A Versatile Stochastic Dissemination Model

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DOI
10.1007/s11009-023-10041-2

Publication date
2023

Document Version
Final published version

Published in
Methodology and Computing in Applied Probability

Citation for published version (APA):
Chan, K. M. D., & Mandjes, M. R. H. (2023). A Versatile Stochastic Dissemination Model. *Methodology and Computing in Applied Probability*, *25*(3), Article 69.
https://doi.org/10.1007/s11009-023-10041-2

Archive for History of Exact Sciences (2023) 77:501–535
https://doi.org/10.1007/s00407-023-00311-3

An early system A-type scheme for Saturn from Babylon

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Received: 20 March 2023 / Accepted: 24 March 2023 / Published online: 20 June 2023

Abstract

In this paper we publish three fragments of a cuneiform tablet that, when complete, contained the dates and zodiacal positions of Saturn's synodic phenomena for roughly 60 years. The text is unique in containing comparisons of computed data with observations. Through an analysis of the preserved data we propose that the dates and positions were computed by an otherwise unknown two-zone System A-type scheme and show that the computed data in the tablet can be dated to the fourth century BC. This early date and the comparisons with observations suggest that the text was produced during the period of active development of the planetary systems.

1 Introduction

Two types of schemes are used for computing the dates and positions in the zodiac of the synodic phenomena of the planets in Babylonian mathematical astronomy: System A, in which the synodic arc and synodic time between consecutive phenomena of the same kind depends upon the planet's position in the zodiac and varies according to a step function, and System B, in which the synodic arc and synodic time are given by linear zigzag functions. Both kinds of schemes are known for Saturn (Ossendrijver 2012: 106–109). System A is a two-zone step function with synodic arcs of 11;43,7,30° and 14;3,45° and zone boundaries at 10° Leo and 30° Aquarius. A variant, System A′,

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Communicated by Alexander Jones.

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which is only attested in one template text, uses the same synodic arcs but shifts the zone boundaries to 20° Leo and 10° Aquarius. Three System B-type schemes are known: Systems B, B$'$, and B$''$. The zigzag functions used in these three schemes have the same difference of 0;12°, but slightly different values for the maximum and minimum.

In Steele (2010), one of us published BM 42878 and BM 45807, two fragments containing lists of the synodic phenomena of Saturn computed according to a previously unattested System A-like scheme. Steele showed that the longitudes were computed using a two-zone scheme with synodic arcs equal to 11;20° and 13;50°. Unfortunately, too little data was preserved on BM 42878 and BM 45807 to allow the boundaries between the fast and slow zones to be identified or for the texts to be dated.

As discussed already by Steele (2010), the scheme does not strictly follow the mathematical rules of System A. In particular, the ratio of the two synodic arcs (13;50: 11;30 = 83:68) is a non-terminating sexagesimal fraction. As a consequence, the ratio between the subdivisions of the synodic arc in the two zones are not precisely equal either to each other or to that of the total synodic arc. More importantly, from a computational perspective, because the ratio of the two synodic arcs is not a terminating sexagesimal fraction, the normal procedure for computing positions which cross zone boundaries cannot be used precisely. In practice, therefore, some approximations must have been made when crossing the zone boundaries, which stands in contrast to the strict mathematical precision and consistency usually found in Babylonian mathematical astronomy. Note also that the subdivision of the synodic arc during Saturn's retrograde motion is symmetrical around acronychal rising. Most schemes for the subdivision of the synodic arc of an outer planet have a shorter time interval and smaller distance between morning station and acronychal rising than between acronychal rising and evening station, in line with what one would expect (Swerdlow 1999; Hollywood and Steele 2004). In assuming a symmetrical division, this Saturn scheme conflates acronychal rising with opposition.

In 2018, Steele identified two further fragments containing the same material. BM 45726, which joins BM 45807, and BM 46004. All three fragments, BM 42878, BM 45726+45807, and BM 46004, are almost certainly part of the same tablet. The new fragments allow for a full, if still provisional, reconstruction of the computational scheme. They also allow the text to be dated to the fourth century BC, right around the time when the System A and System B methods for computing planetary phenomena were actively being developed. But most importantly, some lines in the new fragments contain comparisons between the computed phenomena of Saturn and observations of those same phenomena. Explicit evidence for comparison between computation and observation in order to test computational methods is extremely rare in Babylonian sources, and, indeed, in ancient astronomy generally. We will return to the significance of the comparison of computed and observational data at the end of this paper.

2 The text

Three fragments contain data computed by the early Saturn scheme that is the subject of this study:

A: BM 42878 (81–7–1, 642)
B: BM 45726+45807 (81–7–6, 133+226)
C: BM 46004 (81–7–6, 448)

According to our reconstruction, all three fragments are from the same tablet. The tablet contained four columns on each of the obverse and reverse, with the tablet turning as expected about its horizontal axis. The tablet arrangement is that of a standard multi-column prose text, i.e., columns on the obverse are arranged from left to right whilst those on the reverse are arranged from right to left. Fragment A (BM 42878) preserves a small part towards the bottom of Obv. I. Fragment B (BM 45726+45807) preserves parts of Obv. II, III, and IV. Fragment C (BM 46004) preserves the bottom of Obv. II and III and the top of Rev. II, III, and IV, as well as an uninscribed part of the lower edge. Contrary to normal practice, the curved side of the tablet is the obverse and the flat side is the reverse.[1] Based upon our reconstruction, we estimate that, when complete, each column on the obverse contained about 45 lines and on the reverse about 36 lines.

According to our reconstruction, fragments B (BM 45726+45807) and C (BM 46004) should join or at least very nearly join at the bottom of Obv. III: we would expect that Obv. III B29′ = C1′. However, there is clearly no physical join between the two fragments, which means that there must be an additional line which we do not expect and therefore B30′ = C1′, or even that C1′ follows B30′, making two additional lines.[2]

There are two peculiar features of the physical layout of the tablet. The columns on the obverse are delineated merely by spacing, and sometimes signs from one column overlap into the next column. On the reverse, however, columns are separated by single vertical rulings.[3] Entries in Obv. I, II, and III, and Rev. II, III, and IV are given in one continuous run. However, in Obv. IV horizontal rulings separate the column into sections for individual years.

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[1] Although the vast majority of cuneiform tablets follow the rule that the flat side is the obverse and the curved side is the reverse, there are a number of exceptions (see, for example, ACT 4).

[2] It is of course possible that fragments B and C are not part of the same tablet but fragments of two duplicates. However, the script, layout, and general appearance of the fragments points strongly towards them being part of the same tablet.

[3] This difference in how column boundaries are indicated on the obverse and reverse holds for BM 46004, the only tablet to preserve both sides of the tablet. The lack of column rulings on the obverse of BM 45726+45807 provides additional support to the conclusion that BM 46004 and BM 45726+45807 are part of the same tablet.

The text presents a list of consecutive phenomena of Saturn in several columns in a prose list (not tabular) form. Each entry contains at a minimum the date and the longitude of Saturn when it exhibits a synodic phenomenon. The date is presented first and is given to a whole number of days. The longitude is preceded by the sign *ina* (‘at’) and is given to a precision of 0;10°. For the first and last visibilities, the longitude is followed by an integer number and the sign BE. The earliest entries for Saturn’s stations, i.e., those in Obv. I and II, simply give the computed date and position. Beginning in Obv. III, however, a report of the position of Saturn at the station relative to a Normal Star is inserted after the date and before the computed longitude. Entries for Saturn’s acronychal rising only give the computed date and longitude.

The names of the signs of the zodiac are given using the short forms (e.g., GÍR instead of GÍR.TAB for Scorpio) which are typical in texts of mathematical astronomy (Steele 2018). The writing ABSIN$_0$ (= KI) for Virgo instead of ABSIN is worth noting because this form appears frequently in texts from the fourth or early third century BC (e.g., Atypical Text C (BM 36301), Atypical Text H (MNB 1856), ACT 70 (BM 34934), ADART VII 1 (BM 65156), BM 36822+37022, BM 36599+36941, and BM 36737+47912), but only rarely after this, suggesting a fourth century BC date for the tablet. Similarly, UR instead of A for Leo points to a fourth century BC date. The scribe writes the numeral 9 using the three-wedge cursive form.

We edit the three fragments of this tablet below. Column numbers follow our restoration of the complete tablet. Line numbers are given separately for each fragment with, where known, an estimate of the missing number of lines between fragments indicated. The surfaces of all three fragments, especially BM 45726+45807 and the obverse of BM 46004, are badly abraded and much of the text is illegible. We encourage the reader to take very seriously the uncertain nature of the readings of many damaged signs, especially those where we append a superscript question mark ($^{?}$).

Obv. I

A1′ [… *ina*] ⌈8⌉,[30 ALLA ŠÚ …]

A2′ [… *ina*] 13 ALLA IGI […]

A3′ […] *ina* 21,20 ALLA UŠ […]

A4′ […] *ina* 17,40 ALLA ⌈E⌉ […]

A5′ […] ⌈*ina*⌉ 14 ALLA UŠ […]

A6′ [… *ina* 2]2,20 ALLA ŠÚ […]
A7′ [… *ina* 26,50] ALLA IGI 14[+x BE$^?$ …]
A8′ [… *ina* 5],⸢10$^?$⸣ UR UŠ […]
A9′ [… *ina* 1,30] UR E […]

Obv. II
B1′ […*ina*] ⸢6⸣ [ABSIN$_0$ UŠ …]
B2′ […] *ina* 12,50 ⸢ABSIN$_0$⸣ ŠÚ ⸢15$^?$ BE⸣
B3′ [… *ina* 1]6,30 ABSIN$_0$ IGI 15$^?$ BE
B4′ [… *ina*] 23,20 ABSIN$_0$ UŠ
B5′ […] *ina* 17$^?$,20 ABSIN$_0$ UŠ
B6′ [… *ina*] 24,10 ABSIN$_0$ ⸢ŠU 14$^?$ BE⸣
B7′ [… *ina* 2]⸢7⸣,50 ABSIN$_0$ IGI 15$^?$ BE
B8′ [… *ina* 4],40 RÍN UŠ
B9′ [… *ina*] 1,40 RÍN E
B10′ [… *ina*] ⸢2⸣8,40 ABSIN$_0$ UŠ
B11′ [… *ina*] ⸢5,30 RÍN ŠU 10+x BE⸣
B12′ [… *ina* 9,10 RÍN IGI … BE]
B13′ [… *ina* 16 RÍN UŠ]
B14′ [… *ina* 13 RÍN E]
B15′ [… *ina* 10 RÍN] ⸢UŠ⸣
B16′ […*ina* 16,50 RÍN] ⸢ŠÚ 13$^?$ BE⸣
B17′ [… *ina*] ⸢20,30 RÍN IGI$^?$⸣ 16 ⸢BE⸣
B18′ [… *ina*] 27,20 RÍN UŠ
B19′ […x+]10 ⸢*ina*⸣ 24,20 RÍN E
B20′ […] *ina* 21,20 RÍN ⸢UŠ⸣
B21′ […] *ina* 28,10 RÍN ŠÚ 13 BE?
B22′ [… *ina*] ⸢1,50 GÍR IGI 23$^?$ BE⸣
(7 lines missing)
C1′ [… x+]6 *ina* [14 GÍR UŠ …]
C2′ […x+]5 *ina* 20,50 ⸢GÍR⸣ [ŠU x x]
C3′ [… *ina* 2]4,30 GÍR ⸢IGI$^?$ x BE$^?$ ⸣
C4′ [… *ina*] 1,20 PA ⸢UŠ⸣
C5′ [… *ina*] ⸢2⸣8,20 ⸢GÍR⸣ [E]

Obv. III
B1′ […] ⸢x⸣ […]
B2′ [x] ⸢14$^?$ ⸣ [… UŠ]
B3′ x GAN 13 *ina* 10[+x …]
B4′ ⸢AB⸣ 18 *ina* 28,⸢30$^?$ ⸣ [PA IGI]
B5′ […] x x *ina* ⸢5$^?$⸣,[20 MÁŠ UŠ]
B6′ ⸢ŠU$^?$⸣ 10 *ina* ⸢2⸣,[20 MÁŠ E]
B7′ ⸢IZI 17⸣ [x x x]
B8′ x x x x x x
B9′ ZÍZ$^?$ x x x x
B10′ x x x x x

B11′ GU$_4$ x x […]
B12′ […] x x x x
B13′ […] MÁŠ ⌈UŠ⌉
B14′ ⌈x GAN$^{?}$⌉ [… 17],⌈30⌉ MÁŠ ŠÚ 13 ⌈BE$^{?}$⌉
B15′ x x [x *ina* 21,10] MÁŠ IGI x BE
B16′ […] MÚL IGI *šá* SUḪUR
B17′ [MÁŠ …] x x UŠ$^{?}$ *ina* ⌈29⌉ MÁŠ UŠ
B18′ ⌈ŠU 9$^{?}$⌉ [*ina*] ⌈x,50$^{?}$⌉ MÁŠ E
B19′ ⌈KIN$^{?}$ 1$^{?}$⌉6 x x x MÁŠ$^{?}$
B20′ 22$^{?}$ *ina*$^{?}$ x MÁŠ UŠ
B21′ 28$^{?}$ AB$^{?}$ 5$^{?}$ *ina* 30 MÁŠ ŠÚ 11 BE
B22′ ŠE ⌈5$^{?}$⌉ *ina* [4],⌈30⌉ GU$^{?}$ IGI$^{?}$ x BE
B23′ x x x x x x
B24′ [x] x x *šá* GU x
B25′ [x x] (blank) *ina*$^{?}$ 11$^{?}$,50 GU UŠ$^{?}$
B26′ […] GU E
B27′ […] x MÚL EGIR$^{?}$ *ša* SUḪUR MAŠ$^{?}$
B28′ [… 5],⌈30⌉ GU UŠ
B29′ […] ŠÚ 11 BE
B30′ […] x x x
(B30′ = C1′?; no physical join)
C1′ x […]
C2′ [x x] *ina* 18$^{?}$,1[0+x GU IGI …]
C3′ 30$^{?}$ GU$_4$ 7 2 1/2 K[ÙŠ …]
C4′ ⌈x *ana* NIM x⌉ […]

