{CL}_a^b(b \uparrow p)$.

Validity of the axioms (7a) and (7b) is easy to check. By Facts 4.6, and Lemmas A.2 and A.1, we can easily show validity of the axioms (7c), (7d) and (7e) respectively. □

A.2 Completeness of **DyLeR** *by reduction to a logic* **LACP**

DEFINITION A.2 (Language Φ_{LACP}).
Define the language Φ_{LACP} for Logic for Abilities, Claims and Powers (LACP) as follows, where FA is defined as definition 4.1:

$$\phi ::= p \mid \top \mid \mathrm{AB}_a(a \uparrow p) \mid \mathrm{CL}_a^b(b \uparrow p) \mid \mathrm{PO}_a^b(b \uparrow p) \mid \mathrm{PO}_a^b(a \uparrow p) \mid \neg\phi \mid (\phi \wedge \phi) \mid [\mathsf{FA}]\phi$$

We use LACP to denote the set of valid formulas in Φ_{LACP}.

The language Φ_{LACP} is as expressive as Φ_{DyLeR} by the following result:

LEMMA A.3
For every $\phi \in \Phi_{\mathsf{DyLeR}}$, there is $\phi' \in \Phi_{\mathsf{LACP}}$ such that $\vdash_{\mathsf{DyLeR}} \phi \leftrightarrow \phi'$.

PROOF. Fix a formula ϕ in Φ_{DyLeR}. By Axiom 3a, 3b and 3c, there is ϕ_1 such that $\vdash_{\mathsf{DyLeR}} \phi \leftrightarrow \phi_1$ and ϕ_1 does not contain ?, ; or ∪.

By the axioms for [FA] and [CA] meeting boolean operators, there is ϕ_2 such that $\vdash_{\mathsf{DyLeR}} \phi_1 \leftrightarrow \phi_2$ and ϕ_2 is a boolean combination of some simple propositions and some formulas in the form $[\alpha_1]\dots[\alpha_n]\mathsf{SP}$, where each α_i is a fact-changing action or claim-changing action. Note that SP refers to indicate simple propositions.

Pick a formula ψ in the form $[\alpha_1]\dots[\alpha_n]\mathsf{SP}$. By Axiom 5a and 5b, the axioms for [CA] meeting simple propositions, and the axioms for [FA] and [CA] meeting boolean operators, there is ψ_1 such that $\vdash_{\mathsf{DyLeR}} \psi \leftrightarrow \psi_1$ and ψ_1 is a boolean combination of some simple propositions and some formulas in the form $[\mathsf{FA}_1]\dots[\mathsf{FA}_n]\mathsf{SP}$.

Then there is $\phi_3 \in \Phi_{\mathsf{LACP}}$ such that $\vdash_{\mathsf{DyLeR}} \phi_2 \leftrightarrow \phi_3$. Then $\vdash_{\mathsf{DyLeR}} \phi \leftrightarrow \phi_3$. □

Later we present a sound and strongly complete axiomatic system for LACP, which is contained in the system for DyLeR. Then we can show the strong completeness of DyLeR.

THEOREM A. 2 (Strong completeness of DyLeR).
The axiomatic system for DyLeR is strongly complete.

PROOF. Let I be an index set and $\Gamma = \{\phi_i \mid i \in I\}$ be a set of formulas of Φ_{DyLeR}. Let ψ be a formula of Φ_{DyLeR}. Assume $\Gamma \models \psi$. For every $\phi_i \in \Gamma$, let $\phi_i' \in \Phi_{\mathsf{LACP}}$ be such that $\vdash_{\mathsf{DyLeR}} \phi_i \leftrightarrow \phi_i'$. Let $\psi' \in \Phi_{\mathsf{LACP}}$ be such that $\vdash_{\mathsf{DyLeR}} \psi \leftrightarrow \psi'$. By Lemma A.3, we can do this. Let $\Gamma' = \{\phi_i' \mid i \in I\}$. By soundness of DyLeR, $\Gamma' \models \psi'$. By the strong completeness of LACP, $\Gamma' \vdash_{\mathsf{LACP}} \psi'$. Then $\Gamma' \vdash_{\mathsf{DyLeR}} \psi'$. By Lemma A.3, $\Gamma \vdash_{\mathsf{DyLeR}} \psi$. □

A.3 Sound and complete axiomatization of **LACP**

DEFINITION A.3 (An axiomatic system for LACP).

