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The principle of structural reciprocity: history, properties and design issues

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Summary

This paper deals with the principle of structural reciprocity, considering its origins in both Occidental and Orient culture and aiming to highlight the definition, main peculiarities and interesting aspects of such concept referring to its application to the world of construction. Issues spanning from history, form-finding and morphology, structural behaviour and construction techniques are discussed in the paper, which should be considered as a starting point to stimulate future research and design directions/approaches.

Keywords: *reciprocity, spatial structures, timber constructions, form-finding, morphogenesis, history of construction.*

1. Introduction

The principle of reciprocity in structural design and construction, i.e. the use of load bearing elements to compose a spatial configuration wherein they are mutually supported one another, has been known since the antiquity. Evidence of its application can be initially found in Neolithic pit dwellings, Eskimo tents and Indian tepees, as documented by Popovic Larsen [1][2], as well as during the Roman Empire, when Julius Caesar used it for the construction of a bridge on the Rhine, which was made of interlocked timber elements (described in his Commentaries on Gallic and Civil Wars) with the purpose of simplifying the joints among elements, as also reconstructed and drawn by Palladio [3]. However, apart from these ancient applications, independent one another and developed worldwide, the principle of reciprocity has been then autonomously studied and used, on the basis of different needs and purposes, in both Occidental and Oriental culture for centuries, until it became a research topic for academics. During the last decades, they started a new set of works related to structural, geometrical and constructive issues of 'reciprocal' structures.

In this framework, the present paper aims to present a historical-critical overview on the principle of reciprocity in the world of construction, highlighting its origins in different cultures with differences and similarities, involved research fields and architectural design issues.

2. Different roots and curious similarities of a fragmented history

2.1 Reciprocity in the Occidental culture: timber and short-beams

In Europe, the concept of spanning distances longer than the length of the available timber beams was the main reason for the use and development of the principle of reciprocity until the last century. In spite of the fact that there was other techniques to deal with large span issues and short-beams, such as that used in the roof structure of the 'Square Hall' building in Old Nisa in Mesopotamia, dated around 2000BC and described by Pizzetti and Zorgno [4] (after also used by Guarini for the design of his domes), different architects, scientists, mathematicians and constructors reasoned about this problem without relations and documented continuity on one another. Villard de Honnecourt drew in his sketchbooks some grillage assemblies for the construction of floors inspired to the principle of reciprocity between 1225 and 1250 [5], then

Alexander designed and built the Lincoln cathedral using reciprocal supports between 1220 and 1235, as reported by Hewett [6]. Leonardo Da Vinci explored at least five different spatial configurations based on the principle of reciprocity, as shown his sketches inside his "Codex Atlanticus" [7] [8], practically spanning through the main regular and non-regular 2D and 3D geometrical configurations. Furthermore, Sebastiano Serlio discussed the construction of planar floors with short beams in his first book on architecture (in Italian), dated 1566 [9] [10], and John Wallis wrote in his "Opera Mathematica" [11] about different kind of planar configurations of structures with reciprocally supported elements, also providing examples of calculation worth to be further investigated. Thus, Émy, who was professor of Fortification at the Royal Military School Saint-Cyr, dedicated his efforts in developing a book about Carpentry, the "Traité de l'art de la charpenterie" [12], in which some examples of reciprocal structures can be found, also for the construction of the so-called 'flat-vaults' invented by Joseph Abeille and Sébastien Truchet in 1699 [13], and finally Rondelet [14] dealt with the principle of 'short beam' for the construction of floors in his treatise dated 1810, in a similar way to what was presented in 1837 by Thomas Tredgold in his "Elementary Principles of Carpentry" [15].

2.2 Reciprocity in Orient: interwoven structures and symbolism

In the Oriental culture, the interest in the use of reciprocal structures derived from different inspiration concepts. On the one hand, especially in China, the use of interwoven strips of bamboo for the construction of baskets is an old tradition that has been then transferred to objects of larger scale, such as bridges of which the first example is the so-called 'Rainbow Bridge' in Shandong [16][17][18]. On the other hand, the religious concept of Mandala and the symbolic shape of the 'magic circle' [19] has been used as a direct reference for the development and construction of simple circular 'fans' made of reciprocally supported elements, used in many Buddhist temples all around Asia. Unfortunately, written documents concerning that period can only be found for the period after 1275, in which year Marco Polo arrived in China and therefore at the end of the Song Dynasty. However, in the last decades different Asian architects, such as Kazuhiro Ishii, Yasufumi Kijima and Yoichi Kan, took advantage of this principle for their constructions, as described by Popovic Larsen [2][20] and Gutdeutsch [21].