Obv. IV
B1′ x […]
B2′ x […]
B3′ x […]
B4′ BAR x […]
B5′ ⌈IZI$^{?}$⌉ x x […]
B6′ x x x […]
B7′ x x x […]
B8′ x x x […]

B9′ 2$^{?}$ x BAR 1 x [… ŠÚ]
B10′ GU$_4$ 8$^{?}$ *ina* x [… IGI]
B11′ IZI 27 […]
B12′ 6 KÙŠ x x […UŠ]
B13′ DU$_6$$^{?}$ 25$^{?}$ x [… E]
B14′ GAN 23 x […]
B15′ *ina* 12$^{?}$ [… UŠ]

B16′ 2 DIRI ŠE 27$^{?}$ ⌈*ina*⌉ […]
B17′ 3 GU$_4$ 3 *ina* 20+[x …]

B18′ IZI 23 x […]
B19′ *ša*$^?$ MÚL x […]
B20′ DU$_6^?$ 2⸢6$^?$⸣ […]
B21′ ⸢GAN 18$^?$⸣ […]
B22′ x […]
B23′ x […]

Rev. I
(lost)

Rev. II
C1 x x […]
C2 KIN 10+[x …]
C3 AB 2 […]
C4 *ár* MÚL ⸢x⸣ […]
C5 (blank)
C6 ⸢ŠE⸣ […]

Rev. III

C1 19 DU$_6$ 21 *ina* 15,40$^?$ GÍR ŠÚ ⸢x⸣ [BE]
C2 ⸢APIN$^?$⸣ 24$^?$ *ina* 19,10 GÍR IGI$^?$ x [BE]
C3 ŠE 8 5 KÙŠ ⸢*ina* IGI⸣ KIR$_4$ *šil* PA : *ár* MÚ[L$^?$]
C4 *ina* IGI MÚL x *šá* PA *ina* 26,10 GÍR
C5 UŠ
C6 20 GU$_4$ 16 *ina* ⸢23 GÍR E⸣
C7 ŠU$^?$ 17$^?$ ⸢KIR$_4$⸣ *šil* PA ⸢*ina* 20 GÍR UŠ⸣
C8 traces only

Rev. IV
C1 […] x BE
C2 [… I]GI 20 BE
C3 […x+]1 KÙŠ :
C4 […] *šá* SUḪUR
C5 [MÁŠ …] MÁŠ UŠ
C6 […] ⸢E⸣
C7 [… U]Š

Translation

Obv. I
A1′ [… at] ⸢8⸣,[30 Cancer last appearance …]
A2′ [… at] 13 Cancer first appearance […]
A3′ […] at 21,20 Cancer station […]
A4′ […] at 17,40 Cancer ⸢acronychal rising⸣ […]
A5′ […] ⸢at⸣ 14 Cancer station […]

A6′ [… at 2]2,20 Cancer last appearance […]
A7′ [… at 26,50] Cancer first appearance 14[+x BE$^{?}$ …]
A8′ [… at 5],⌈10$^{?}$⌉ Leo station […]
A9′ [… at 1,30] Leo station […]

Obv. II
B1′ [… at] ⌈6⌉ [Virgo station …]
B2′ […] at 12,50 ⌈Virgo⌉ last appearance ⌈15$^{?}$ BE⌉
B3′ [… at 1]6,30 Virgo first appearance 15$^{?}$ BE
B4′ [… at] 23,20 Virgo station
B5′ […] at 17$^{?}$,20 Virgo station
B6′ [… at] 24,10 Virgo ⌈last appearance 14$^{?}$ BE⌉
B7′ [… at 2]⌈7⌉,50 Virgo first appearance 15$^{?}$ BE
B8′ [… at 4],40 Libra station
B9′ [… at] 1,40 Libra acronychal rising
B10′ [… at] ⌈2⌉8,40 Virgo station
B11′ [… at] ⌈5,30 Libra last appearance 10+x BE⌉
B12′ [… at 9,10 Libra first appearance … BE]
B13′ [… at 16 Libra station]
B14′ [… at 13 Libra acronychal rising]
B15′ [… at 10 Libra] ⌈station⌉
B16′ [… at 16,50 Libra] ⌈station 13$^{?}$ BE⌉
B17′ [… at] ⌈20,30 Libra first appearance⌉ 16 ⌈BE⌉
B18′ [… at] 27,20 Libra station
B19′ […x+]10 ⌈at⌉ 24,20 Libra acronychal rising
B20′ […] at 21,20 Libra ⌈station⌉
B21′ […] at 28,10 Libra last appearance 13 BE$^{?}$
B22′ [… at] ⌈1,50 Scorpio first appearance 23$^{?}$ BE⌉
(7 lines missing)
C1′ [… x+]6 at [14 Scorpio station …]
C2′ […x+]5 at 20,50 ⌈Scorpio⌉ [last appearance x x]
C3′ [… at 2]4,30 Scorpio ⌈first appearance$^{?}$ x BE$^{?}$⌉
C4′ [… at] 1,20 Sagittarius ⌈station⌉
C5′ [… at] ⌈2⌉8,20 ⌈Scorpio⌉ [acronychal rising]

Obv. III
B1′ […] ⌈x⌉ […]
B2′ [x] ⌈14$^{?}$⌉ [… UŠ]
B3′ x Month IX 13 at 10[+x …]
B4′ ⌈Month X⌉ 18 at 28,⌈30$^{?}$⌉ [Sagittarius first appearance]
B5′ […] … at ⌈5$^{?}$⌉,[20 Capricorn station]
B6′ ⌈Month IV⌉ 10 at ⌈2⌉,[20 Capricorn acronychal rising]
B7′ ⌈Month V 17⌉ […]
B8′ …
B9′ Month XI …
B10′ …

B11′ Month II … […]
B12′ […] …
B13′ […] Capricorn ⸢station⸣
B14′ ⸢… Month IX$^{?}$⸣ [… 17],⸢30⸣ Capricorn last appearance 13 ⸢BE$^{?}$⸣
B15′ … [… at 21,10] Capricorn first appearance x BE
B16′ […] The Front Star of the Goat-[fish (γ Cap)
B17′ […] … station$^{?}$ at ⸢29⸣ Capricorn station
B18′ ⸢Month IV 9$^{?}$⸣ [at] ⸢x,50$^{?}$⸣ Capricorn acronychal rising
B19′ ⸢Month VI$^{?}$ 1$^{?}$⸣6 x x x [Goat]-fish$^{?}$
B20′ 22$^{?}$ at$^{?}$ x Capricorn station
B21′ 28$^{?}$ Month X$^{?}$ 5$^{?}$ at 30 Capricorn last appearance 11 BE
B22′ Month XII ⸢5$^{?}$⸣ at [4],⸢30⸣ Aquarius$^{?}$ first appearance$^{?}$ x BE
B23′ …
B24′ […] … of the Great One x
B25′ […] (blank) at$^{?}$ 11$^{?}$,50 Aquarius station$^{?}$
B26′ […] Aquarius acronychal rising
B27′ […] x The Rear$^{?}$ Star of the Goat-fish$^{?}$ (δ Cap)
B28′ [… 5],⸢30⸣ Aquarius station
B29′ […] last appearance 11 BE
B30′ […] …
(B30′ = C1′?; no physical join)
C1′ … […]
C2′ […] at 18$^{?}$,1[0+x Aquarius first appearance …]
C3′ 30$^{?}$ Month II 7 2 1/2 cu[bits …]
C4′ ⸢… to the east …⸣ […]

Obv. IV
B1′ … […]
B2′ … […]
B3′ … […]
B4′ Month I … […]
B5′ ⸢Month V$^{?}$⸣ … […]
B6′ … […]
B7′ … […]
B8′ … […]

B9′ 2$^{?}$ … Month I 1 … [… last appearance]
B10′ Month II 8$^{?}$ at … [… first appearance]
B11′ Month V 27 […]
B12′ 6 cubits … [… station]
B13′ Month VII$^{?}$ 25$^{?}$ … [… acronychal rising]
B14′ Month IX 23 … […]
B15′ at 12$^{?}$ [… station]

B16′ 2 Month XII$_{2}$ 27$^{?}$ ⸢at⸣ […]
B17′ 3 Month II 3 at 20+[x …]

B18′ Month V 23 x […]
B19′ of$^{?}$ The Star … […]
B20′ Month VII$^{?}$ 2⸢6$^{?}$⸣ […]
B21′ ⸢Month IX 18$^{?}$⸣ […]
B22′ … […]
B23′ … […]

Rev. I
(lost)

Rev. II
C1 … […]
C2 Month VI 10+[x …]
C3 Month X 2 […]
C4 behind the Star ⸢…⸣ […]
C5 (blank)
C6 ⸢Month XII⸣ […]

Rev. III

C1 19 Month VII 21 at 15,40$^{?}$ Scorpio last appearance ⸢…⸣ […]
C2 ⸢Month VIII$^{?}$⸣ 24$^{?}$ at 19,10 Scorpio first appearance$^{?}$ …
C3 Month XII 8 5 cubits ⸢in front of⸣ The Tip of Pabilsag's Arrow (θ Oph) : behind The S[tar$^{?}$]
C4 in front of the Star … of Pabilsag at 26,10 Scorpio
C5 station
C6 20 Month II 16 at ⸢23 Scorpio acronychal rising⸣
C7 Month IV$^{?}$ 17$^{?}$ ⸢The Tip⸣ Pabilsag's Arrow (θ Oph) ⸢at 20 Scorpio station⸣
C8 …

Rev. IV
C1 […] … BE
C2 [… fi]rst appearance 20 BE
C3 […x+]1 cubits :
C4 […] of the Goat-
C5 [fish …] Capricorn station
C6 […] ⸢acronychal rising⸣
C7 [… stat]ion

Critical apparatus

Obv. I A8′	Only the faintest traces of the 10 remain. The damage here is particularly unfortunate because this line when combined with Obv. I A2′ provides the only direct evidence for the value of the synodic arc in the fast zone
Obv. II B4′–5′	The scribe has omitted an entry for Saturn's acronychal rising, skipping straight from western station in B4′ to eastern station in B5′

Obv. III B2′	The wedges for 14 are preserved. There appear to be traces before the first wedge which could be either the remains of another winkelhaken, making the number 24, or they could just be damage
Obv. III B3′	Following the *ina* sign there appears to be a small vertical wedge, which is in turn followed by at least one, and perhaps several, winkelhaken. We are tempted to read *ina* 1,40, but this does not fit in with the positions given in the surrounding lines. We therefore assume that what appears to be a vertical wedge is simply damage and read *ina* 10 + [x]
Obv. III B15′	A few traces can be made out before the MÁŠ sign. The traces do not look particularly like the expected 21,10
Obv. III B19′	The day number could be either 16 or 26
Obv. III B28′	Only the final winkelhaken of the 30 remains
Obv. III B29′–C1′	From the content, we would expect that line B29′ would join C1′, but this is not physically possible. It is possible that B30′ and C1′ are the same line, but there is not a physical join and the placement of the fragments looks a little tight. Alternatively, C1′ may follow B30′, which would be quite possible from the physical fragments. We cannot explain, however, what would have been written in lines B30′ and C1′ in either case; we would not expect anything in these lines
Obv. III C2′	The number after *ina* is probably 18,20
Obv. IV B9′	The second sign looks somewhat like an A sign, but there seem to be some additional traces towards the bottom of the sign
Obv. IV B15′	The number following *ina* could be either 12 or 13
Obv. IV B20′	The tablet breaks off immediately following the two winkelhaken of the 20. Any number between 20 and 29 is possible
Rev. III C1′	We only see three wedges of the 40, but the spatial arrangement of those wedges is what we would expect for 40 not 30

3 Date

Several lines begin with what appear to be year numbers: Obv. III B21′ and C3′, Obv. IV B9′, B16′, and B17′, and Rev. III C1 and C6. Unfortunately, the reading of the year numbers in Obv. III are all uncertain and, as we will discuss below, the most likely readings do not fit the longitude scheme. Similarly, the year numbers in Obv. III and Rev. III do not fit the dates implied by the longitude scheme. We will return to this problem below.

The most secure year numbers seem to be those preserved in Obv. IV. Obv. IV B9′ and B16′ mention a year 2 and B17′ mentions a year 3. Furthermore, B16′ indicates that year 2 contained an intercalary month XII. Over the whole of the Late Babylonian period, an intercalary month XII in the second year of a king's reign is attested only for Nabopolassar (624/623 BC), Darius II (422/421 BC), Artaxerxes II (403/402 BC), and Artaxerxes III (357/356 BC). Although Obv. IV is badly damaged, most of the dates are preserved, as well as indications of which entries include reports of observations of positions relative to Normal Stars, which must therefore be the stations. A quick comparison of the dates of the phenomena with computed data for the second and third years of Nabopolassar, Darius II, Artaxerxes II and Artaxerxes III shows that only Artaxerxes III's reign is a possibility.

On the assumption that these lines concern Artaxerxes III, we can then fix the date of the longitude scheme for the whole tablet. It likely covered a 59-year period beginning in about year 16 of Artaxerxes II (389 BC) and ending in about year 1 of Alexander III (330 BC). Comparison of the reconstructed longitude data with modern computation shows excellent agreement (see Sect. 5 below), which confirms our dating.