Axioms:

1. Axioms for the Propositional Logic
2. Axioms for [FA]:

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(a) $[\mathsf{FA}](\phi \to \psi) \to ([\mathsf{FA}]\phi \to [\mathsf{FA}]\psi)$
(b) $\mathsf{FA}^{\mathbf{avai}} \leftrightarrow \langle \mathsf{FA} \rangle \top$
(c) $\langle \mathsf{FA} \rangle \phi \to [\mathsf{FA}]\phi$
(d) $(\bigwedge \Delta \wedge \bigwedge \Gamma) \to [\mathsf{FA}](\bigwedge \Delta \wedge \bigwedge \neg \Gamma)$, where Δ is a finite set of literals such that none of their atomic propositions occur in FA, and Γ is a finite set of literals such that all of their atomic propositions occur in FA
(e) $((\mathsf{FA}_1; \ldots; \mathsf{FA}_n)^{\mathbf{avai}} \wedge (\mathsf{FA}'_1; \ldots; \mathsf{FA}'_m)^{\mathbf{avai}}) \to ([\mathsf{FA}_1] \ldots [\mathsf{FA}_n]\phi \leftrightarrow [\mathsf{FA}'_1] \ldots [\mathsf{FA}'_m]\phi)$, where $\mathsf{FA}_1; \ldots; \mathsf{FA}_n$ and $\mathsf{FA}'_1; \ldots; \mathsf{FA}'_m$ are alternatives to each other
(f) $\phi \to [\mathsf{FA}_1] \ldots [\mathsf{FA}_n]\phi$, where $\mathsf{FA}_1; \ldots; \mathsf{FA}_n$ is a skip action

Inference rules:

1. Modus ponens: From $\phi \to \psi$ and ϕ, we can get ψ;
2. Action generalization: From ϕ, we can get $[\mathsf{FA}]\phi$.

We claim that this system is sound and strongly complete.

Since this system is contained in the system for DyLeR, its soundness is implied by the soundness of DyLeR.

In order to show the strong completeness of LACP, it suffices to show that for every consistent set of formulas Λ, there is a pointed model $(\mathtt{M}, w)$ for Φ_{DyLeR} such that for all $\phi \in \Lambda$, $\mathtt{M}, w \Vdash \phi$.

A.3.1 Maximally consistent sets of formulas and canonical relations between them Maximally consistent sets of formulas (**MCS**s) are defined as usual and their usual properties are assumed.

We use **LT** to express the set of all literals. For every **MCS** w and fact-changing action FA, define

$$\mathbf{L}^+(w, \mathsf{FA}) = \{l \in \mathbf{LT} \mid l \in w \textit{ and its atomic proposition occurs in } \mathsf{FA}\}$$

and

$$\mathbf{L}^-(w, \mathsf{FA}) = \{l \in \mathbf{LT} \mid l \in w \textit{ and its atomic proposition does not occur in } \mathsf{FA}\}.$$

For every fact action FA, define a relation $\xrightarrow{\mathsf{FA}}$ between **MCS**s as follows:
$w \xrightarrow{\mathsf{FA}} u$ if and only if (1) $\mathsf{FA}^{\mathbf{avai}} \in w$ and (2) $\mathbf{L}^-(w, \mathsf{FA}) \cup \neg\mathbf{L}^+(w, \mathsf{FA}) \cup \{\phi \mid [\mathsf{FA}]\phi \in w\} \subseteq u$.

LEMMA A.4
Let $w \xrightarrow{\mathsf{FA}} u$. Then for every $\phi \in u$, $\langle \mathsf{FA} \rangle \phi \in w$.