2.3 The bases of a scientific approach

It should be underlined that Leonardo Da Vinci was the only one in Occident who studied reciprocal structures in their potential of being elaborated as complex geometrical configurations, also in the three-dimensional space [7]. John Wallis is also worth to be cited as the first author who approached the principle of reciprocity with a modern scientific approach [11]. He considered a precise and well-defined problem related to covering a long span with a flat grillage of reciprocal short-beams, and he proposed a rational and logical calculation of the forces acting on each element of the proposed configurations. Therefore, he dealt with the problem in mathematical and structural terms, but also giving inspiration and focusing on practical aspects of construction of timber frames.

In summary, the principle of reciprocity has not had a linear history and the evidences of its knowledge and use around the world seems to be largely unrelated to one another. However, a common point in the development and application of the principle of reciprocity is the strict relationship with the use of timber as constructive material, in both Occidental and Oriental culture, but it was more a practical and constructive issue in Europe, aimed to the development of flat configurations, while it was also a spiritual reason in Asia, with the realization of three-dimensional structures recalling magical and religious symbolic shapes.

2.4 The years of patents and the development of research

In the last century, the principle of reciprocity continued to stimulate the interest of designers, and for the first time it became a topic of interest in the field of scientific research. Ten different patents based on the principle of MSE was registered between 1965 and 1992, starting with the foldable structure by Piñero (filed in 1961 in USA, published in USA and Canada in 1965) presenting a MSE spatial configuration of bars [22][23][24]. Thus, the reinforced concrete ceiling made of prefabricated units hinged together by Walle and Prinz (filed in 1971 and published in 1975) [25]; the MSE geodesic construction proposed by Bijnen in 1977 [26]; the building element for construction of interlocking grids by Daniel Gat in 1981 [27], also described in a scientific article previously appeared in 1978 in "Architectural Science Review" [28]; the interwoven construction developed by Melvin Crooks in 1978 (and patented in 1980) made of flexible so-called 'A'-shaped elements, intended to be used for three-dimensional structures such as domes [29]; the three-dimensional structure of reciprocally supported timber beams connected with notched joints registered by Graham Brown in UK, Canada and as international patent between 1989 and 1993 [30][31][32], and finally the Rotegrity system, conceived as a interwoven structure made of flexible elements based on the principle of reciprocity, developed and patented in 1992 by the Simple Science Toys based in Eugene, Oregon, USA.

Focusing on scientific publications in the last decades, an increasing number of papers started to appear in conference proceedings and journals. As already mentioned, the first was written by Daniel Gat referring to his SIGMA system [28]. Thus, different researches based on the principle of reciprocity have been developed and all of them are referring to the initial investigation by Chilton et al., published in 1992, 1994 and 1995 [33][34][35][36][37]. These authors were the first using the term 'Reciprocal Frame' in a scientific paper (a first introduction to terminological heterogeneity in the scientific research on reciprocity is provided by Popovic Larsen [2]) and they also provided a precise definition and description of the principle restricted to closed circuits of elements. In fact, in that period John Chilton started a collaboration with Graham Brown at Nottingham University, who recently patented a reciprocal three-dimensional structure, previously mentioned, and decided to increase his level of understanding of its structural behaviour. At the same university, the first PhD thesis on that topic was published. It was written by Olga Popovic and completed in 1996 [1]. Chilton was the advisor of the research involving historical references and architectures based on the RF principle, also focusing on geometrical, structural and construction considerations.

The following papers deal with different aspects of structures designed on the principle of reciprocity, and can be roughly classified in: (1) related to morphology and geometry of spatial structures based on reciprocity [38][39], also in their polyhedrical configurations [40][41][42][43]; (2) dealing with form-finding and morphogenesis of such structures with computational tools [44][45][46], recently also combined with the rapid prototyping of the timber elements[47]; (3) concerning joints and connections of reciprocal structures [48][49]; (4) focusing on the structural behaviour [50][51][46]; (5) investigating the kinematic potential in order to generate adaptive structures/architectures [52][53][54][55][56] mainly inspired from Leonardo Da Vinci's work [57] and on the Piñero's patents [22][23]; (6) working on material and sections for their realization [58], (7) or presenting and discussing architectures, constructions and prototypes [1][2][18][59].

2.5 Reciprocity in contemporary art, architecture and industrial design

Considering realized architectures in the last century, some famous and unrelated examples implementing the principle of reciprocity have been built in order to covering long spans, such as in the Mill Creek Public Housing project by Kahn designed in 1952-53, in the Berlin Philarmonie by Scharoun, built in 1960-63, and in a salt storage building in Lausanne by Atelier Gamme Architecture, realized in 1989. Some structures have been also realized with the aim of giving shape to spiritual philosophies, therefore involving symbolic meaning, such as in the Casa Negre by Josep Maria Jojul (1915-26) and in all Graham Brown and previously cited Japanese constructions.