Given the strength of the agreement between the longitude scheme and modern computation, and the fact that the implied date agrees with the year numbers in Obv. IV, we are confident in the dating of the calculated phenomena. As mentioned above, year numbers in Obv. III and Rev. III do not fit, however. In Obv. III B21′ and C3′ respectively we have what seem to be the year numbers $28^?$ and $30^?$; according to the longitude scheme, these should correspond to Artaxerxes II years 40 and 42. It seems that the scribe made a 12 year error somewhere in moving back from the preserved year number in Obv. IV to those in Obv. III[4]; the obvious place where such an error could be introduced is at the reign transition between Artaxerxes II and Artaxerxes III. In Rev. III C1 and C6, we have the year numbers 19 and 20 (unlike the numbers in Obv. III, the reading of these numbers is certain), but the longitude data is for Artaxerxes III years 17 and 18. Thus, between Obv. IV and Rev. III, a 2 year error (in the opposite direction to that between Obv. III and Obv. IV) has occurred. We can offer no explanation for these errors. It seems simply that the scribe was careless in keeping track of the years. While the presence of these errors is of some concern, we remain confident in our dating of the phenomena based upon the reconstructed longitude scheme.

The inclusion of observational data in this tablet means that it must have been compiled after the data of the last observation it contains. This would situate the composition of the tablet in the late fourth century BC. This date is in line with the use of UR and ABSIN_0 to write the names of the zodiacal signs Leo and Virgo and puts the tablet right in the time period where the various systems of mathematical astronomy were being actively developed.

4 The system A_0 scheme underlying the Saturn ephemeris

The three fragments BM 42878, BM 45726+45807 and BM 46004 are part of a tablet that originally contained a list of longitudes of Saturn at its consecutive synodic phenomena, probably from 389 to 330 BC covering one full 59-year period of Saturn.

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[4] If what we have read as $28^?$ and $30^?$ are in fact damaged 38 and 40 respectively, which cannot be ruled out but which seems unlikely based upon the preserved traces, then the error would reduce to 2 years.

Table 1 Synodic intervals and their ratios for the System A_0 model of Saturn

Intervals (Pushes)	Slow zone [°]	Fast zone [°]	Fast/Slow		Slow/Fast	
			Ratio	Sexagesimal	Ratio	Sexagesimal
S2 → LA	+6;50	+8;20	50/41	1;13,10,…	41/50	0;49,12
LA → FA	+3;40	+4;30	27/22	1;13,38,…	22/27	0;48,53,20
FA → S1	+6;50	+8;20	50/41	1;13,10,...	41/50	0;49,12
S1 → AR	−3;00	−3;40	11/9	1;13,20	9/11	0;49,05,…
AR → S2	−3;00	−3;40	11/9	1;13,20	9/11	0;49,05,…
Synodic Arc	11;20	13;50	83/68	1;13,14,…	68/83	0;49,09,…
(i)	(ii)	(iii)	(iv)		(v)	

The longitudes of Saturn appear to have been computed according to a System A model, that we will call System A_0 to distinguish it from the previously known Systems A and A′ (for a recent summary see Ossendrijver 2012, 106–108).

In the earlier paper based on fragments BM 42878 and BM 45807, Steele (2010) was able to derive the following properties of System A_0:

1. The variable motion of Saturn is approximated by a step function with two zones: a slow zone where the synodic arc equals 11;20° and a fast zone where the synodic arc equals 13;50°.
2. In both zones the synodic arc is split up in a number of different intervals (pushes[5]) when Saturn moves from one synodic phase to the next one. These intervals are listed in Table 1.

As we have discussed above a peculiar feature of the Babylonian Saturn ephemeris BM 42878+ is the fact that the longitudes are not presented in the typical way of later synodic tables, i.e., for each synodic phase separately, but instead in the order in which they are observed, from one synodic phase to the next one; this feature suggests that the data was computed from one synodic phase to the next. The intervals in Table 1 in principle allow us to do so, once we know the longitudes of the two transitions between the slow and the fast zones. As pointed out by Steele (2010) the longitudes preserved on fragments BM 42878 and BM 45807 are not sufficient to determine the zone boundaries but, as we will show below, including the two additional fragments BM 45726 and BM 46004, the tablet now contains just enough information to reconstruct the full System A_0 scheme.

Our reconstruction of the longitudes of Saturn preserved on BM 42878+ is shown in Table 2 where we list computed positions of Saturn at 205 successive synodic phases while it moves almost one and a half times through the zodiac from Cancer to Cancer to Sagittarius. Slow-to-fast and fast-to-slow boundary crossings are indicated by dashed lines in Table 2. The lengths of the zones and the longitudes of the zone boundaries in this reconstruction were determined as follows.

We first note that the interval from 6;00° Virgo (line 28 in Table 2) to 21;10° Capricorn (line 85) lies in the slow zone and that the interval from 30;00° Capricorn (line 89) to 18;20° Aquarius (line 95) lies in the fast zone because all preserved

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[5] For the term "pushes" see Ossendrijver (2012, 63).

Table 2 Reconstruction of the longitudes of Saturn at its synodic phases preserved on BM 42878+

nr	Syn	Longitude			Preserved Text		
1	S1	7	30	Cnc			
2	AR	3	50	Cnc			
3	S2	0	10	Cnc			
4	LA	8	30	Cnc	˹8˺	[30]	Cnc
5	FA	13	0	Cnc	13		Cnc
6	S1	21	20	Cnc	21	20	Cnc
7	AR	17	40	Cnc	17	40	Cnc
8	S2	14	0	Cnc	14		Cnc
9	LA	22	20	Cnc	2]2	20	Cnc
10	FA	26	50	Cnc	[26	50]	Cnc
11	S1	5	10	Leo	5]	10	Leo
12	AR	1	30	Leo	1	30]	Leo
13	S2	27	50	Cnc			
14	LA	6	10	Leo			
15	FA	10	40	Leo			
16	S1	19	0	Leo			
17	AR	15	20	Leo			
18	S2	11	40	Leo			
19	LA	20	0	Leo			
20	FA	23	50	Leo			
21	S1	0	40	Vir			
22	AR	27	40	Leo			
23	S2	24	40	Leo			
24	LA	1	30	Vir			
25	FA	5	10	Vir			
26	S1	12	0	Vir			
27	AR	9	0	Vir			
28	S2	6	0	Vir	˹6˺		Vir
29	LA	12	50	Vir	12	50	Vir
30	FA	16	30	Vir	[1]6	30	Vir
31	S1	23	20	Vir	23	20	Vir
32	AR	20	20	Vir			
33	S2	17	20	Vir	17?	20	Vir
34	LA	24	10	Vir	24	10	Vir
35	FA	27	50	Vir	[2]˹7˺	50	Vir
36	S1	4	40	Lib	[4]	40	Lib
37	AR	1	40	Lib	1	40	Lib
38	S2	28	40	Vir	˹2˺8	40	Vir
39	LA	5	30	Lib	5	30	Lib
40	FA	9	10	Lib	9	10	Lib
41	S1	16	0	Lib	[16]		Lib

(i)

nr	Syn	Longitude			Preserved Text		
42	AR	13	0	Lib	13		Lib
43	S2	10	0	Lib	10		Lib
44	LA	16	50	Lib	16	50	Lib
45	FA	20	30	Lib	˹20˺	˹30˺	Lib
46	S1	27	20	Lib	27	20	Lib
47	AR	24	20	Lib	24	20	Lib
48	S2	21	20	Lib	21	20	Lib
49	LA	28	10	Lib	28	10	Lib
50	FA	1	50	Sco	˹1˺	˹50˺	Sco
51	S1	8	40	Sco			
52	AR	5	40	Sco			
53	S2	2	40	Sco			
54	LA	9	30	Sco			
55	FA	13	10	Sco			
56	S1	20	0	Sco			
57	AR	17	0	Sco			
58	S2	14	0	Sco	14		Sco
59	LA	20	50	Sco	20	50	Sco
60	FA	24	30	Sco	[2]4	30	Sco
61	S1	1	20	Sag	1	20	Sag
62	AR	28	20	Sco	˹2˺8	20	Sco
63	S2	25	20	Sco			
64	LA	2	10	Sag			
65	FA	5	50	Sag			
66	S1	12	40	Sag			
67	AR	9	40	Sag			
68	S2	6	40	Sag			
69	LA	13	30	Sag			
70	FA	17	10	Sag			
71	S1	24	0	Sag			
72	AR	21	0	Sag			
73	S2	18	0	Sag			
74	LA	24	50	Sag			
75	FA	28	30	Sag	28	˹30?˺	Sag
76	S1	5	20	Cap	˹5?˺	[20]	Cap
77	AR	2	20	Cap	˹2˺	[20]	Cap
78	S2	29	20	Sag			
79	LA	6	10	Cap			
80	FA	9	50	Cap			
81	S1	16	40	Cap			
82	AR	13	40	Cap			

(ii)

nr	Syn	Longitude			Preserved Text		
83	S2	10	40	Cap	x		Cap
84	LA	17	30	Cap	[17]	˹30˺	Cap
85	FA	21	10	Cap	[21]	[10]	Cap
86	S1	29	0	Cap	˹29?˺		Cap
87	AR	25	20	Cap	˹x˺	˹50?˺	Cap
88	S2	22	0	Cap	22?		
89	LA	0	0	Aqr	30		Cap
90	FA	4	30	Aqr	[4]	˹30˺	Aqr
91	S1	12	50	Aqr	11?	50	Aqr
92	AR	9	10	Aqr			
93	S2	5	30	Aqr	[5]	30?	Aqr
94	LA	13	50	Aqr			
95	FA	18	20	Aqr	18?	2[0]	Aqr
96	S1	26	40	Aqr			
97	AR	23	0	Aqr			
98	S2	19	20	Aqr			
99	LA	27	40	Aqr			
100	FA	2	10	Psc			
101	S1	10	30	Psc			
102	AR	6	50	Psc			
103	S2	3	10	Psc			
104	LA	11	30	Psc			
105	FA	16	0	Psc			
106	S1	24	20	Psc			
107	AR	20	40	Psc			
108	S2	17	0	Psc			
109	LA	25	20	Psc			
110	FA	29	50	Psc			
111	S1	8	10	Ari			
112	AR	4	30	Ari			
113	S2	0	50	Ari			
114	LA	9	10	Ari			
115	FA	13	40	Ari			
116	S1	22	0	Ari			
117	AR	18	20	Ari			
118	S2	14	40	Ari			
119	LA	23	0	Ari			
120	FA	27	30	Ari			
121	S1	5	50	Tau			
122	AR	2	10	Tau			
123	S2	28	30	Ari			

(iii)

nr	Syn	Longitude			Preserved Text		
124	LA	6	50	Tau			
125	FA	11	20	Tau			
126	S1	19	40	Tau			
127	AR	16	0	Tau			
128	S2	12	20	Tau	12	[20]	
129	LA	20	40	Tau			
130	FA	25	10	Tau	2[5	10]	
131	S1	3	30	Gem			
132	AR	29	50	Tau			
133	S2	26	10	Tau			
134	LA	4	30	Gem			
135	FA	9	0	Gem			
136	S1	17	20	Gem			
137	AR	13	40	Gem			
138	S2	10	0	Gem			
139	LA	18	20	Gem			
140	FA	22	50	Gem			
141	S1	1	10	Cnc			
142	AR	27	30	Gem			
143	S2	23	50	Gem			
144	LA	2	10	Cnc			
145	FA	6	40	Cnc			
146	S1	15	0	Cnc			
147	AR	11	20	Cnc			
148	S2	7	40	Cnc			
149	LA	16	0	Cnc			
150	FA	20	30	Cnc			
151	S1	28	50	Cnc			
152	AR	25	10	Cnc			
153	S2	21	30	Cnc			
154	LA	29	50	Cnc			
155	FA	4	20	Leo			
156	S1	12	40	Leo			
157	AR	9	0	Leo			
158	S2	5	20	Leo			
159	LA	13	40	Leo			
160	FA	18	10	Leo			
161	S1	25	30	Leo			
162	AR	22	30	Leo			
163	S2	19	20	Leo			
164	LA	26	20	Leo			

(iv)

nr	Syn	Longitude			Preserved Text		
165	FA	0	0	Vir			
166	S1	6	50	Vir			
167	AR	3	50	Vir			
168	S2	0	50	Vir			
169	LA	7	40	Vir			
170	FA	11	20	Vir			
171	S1	18	10	Vir			
172	AR	15	10	Vir			
173	S2	12	10	Vir			
174	LA	19	0	Vir			
175	FA	22	40	Vir			
176	S1	29	30	Vir			
177	AR	26	30	Vir			
178	S2	23	30	Vir			
179	LA	0	20	Lib			
180	FA	4	0	Lib			
181	S1	10	50	Lib			
182	AR	7	50	Lib			
183	S2	4	50	Lib			
184	LA	11	40	Lib			
185	FA	15	20	Lib			
186	S1	22	10	Lib			
187	AR	19	10	Lib			
188	S2	16	10	Lib			
189	LA	23	0	Lib			
190	FA	26	40	Lib			
191	S1	3	30	Sco			
192	AR	0	30	Sco			
193	S2	27	30	Lib			
194	LA	4	20	Sco			
195	FA	8	0	Sco			
196	S1	14	50	Sco			
197	AR	11	50	Sco			
198	S2	8	50	Sco			
199	LA	15	40	Sco	15	40?	Sco
200	FA	19	20	Sco	19	10?	Sco
201	S1	26	10	Sco	26	10	Sco
202	AR	23	10	Sco	˹23˺		Sco
203	S2	20	10	Sco	20		Sco
204	LA	27	0	Sco			
205	FA	0	40	Sag			

(v)

longitudes in these intervals can be reconstructed by applying the pushes listed for the slow or the fast zones in Table 1. This implies that the boundary between the slow and the fast zone must lie somewhere between 21;10° and 30;00° Capricorn.