PROOF. Let $\phi \in u$. Assume $\langle \mathsf{FA} \rangle \phi \notin w$. Then $\neg \langle \mathsf{FA} \rangle \phi \in w$. Then $[\mathsf{FA}]\neg\phi \in w$. Then $\neg\phi \in u$. Then u is not consistent, which is impossible. □

LEMMA A.5
Let $\mathsf{FA}^{\mathbf{avai}} \in w$. Then there is one and only one u such that $w \xrightarrow{\mathsf{FA}} u$.

PROOF. We firstly show the existence. It suffices to show that $\mathbf{L}^-(w, \mathsf{FA}) \cup \neg\mathbf{L}^+(w, \mathsf{FA}) \cup \{\phi \mid [\mathsf{FA}]\phi \in w\}$ is consistent. Assume not. Then there is a finite subset X of $\mathbf{L}^-(w, \mathsf{FA})$, a finite subset Y of $\mathbf{L}^+(w, \mathsf{FA})$, and a finite subset Z of $\{\phi \mid [\mathsf{FA}]\phi \in w\}$ such that $\vdash_{\mathsf{LACP}} (\bigwedge X \wedge \bigwedge \neg Y \wedge \bigwedge Z) \to \bot$. By the inference rule of action generalization, $\vdash_{\mathsf{LACP}} [\mathsf{FA}](\bigwedge X \wedge \bigwedge \neg Y \wedge \bigwedge Z \to \bot)$. By use of Axiom 2a, we can get $\vdash_{\mathsf{LACP}} ([\mathsf{FA}](\bigwedge X \wedge \bigwedge \neg Y) \wedge [\mathsf{FA}] \bigwedge Z) \to [\mathsf{FA}]\bot$. By Axiom 2d, $[\mathsf{FA}](\bigwedge X \wedge \bigwedge \neg Y) \in w$. By Axiom 2a, $[\mathsf{FA}] \bigwedge Z \in w$. Then $[\mathsf{FA}]\bot \in w$. By Axiom 2d, $\neg\mathsf{FA}^{\mathbf{avai}} \in w$. Then w is not consistent, which is impossible.

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Now we show the uniqueness. Assume that there are u_1 and u_2 such that $w \xrightarrow{\mathsf{FA}} u_1$ and $w \xrightarrow{\mathsf{FA}} u_2$. Let $\phi \in u_1$. By Lemma A.4, $\langle \mathsf{FA} \rangle \phi \in w$. By Axiom 2c, $[\mathsf{FA}]\phi \in w$. Then $\phi \in u_2$. Then $u_1 \subseteq u_2$. As both u_1 and u_2 are **MCS**s, $u_1 = u_2$. □

LEMMA A.6 (Existence lemma).
Let $\langle \mathsf{FA} \rangle \phi \in w$. Then there is u such that $w \xrightarrow{\mathsf{FA}} u$.

PROOF. It suffices to show that $\mathsf{FA}^{\mathbf{avai}} \in w$ and $\mathbf{L}^-(w, \mathsf{FA}) \cup \neg\mathbf{L}^+(w, \mathsf{FA}) \cup \{\psi \mid [\mathsf{FA}]\psi \in w\} \cup \{\phi\}$ is consistent.

It can be shown $\vdash_{\mathsf{LACP}} \langle \mathsf{FA} \rangle \phi \to \langle \mathsf{FA} \rangle \top$. Then $\langle \mathsf{FA} \rangle \top \in w$. By Axiom 2b, $\mathsf{FA}^{\mathbf{avai}} \in w$.