Also some experimental pavilions have been constructed, such as the Forest Park structure by Shigeru Ban and ARUP AGU, or the two interesting experimentations uniquely designed by Cecil Balmond and ARUP AGU - the H-edge pavilion realized together with the students at Penn University and the Serpentine Gallery 2005, designed in collaboration with Siza. In both examples, the principle of reciprocity has been explored in its architectural potential of being used with different materials, element sections, joints and planar or three-dimensional configurations, providing a fundamental evidence of much this pseudo-typology should be still explored by architectural designers.

In the field of industrial design, Pino Pizzigoni realized during the Fifties timber and marble chairs and tables made of reciprocally supported elements, called by the architect as structures with 'interlocked members' emphasizing the attention on the joints (See Pizzigoni Archive: PIZ N, 1948) [60]. Finally, the sculptors George Hart and Rinus Roefols [61] have been interested in the principle of reciprocity, using it for the conception of some of their geodesic domes and complex three dimensional sculptures.

3. Peculiarities and interesting aspects of reciprocity

The term reciprocity derives from the Latin 'reciprocus' which is composed by the two parts 'recus', meaning backwards, and 'procus', translated as forwards. The etymological significance of reciprocity is therefore 'moving backwards and forwards', implying the practice of exchanging things with others for mutual benefits. Such definition stresses the obliged, forced return of a certain action, slightly differing from 'mutuality', which refers to a voluntary exchange, or 'alternative', which focuses more on the time aspect of acting at regular intervals.

In the world of construction, the application of the principle of reciprocity requires:

- the presence of at least two elements allowing the generation of a certain forced interaction;

- that each element of the composition must support and be supported by another one;

- that every supported element must meet its support along the span and never in the vertices (in order to avoid the generation of a space grid with pin-joints).

A structure or spatial configuration respecting such conditions has some peculiar characteristics and interesting aspects which have to be taken into account during design but also stimulate the exploration of their expressive potential in architectural terms.

3.1 Reciprocity vs. hierarchy

According to the second requirement of a structure developed on the principle of reciprocity, each element of the configuration must work, at the same time, as support and supporter of other elements, generating, with the repetition of such scheme, a pattern wherein all the components of the system play the same role, without differences in terms of structural hierarchy. This situation is clearly opposite to that of other structures wherein the elements have distinct functions depending on their respective positions and relations. For instance, a frame is 'read' as girders and columns, in a truss the upper and lower chords transfer bending while the web elements transfer shear, even the well-known theory of shells shaped as revolution surfaces refers to parallels and meridians to interpret the tie effect of the formers on the latter, giving the possibility to the designer of 'reading' the structural hierarchy of the composition, and thereby distinguishing between main and secondary elements, vertical and lateral systems, bracing, slabs, etc. In structures based on the principle of reciprocity the possibility of an intuitive understanding of the structural behaviour is intrinsically lost. It was already clear in the past, when Wallis was studying reciprocal grillages [11], that following the load paths inside the structure is a good starting point to evaluate the internal forces acting in each element. However, it was equally clear that the path choice, i.e. the sequence of elements to use for computing the internal reactions is totally arbitrary. Recently such issue has been also investigated by Kohlhammer and Kotnik [51], confirming previous studies. Reciprocal structures seem to have an intrinsic 'generative' property, being understandable more as a reproduction of elementary/minimum patterns (fans) rather than instantiations of higher level abstract schemes.

The prevalence of reciprocity on hierarchy has consequences both on the architectural conception and structural design levels. The absence of a hierarchical logic in the composition of structural elements is relatively far from the Occidental culture, in which from gothic cathedrals to Calatrava’s bridges the discovery of load paths and the compression of the load bearing mechanisms are an integral part of the architectural experience, and many aspects of the mechanical behaviour, such as redundancy and robustness are related to this aspect, as explained in the paper by Balfroid et al. [62].

3.2 Form-finding/morphogenesis techniques and design approaches

The definition of simple two- and three-dimensional configurations of reciprocal structures, as well as of those based on regular polyhedra, can be found by means of different geometrical techniques, as clearly described by Popovic Larsen [2], Baverel [44] and Stacchetti [63]. However, the use of physical models is probably the most diffused tool to 'find' the form of reciprocal configuration due to its more direct interaction with the designer who can better see the assembly of elements in space. However, the adaptation of the principle of reciprocity to more general reference shapes requires the use of some computational aid in order to predict the geometrical configuration of the frame.

Furthermore, another degree of difficulty in the description of its final geometry could be given by the use of non-regular frames, characterized by different unit cells in shape and dimensions, or non-regular frames with non-regular topology, that also implies a variation in the dimensions of bars and position of joints.