Longitudes of Saturn when it passes zone boundaries, while moving from one synodic phase to the next one, can be calculated by applying the usual interpolation algorithm for Babylonian System A step functions (see e.g. Ossendrijver 2012, 48):

$$\lambda_{i+1} = (\lambda_i + \Delta\lambda_i - \lambda_b) \cdot r + \lambda_b, \tag{1}$$

where λ_i is the initial longitude of Saturn at synodic phase i, $\Delta\lambda_i$ is the interval from synodic phase (i) to (i + 1) in the zone in which λ_i is located, λ_b is the longitude of the zone boundary crossed, r is the interpolation factor and λ_{i+1} is the longitude of Saturn at the synodic phase (i + 1) located in the next zone.

To be able to apply this algorithm we must know the value of the interpolation factor r. In the traditional Babylonian System A theory this interpolation factor is equal to the ratio of the step function amplitudes (synodic arcs), usually chosen such that it results in simple ratios of "nice" numbers like 2/3, 3/4, 4/5, 5/6 for computational convenience.[6] In the System A_0 model of Saturn the situation is different and numerically more complicated because the synodic arc is split up in different intervals and the ratios of these intervals are slightly different for each set of intervals and not equal to simple ratios of "nice" numbers. In fact, the numbers in columns (iv) and (v) of Table 1 show that the adopted ratios are virtually identical (within less than 1%) for all intervals and that only three correspond to terminating sexagesimal numbers: 1;13,20 for the fast/slow zone transition and 0;48,53,20 and 0;49,12 for the slow/fast transition. It seems plausible to assume that 0;49,12 and 1;13,20 were the values adopted for the interpolation factors r used to compute the longitudes in our text.

Using these values of the interpolation factors in Eq. (1) we then find from the preserved longitudes of Saturn in lines 85–89 of Table 2 three values for the longitude of the zone boundary λ_b of 23;30° Capricorn (lines 85–86), 23;31° Capricorn (lines 87–88) and 23;30° Capricorn (lines 88 to 89). Since the small difference of 0;01° between these values can be attributed to the fact that all preserved longitudes on BM 42878+ are rounded off to an accuracy of 0;10° we find that the transition of the slow to the fast zone occurs at 23;30° Capricorn.

We next turn to the transition from the fast to the slow zone. From the data in Table 2 we find that this transition must be located somewhere between 5;10° Leo (line 11 in Table 2) and 6;00° Virgo (line 28). The exact value can be found by numerically experimenting with different values of the fast to slow boundary longitude. This trial-and-error approach leads in a few steps to a boundary value of 20;30° Leo, resulting in fast and slow zone lengths of 207° and 153°. We further find that going from the fast to the slow zone the boundary value is crossed once between lines 19 and 20 and three times between lines 160 and 161, lines 162 and 163 and lines 163 to 164 in Table 2. While the boundary values of the slow and the fast zone are not "nice" integer values as they are in the usual Babylonian System A models of the planets, we note that the amount of computational effort required to generate the longitudes of Saturn on BM

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[6] These ratios are attested in the System A schemes of the outer planets (see e.g. de Jong 2019a, Table 2).

Table 3 Babylonian system A parameters for Saturn

System	Period [years]	Synodic events	Orbital rotations	mean Δt [tithis]	mean $\Delta\lambda$ [°]	c	Zone	w_s, w_f [°]	r_{sf}, r_{fs}	λ_{sf}, λ_{fs} [°]	z_s, z_f [°]
A	265	256	9	12 mon + 24;06,43,…	12;39,22,30	11;27,20	slow	11;43,07,30	5/6	130	200
							fast	14;03,45	6/5	330	160
A'	6529	6304	225	12 mon + 24;18,38,…	12;50,56,…	11;27,42	slow	11;43,07,30	5/6	140	170
							fast	14;03,45	6/5	310	190
A_0	4891	4725	166	12 mon + 24;06,11,…	12;38,51,…	11;27,20	slow	11;20	68/83	140;30	153
							fast	13;50	83/68	293;30	207
(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)	(ix)	(x)	(xi)	(xii)

42878+ is minimal. Assuming that the tablet originally covered one 59-year period of Saturn the computation of the full run of data involves only about ten boundary crossings.

Using the boundary values of 20;30° Leo and 23;30° Capricorn between the slow and fast zones of the A_0 step function and the interpolation factors 0;49,12 and 1;13,20 discussed above, we computed the tabulated values of the longitudes in the text of BM 42878+ as shown in Table 2. This computation requires minimal arithmetic effort because it involves simple adding and subtraction of intervals and interpolation at only seven boundary crossings. Rounding errors which occur at the boundary crossings go both ways so that they do not accumulate but will statistically average out.

A comparison of the reconstructed longitudes of Saturn with those preserved on BM 42878+ shows overall excellent agreement. The differences in lines 87 and 91 of Table 2 can be attributed to uncertain readings and the discrepancies in lines 200 and 203 are probably due to rounding errors generated in the computation of the longitudes at the transition between the fast and the slow zones in lines 160–164 since they differ by 0;10° from the reconstructed values. All together it appears that the computation of the longitudes of Saturn's phenomena was carried out by a quite competent Babylonian scholar.

In Table 3 we list the parameters characterizing the System A_0 step function together with those of the previously known Systems A and A′ of Saturn. The parameters of the canonical system A of Saturn are derived from a period relation which has a clear relation to astronomical reality.[7] In 265 years Saturn experiences 256 synodic events while it completes 9 passages of Normal Stars in the sky (orbits around the Sun). This set of parameters results in a mean synodic arc of $\Delta\lambda = 9 \times 360°/256 = 12;39,22,30°$ (exactly). If system A_0 is similarly formulated in terms of a period relation the parameters turn out to be unrealistically large (see Table 3): in 4891 years Saturn experiences 4725 synodic events while it completes 166 passages in the sky resulting in a mean synodic arc of $\Delta\lambda = 166 \times 360°/4725 = 12;38,51,\ldots°$. Satisfying a period relation is equivalent to ensuring that the parameters reproduce the correct average synodic arc so that the model does not derail over long time intervals. However, this can also – and even more simply – be done by making sure that the combination of amplitudes and zone lengths of the step function reproduces the mean synodic arc

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[7] For a recent study of Babylonian planetary theory see de Jong (2019a,b; 2021).

derived from an observed period relation. The mean synodic arc of system A_0 is very close to the value of the canonical system A (see Table 3) so that it is quite possible that the scholar who constructed system A_0 started out with an accurate value of the mean synodic arc, chose two synodic arcs (one for each zone) based on a direct comparison with observations of Saturn near Normal Stars at its stations (see de Jong 2019a) and experimented with different zone lengths to find a combination that reproduced the mean synodic arc.[8]

Notice that in system A_0 the slow arc is reduced in size by almost 50° compared to system A (and the fast arc similarly enlarged), and that the symmetry axes of both systems differ by 13° (37°–217° in System A_0 compared to 50°–230° in System A).

System A_0 differs from most other system A models of the planets in the choice of the amplitudes of its step function with the rather awkward ratio of 68/83. While in practice computation with this ratio may be avoided by using the somewhat more user friendly values of 41/50 and 9/11, it is clear that the author of System A_0 gave higher priority to selecting "nice" sexagesimal numbers for the amplitudes of the step function (the synodic arcs) and the pushes than to "nice" numbers for the amplitude ratio which would have simplified the interpolation at the crossing of zone boundaries in the computation of the ephemeris. This is actually a quite sensible policy because, as shown above, the number of zone boundary crossings in computing an ephemeris of Saturn is quite limited.

While we can understand the choices of the numerical values of the amplitudes and of the zone lengths of the System A_0 step function, the reason for choosing non-integer values of 20;30° Leo and 23;30° Capricorn for the zone boundaries is unexpected because in all but one of the presently known System A-type models of the planets the longitudes of the zone boundaries are integer values (Neugebauer 1975, 423). Moreover, since the accuracy of the System A ephemeris of Saturn has been shown to be quite insensitive to shifting the position of the zones by ± 10° in the zodiac (de Jong 2019a, 30–32), the choice of non-integer values for the zone boundaries in the System A_0 step function is puzzling. In an attempt to come up with an explanation we first note that the choice of non-integer values for the zone boundaries in System A_0 does not affect the computational effort of calculating the longitude date on BM 42878+ because the total number of boundary crossings is limited to about ten. We further note that, once a run of longitudes from a System A scheme has been computed, it may be shifted in longitude by any number of (fractional) degrees as long as the computed longitudes and the zone boundaries are shifted by the same amount. With this in mind we suggest that the choice of non-integer boundary values may have been driven by shifting a previously computed run by a small amount to anchor it to a specific observation of Saturn at one of its stations. This procedure is known from several Babylonian planetary tables that appear to be anchored to a specific observation of the planet at one of its stations when it happened to be particularly close to one of the Normal Stars so that its position in the Babylonian zodiac could be accurately determined (de Jong 2019a,b; 2021).

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[8] In fact, it is straightforward to show that the choice of 153° for the slow zone and 207° for the fast zone is the pair of zone lengths with integer values that gets closest to producing a mean synodic arc of 12;39°.

System A′, known from tablet BM 78080 (Aaboe and Sachs 1966, Text C; for the parameters see Table 3) is closely related to System A because it employs exactly the same values of the amplitudes (synodic arcs) in the slow and the fast zones but uses different zone lengths and boundaries. System A′ results in a very inaccurate ephemeris of Saturn because the value of the mean synodic arc is about 11 arcminutes too large. Since Saturn experiences roughly one synodic event per year this implies a runaway of the computed longitudes of some 5° every thirty years. Aaboe and Sachs suggest that the scribe may have made a mistake by accidentally taking the beginning of the slow zone at 20° Leo instead of 20° Cancer.

However, it may not be accidental but a deliberate choice to put the value of the beginning of the slow zone in model A′ at 20° Leo, almost identical to that in model A_0. In that case we may consider model A′ as a failed attempt to model the synodic phases of Saturn using parameters which are a mixture of those in the early model A_0 and the final model A. This could be understood if the Babylonian scholar(s) went through a phase (sometime during the fourth century BC) where they were experimenting with different approaches to model Saturn's motion, going from models based on selecting values (with "nice" numbers) for the amplitudes (model A_0), to the final system A models in which the emphasis was on choosing numerically convenient amplitude ratios.[9] Such a scenario is consistent with the usage of the logogram ABSIN_0 for Virgo in both BM 42878+ and in BM 78080, a writing habit known to have been *en vogue* during the fourth century BC.

5 Reconstruction of tablet BM 42878+

In Sect. 3 we argued that the regnal years preserved in Obv. IV indicated that the data were computed for dates during the fourth century BC and that it may have covered one full 59-year period of Saturn. Having determined the System A_0 parameters underlying the computation of the ephemeris in Sect. 4 we now attempt to reconstruct the layout of the tablet and to place the different fragments in the reconstructed tablet. The results are shown in Tables 4 and 5 where we list dates in the Julian calendar and zodiacal longitudes of successive synodic phases of Saturn distributed over 4 columns on the obverse (from left to right) and over 4 columns on the reverse (from right to left) of tablet BM 42878+. The successive synodic phases of Saturn are indicated by the acronyms: S2 for second station, LA for last appearance in the west, FA for first appearance in the east, S1 for first station and AR for acronychal rising. The longitudes of Saturn at each of its synodic phases are longitudes in the fixed Babylonian zodiac so that they can be directly compared to the longitudes preserved on the three fragments A, B and C of the tablet shown in the shaded areas.

The dates and longitudes in Tables 4 and 5 of Saturn at its first and last appearance and at its stations are taken from the database of synthetic observations of Saturn during the fourth century BC generated by de Jong (2019a). Dates and longitudes for Saturn at its acronychal rising during the fourth century BC were newly computed

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[9] This approach is also known from the early Mercury text BM 36551+ (Aaboe et al. 1991, Text M) which dates from around 400 BC (see de Jong 2021).