Assume that $\mathbf{L}^-(w, \mathsf{FA}) \cup \neg\mathbf{L}^+(w, \mathsf{FA}) \cup \{\psi \mid [\mathsf{FA}]\psi \in w\} \cup \{\phi\}$ is not consistent. Then there is a finite subset X of $\mathbf{L}^-(w, \mathsf{FA})$, a finite subset Y of $\mathbf{L}^+(w, \mathsf{FA})$, and a finite subset Z of $\{\phi \mid [\mathsf{FA}]\phi \in w\}$ such that $\vdash_{\mathsf{LACP}} (\bigwedge X \wedge \bigwedge \neg Y \wedge \bigwedge Z) \to \neg\phi$. By use of a similar argument with one in the proof for Lemma A.5, we can get $[\mathsf{FA}]\neg\phi \in w$. Then $\neg\langle \mathsf{FA} \rangle \neg\phi \in w$ and there is a contradiction. □

A.3.2 Generated trees We say that a **MCS** w can reach a **MCS** u if $w = u$ or there is a sequence of fact-changing actions $\mathsf{FA}_1, \ldots, \mathsf{FA}_n$ and a sequence of **MCS**s $x_1, \ldots, x_{n-1}$ such that $w \xrightarrow{\mathsf{FA}_1} x_1 \ldots x_{n-1} \xrightarrow{\mathsf{FA}_n} u$. For every w, we use $\mathtt{T}_w$ to denote the set of all **MCS**s reachable from w.

We say that a set of literals $\mathbf{F}$ is a *full list of atomic facts* if every p occurs in one and only one literal in $\mathbf{F}$. For every **MCS** w, we use $\mathbf{F}_w$ to denote the full list of atomic facts contained in w.

LEMMA A.7
For all x and y in $\mathtt{T}_w$, if $\mathbf{F}_x = \mathbf{F}_y$, then $x = y$.

PROOF. Assume that there are x and y in $\mathtt{T}_w$ such that $\mathbf{F}_x = \mathbf{F}_y$ but $x \neq y$. Then there is ϕ such that $\phi \in x$ and $\phi \notin y$. We respectively consider four possible cases.

(a) $x = w$ and $y = w$. This case is impossible.

(b) $x = w$ and $y \neq w$. Then there is a sequence $\mathsf{FA}_1, \ldots, \mathsf{FA}_n$ and a sequence $u_1, \ldots, u_{n-1}$ such that $w \xrightarrow{\mathsf{FA}_1} u_1 \ldots u_{n-1} \xrightarrow{\mathsf{FA}_n} y$. As $\mathbf{F}_w = \mathbf{F}_y$, $\mathsf{FA}_1, \ldots, \mathsf{FA}_n$ must be a skip action. By Axiom 2f, $[\mathsf{FA}_1] \ldots [\mathsf{FA}_n]\phi \in w$. Then $\phi \in y$. We have a contradiction.

(c) $x \neq w$ and $y = w$. This case is similar to the previous case.

(d) $x \neq w$ and $y \neq w$. Then there is a sequence $\mathsf{FA}_1, \ldots, \mathsf{FA}_n$ and a sequence $u_1, \ldots, u_{n-1}$ such that $w \xrightarrow{\mathsf{FA}_1} u_1 \ldots u_{n-1} \xrightarrow{\mathsf{FA}_n} x$, and there is a sequence $\mathsf{FA}'_1, \ldots, \mathsf{FA}'_m$ and a sequence $v_1, \ldots, v_{m-1}$ such that $w \xrightarrow{\mathsf{FA}'_1} v_1 \ldots v_{m-1} \xrightarrow{\mathsf{FA}'_m} y$. As $\mathbf{F}_x = \mathbf{F}_y$, $\mathsf{FA}_1; \ldots; \mathsf{FA}_n$ and $\mathsf{FA}'_1; \ldots; \mathsf{FA}'_m$ must be alternatives to each other. As $\phi \in x$, $\langle \mathsf{FA}_1 \rangle \ldots \langle \mathsf{FA}_n \rangle \phi \in w$. By Axiom 2c, $[\mathsf{FA}_1] \ldots [\mathsf{FA}_n]\phi \in w$. By Axiom 2e, $[\mathsf{FA}'_1] \ldots [\mathsf{FA}'_m]\phi \in w$. Then $\phi \in y$. We have a contradiction. □