At present, three main form-finding techniques has been studied in order to deal with these problems. The first is known as Iterative Additions and is based on the addition of set of bars to the frame, starting from the basic configurations of three or four elements [47]; the second is related to the use of optimization strategies, such as Genetic Algorithms [44] or Relaxation [45], which iteratively find the spatial definition of the frame (the solution) by reducing a measure of geometrical errors (fitness function) from a non-compatible configurations (tentative individuals, population); the third technique implies the modelling of the behaviour of kinematically undetermined configurations in order to define their final shape.

It should be underlined that the definition of reciprocal spatial configurations on the basis of physical models is always based on taking advantage of the main characteristics of reciprocity, allowing the designer to follow a so-called 'bottom-up' approach bringing to spatial complexity from the composition and modification/deformation of elementary/minimum configurations. On the contrary, the use of computational techniques can only start from an over-imposed reference geometry/shape, in a forced 'top-down' approach in which the final configuration is not a result of the peculiar properties of reciprocity and could then, from the ontological point of view, by another structural typology such as grid-shell. However, such discussion about design approaches should be developed in a more general way, comparing for instance the design of reciprocal configurations with that of more consolidated typologies such as grid-shells with planar quadrilateral elements. Also in that case, the use of geometric rules of translation and scaling of curves, defined by Schaich and Shober, could be considered as a bottom-up approach, while the 'a priori' definition of a reference surface, and then the application of an optimization procedure, is always a top-down design process. In addition to this geometrical problems of form-finding of reciprocal structures, a new set of questions and potential experimentation fields are arising from the development of digital fabrication techniques and CNC machines, which require a completely different approach to the design of the frame elements since the new way of component production [47].

3.3 The use of timber: connections and joints

A structural system where all the elements work as beams subjected to shear and bending is naturally related with timber constructions and technologies, which have been always preferred, in the Occidental tradition, for the application in works where the functionality is the prevailing aspect, such as simple bridges, cranes and machinery, slabs and roofs, instead of masonry, which has been mainly used, on the contrary as reference material for significant buildings and infrastructures. In this context, the principle of reciprocity has been mainly conceived and used to solve practical construction problems, and by means of that, for instance, simple bridges can be rapidly built without the need of iron keys or complex joints. In all cases, the superposition joint, with or without a friction component, is an essential part of the timber system, and its design can be refined, as when a small over-thickness is suggested to correct slab deflections with a suitable camber [11].

The strict relationship between timber elements and reciprocal joints deserves to be compared with current glulam construction, where joints are frequently realized with steel plates, clamps, or pins, connected to timber elements by bolts or needles. Such kind of constructions, efficient form the point of view of constructability and structural behaviour, keep the joints separated from the base material, suggesting that it could be replaced by other materials, as for instance the architect Shigeru Ban does by using cardboard pipes. In reciprocal structures, connections are parts of the members themselves and the design and detailing cannot be separated by the whole conception. However, the mechanical behaviour of joints, in terms of degrees of freedom, presence of friction, local deformability, displacement capability, etc, directly contribute to define the global behaviour of the structure [62].

3.4 Static and kinematical determination

According to the previously stated third requirement of reciprocity, in this kind of structures supported elements cannot meet their supports in the vertices. Such characteristic allows to define the constraint between supporting and supported members in a wider way, considering sliding hinges and prismatic joints as possible substitutes of rotation hinges, and offering to configurations obtained with such a modelling of constraints unique kinematic properties which are difficult to intuitively predict. A preliminary research, carried through the analysis of the kinematic matrix, showed how different constraint patterns can lead to unexpected kinematic behaviour, which is worth of interest in the context of this kind of structures [52]. More recent developments [54][55] propose a kinetic system based on the concept of reciprocity called Kinetic Reciprocal System (KRS), i.e. a specific morphogenetic process, aimed to the generation of suitable KRFs with assigned overall behaviour starting from sets of kinematic parameters, that involve the creation of complex geometries of intersecting curves and lines which cannot be predicted directly from the input parameters.

4. Conclusions and further developments

The principle of reciprocity has been studied and applied worldwide, starting from different inspiration concepts and intended to achieve distinct purposes. However, some relevant historical references still need to be further investigated for their scientific approach to the topic, such as the "Opera Mathematica" by Wallis [11], and a set of architectural issues characterizing this principle can offer new ideas for future research projects, such as the question of connections and joints in timber reciprocal configurations, which stimulate the use of the material as a 'whole' in the construction, in terms of defining elements and connections, and the possibility of studying reciprocal structures as assemblies of elements with a high interest kinematic potential.

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