Table 4 Reconstruction of BM 42878+ and the placement of fragments based on a comparison of the longitudes in the text with synthetic observations of Saturn–Obverse

Syn	Julian date			λ_{Bab}	Line #	Degrees		Sign	$\delta\lambda$
S2	30	11	-388	2,4					
LA	14	3	-387	10,6					
FA	29	4	-387	16,5					
S1	2	8	-387	23,4					
AR	9	10	-387	19,8					
S2	14	12	-387	16,4					
LA	29	3	-386	24,8					
FA	13	5	-386	30,6					
S1	17	8	-386	37,7					
AR	23	10	-386	34,2					
S2	28	12	-386	30,7					
LA	13	4	-385	39,2					
FA	26	5	-385	44,7					
S1	1	9	-385	52,1					
AR	7	11	-385	48,6					
S2	12	1	-384	45,1					
LA	27	4	-384	53,7					
FA	7	6	-384	58,9					
S1	14	9	-384	66,6					
AR	20	11	-384	63,1					
S2	26	1	-383	59,6					
LA	12	5	-383	68,1					
FA	20	6	-383	73,1					
S1	29	9	-383	81,0					
AR	5	12	-383	77,4					
S2	9	2	-382	74,0					
LA	27	5	-382	82,4					
FA	3	7	-382	87,1					
S1	13	10	-382	95,2					
AR	19	12	-382	91,7					
S2	24	2	-381	88,2					
LA	10	6	-381	96,5	A1'	8	30	Cnc	2,0
FA	17	7	-381	101,1	A2'	13		Cnc	1,9
S1	27	10	-381	109,1	A3'	21	20	Cnc	2,2
AR	2	1	-380	105,6	A4'	17	40	Cnc	2,0
S2	10	3	-380	102,2	A5'	14		Cnc	1,8
LA	23	6	-380	110,2	A6'	22	20	Cnc	2,1
FA	29	7	-380	114,7	A7'	26	50	Cnc	2,1
S1	9	11	-380	122,7	A8'	5	10	Leo	2,4
AR	14	1	-379	119,3	A9'	1	30	Leo	2,2
S2	24	3	-379	115,8					
LA	5	7	-379	123,5					
FA	12	8	-379	128,2					
S1	22	11	-379	136,0					
AR	28	1	-378	132,6					

Obverse I

Syn	Julian date			λ_{Bab}	Line #	Degrees		Sign	$\delta\lambda$
S2	7	4	-378	129,1					
LA	18	7	-378	136,5					
FA	26	8	-378	141,3					
S1	5	12	-378	148,9					
AR	10	2	-377	145,5					
S2	21	4	-377	142,1					
LA	29	7	-377	149,0					
FA	8	9	-377	153,9					
S1	17	12	-377	161,4					
AR	23	2	-376	158,1					
S2	3	5	-376	154,6	B1'	6		Vir	1,4
LA	10	8	-376	161,5	B2'	12	50	Vir	1,4
FA	19	9	-376	166,2	B3'	16	30	Vir	0,3
S1	29	12	-376	173,6	B4'	23	20	Vir	-0,3
AR	7	3	-375	170,2					
S2	16	5	-375	166,8	B5'	17	20	Vir	0,5
LA	23	8	-375	173,6	B6'	24	10	Vir	0,6
FA	1	10	-375	178,2	B7'	27	50	Vir	-0,3
S1	9	1	-374	185,4	B8'	4	40	Lib	-0,8
AR	19	3	-374	182,1	B9'	1	40	Lib	-0,5
S2	29	5	-374	178,7	B10'	28	40	Vir	-0,1
LA	4	9	-374	185,3	B11'	5	30	Lib	0,2
FA	13	10	-374	189,9	B12'	9	10	Lib	-0,7
S1	21	1	-373	197,0	B13'	16		Lib	-1,0
AR	31	3	-373	193,8	B14'	13		Lib	-0,8
S2	11	6	-373	190,4	B15'	10		Lib	-0,4
LA	16	9	-373	196,9	B16'	16	50	Lib	0,0
FA	24	10	-373	201,2	B17'	20	30	Lib	-0,7
S1	2	2	-372	208,4	B18'	27	20	Lib	-1,1
AR	11	4	-372	205,1	B19'	24	20	Lib	-0,8
S2	22	6	-372	201,8	B20'	21	20	Lib	-0,4
LA	27	9	-372	208,2	B21'	28	10	Lib	-0,1
FA	3	11	-372	212,5	B22'	1	50	Sco	-0,6
S1	12	2	-371	219,6					
AR	23	4	-371	216,4					
S2	4	7	-371	213,0					
LA	9	10	-371	219,5					
FA	15	11	-371	223,7					
S1	24	2	-370	230,8					
AR	5	5	-370	227,5					
S2	16	7	-370	224,2	C1'	14		Sco	-0,2
LA	21	10	-370	230,7	C2'	20	50	Sco	0,2
FA	26	11	-370	234,8	C3'	24	30	Sco	-0,3
S1	7	3	-369	241,9	C4'	1	20	Sag	-0,5
AR	17	5	-369	238,6	C5'	28	20	Sco	-0,2

Obverse II

Syn	Julian date			λ_{Bab}	Line #	Degrees		Sign	$\delta\lambda$
S2	27	7	-369	235,3					
LA	2	11	-369	241,8					
FA	8	12	-369	245,9					
S1	18	3	-368	253,0					
AR	28	5	-368	249,7					
S2	7	8	-368	246,4					
LA	12	11	-368	252,9					
FA	18	12	-368	257,0					
S1	30	3	-367	264,1					
AR	9	6	-367	260,8	B1'				
S2	19	8	-367	257,5	B2'				
LA	24	11	-367	264,2	B3'				
FA	31	12	-367	268,4	B4'	28	30	Sag	0,1
S1	11	4	-366	275,5	B5'	5	20	Cap	-0,1
AR	21	6	-366	272,2	B6'	2	20	Cap	0,2
S2	30	8	-366	268,8	B7'				
LA	6	12	-366	275,6	B8'				
FA	13	1	-365	280,0	B9'				
S1	23	4	-365	287,0	B10'				
AR	3	7	-365	283,7	B11'				
S2	11	9	-365	280,3	B12'				
					B13'			Cap	
LA	18	12	-365	287,2	B14'	17	30	Cap	0,3
FA	27	1	-364	291,9	B15'	21	10	Cap	-0,7
S1	5	5	-364	298,7	B16'	29		Cap	0,3
					B17'				
AR	15	7	-364	295,4	B18'				
S2	22	9	-364	292,1	B19'	22		Cap	-0,1
					B20'				
LA	30	12	-364	299,1	B21'	30		Cap	0,9
FA	10	2	-363	304,1	B22'	4	30	Aqr	0,4
S1	18	5	-363	310,8	B23'	12	50	Aqr	2,1
					B24'				
					B25'				
AR	28	7	-363	307,4	B26'			Aqr	
S2	4	10	-363	304,1	B27'	5	30	Aqr	1,4
LA	11	1	-362	311,2	B28'				
					B29'				
					B30'				
					C1'				
FA	26	2	-362	316,7	C2'	18	20	Aqr	1,6
S1	31	5	-362	323,2	C3'				
AR	10	8	-362	319,7	C4'				

Obverse III

Syn	Julian date			λ_{Bab}	Line #	Degrees		Sign	$\delta\lambda$
S2	17	10	-362	316,4					
LA	24	1	-361	323,7					
FA	14	3	-361	329,7					
S1	14	6	-361	335,9					
AR	23	8	-361	332,5					
S2	29	10	-361	329,1					
LA	7	2	-360	336,7					
FA	27	3	-360	342,8					
S1	27	6	-360	349,1					
AR	5	9	-360	345,6					
S2	11	11	-360	342,2					
LA	20	2	-359	350,0					
FA	10	4	-359	356,1					
S1	11	7	-359	2,6					
AR	18	9	-359	359,2					
S2	24	11	-359	355,7					
LA	7	3	-358	3,8					
FA	23	4	-358	9,7					
S1	26	7	-358	16,5					
AR	2	10	-358	13,0	B1'				
S2	8	12	-358	9,6	B2'				
LA	22	3	-357	17,9	B3'				
FA	7	5	-357	23,7	B4'				
S1	10	8	-357	30,7	B5'				
					B6'				
AR	17	10	-357	27,1	B7'				
S2	22	12	-357	23,7	B8'				
LA	5	4	-356	32,1	B9'				
FA	19	5	-356	37,8	B10'				
S1	24	8	-356	45,0	B11'				
					B12'				
AR	30	10	-356	41,5	B13'				
S2	4	1	-355	38,0	B14'	12	20		4,3
					B15'				
LA	21	4	-355	46,7	B16'				
FA	1	6	-355	51,9	B17'	25	10		3,2
S1	8	9	-355	59,5	B18'				
					B19'				
AR	14	11	-355	55,9	B20'				
S2	19	1	-354	52,5	B21'				
LA	6	5	-354	61,1	B22'				
FA	14	6	-354	66,1	B23'				
S1	22	9	-354	73,9					
AR	28	11	-354	70,4					
S2	3	2	-353	66,9					

Obverse IV

Table 5 Reconstruction of BM 42878+ and the placement of fragments based on a comparison of the longitudes in the text with synthetic observations of Saturn–Reverse

Syn	Julian date			λ_{Bab}	Line #	Degrees		Sign	δλ
LA	12	12	-336	281,7	C1				
FA	20	1	-335	286,3	C2				
S1	30	4	-335	293,2	C3			Cap	
					C4				
					C5				
AR	10	7	-335	289,8	C6				
S2	17	9	-335	286,5	C7				
LA	25	12	-335	293,5					
FA	3	2	-334	298,3					
S1	12	5	-334	305,1					
AR	22	7	-334	301,7					
S2	29	9	-334	298,4					
LA	6	1	-333	305,5					
FA	19	2	-333	310,8					
S1	26	5	-333	317,3					
AR	4	8	-333	313,9					
S2	11	10	-333	310,6					
LA	19	1	-332	317,9					
FA	6	3	-332	323,6					
S1	7	6	-332	329,9					
AR	16	8	-332	326,5					
S2	23	10	-332	323,1					
LA	31	1	-331	330,6					
FA	21	3	-331	336,6					
S1	21	6	-331	342,9					
AR	30	8	-331	339,4					
S2	5	11	-331	336,1					
LA	14	2	-330	343,8					
FA	4	4	-330	349,9					
S1	5	7	-330	356,3					
AR	12	9	-330	352,8					
S2	18	11	-330	349,4					
LA	1	3	-329	357,4					
FA	18	4	-329	3,4					
S1	20	7	-329	10,0					
AR	26	9	-329	6,5					
Reverse IV									

Syn	Julian date			λ_{Bab}	Line #	Degrees		Sign	δλ
LA	16	10	-341	225,3	C1	15	40	Sco	0,3
FA	21	11	-341	229,4	C2	19	10	Sco	-0,3
S1	2	3	-340	236,6	C3	26	10	Sco	-0,4
					C4				
					C5				
AR	11	5	-340	233,3	C6	23		Sco	-0,3
S2	21	7	-340	230,0	C7	20		Sco	0,0
					C8				
LA	27	10	-340	236,5					
FA	2	12	-340	240,6					
S1	13	3	-339	247,7					
AR	23	5	-339	244,4					
S2	2	8	-339	241,1					
LA	7	11	-339	247,6					
FA	13	12	-339	251,7					
S1	25	3	-338	258,8					
AR	4	6	-338	255,5					
S2	14	8	-338	252,2					
LA	19	11	-338	258,8					
FA	25	12	-338	263,0					
S1	6	4	-337	270,1					
AR	16	6	-337	266,8					
S2	25	8	-337	263,5					
LA	1	12	-337	270,2					
FA	7	1	-336	274,5					
S1	17	4	-336	281,5					
AR	27	6	-336	278,2					
S2	5	9	-336	274,9					
Reverse III									

Syn	Julian date			λ_{Bab}	Line #	Degrees	Sign	δλ
LA	5	8	-347	155,5	C1			
FA	14	9	-347	160,3	C2			
S1	23	12	-347	167,7	C3			
					C4			
					C5			
AR	1	3	-346	164,3	C6			
S2	10	5	-346	160,9				
LA	17	8	-346	167,7				
FA	26	9	-346	172,4				
S1	4	1	-345	179,7				
AR	14	3	-345	176,3				
S2	23	5	-345	173,0				
LA	30	8	-345	179,7				
FA	8	10	-345	184,2				
S1	16	1	-344	191,4				
AR	25	3	-344	188,1				
S2	4	6	-344	184,8				
LA	10	9	-344	191,3				
FA	18	10	-344	195,7				
S1	27	1	-343	202,9				
AR	6	4	-343	199,7				
S2	16	6	-343	196,3				
LA	22	9	-343	202,8				
FA	30	10	-343	207,2				
S1	7	2	-342	214,2				
AR	18	4	-342	211,0				
S2	29	6	-342	207,6				
LA	4	10	-342	214,1				
FA	10	11	-342	218,3				
S1	19	2	-341	225,4				
AR	30	4	-341	222,2				
S2	10	7	-341	218,8				
Reverse II								

Syn	Julian date			λ_{Bab}	Line #	Degrees	Sign	δλ
LA	20	5	-353	75,4				
FA	27	6	-353	80,2				
S1	7	10	-353	88,2				
AR	12	12	-353	84,8				
S2	17	2	-352	81,2				
LA	3	6	-352	89,6				
FA	10	7	-352	94,3				
S1	20	10	-352	102,3				
AR	25	12	-352	98,9				
S2	3	3	-351	95,3				
LA	17	6	-351	103,5				
FA	23	7	-351	108,1				
S1	3	11	-351	116,1				
AR	8	1	-350	112,7				
S2	18	3	-350	109,2				
LA	30	6	-350	117,1				
FA	6	8	-350	121,6				
S1	16	11	-350	129,5				
AR	22	1	-349	126,1				
S2	1	4	-349	122,6				
LA	12	7	-349	130,1				
FA	20	8	-349	134,9				
S1	29	11	-349	142,6				
AR	4	2	-348	139,2				
S2	14	4	-348	135,8				
LA	23	7	-348	142,9				
FA	2	9	-348	147,9				
S1	11	12	-348	155,3				
AR	16	2	-347	152,0				
S2	27	4	-347	148,5				
Reverse I								

based on the assumption that here acronychal rising is equivalent to exact opposition of Saturn with the Sun, consistent with the algorithm employed in the computation of the ephemeris: S1 → AR = AR → S2 (see discussion above in Sect. 1 and Table 1).[10]

Our reconstruction of the tablet and the placement of the different fragments in time is based on a comparison of the preserved longitudes in the text with the computed longitudes of Saturn in the synthetic observational database. The exact first entry in column I on the Obverse and the exact last entry in column IV of the Reverse cannot be determined but assuming that the tablet originally covered one full 59-year period of Saturn we expect in total 5 (number of synodic phenomena in one synodic period of Saturn) × 57 (number of similar synodic events in 59 years) = 285 entries distributed over eight columns, or on average about 35.5 entries in eight columns. According to our reconstruction the obverse of the tablet contained 45 lines with 36 to 45 synodic events per column, and the reverse contained 36 lines with 30 to 34 synodic events per column. The number of synodic events tabulated per column varies depending on the number of Normal Star observations included for comparison in each column because these require two to three lines per observation. The reconstruction is further constrained by the requirement that fragment C contains the last lines of columns III and IV on the obverse and the first lines of columns II, III and IV on the reverse side of the tablet. As mentioned above the precise beginning of the tabulated events in Obv. I and the precise end in Rev. IV cannot be determined but in our reconstruction these are based on the assumption that a full 59-year period of Saturn was tabulated so that the tablet covered two runs of Saturn through the zodiac, starting around 0° Aries.