A.3.3 Canonical p-models For every **MCS** w, a structure $\mathtt{M}_w = (\mathtt{W}, \mathtt{L}, \mathtt{AF}, \mathtt{CF}, \mathtt{PF}_O, \mathtt{PF}_I)$ is called the *canonical p-model* with respect to w if the following conditions are met:

- $\mathtt{W} = \mathtt{T}_w$;
- $\mathtt{L}(u) = \{p \mid p \in u\}$;
- $\mathtt{AF}(u, a) = \{p \mid \mathrm{AB}_a(a \uparrow p) \in u\}$;
- $\mathtt{CF}(u, a, b) = \{p \mid \mathrm{CL}_a^b(b \uparrow p) \in u\}$;
- $\mathtt{PF}_O(u, a, b) = \{p \mid \mathrm{PO}_a^b(b \uparrow p) \in u\}$;
- $\mathtt{PF}_I(u, a, b) = \{p \mid \mathrm{PO}_a^b(a \uparrow p) \in u\}$.

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LEMMA A.8 (Truth lemma).
Let $\mathtt{M}_w$ be the canonical p-model with respect to w. Then for every $\phi \in \Phi_{\mathsf{LACP}}$, $\mathtt{M}_x, u \Vdash \phi$ if and only if $\phi \in u$.

PROOF. We put an induction on ϕ. We only consider the case $\phi = [\mathsf{FA}]\psi$ and skip other cases, which are easy.

Assume $[\mathsf{FA}]\psi \notin u$. Then $\neg[\mathsf{FA}]\psi \in u$. Then $\langle \mathsf{FA} \rangle \neg\psi \in u$. By the existence lemma, there is v such that $u \xrightarrow{\mathsf{FA}} v$ and $\neg\psi \in v$. Then $\psi \notin v$. As u is reachable from w and $u \xrightarrow{\mathsf{FA}} v$, v is reachable from w. Then $v \in \mathtt{T}_w$. By the inductive hypothesis, $\mathtt{M}, v \not\Vdash \psi$. Then $\mathtt{M}, u \not\Vdash [\mathsf{FA}]\psi$.

Assume $\mathtt{M}, u \not\Vdash [\mathsf{FA}]\psi$. Then there is v such that $u \xrightarrow{\mathsf{FA}} v$ and $\mathtt{M}, v \not\Vdash \psi$. By the inductive hypothesis, $\psi \notin v$. Then $[\mathsf{FA}]\psi \notin u$. □

LEMMA A.9
$u \xrightarrow{\mathsf{FA}} v$ if and only if (1) FA is available at u, and (2) for every p, if p occurs in FA, then $p \in \mathtt{L}(v)$ if and only if $p \notin \mathtt{L}(u)$, and if p does not occur in FA, then $p \in \mathtt{L}(v)$ if and only if $p \in \mathtt{L}(u)$.

This result can be seen from the definition of $\xrightarrow{\mathsf{FA}}$. It implies that fact-changing actions are interpreted as expected.

By Lemma A.7, we can get that for every $x, y \in \mathtt{W}$, $\mathtt{L}(x) \neq \mathtt{L}(y)$. Then $\mathtt{L}$ is an injective function from $\mathtt{W}$ to $\mathcal{P}(\Phi_0)$. But it might not be surjective. So $\mathtt{M}$ might not be a model for Φ_{DyLeR}.

Note that $\mathtt{M}_w$ meets the following condition: For every $x \in \mathtt{T}_x$ and FA, if FA is available at x, then there is $y \in \mathtt{T}_x$ such that $x \xrightarrow{\mathsf{FA}} y$. By a result given below, we can extend it to a model $\mathtt{M}'$ such that for every x of $\mathtt{M}$, $\mathtt{M}, x \Vdash \phi$ if and only if $\mathtt{M}', x \Vdash \phi$ for every $\phi \in \Phi_{\mathsf{LACP}}$. Then we can get the strong completeness of LACP.

LEMMA A.10
The axiomatic system for LACP is strongly complete.