Notice that based on the preserved longitudes our reconstruction shows that fragments B and C should join in column III on the obverse and that the horizontal rulings in column IV of the obverse indeed indicate the separation between successive Babylonian years. As discussed above in Sect. 2, there is no physical join between fragments B and C, indicating that there must be at least one or perhaps two extra and unexpected lines of text here. We can offer no explanation of why this is the case or what that text might have been.

In Tables 4 and 5 we also list values of $\delta\lambda$, the difference between the longitudes of Saturn preserved on the tablet and the longitudes in the synthetic observational database. Inspection of these values shows that the System A_0 ephemeris BM 42878+ is quite accurate over large sections of the zodiac. The largest errors seem to occur in Pisces and Cancer.

The synthetic observational data in Tables 4 and 5 allow us to investigate the accuracy of the System A_0 model of Saturn in more detail. Computing synodic arcs for all observations listed in Tables 4 and 5 and plotting them as a function of initial longitude, we show in Fig. 1 a graphical representation of the results for each synodic phase separately. These graphs can be directly compared with those for System A of Saturn, discussed by de Jong (2019a; see his Fig. 3).[11] There it is also explained why the graphs for all four synodic phenomena are quite similar (primary synodic

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[10] All longitudes in the database are given in the Babylonian fixed zodiac. They are converted from ecliptic longitudes using the relation derived by Huber (1958; see de Jong 2019a).

[11] In Fig. 3 of de Jong (2019a) the graph for acronychal rising is not included because the observational criterion used by the Babylonian observers to determine the date of acronychal rising is unclear (Hollywood and Steele 2004) so that no reliable synthetic observational data can be generated. In BM 42878+ acronychal

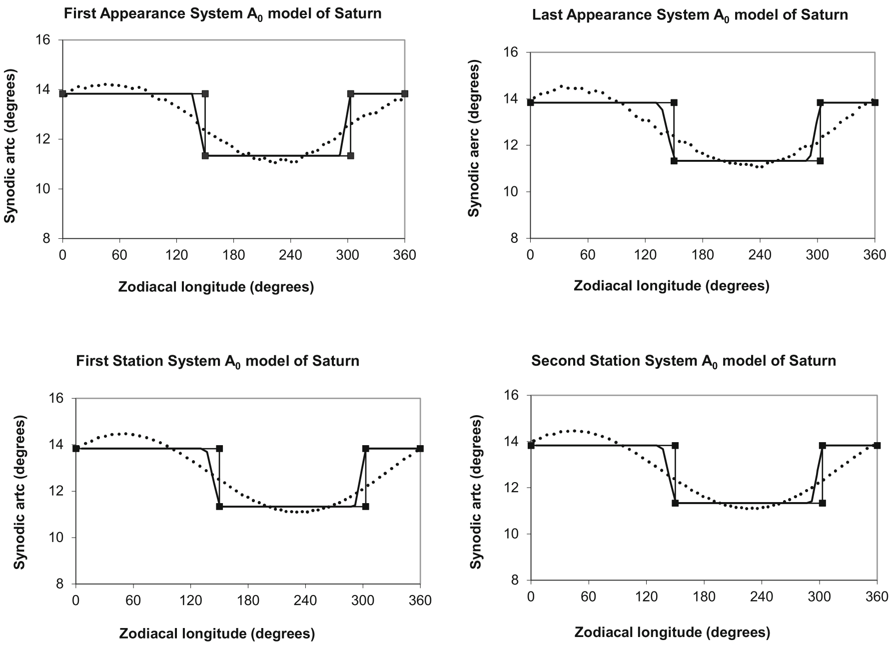

Fig. 1 Variation of the synodic arc at first appearance, first station, second station and last appearance of Saturn as a function of Babylonian zodiacal longitude. Dots represent values derived from synthetic observations of Saturn in the fourth century BC. Also shown is the Babylonian system A_0 model step function for the synodic arc (thin line marked by squares) and the interpolated model (thick line)

phenomena, see also Ossendrijver 2012, 59) and how the System A step functions were constructed based on observations of Saturn at one of its stations with respect to nearby Normal Stars.

A comparison of System A_0 with System A shows that the System A step function of Saturn provides a better fit to the observational data from Aries to Cancer (0°–120°) and from Aquarius to Pisces (300° to 360°) than the early variant System A_0. This is also reflected in the standard deviations of the longitude differences $\delta\lambda$ averaged over one century which for System A_0 amount to ± 1.6° (FA), ± 2.2° (S1), ± 1.8° (S2) and ± 1.8° (LA), compared to the smaller values (better accuracy) for system A in column (i) of Table 4 of de Jong (2019a) where we find ± 1.0° (FA), ± 1.3° (S1), ± 1.1° (S2) and ± 1.0° (LA).

6 The dates and the BE values

The full dates (day, month, and year) of 25 synodic phenomena are preserved or can be restored from surrounding entries. In Table 6 we compare these dates listed in column (iv) with computed dates of these phenomena in the synthetic observation database

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Footnote 11 continued
rising is put identical to exact opposition with the Sun so that the System A_0 graph is virtually identical to the ones for first and second station.

Table 6 Comparison of preserved dates of synodic phases of Saturn in BM 42878+ with synthetic observation dates

Text line nr.		Syn.	Synthetic date				Text Date			δDay (Text	Comments
		Phase	Reignal Year		Mon	Day	Year	Mon	Day	− Synthetic)	
Obv. III	B3′	LA	Artaxerxes II	37	IX	1	[25]	IX	13	12	
	B4′	FA			X	8		⸢X⸣	18	10	
	B6′	AR		38	III	2	[26]	⸢IV⸣	10	8	Month III adopted
	B7′	S2			V	13		⸢V⸣	⸢17⸣	4	
	B18′	AR		40	IV	20	[28]	⸢IV?⸣	⸢9?⸣	-11	
	B19′	S2			VII	30		⸢VI?⸣	⸢16?⸣	-16	
	B21′	LA			X	11	28	⸢X?⸣	5?	-6	
	B22′	FA			XI	22	[28]	⸢XII?⸣	⸢5?⸣	-13	
	C3′	S1		42	II	25	30?	II	7	-18	
Obv. IV	B9′	LA	Artaxerxes III	2	I	7	2	I	1	-6	
	B10′	FA			II	22		II?	8?	-14	
	B11′	S1			V	29		V	27	-2	
	B13′	AR			VIII	8		VII?	25?	-13	
	B14′	S2			X	13		IX	23	-20	
	B16′	LA		3	I	4		XII?	27?	-7	
	B17′	FA			II	14	3	II	3	-11	
	B18′	S1			V	25		V	23	-2	
	B20′	AR			VIII	3		VII?	26?	-7	
	B21′	S2			X	9		IX?	18?	-21	
Rev. II	C3	S1		11	IX	10	[…]	X	2	22	
	C1	LA		17	VII	8	19	VII	21	13	
	C2	FA			VIII	14		VIII?	24?	10	
	C3	S1			XI	28		XII	8	10	
	C6	AR		18	II	8	20	II	16	8	
	C7	S2			IV	21		IV	17	-4	
(i)		(ii)	(iii)				(iv)			(v)	(vi)

converted into the Babylonian calendar in column (iii). The number of days difference between the dates given in the text and the synthetic dates in column (v) have been computed ignoring the year numbers and assuming that months have 30 days. Similar to the synthetic longitudes, for achronycal rising we list the date of opposition. We have assumed that the scribe made a one-month error in the date of acronychal rising given at Obv. III B6′, writing month IV instead of month III. Support for this date being an error is provided by the implied interval between this and following synodic phenomena. If we assume that the date is correct, the time interval between this acronychal rising and the following evening station would be 37 days, which is far too short. If we correct the date of the acronychal rising from Month IV to Month III, the time interval becomes 67 days, which is more in line with other values for this interval found in the text. The error can be understood by noting that the time interval that should be added to the date of the previous acronychal rising (or the preceding morning station) to obtain the date of the acronychal rising recorded at Obv. III B6′ contains an intercalary second Addaru (Month XII_2) which may have been overlooked.

Inspection of Table 6 reveals several things. First, there is general agreement between the preserved months and days of the phenomena with observation. However, as mentioned above, there are serious problems with the year numbers in Obv. III and Rev. III. In Obv. III, the year numbers seem to be 12 years too early; in Rev. III, they are two years too late. We cannot provide a good explanation for this miscounting of the years. Secondly, we see that there are two abrupt changes in the difference between

Table 7 Time intervals between the synodic phases of Saturn

	LA → FA [tithis]	FA → S1 [tithis]	S1 → AR [tithis]	AR → S2 [tithis]	S2 → LA [tithis]
Text	33 - 37	105 - 110	58 - 66	58 - 67	109 - 124
Synthetic					
Slow zone	37 - 42	101 - 104	68 - 72	70 - 73	99 - 102
Fast zone	37 - 50	93 - 105	67 - 72	67 - 71	101 - 109
Algorithm					
Slow zone	35;47,20	114;00	58;00	62;00	113;00
(i)	(ii)	(iii)	(iv)	(v)	(vi)

the dates given in the text and the computed dates: From the earliest preserved dates up to those in Obv. III B7′, the dates given in the text are systematically later than those given by modern computation. However, for the remainder of Obv. III and Obv. IV, the dates switch to being systematically early. The entries on the reverse are systematically late again. One possible explanation for this pattern in the recorded dates is that the scribe has incorrectly intercalated somewhere between these preserved runs of dates. Given the general confusion in the year numbers, omitting/adding an extra intercalary month seems quite plausible.

There are two important questions that need to be addressed: (1) are the dates in the text computed or observed, and (2) what is the meaning of the BE values listed for a number of first and last appearances of Saturn? In an attempt to provide an answer to the first of these questions we show in the upper part of Table 7 values of the time intervals between successive synodic phases of Saturn derived from the preserved dates in the text listed in Table 6.[12] Due to the scarcity of data we have not discriminated between time intervals for Saturn in the slow and the fast zone of the zodiac.

For comparison we also show in the middle section of Table 7 the time intervals between successive synodic phases of Saturn in its fast and slow zone computed from the synthetic observations of Saturn between 389 and 330 BC listed in Tables 4, 5. These time intervals do not include the variation in the observed dates of the first and last appearance of Saturn due to variable atmospheric conditions and the observational uncertainty in the dates of the stations due to the difficulty of exactly determining the date of standstill of the planet. These effects cause an additional spread in the observed dates of first and last appearance of Saturn of about ± 3 days and in the station dates of at least ± 1 week, which should be added to the range of values of the synthetic synodic intervals displayed in the middle section of Table 7.

The data in Table 7 allow two conclusions: (1) the spread in the synodic time interval values in the text are significantly smaller than expected for observed values so that they most probably are computed rather than observed, and (2) the time intervals of FA

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[12] As already noted in Sect. 3 for several dates the regnal years in the text are inconsistent with the associated longitudes of Saturn. We have still used those dates for our analysis because the year shifts are the same for successive dates so that the time intervals are probably not affected.

Table 8 Synthetic dates of synodic phases of Saturn and dates of observations in Diaries and Planetary Texts

Synthetic observations								Diary data			Reference	Difference	Text	
Syn	Julian date			Babylonian date				Bab date		Sign/		Observed -	Bab date	
phase	day	mon	year	Regnal year		mon	day	mon	day	Star	ADART	Synthetic	mon	day
FA	3	7	-382	Artx II	22	III	18	III	16-20	β Gem	Vol. I No. -382	-2/2		
AR	14	1	-379		24	X	28	X	26	Leo	Vol. I No. -380	2		
LA	18	7	-378		26	IV	18	IV	22	Leo	Vol. V No. 62	4		
S1	5	12	-378		26	IX	9	IX	21	Leo	Vol. V No. 62	12		
AR	10	2	-377		26	XI	18	XI	20	–	Vol. V No. 62	2		
S2	21	4	-377		26	XII2	28	XII2	15	Leo	Vol. V No. 62	-13		
S2	22	6	-372		32	III	28	III	12	–	Vol. I No. -372	-16		
LA	21	10	-370		34	VII	23	VII	23-27	Sco	Vol. I No. -370	0/4		
S2	30	8	-366		38	V	13	IV	29	Sag	Vol. I No. -366	-14	V	17
S1	4	1	-345	Artx III	12	X	3	X	17	α Vir	Vol. I No. -346	14		
AR	18	4	-342		16	I	23	I	21	–	Vol. I No. -342	-2		
AR	4	8	-333	Dar III	2	IV	21	IV	19	–	Vol. I No. -333	-2		
AR	16	8	-332		3	V	15	V	16	–	Vol. I No. -332	1		
S2	23	10	-332		3	VII	24	VII	10	Aqr	Vol. I No. -332	-14		
(i)	(ii)			(iii)		(iv)		(v)			(vi)	(vii)	(viii)	

→ S1 and of S2 → LA in the text are on average systematically about 10 days larger than the actual values. This implies that the dates when Saturn reaches its stations are systematically about 10 days late for the morning station (S1) and about 10 days early for the evening station (S2). Apparently, the Babylonian observers determined the date of standstill for the morning station of Saturn as the first day on which it was observed to start moving backwards (1–2 weeks after standstill) and the date of the evening station as the first day on which it was observed to halt its backward motion (1–2 weeks before standstill).