PROOF. Fix a consistent set of formulas Γ. Let w be a **MCS** containing Γ. Let $\mathtt{M}_w$ be the canonical p-model with respect to w. By the truth lemma, $\mathtt{M}_w, w \Vdash \phi$ for every $\phi \in \Gamma$. By Lemma A.11, we can get a model $\mathtt{M}'$ for Φ_{DyLeR}, whose domain contains the domain of $\mathtt{M}_w$, such that for every $\phi \in \Gamma$, $\mathtt{M}', w \Vdash \phi$. □

A.3.4 Some transformation of structures Let $\mathtt{M} = (\mathtt{W}, \mathtt{L}, \mathtt{AF}, \mathtt{CF}, \mathtt{PF}_O, \mathtt{PF}_I)$ be a structure such that (1) it is almost like a model except $\mathtt{L}$ is just injective, and (2) for every $x \in \mathtt{W}$ and FA, if FA is available at x, then there is $y \in \mathtt{W}$ such that $x \xrightarrow{\mathsf{FA}} y$.

Let $\mathtt{D}$ be a set of states which is disjoint with $\mathtt{W}$ and has the same cardinality with $\mathcal{P}(\Phi_0) - \{\mathtt{L}(x) \mid x \in \mathtt{W}\}$. Let f be a bijective function from $\mathtt{D}$ to $\mathcal{P}(\Phi_0) - \{\mathtt{L}(x) \mid x \in \mathtt{W}\}$. Define a structure $\mathtt{M}' = (\mathtt{W}', \mathtt{L}', \mathtt{AF}', \mathtt{CF}', \mathtt{PF}'_O, \mathtt{PF}'_I)$ as follows:

- $\mathtt{W}' = \mathtt{W} \cup \mathtt{D}$;
- $\mathtt{L}' = \mathtt{L} \cup f$;
    - $\mathtt{AF}'(x, a) = \mathtt{AF}(x, a)$, where $x \in \mathtt{W}$;
    - $\mathtt{AF}'(x, a) = \emptyset$, where $x \in \mathtt{D}$;
    - $\mathtt{CF}'(x, a, b) = \mathtt{CF}(x, a, b)$, where $x \in \mathtt{W}$;
    - $\mathtt{CF}'(x, a, b) = \emptyset$, where $x \in \mathtt{D}$.

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- $\mathtt{PF}'_O(x,a,b) = \mathtt{PF}_O(x,a,b)$, where $x \in \mathtt{W}$;
- $\mathtt{PF}'_O(x,a,b) = \emptyset$, where $x \in \mathtt{D}$;
- $\mathtt{PF}'_I(x,a,b) = \mathtt{PF}_I(x,a,b)$, where $x \in \mathtt{W}$;
- $\mathtt{PF}'_I(x,a,b) = \emptyset$, where $x \in \mathtt{D}$;

It is easy to see that $\mathtt{L}'$ is a bijective function from $\mathtt{W}'$ to $\mathcal{P}(\Phi_0)$. Then $\mathtt{M}'$ is a model of Φ_{DyLeR}.

Define $\xrightarrow{\mathsf{FA}}{}'$ on $\mathtt{W}'$ as before: $u \xrightarrow{\mathsf{FA}}{}' v$ if and only if (1) FA is available at u, and (2) for every p, if p occurs in FA, then $p \in \mathtt{L}(v)$ if and only if $p \notin \mathtt{L}(u)$, and if p does not occur in FA, then $p \in \mathtt{L}(v)$ if and only if $p \in \mathtt{L}(u)$.

It can be verified that for every x and y in $\mathtt{W}$, $x \xrightarrow{\mathsf{FA}}{}' y$ if and only if $x \xrightarrow{\mathsf{FA}} y$.

The following result can be easily shown:

LEMMA A.11
For every x in $\mathtt{W}$, $\mathtt{M}, x \Vdash \phi$ if and only if $\mathtt{M}', x \Vdash \phi$ for every $\phi \in \Phi_{\mathsf{DyLeR}}$.

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