This observational practice is confirmed by a comparison of the dates of synthetic observations of Saturn between 389 and 330 BC, the period covered by the tablet, with records of preserved observations of Saturn in the Diaries. The results are summarized in Table 8. Observed dates of first and last appearances of Saturn differ by up to three days from the expected (synthetic) dates while the observed dates of the morning station S1 of Saturn are about 2 weeks late and those of the evening station (S2) are about 2 weeks early. A similar conclusion about the station dates was reached by Steele and Meszaros (2021) who studied all observations of Saturn at its stationary points preserved in the Astronomical Diaries.[13]

There is one observation in Table 8 which unambiguously proves that the dates in BM 42878+ are computed rather than observed: the observation of the evening station of Saturn in 367 BC. The exact standstill of Saturn occurred on August 30 of that year, or day 13 of month V in year 38 of Artaxerxes II according to the Babylonian calendar. In the Diary of that year (No. − 366) the Babylonian observer(s) recorded that Saturn reached its evening station on day 29 of month IV, 14 days earlier, consistent with the Babylonian observing practice discussed above. By a lucky coincidence it so happens that among the roughly 20 preserved dates on BM 42878+ the text gives for this evening station day 17 of month V, 4 days later instead of 14 days earlier than the actual date of standstill. The only reasonable explanation for this discrepancy is that this date and by analogy all dates in the text are computed. As we shall see not only this date but all dates in this part of the text are shifted to later dates by about 2 weeks.

Given that the dates in BM 42878+ are indeed computed the question arises whether the underlying algorithm can be reconstructed from the preserved dates and BE values. In an attempt to answer this question, we have computed dates of the synodic phases (FA, S1, AR, S2 and LA) of Saturn by applying the standard system A algorithm which prescribes that successive dates of each synodic phase can be found by adding a synodic time interval $\Delta t = \Delta\lambda + c$, where $\Delta\lambda$ is the synodic arc and the parameter c is a constant. For system A_0 of Saturn we have c = 11;27,20 (see Table 3) so that $\Delta t = 360 + 11;20 + 11;27,20 = 382;47,10$ tithis in the slow zone and $\Delta t = 360 + 13;50 + 11;27,20 = 385;17,10$ tithis in the fast zone. As a working hypothesis, we will assume that the BE values correspond to the difference between the date computed by the System A_0 scheme and the observed date of the phenomenon. Since all but one of the preserved BE values and four out of twenty-five preserved dates fall between May 377 BC and January 365 BC when Saturn moved from 6;00° Virgo to 21;10° Capricorn in the slow zone we restrict the computation to this period. Initial dates in 377/376 BC for S2, LA, FA, S2 an AR were chosen in such a way that the preserved dates and the BE values are on average reproduced. The results of the computation are shown in Table 9.

The data in Table 9 cover the period from May 377 BC up to and including January 365 BC with a gap between 373 and 368 BC when no data are preserved on the tablet. Synthetic observation dates of Saturn at its successive synodic phases in the Julian calendar in column (ii) are converted to the Babylonian lunar calendar in column (iii). In column (iv) we list the dates of the synodic phases of Saturn computed according to the Babylonian date algorithm and column (v) shows computed BE numbers, defined as the difference in days between dates computed according to the algorithm and the synthetic observational dates. Column (vi) contains the line numbers of the text and the dates and BE numbers preserved on the tablet. In column (vii) we show the difference in days between the dates in the text and those computed according to the

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[13] As noted by Steele and Meszaros (2021), this trend in the dates of the stations of Saturn differs from those for Jupiter and Mars, for which both the morning and evening stations are systematically early, implying that they are recorded for the moment when the planet was seen to stop moving. They, and we, can offer no explanation for the difference between the Babylonian observations of Saturn and of the other planets.

Table 9 Synthetic and computed dates of the synodic phases of Saturn compared to preserved data on BM 42878+

Synthetic observations							Algorithm			Preserved text				Text - Algorithm		Comments
Synodic	Julian date			Babylonian date			Babylon date		BE	Line nr.	Date		BE	Date	BE	
phase	day	month	year	Artax II	month	day	month	day	days	Obv. II	month	day	days	days	days	
S2	3	5	-376	28	I	23	I	27	4	B1'						
LA	10	8	-376	28	V	4	V	20	16	B2'			⸢15?⸣		-1	
FA	19	9	-376	28	VI	13	VI	26	13	B3'			15?		2	
S1	29	12	-376	28	IX	26	X	20	24	B4'						
S2	16	5	-375	29 a	II	17	II	20	3	B5'						
LA	23	8	-375	29 a	V	28	VI	13	15	B6'			⸢14?⸣		-1	
FA	1	10	-375	29 a	VII	6	VII	19	13	B7'			15?		2	
S1	9	1	-374	29 a	X	18	XI	13	25	B8'						
AR	19	3	-374	29 a	XII	28	XII2	11	13	B9'						
S2	29	5	-374	30	II	11	II	13	2	B10'						
LA	4	9	-374	30	V	21	VI	6	15	B11'			⸢10+x⸣		x - 5	
FA	13	10	-374	30	VI	29	VII	12	13	B12'						
S1	21	1	-373	30	X	11	XI	6	25	B13'						
AR	31	3	-373	30	XII	21	I	4	13	B14'						
S2	11	6	-373	31	III	5	III	6	1	B15'						
LA	16	9	-373	31	VI	14	VI	29	15	B16'			⸢13?⸣		-2	
FA	24	10	-373	31	VII	21	VIII	4	13	B17'			16		3	
S1	2	2	-372	31	XI	4	XI	28	24	B18'						
AR	11	4	-372	32 a	I	14	I	26	12	B19'						
S2	22	6	-372	32 a	III	28	III	28	0	B20'						
LA	27	9	-372	32 a	VII	7	VII	21	14	B21'			13		-1	
FA	3	11	-372	32 a	VIII	14	VIII	27	13	B22'			23?		10	23? uncertain reading. Smaller number expected
										Obv. III						
AR	9	6	-367	37 a	III	10	III	20	10	B1'						
S2	19	8	-367	37 a	V	21	V	22	1	B2'						
LA	24	11	-367	37 a	IX	1	IX	15	14	B3'	IX	13		-2		
FA	31	12	-367	37 a	X	8	X	21	13	B4'	⸢X⸣	18		-3		
S1	11	4	-366	37 a	XII2	21	I	15	24	B5'						
AR	21	6	-366	38	III	3	III	13	10	B6'	⸢IV⸣	10		-3		Month ⸢IV⸣ scribal error? Month III adopted (see text)
S2	30	8	-366	38	V	13	V	15	2	B7'&B8'	⸢V	17⸣		2		
LA	6	12	-366	38	VIII	24	IX	8	14	B9'	XI					Month XI scribal error? Month IX expected
FA	13	1	-365	38	X	2	X	14	12	B10'						
S1	23	4	-365	39	I	14	II	8	24	B11'	II					
AR	26	3	-365	39	III	26	IV	6	10	B12'						
S2	11	9	-365	39	VI	7	VI	8	1	B13'						
LA	18	12	-365	39	IX	17	X	1	14	B14'	⸢IX?⸣		13		-1	
FA	27	1	-364	39	X	26	XI	7	11	B15'						
(i)	(ii)			(iii)			(iv)		(v)	(vi)				(vii)	(viii)	(ix)

algorithm and in column (viii) the differences between the BE numbers in the text and the computed ones in column (v).

Our reconstruction of the dates and the BE values is still somewhat preliminary because it is based on only four preserved dates and ten BE values. In spite of the preliminary nature of our reconstruction, the data in Table 9 suggest that the Babylonian scholars may have used a more refined algorithm than the straightforward system A algorithm used here because the computed dates and the BE values differ by up to −3/+2 days from the dates preserved in the text (see column (vii)). We know from several ACT texts that the Babylonian scholars occasionally used more refined algorithms to compute dates of the synodic phenomena. A well-known example is ACT 300a where a more complicated algorithm is applied to compute the dates of Mercury at its last appearance (Ossendrijver 2012, 72).

Based on the choice of the initial dates and implicit in the computation of the dates in Table 9 are the time intervals to go from one synodic phase to the next one: LA + 35;47,20 tithis → FA + 114 tithis → S1 + 58 tithis → AR + 62 tithis → S2 + 113 tithis → LA (listed in the bottom section of Table 7). These intervals add up to 382;47,20 tithis, as they should in the slow zone. Notice that these intervals are in reasonable agreement with the ones derived from the preserved dates in the text also displayed Table 7. Also notice the asymmetry in the intervals S1 → AR and AR → S1 which implies that the dates of acronychal rising fall two days before opposition, consistent with Babylonian observational practice (see Hollywood and Steele 2004).[14]

The BE values in the text are a few days larger than predicted for FA and a few days smaller for LA (see column (viii) of Table 9). This is to be expected because the predicted values of BE are based on synthetic observations computed for an average atmospheric extinction of 0.27 magnitudes per airmass (de Jong 2019a). Under realistic atmospheric conditions with variations in the atmospheric extinction from day to day first appearance will often be observed a few days earlier than predicted and last appearance a few days later (see de Jong 2012).

Taking all dates in Table 9 together, both of the stations and of the first and last appearances of Saturn, we find that the whole scheme is on average about 14 days late (see column (v)). The reason for this is unclear. One possibility is that the scribe of BM 42878+ anchored his computation of the ephemeris to an observation of Saturn at one of its stations, a procedure known from several later System A ephemerides (see de Jong 2019a,b; 2021). Indeed, as shown by the observational data in Table 8, if the author of BM 42878+ used an observation of Saturn at its morning station as initial condition the whole scheme would be about 2 weeks late. On the other hand, it may be just due to some scribal error because continuing the computation of the dates after the period covered in Table 9, using a similar slightly adapted algorithm for the fast zone, we encountered differences between computed and preserved dates in the text running up to 22 days (see above and column (v) of Table 6).

It is worth noticing that the system A modelling of Saturn in BM 42878+, apart from the systematic offset in the dates and in spite of the approximate nature of the date

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[14] At first sight this asymmetry seems inconsistent with the longitude algorithm of system A_0 (see Sect. 4) in which the position of Saturn at its acronychal rising is computed as if it was at opposition with the Sun. However, since Saturn moves only about -0.1° per day at opposition this inconsistency is unnoticeable in Babylonian observational practice where positions are determined with an accuracy of about 1°.

algorithm, is fairly successful. In the previous section we have seen that the computed longitudes are on average accurate to within 1°–2°, while the present analysis shows that the computed dates in the text are on average off by only 2–3 days (column (vii)) and that the BE values are on average correct to within ± 2 days (column (vii)). However, in later sections of the text the scribe appears to have made gross errors in the computation of the dates and in the comparison with observational data (see above and Sect. 7 below).

We conclude that: (1) in tablet BM 42878+ the dates of Saturn at its five synodic phases were computed, that (2) the computation was probably anchored to an initial observation of Saturn at is morning station (S1), and that (3) at the first and last appearances of Saturn the computed dates were compared to observed dates.

7 Normal Star observations

The text BM 42878+ is unique in illustrating that during the early phase in the development of Babylonian planetary theory the results of the computations were compared to observational data to check on the quality of the System A_0 model of Saturn and on the accuracy of the predicted longitudes and dates. In the previous section we have shown that computed dates of Saturn at its first and last appearances were compared to observed dates and that the differences were listed as BE numbers in the text. In this section we discuss the Normal Star observations which are occasionally included in the text in lines which contain computed values of the date and of the longitude of Saturn at its evening or morning station.

We have identified 14 preserved entries of computed dates and longitudes of Saturn at either one of its two stations where Normal Star observations are, or possibly were, included in the text for comparison. The relevant data are collected in Table 10. Below we discuss and comment on each of these Normal Star observations.[15]

1. Obv. III, lines B12′-B13′. Second (evening) station of Saturn on 11 September 366 BC. Two lines of text suggesting room for a Normal Star Observation. No textual information. On this date Saturn was 1.0° behind The Horn of the Goat-fish (β Cap).
2. Obv. III, lines B16′-B17′. First (morning) station of Saturn on 5 May 365 BC. Two lines of text with reference to an observation of Saturn with respect to The Front Star of the Goat-fish (γ Cap). On this date Saturn was 1.8° behind γ Cap but exactly above The Rear Star of the Goat-fish (δ Cap) at a distance of 1.6°. It is not clear why the observational record apparently prefers γ Cap over δ Cap as reference star. According to the list of Normal Stars on BM 36609+ (Roughton et al. 2004), the Front Star of the Goat-fish was located at 28;30° Capricorn, quite close to the computed longitude of Saturn of 29;00° Capricorn given in the text.
3. Obv. III, lines B19′-B20′. Second (evening) station of Saturn on 22 September 365 BC. Two lines of text with reference to an observation of Saturn with respect

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[15] For a discussion of the Babylonian Normal Stars and their use in the Astronomical Diaries and Related Texts see Sachs and Hunger (1988, 16–19) and Jones (2004).

Table 10 Normal Star observations preserved on BM 42878+

	Synthetic Observation of Saturn									BM 42787+		Text Date		Dif	Text Long		Dif
NS obs.	Syn. phase	Julian date Day	Mon	Year	Babylonian date Regnal year		Mon	Day	Long [°]	Line #	Preserved Text	Mon	Day	δDay	Long [°]	Sign	δλ [°]
1	S2	11	9	-365	39		VI	7	280,3	Obv. III, B12'							
										B13'	[…] Capricorn ⸢station⸣						
2	S1	5	5	-364	40	a	II	8	298,7	B16'	[…] The Front Star of the Goat-[fish (γ Cap)				29;00	Cap	-0,3
										B17'	[…] … station$^?$ at ⸢29⸣ Capricorn station						
3	S2	22	9	-364	40	a	VI	30	292,1	B19'	⸢Month VI$^?$ 2$^?$⸣6 x x x [Goat]-fish$^?$	VI	26	-4	22;00	Cap	0,1
										B20'	22? at? x Capricorn station						
4	S1	18	5	-363	41		II	2	310,8	B23'	…				11;50	Aqr	-1,0
										B24'	[…] … of the Great One x						
										B25'	[…] (blank) at$^?$ 11$^?$,50 Aquarius station$^?$						
5	S2	4	10	-363	41		VI	23	304,1	B27'	[…] x The Rear$^?$ Star of the Goat-fish$^?$ (δ Cap)				5;30	Aqr	-1,4
										B28'	[… 5],⸢30⸣ Aquarius station						
6	S1	31	5	-362	42		II	25	323,2	C3'	30$^?$ Month II 7 2 1/2 cu[bits …]	II	7	-18			
										C4'	⸢… to the east …⸣ […]						
7	S1	10	8	-357	1		V	3	30,7	Obv. IV, B5'	⸢Month V$^?$⸣ … […]						
										B6'	… […]						
8	S1	24	8	-356	2	a	V	29	45,0	B11'	Month V 27 […]	V	27	-2			
										B12'	6 cubits … [… station]						
9	S2	4	1	-355	2	a	X	13	38,0	B14'	Month IX 23 … […]	IX	23	-20	12;00	Tau	-4,0
										B15'	at 12$^?$ [… station]						
10	S1	8	9	-355	3		V	25	59,5	B18'	Month V 23 x […]	V	23	-2			
										B19'	of$^?$ The Star … […]						
11	S1	23	12	-347	11		IX	10	167,7	Rev. II, C3	Month X 2 […]	X	2	22			
										C4	behind the Star ⸢…⸣ […]						
										C5	(blank)						
12	S1	2	3	-340	17		XI	28	236,6	Rev. III, C3	Month XII 8 5 cubits ⸢in front of⸣ The Tip of	XII	8	10	26;10	Sco	0,5
										&C3	Pabilsag's Arrow (θ Oph); behind The S[tar?]						
										C4	in front of the Star … of Pabilsag at 26,10 Scorpio						
										C5	station						
13	S2	21	7	-340	18	a	IV	21	230,0	C7	Month IV$^?$ 17$^?$ ⸢The Tip⸣ Pabilsag's Arrow (θ Oph)	IV	17	-4	20;00	Sco	0,0
										&C7	⸢at 20 Scorpio station⸣						
										C8	…						
14	S1	30	4	-335	2		I	22	293,2	Rev. IV, C3	[…x+]1 cubits;						
										C4	[…] of the Goat-						
										C5	[fish …] Capricorn station						
(i)	(ii)	(iii)			(iv)				(v)	(vi)	(vii)	(viii)			(ix)		

to a star in the Goat-fish. On this date Saturn was 4.9° in front of The Front Star of the Goat-fish (γ Cap).

4. Obv. III, lines B23′-B25′. First (morning) station of Saturn on 5 May 365 BC. Two lines of text with reference to a star in the Great One (Aquarius). On this date Saturn was 11.7° in front of φ Aqr and 6.1° in front of λ Aqr. Both stars are attested to have been used as Normal Star in exceptional cases: the star φ Aqr as The Front Basket of the Great One (Jones 2004, 489) and λ Aqr as The Bright Star of … in Diary No. -567 (Sach and Hunger 1988, 51; Month XII, line 18´). We prefer λ Aqr as the Normal Star used in this report because it is the closest candidate star (see also nr. 6 below).
5. Obv. III, lines B27′-B28′. Second (evening) station of Saturn on 4 October 364 BC. Two lines of text with reference to an observation of Saturn with respect to The Rear Star of the Goat-fish (δ Cap). On this date Saturn was 5.3° behind δ Cap. According to the list of Normal Stars on BM 36609+, the Rear Star of the Goat-fish is located at 30° Capricorn. For the position of Saturn computed according to System A_0 the text gives 5;30° Aquarius.
6. Obv. III, lines C3′-C4′. First (morning) station of Saturn on 31 May 363 BC. Two lines of text with reference to an observation of Saturn at its morning station ("… to the east …") at 2½ cubits (about 5°) from some Normal Star. On this date Saturn was 1.0° behind the star φ Aqr and 6.4° behind λ Aqr, the same two stars that function as possible candidate Normal Stars in observation nr. 4 above. Based on the preserved 2½ cubits we prefer again (see observation nr. 4) the star λ Aqr (The Bright Star of …) over φ Aqr (The Front Basket of the Great One) as the most probable Normal Star used in this observational report.
7. Obv. IV, lines B5′-B6′. First (morning) station of Saturn on 10 Augustus 358 BC. Two lines of text suggesting room for a Normal Star Observation. No textual information. On this date Saturn was 4.6° in front of The Bristle (η Tau).
8. Obv. IV, lines B11′-B12′. First (morning) station of Saturn on 24 Augustus 357 BC. Two lines of text with reference to an observation of Saturn at 6 cubits (about 12°) from some Normal Star. On this date Saturn was 10° behind The Bristle (η Tau) but exactly above the Jaw of the Bull (α Tau) at a distance of only 3.3°. It is not clear why the observational record apparently prefers η Tau over α Tau as reference star.
9. Obv. IV, lines B14′-B15′. Second (evening) station of Saturn on 4 January 356 BC. Two lines of text suggesting room for a Normal Star Observation. No textual information. On this date Saturn was 2.7° in front of The Bristle (η Tau).
10. Obv. IV, lines B18′-B19′. First (morning) station of Saturn on 8 September 356 BC. Two lines of text with reference to an observation of Saturn with respect to … of The Star…. On this date Saturn was 0.6° in front of The Southern Star of the Chariot (ζ Tau).
11. Rev. II, lines C3-C5. First (morning) station of Saturn on 23 December 348 BC. Three lines of text with reference to an observation of Saturn behind the Star … On that date Saturn was 2.2° behind The Single Star in front of the Furrow (γ Vir).
12. Rev. III, lines C3-C5. First (morning) station of Saturn on 2 March 341 BC. Three lines of text with reference to an observation of Saturn 5 cubits (10°) in

front of The Tip of Pabilsag's Arrow, and behind The Star in front of the Star … of Pabilsag. On this date Saturn was 0.2° in front of The Tip of Pabilsag's Arrow (θ Oph), in conflict with the text. We propose that the Normal Star observation quoted here in the text for comparison with the computed position of Saturn was the one from one year later when Saturn reached its first station on 13 March 340 BC. This error is consistent with the fact that the regnal year numbers in this section of the preserved text of BM 42878+ seem to be shifted to later years.[16] On 13 March 340 BC Saturn was 11.0° behind (and *not* in front of, sic!) θ Oph, and it was 1.0° in front of the star μ Sag and 4.0° in front of the star λ Sag. These latter two stars have been suggested as candidates for Normal Stars used in an early text with observations of Saturn dating from 647 to 634 BC by Hunger (1999) and de Jong (2002). Either one of these two stars might be referred to in the second half of the observational report quoted in these lines although the text has again "behind" rather than the actual "in front of".

13. 13. Rev. III, lines C7-C8. Second (evening) station of Saturn on 21 July 341 BC. Three lines of text with reference to an observation of Saturn with respect to The Tip of Pabilsag's Arrow (θ Oph). This observational record may again have been erroneously inserted here since it better fits an observation of Saturn one year later when Saturn reached its first station on 2 August 340 BC. On this date Saturn was 4.4° behind θ Oph rather than 6.7° in front of θ Oph. However, both dates are possible given the lack of detail in the observational record.
14. 14. Rev. IV, lines C3-C5. First (morning) station of Saturn on 30 April 336 BC. Two lines of text with reference to an observation of Saturn with respect to a star in the Goat-fish. On that date Saturn was 3.8° in front of The Front Star of the Goat-fish (γ Cap).

On the basis of our analysis of the Normal Star observations inserted in the text of BM 42878+ we may conclude that these observations will have assisted the author of the text in verifying that the positions of Saturn at its stations computed according to his System A_0 model were overall in agreement with the positions of Saturn derived from Normal Star observations. This was to be expected because we have seen in Sect. 5 above that the accuracy of the longitudes of Saturn computed according to the System A_0 model is of order of a few degrees. This accuracy is also reflected in the δλ-values displayed for the stations of Saturn in column (ix) of Table 10, as far as they are preserved.

In this section we have seen that the author of BM 42878+ apparently compared the computed longitudes of Saturn at its stations with observations of Saturn at its stations with respect to nearby Normal Stars. This early Saturn text is unique in showing a Babylonian astronomer at work in the construction and verification of the System A_0 model of Saturn.

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[16] But note that the text has a 2-year shift in the year number. If we accept the year number 19 in line C1, then the date of this entry corresponds to 16 March 339 BC. That year Saturn reached its morning station on 25 March at a longitude of about 19° Sagittarius, about 22° behind θ Oph.

8 Conclusion

BM 42878+ contains the positions and dates of the synodic phenomena for Saturn computed according to a previously unknown two-zone System A-type scheme which we name System A_0. The computed values correspond to dates from roughly 390 to 330 BC and the data were likely computed around the end of this period, i.e., in the late fourth century BC. This date places BM 42878+ around the time when the various System A and System B planetary schemes seem to have been actively in development. The scheme on BM 42878+ differs from other System A-type schemes in that it apparently prioritizes ease of calculation of one synodic phenomenon to the next by means of nice sexagesimal values for the subdivisions of the synodic arc and the synodic arcs themselves rather than obeying the normal System A rule of a nice value for the ratio of (the subdivisions of) the synodic arc in the two zones.

A similar approach is encountered in Text M, an early Mercury ephemeris first discussed by Aaboe et al. (1991). This text, which probably dates from around 400 BC, was recently rediscussed by de Jong (2021) who suggested that the choice of "nice" sexagesimal values for the synodic arcs (the amplitudes of a System A step function) in different zones of the zodiac is a typical feature in the early development of Babylonian planetary theory. In the canonical System A planetary theory the values of the synodic arcs are chosen in such a way that their ratios are "nice" sexagesimal terminating fractions. This choice significantly simplifies the numerical computation of planetary ephemerides and apparently became standard procedure in the computation of planetary ephemerides from about 300 BC onwards.

BM 42878+ is of particular interest not only because it attests to this newly identified System A_0 but also – indeed more importantly – because it appears to show evidence of the scribe testing the accuracy of the computed data against observations. This testing was performed in two ways: (i) determining the difference between the computed dates of first and last appearance and those found by observation, with this difference noted in the text as a value followed by the term BE, and (ii) comparing the computed longitudes of Saturn at its stations with observations of the position of Saturn relative to a Normal Star at that station; these positions could be compared by converting between a longitude and a Normal Star position using the known position of the Normal Stars in the zodiac.[17] The scribe demonstrated good judgement in choosing these two types of comparison. The dates of first and last appearance are by definition precise determinations (either the planet is seen or it is not), even if these dates are (from a modern perspective) inherently uncertain because of variable atmospheric conditions.[18] The longitude of a planet at its first or last appearance, on the other hand, is difficult to determine because few if any stars may be visible near the planet due to the sky brightness caused by the sun being only a little below the horizon.[19] By contrast,

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[17] For examples of known positions of the Normal Stars, see the two lists of Normal Stars with zodiacal positions found on BM 36609+ and BM 46083 (Roughton et al. 2004).

[18] The Babylonian observers were clearly aware of this problem, sometimes correcting the date of first or last appearance by a few days if the planet seems too high or bright on the day of its first or last appearance of if the interval between its rising/setting or sunrise/sunset was too big.

[19] This point is discussed in more detail in a series of papers on the development of Babylonian planetary theory by de Jong (2019a,b; 2021).

the dates of planetary stations, especially for Saturn which moves so slowly, are very difficult to determine whereas its position at a station can be measured precisely (and repeatedly on several nights) because the planet will be well above the horizon and moves imperceptibly for many days (Steele and Meszaros 2021). The scribe was clearly fully aware of these issues and chose the most reliable observational data at his disposal to test the computed data. Unfortunately for the scribe of this tablet, however, the value of these comparisons was to some extent vitiated by the errors that he made in computing the dates of the phenomena, especially dates in the year count, which seem to have led to at least one case of the scribe comparing a computed station with the observation of a station in a different year.

We only know of one other possible example of the testing of astronomical computation against observation from Babylonia: the so-called Text S preserved on two tablets, BM 36910+ and BM 34597 (Aaboe and Sachs 1969; Britton 1989). This text contains the values of various lunar functions for the dates of solar eclipse possibilities along with what seem to be the details of those eclipses as predicted by Goal-Year methods.[20] The computed data in Text S refer to the early fifth century BC but the tablets were probably written during the fourth century. If Text S does indeed include comparisons between solar eclipses computed by mathematical astronomy and solar eclipses computed using goal-year methods, then we would appear to have two texts demonstrating an interest in testing systems of mathematical astronomy from the fourth century BC, a period from which we have considerable other evidence for the development of these systems into their final forms.

BM 42878+ is therefore of considerable interest in providing a rare insight into the process by which Babylonian astronomers tested systems of mathematical astronomy during the process of their development.

Acknowledgements We thank the Trustees of the British Museum for permission to study and publish the cuneiform tablets discussed in this article. We also express our thanks to Mathieu Ossendrijver and Alexander Jones for their valuable comments on a draft of this article. We also wish to thank Christopher Walker for checking tablet joins.

Declarations

Conflict of interest The authors state that there are no conflicts of interest.

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[20] For the Goal-Year methods for predicting eclipses, which rely upon the Saros cycle, see Steele (2000) and Brack-Bernsen and Steele (2005).

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